All-loop cross sections in the Reggeon Field Theory via the stochastic approach.

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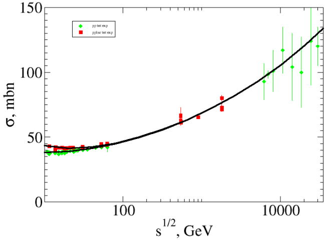
Outline

Faces of the Regge theory

1. t-channel consideration (a real theory)
   Analytical continuation of t-channel unitarity equations
   - hard to understand;
   - gives t-channel quantum number classification
2. s-channel approach (Regge field theory)
   Analysis of Feynman diagrams using Regge poles as input
   - reproduces old results
   - allows to get new ones
   - has a simple interpretation
3. Space-time interpretation (human face)
   An interpretation only!
   - very picturesque
   - easy to accept
   - gives an intuitive understanding

Power-like contributions to the amplitude



PDG fit:

$$\sigma_{tot}^{pp(\bar{p})} = 18.3s^{0.095} + 60.1s^{-0.34} \pm 32.8s^{-0.55}$$

Optical theorem:

$$\sigma_{tot} = \frac{1}{s} 2\mathrm{Im} A_{el}(q=0) \equiv 2\mathrm{Im} T_{el}(q=0)$$

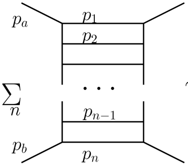
Indication: High energy elastic scattering goes via quasiparticle, "Reggeon", exchanges with powerlike asymptotic in c.m.energy.
Leading contirbution – Pomeron, $T_{\mathbb{P}} \sim s^{\Delta}$, $\Delta > 0$.
Caveat: Single Pomeron exchange violates Froissart bound ($\sigma_{tot} \lesssim C \ln^2 s$)

s-channel ($s \to \infty$, $t = Q^2$ small) dominant contributions

Analiticity&unitarity:

- Power-like terms come from poles in the complex L plane of the t-channel amplitude, Pomeron = the rightmost singularity

Field theories (φ^3, QCD):

- $\sum_n$ 

$\sim s^{\alpha(t)}$

$$p_1^+ \gg p_2^+ \gg \ldots \gg p_n^+$$
$$p_1^- \ll p_2^- \ll \ldots \ll p_n^-$$
$$p^\pm = p^0 \pm p^3$$

For phenomenological applications: $\mathbb{R}/\mathbb{P}$ = exchange of a “ladder” structure in the t-channel with ordering of the ladder rungs in rapidity $y = 1/2 \ln p_+/p_-$

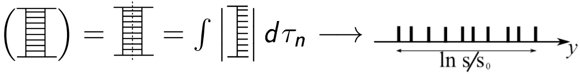
Contributions to σ_{tot}

Contributions to imaginary part (Cutkosky rules):

- Cut the diagram for the elastic scattering amplitude
- Put cut lines on the mass shell, integrate over the phase space

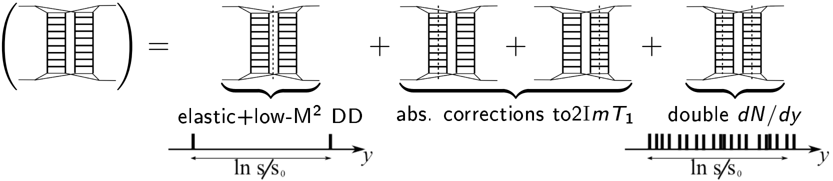
Single "ladder" exchange – uniform rapidity distribution

$2\mathrm{Im}\,T_1 = 2\mathrm{Im}$



Double "ladder"

$2\mathrm{Im}$



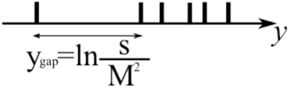
Iterating ladders slows the growth:

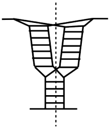
from $\sigma_{\mathrm{tot}} \sim s^{\Delta}$ down to $\sigma_{\mathrm{tot}} \sim \ln^2 s$.

Contributions to σ_{tot}

Rapidity gaps – splitting of the “ladder”:

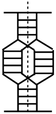
Single diffraction dissociation





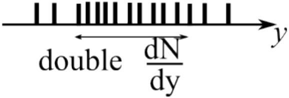
+ abs. corrections

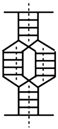
Double diffraction dissociation



+ abs. corrections

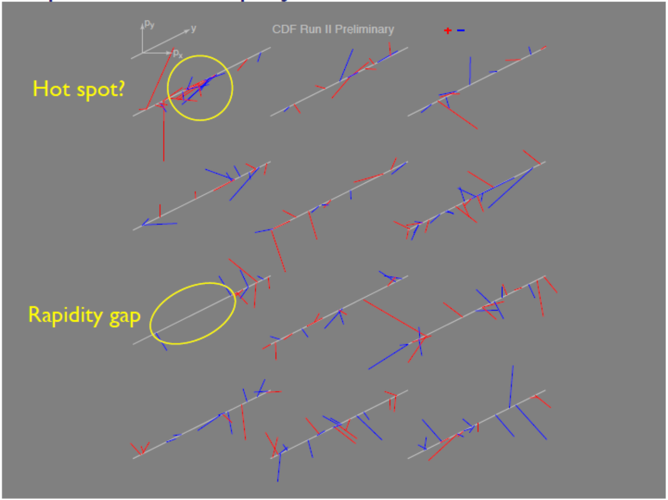
other cut of same graph





+ abs. corrections

Example Event Displays from CDF Run II



Picture taken from a talk by Chris Quigg at Nordic particle physics meeting "Spaatind 2012"

Obvious observation:

- Cross sections for the events with rapidity gaps in *pp* is the main source of information about the value of effective coupling for “ladder” splitting.
- Pomeron (“ladder”) splitting and fusion must be especially important for large number of exchanged Pomerons.

RFT

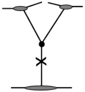
Reggeon Field Theory = the theory of the Pomeron (Reggeon) exchanges and interactions. The underlying principles of the RFT are analyticity and t-channel unitarity of the elastic amplitude.

- Gives reliable predictions of hadronic X-sections
  - The $\sigma_{tot} \lesssim C \ln^2 s$ comes out quite naturally (taking into account multiple Pomeron exchanges)
- Cuts of the RFT diagrams define X-sections of various inelastic processes via AGK rules (a special case of Cutkosky rules)
- Provides a baseline for describing the events with rapidity gaps (single and double diffraction). At higher energies the loop contributions become increasingly important.

Account of loop contirbution is an untrivial task and is under investigation by several groups (Ostapchenko, Khoze et al., Poghosyan; also Lund group non-RFT approach).

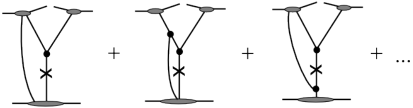
Contribution of diffractive cut

Lowest order contribution:



$$\frac{d^2\sigma_{SD}}{dtd(M^2/s)} \sim \left(\frac{M^2}{s}\right)^{-1-\Delta} s^{\Delta} \Rightarrow \sigma_{SD}(M^2/s < \alpha) \sim s^{\Delta}$$

Absorptive corrections:



Alternatives:

- Introduce reg. scale and compute order by order
- Use specific models with tuned $m\mathbb{P} \to n\mathbb{P}$ vertices $\to$ transforms power-like behaviour of Pomeron propagator to $\sim \ln^2$.
- Use effective approaches.

RFT

The elastic amplitude $T = A/(8\pi s)$ is written as (Regge factorization):

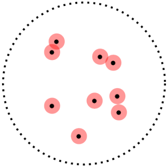
$$T = \sum_{n,m} V_n \otimes G_{nm} \otimes V_m$$

Green functions G_{mn} are obtained within the effective field theory, process independent

$$\mathcal{L} = \frac{1}{2}\phi^\dagger(\overleftarrow{\partial_y} - \overrightarrow{\partial_y})\phi - \alpha'(\nabla_{\mathbf{b}}\phi^\dagger)(\nabla_{\mathbf{b}}\phi) + \Delta\phi^\dagger\phi + \mathcal{L}_{int}.$$

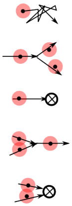
For $\mathcal{L}_{int} = i\, r_{3P}\phi^\dagger\phi(\phi^\dagger + \phi) + \chi{\phi^\dagger}^2\phi^2$
it is possible to use reaction-diffusion (or "stochastic") models for obtaining the Green functions with account of all loops.
[Grassberger&Sundermeyer'78; Boreskov'01]

The stochastic model.



Consider a system of classic “partons” in the transverse plane with:

- Diffusion (chaotical movement) D;
- Splitting (λ – prob. per unit time)
- Death (m_1)
- Fusion ($\sigma_\nu \equiv \int d^2b\, p_\nu(b)$)
- Annihilation ($\sigma_{m_2} \equiv \int d^2b\, p_{m_2}(b)$)



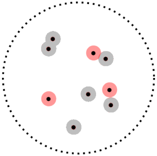
Parton number and positions are described in terms of

probability densities $\rho_N(y, \mathcal{B}_N)$ $(N = 0, 1, \ldots; \mathcal{B}_N \equiv \{b_1, \ldots, b_N\})$

with normalization $p_N(y) \equiv \frac{1}{N!} \int \rho_N(y, \mathcal{B}_N) \prod d\mathcal{B}_N;\quad \sum_0^\infty p_N = 1.$

Inclusive distributions

S-parton inclusive distributions:



$$f_s(y;\mathcal{Z}_s) = \sum_N \frac{1}{(N-s)!}\int d\mathcal{B}_N\ \rho_N(y;\mathcal{B}_N)\prod_{i=1}^{s}\delta(\mathbf{z}_i-\mathbf{b}_i);$$

$\int d\mathcal{Z}_s f_s(y;\mathcal{Z}_s) = \sum \frac{N!}{(N-s)!}\, p_N(y) \equiv \mu_s(y)$. – factorial moments.

Example: Start with a single parton with only diffusion and splitting allowed.

$$f_1^{1\ \mathrm{parton}}(y,b) = \frac{\exp(\lambda y)\exp(-b^2/4Dy)}{4\pi D y}.$$

– the bare Pomeron propagator.

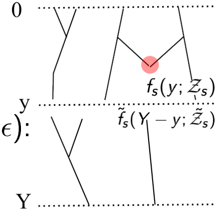
The set of evolution equations for $f_s(\mathcal{Z}_s)$, $(s=1,\ldots)$ coincides with the set of equations for the exact Green functions of the RFT

The amplitude.

Green functions:

$f_s(y;\mathcal{Z}_s) \propto \sum_m \int d\mathcal{X}_m\, V_m(\mathcal{X}_m) G_{mn}(0;\mathcal{X}_m|y;\mathcal{Z}_n)$;

$f_m(y = 0, \mathcal{X}_m) \propto V_m(\mathcal{X}_m)$ – particle–mPomeron vertices



The amplitude ($g(b)$ assumed narrow; $\int g(b)d^2b \equiv \epsilon$):

$T(Y) = \langle A|T|\tilde{A}\rangle =$

$$= \sum_{s=1}^{\infty} \frac{(-1)^{s-1}}{s!} \int d\mathcal{Z}_s d\tilde{\mathcal{Z}}_s f_s(y;\mathcal{Z}_s)\tilde{f}_s(Y-y;\tilde{\mathcal{Z}}_s) \prod_{i=1}^{s} g(\mathbf{z}_i - \tilde{\mathbf{z}}_i - \mathbf{b}).$$

It does not depend on the linkage point y (“boost invariance”) **if**

$$\lambda \int g(b)d^2b = \int p_{m_2}(b)d^2b + \frac{1}{2}\int p_\nu(b)d^2b \ ,$$

Correspondence RFT–Stochastic model

We use the simplest form of $g(b)$, $p_{m_2}(b)$ and $p_\nu(b)$:

$$p_{m_2}(\mathbf{b}) = m_2\ \theta(a - |\mathbf{b}|); \quad p_\nu(\mathbf{b}) = \nu\ \theta(a - |\mathbf{b}|);$$
$$g(\mathbf{b}) = \theta(a - |\mathbf{b}|);.$$

with a – some small scale; $\epsilon \equiv \pi a^2$.

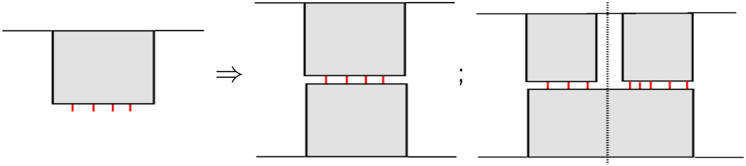
RFT	stochastic model
Rapidity y	Evolution time y
Slope α'	Diffusion coefficient D
$\Delta = \alpha(0) - 1$	$\lambda - m_1$
Splitting vertex r_{3P}	$\lambda\sqrt{\epsilon}$
Fusion vertex r_{3P}	$(m_2 + \frac{1}{2}\nu)\sqrt{\epsilon}$
Quartic coupling χ	$\frac{1}{2}(m_2 + \nu)\epsilon$

Boost invariance $(\lambda = m_2 + \frac{\nu}{2}) \Leftrightarrow$ equality of fusion and splitting vertices.

Summary of the stochastic approach

The approach allows to compute numerically (via the explicit evolution of the stochastic system) the RFT Green functions in their convolutions which correspond to

- the elastic scattering amplitude
- the single diffractive cut of the amplitude.



Peculiarities of the stochastic approach to the RFT:

- Presence of the triple and $2 \to 2$ couplings
- Regularization scale (equivalient to the cutoff or the Pomeron size in RFT) enters via functions $g(b)$, $p_{m_2}(b)$ and p_ν.
- Neglect of the real part of the $\mathbb{P}$ exchange amplitude.

Fitting the cross sections

The calculation method for elastic amplitude is described in detail in R.K., K.Boreskov and L.Bravina, Eur. Phys. J. C **71** (2011) 1757 [arXiv:1105.3673 [hep-ph]]. Compared to that now we:

- Implement two-channel eikonal p–$n\mathbb{P}$ vertices to incorporate low-M^2 diffraction
- Account the secondary Reggeons contribution in the lowest order

Other assumptions:

- Neglect the real part of the Pomeron exchange amplitude (keeping it for the secondary Reggeons)
- Neglect central diffraction in calculation of SD cross sections.

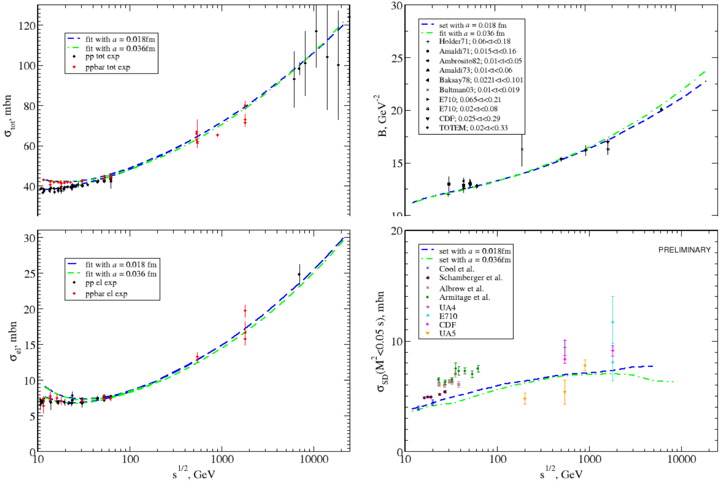
Model parameters

$r_{3\mathbb{P}}$ – fixed [Kaidalov'79]
a – regularization scale
$1 + \Delta$ – bare Pomeron intercept
α' – Pomeron slope
$|p\rangle = \beta_1|1\rangle + \beta_2|2\rangle; \quad |\beta_1|^2 \equiv C_1;\ |\beta_2|^2 \equiv C_2 = 1 - C_1.$
$\mathbb{P}$ couplings to $|1\rangle$ and $|2\rangle$: $g_{1/2} = g_0(1 \pm \eta)$
R – size of the p–$\mathbb{P}$ vertex

Strategy:

1. Eikonal fit to σ_{tot}, σ_{el}, B and low-M^2 σ_{SD} @ 14GeV/c (~ 2.2 mbn)
2. All-loop fit to σ_{tot}, σ_{el}, B starting with parameter set from [1]
3. Calculation of diffractive cross sections with parameters obtained at [2]

Results on X-sections and slope $\left(B = \frac{d}{dt} \ln \frac{d\sigma_{el}}{dt}\Big|_{t=0}\right)$



Fit parameters: 1) a = 0.018fm=0.09GeV^{-1}, $\Delta = 0.255$; 2) a = 0.036fm=0.18GeV^{-1}, $\Delta = 0.24$

$r_{3\mathbb{P}} = 0.087\text{GeV}^{-1}$ [Kaidalov'79], $\alpha' = 0.0035\text{fm}^2$=0.09GeV^{-2} for both.

Conclusions

- Total, elastic and SD cross sections are computed within a single approach.
- All-loop RFT fit to total and elastic cross setions with all enhanced and loop contributions taken into account on equal basis.
- Slow growth of all-loop high-M^2 SD cross sections is consistent with the experimental data.

Backup – cross sections definitions

$$\sigma^{\rm tot}(Y) = 2\,{\rm Im}M(Y,\mathbf{q}=0), \quad \sigma^{\rm el} = \int \frac{d^2q}{(2\pi)^2}\,|M(Y,\mathbf{q})|^2 \ ,$$

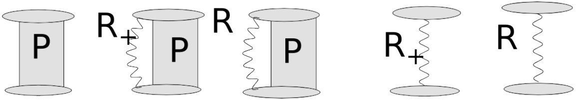
$$f(Y,\mathbf{b}) = \frac{1}{(2\pi)^2}\int d^2q\, e^{-i\mathbf{qb}} M(Y,\mathbf{q}) \ .$$

$$\sigma^{\rm tot}(Y) = 2\int d^2b\,{\rm Im}f(Y,\mathbf{b}) \ , \quad \sigma^{\rm el} = \int d^2b\,|f(Y,\mathbf{b})|^2.$$

$$f(Y,\mathbf{b}) \simeq iT(Y,\mathbf{b}), \quad T \equiv {\rm Im}f$$

$$B = -\frac{d}{dt}\ln\frac{d\sigma^{el}}{dt}\bigg|_{t=0} = \frac{\int b^2{\rm Im}A(b)d^2b\int{\rm Im}A(b)d^2b + \int b^2\Re A(b)d^2b\int\Re A(b)d^2b}{2\Big((\int{\rm Im}A(b)d^2b)^2 + (\int\Re A(b)d^2b)^2\Big)}$$

Backup – secondary trajectories



pp: $\mathrm{Im} f_{pp}(b) = \left(\mathrm{Im} A_P(b) + 1\right)\left(1 + \mathrm{Im} A_{R_+}(b) + \mathrm{Im} A_{R_-}(b)\right) - 1$

$\Re f_{pp}(b) = (\mathrm{Im} A_P(b) + 1)(\Re A_{R_+} + ReA_{R_-})$

$p\bar{p}$: $\mathrm{Im} f_{p\bar{p}}(b) = \left(\mathrm{Im} A_P(b) + 1\right)\left(1 + \mathrm{Im} A_{R_+}(b) - \mathrm{Im} A_{R_-}(b)\right) - 1$

$\Re f_{p\bar{p}}(b) = (\mathrm{Im} A_P(b) + 1)(\Re A_{R_+} - ReA_{R_-})$

Δ_+	R^2_+, GeV^{-2}	α'_+, GeV^{-2}	Δ_-	R^2_-, GeV^{-2}	α'_-, GeV^{-2}
-0.34	4.5	0.70	-0.55	10.0	1.0

Backup - calculation method

Taking an explicit note of the initial parton distributions

$$T = \sum_{n,k} P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{Z}}) \otimes \tilde{f}_{ks}(\tilde{\mathcal{X}}|\tilde{\mathcal{Z}})}_{T_{sample}} \otimes \tilde{P}_k(\tilde{\mathcal{X}}).$$

Main idea: simulate a sample of 2 parton sets which correspond to f_s and $\tilde{f}_s$ on the average, compute T_{sample} and make its MC average.
For N partons with fixed positions

$$f_s(\mathcal{Z}_s) = \sum_{\{\hat{\mathbf{x}}_{i_1},..,\hat{\mathbf{x}}_{i_s}\} \in \hat{\mathcal{X}}_N} \delta(\mathbf{z}_1 - \hat{\mathbf{x}}_{i_1}) \ldots \delta(\mathbf{z}_s - \hat{\mathbf{x}}_{i_s})$$

$$T_{\text{sample}} = \sum_{s=1}^{N_{min}} (-1)^{s-1} \sum_{i_1 < i_2 \ldots < i_s} \sum_{j_1 < \ldots < j_s} g_{i_1 j_1} \cdots g_{i_s j_s}\,.$$

- expansion of T_{sample} in the number of **P** exchanges s;
- works for any position of the linkage point y.

Backup – calculation method 2

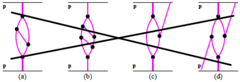
Setting the linkage point to full rapidity interval $y = Y$ simplifies the calculation: $\tilde{f}_s(y = 0, \mathcal{Z}_s) = N_s(\mathcal{Z}_s)/\epsilon^{s/2}$ and the MC average involves evolution from only one side:

$$T = \sum_n P_n(\mathcal{X}) \otimes \underbrace{\sum_s \frac{(-1)^{s-1}}{s!} f_{ns}(\mathcal{X}|\mathcal{Z}) \otimes \prod g(\mathcal{Z} - \tilde{\mathcal{X}}) \otimes \tilde{P}_s(\tilde{\mathcal{X}}).}_{T_{sample}}$$

Other RFT calculations with enhanced contributions

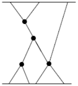
Ostapchenko

- Infinite number of $n \to m$ couplings: $G^{(n,m)} = G\gamma^{n+m}$
- Fine-tuned hard Pomeron
- Some contributions neglected



Khoze, Martin, Ryskin & Luna

- Infinite number of couplings
  $G^{(n,m)} = G\gamma^{n+m}$ or $G^{(n,m)} = mnG\gamma^{n+m}$
- Special set of graphs, no loops (as I understand)



Poghosyan & Kaidalov

- Enhanced and loop contributions are neglected in the elastic amplitude, one loop is kept for the double diffractive cut
- Infinite number of $n \to m$ Pomerons couplings
  $G^{(n,m)} = G\gamma^{n+m} \exp(-R_\pi^2 \sum q_i^2)$
- Explicit account of G_{PPR}, G_{RRP} *etc.* couplings