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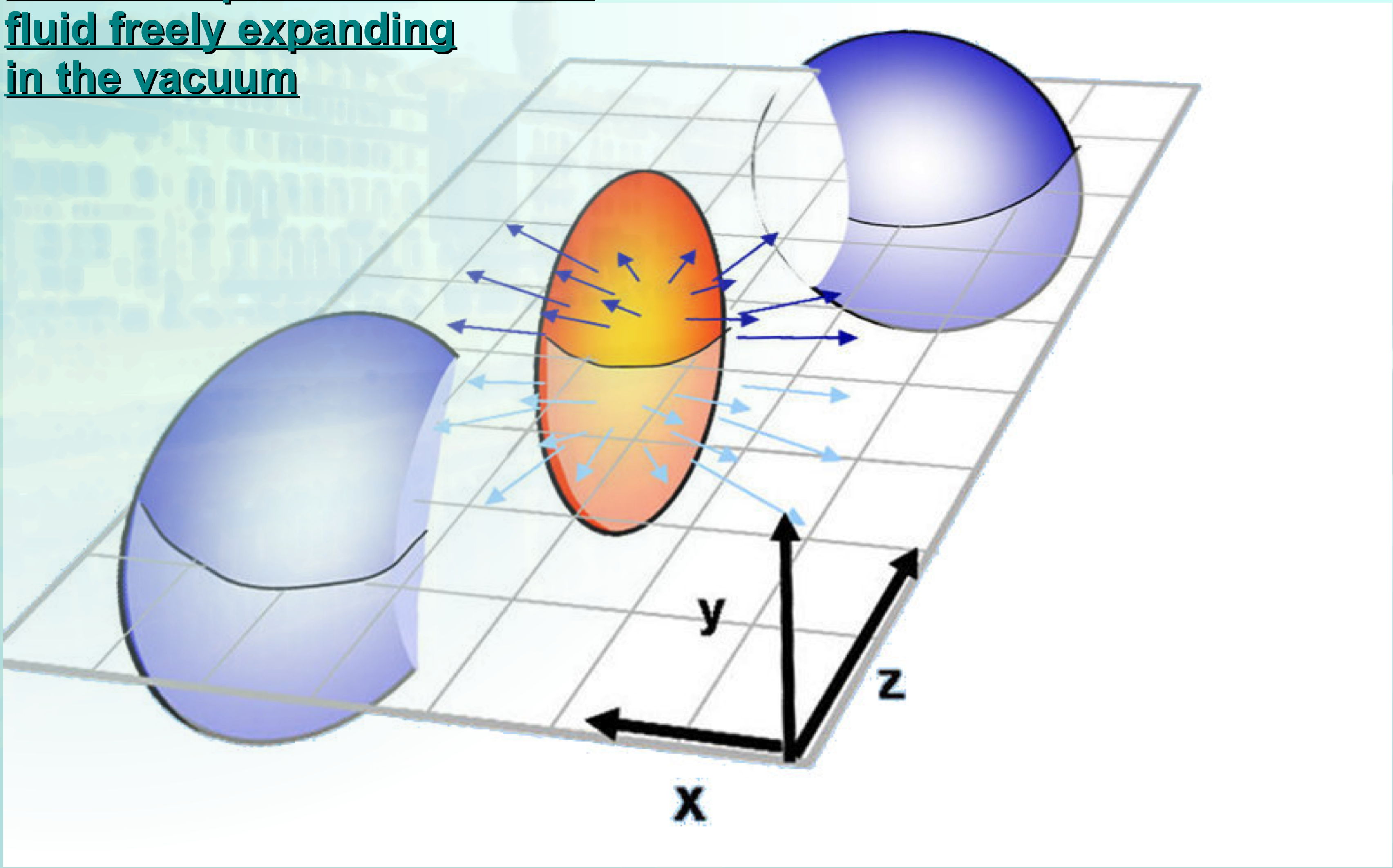
Quantum spin tensor and transport coefficients in a relativistic fluid

- Motivations and introduction
- Thermodynamical inequivalence of microscopic tensors
- Linear response

Motivations

In heavy ion collision Quark Gluon Plasma has been successfully described as an almost perfect relativistic fluid freely expanding in the vacuum



What are the consequences of the presence of other degrees of freedom (spin)? Especially in peripheral collision?

Relativistic fluids with spin

We describe relativistic fluids with two tensor, the stress-energy $T^{\mu\nu}$ and spin tensor $\mathcal{S}^{\lambda,\mu\nu}$ fulfilling the equations:

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\lambda \mathcal{S}^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \end{cases}$$

Which imply the total four-momentum and angular momentum conservation

$$\int_V T^{0\mu} d^3\boldsymbol{x} = P^\mu$$ **Total four-momentum**

$$\int_V \left(\mathcal{S}^{0,\mu\nu} + x^\mu T^{0\nu} - x^\nu T^{0\mu}\right) d^3\boldsymbol{x} = J^{\mu\nu}$$ **Total angular momentum**

Provided that the flux at the boundary vanishes

$$\int_{\partial V} \mathrm{d}\Sigma_i T^{i\mu} = 0$$

$$\int_{\partial V} \mathrm{d}\Sigma_i\, \mathcal{J}^{i,\mu\nu} = 0$$

General approach to the study of relativistic fluids

Macroscopic (classical) properties stem from a microscopic (quantum) description

$$\mathcal{O}_{\rm cl.} = {\rm tr}\left(\widehat{\rho} : \widehat{O} :\right)$$

Therefore we would like to take as the (macroscopic) stress-energy and spin tensors:

$$T^{\mu\nu}(x) = {\rm tr}\left(\widehat{\rho} \; : \widehat{T}^{\mu\nu}(x) :\right)$$

$$\mathcal{S}^{\lambda,\mu\nu}(x) = {\rm tr}\left(\widehat{\rho} \; : \widehat{\mathcal{S}}^{\lambda,\mu\nu}(x) \; :\right)$$

Noether's theorem give us canonical stress-energy and spin operators

From space-time translation invariance:

$$\widehat{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \widehat{\phi}_i\right)} \partial^\nu \widehat{\phi}_i - \mathcal{L} g^{\mu\nu} \qquad \partial_\mu \widehat{T}^{\mu\nu} = 0$$

and Lorentz group:

$$\widehat{\mathcal{J}}_{\lambda,\mu\nu} = -i \frac{\partial \mathcal{L}}{\partial \left(\partial^\lambda \widehat{\phi}_i\right)} \left(\Sigma_{\mu\nu}\right)_i^j \widehat{\phi}_j + x_\mu \widehat{T}_{\lambda\nu} - x_\nu \widehat{T}_{\lambda\mu}$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = 0$$

$$\widehat{\mathcal{S}}_{\lambda,\mu\nu} = -i \frac{\partial \mathcal{L}}{\partial \left(\partial^\lambda \widehat{\phi}_i\right)} \left(\Sigma_{\mu\nu}\right)_i^j \widehat{\phi}_j$$

Pseudo-gauge transformations

$$\widehat{T'}^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha\left(\widehat{\phi}^{\alpha,\mu\nu} - \widehat{\phi}^{\mu,\alpha\nu} - \widehat{\phi}^{\nu,\alpha\mu}\right)$$

$$\widehat{\mathcal{S}'}^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\phi}^{\lambda,\mu\nu}$$

provided that the boundary integrals vanish

$$\int_{\partial V} dS\left(\widehat{\phi}^{0,i\nu} - \widehat{\phi}^{i,0\nu} - \widehat{\phi}^{\nu,0i}\right)n_i = 0 \quad \int_{\partial V} dS\left[x^\mu\left(\widehat{\phi}^{0,i\nu} - \widehat{\phi}^{i,0\nu} - \widehat{\phi}^{\nu,0i}\right) - x^\nu\left(\widehat{\Phi}^{0,i\mu} - \widehat{\phi}^{i,0\mu} - \widehat{\phi}^{\mu,0i}\right)\right]n_i = 0$$

the new couple fulfill the same equations and give same generators of the Poincaré group.

An important example is the Belinfante symmetrization, where $\widehat{\phi}^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu}$

and

$$\widehat{T}^{\mu\nu}_{\text{B.}} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha\left(\widehat{\mathcal{S}}^{\alpha,\mu\nu} - \widehat{\mathcal{S}}^{\mu,\alpha\nu} - \widehat{\mathcal{S}}^{\nu,\alpha\mu}\right)$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu}_{\text{B.}} = 0$$

where the conservation laws are

$$\begin{cases} \partial_\mu \widehat{T}^{\mu\nu}_{\text{B.}} = 0 \\ 0 = \widehat{T}^{\nu\mu}_{\text{B.}} - \widehat{T}^{\mu\nu}_{\text{B.}} \end{cases}$$

If the new couple were actually equivalent, we would expect the same average values for observable quantities.

<u>*Four momentum density and angular momentum density are in principle observable*</u>

$$
\begin{aligned}
T'^{0\mu}(x) &= T^{0\mu}(x) \\
\mathcal{J}'^{0,ij}(x) &= \mathcal{J}^{0,ij}(x)
\end{aligned}
$$

to be true in any inertial frame, it means

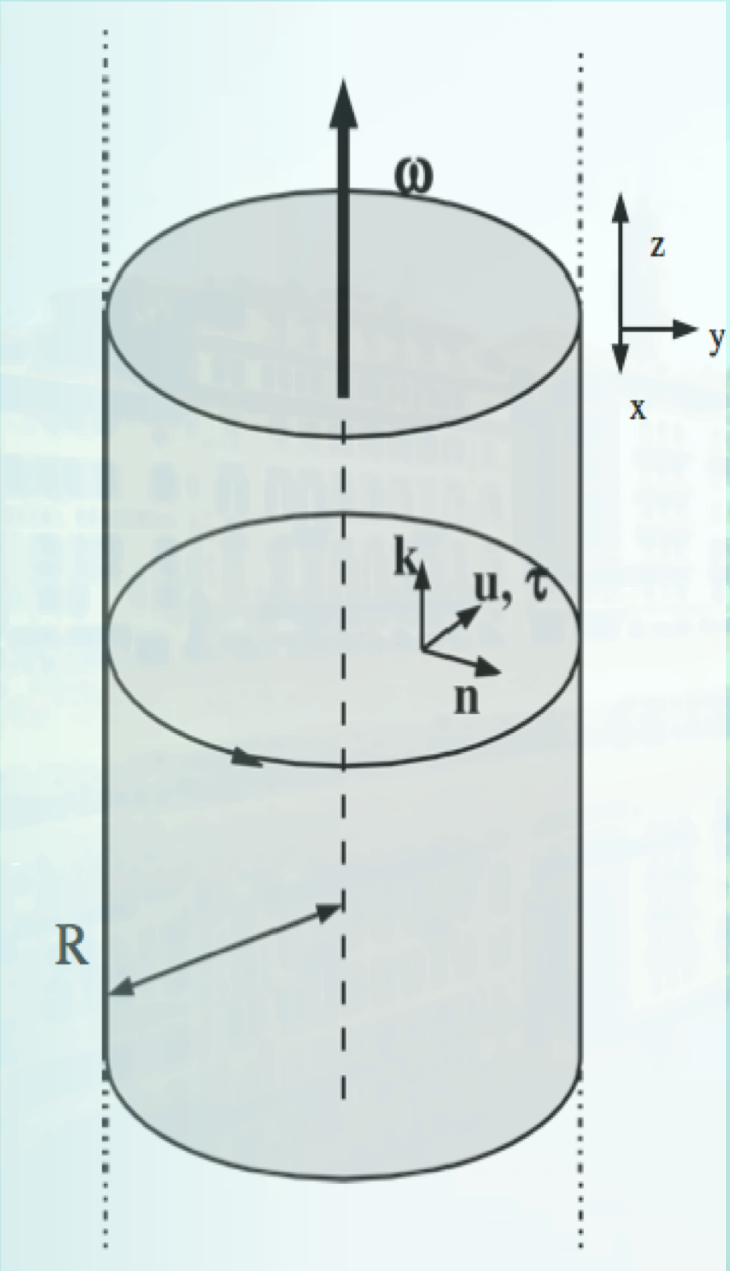
$$
\begin{aligned}
&T'^{\mu\nu} = T^{\mu\nu} \\
&\mathcal{J}'^{\lambda,\mu\nu} = \mathcal{J}^{\lambda,\mu\nu} + g^{\lambda\mu} K^{\nu} - g^{\lambda\nu} K^{\mu}
\end{aligned}
$$

the method used to get the new primed tensors constrains the possible values of the vector field:

$$\Rightarrow \partial^{\nu} K^{\mu} = 0$$

- F.Becattini, L. Tinti, Phys. Rev. D 84, 025013 (2011)

Are these conditions fulfilled?



Being the conditions on the average values, the symmetry of the state of the system plays a fundamental role. In the *grand-canonical ensemble* we proved that any microscopic tensor give the same macroscopic results

$$\widehat{\rho} = \frac{1}{Z}\exp(-\widehat{H}/T + \mu\widehat{Q}/T)$$
$$Z = \mathrm{tr}[\exp(-\widehat{H}/T + \mu\widehat{Q}/T)]$$

The situation is completely different in a less symmetric case

$$\widehat{\rho} = \frac{1}{Z_\omega}\mathrm{P}_V\exp(-\widehat{H}/T + \boldsymbol{\omega}\cdot\widehat{\boldsymbol{J}}/T + \mu\widehat{Q}/T)$$
$$Z_\omega = \mathrm{tr}[\mathrm{P}_V\exp(-\widehat{H}/T + \boldsymbol{\omega}\cdot\widehat{\boldsymbol{J}}/T + \mu\widehat{Q}/T)]$$

where the average value is less constrained by the symmetry of the system

$$\phi^{\lambda,\mu\nu} = \langle\widehat{\phi}^{\lambda,\mu\nu}\rangle$$

and the the new primed tensors don't fulfill the equivalence conditions by construction any longer.

Thermodynamical inequivalence of stress-energy and spin tensor

An example, the free Dirac field:

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}_\mu \Psi - m\overline{\Psi}\Psi$$

$$\widehat{T}^{\mu\nu} = \frac{i}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{1}{2}\overline{\Psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi = \frac{i}{8}\overline{\Psi}\{\gamma^\lambda, [\gamma^\mu, \gamma^\nu]\}\Psi$$

Are the tensor given by Belinfante symmetrization equivalent like in the grand-canonical ensemble?

$$\widehat{T}^{\mu\nu}_{\mathrm{B.}} = \frac{i}{4}\left[\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi + \overline{\Psi}\gamma^\nu \overleftrightarrow{\partial}^\mu \Psi\right]$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu}_{\mathrm{B.}} = 0$$

i.e. does the spin tensor operator represent a suitable transformation ?

For symmetry reasons the average value of $\widehat{\mathcal{S}}^{\lambda,\mu\nu}$, decomposed using the basis:

$$u = (\gamma, \gamma\mathbf{v}) \qquad \tau = (\gamma v, \gamma\hat{\mathbf{v}})$$

$$n = (0, \hat{\mathbf{r}}) \qquad k = (0, \hat{\mathbf{k}})$$

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{x}$$

$$\mathcal{S}^{\lambda,\mu\nu} = D(r)[(n^\mu\tau^\nu - n^\nu\tau^\mu)u^\lambda + (n^\lambda\tau^\mu - n^\mu\tau^\lambda)u^\nu - (n^\lambda\tau^\nu - n^\nu\tau^\lambda)u^\mu]$$

is fully described by a function:

$$\mathcal{S}^{0,12} = \mathrm{tr}\left(\widehat{\rho}\,\widehat{\mathcal{S}}^{0,12}\right) = D(r)$$

Free rotating Dirac field in a cylindrical region

$$T^{0i}_{\rm Belinfante} = T^{0i}_{\rm canonical} - \frac{1}{2}\frac{{\rm d}D(r)}{{\rm d}r}\hat{v}^i$$

$$\boldsymbol{\mathcal{J}}_{\rm Belinfante} = \boldsymbol{\mathcal{J}}_{\rm canonical} - \left(\frac{1}{2}r\frac{{\rm d}D(r)}{{\rm d}r} + D(r)\right)\hat{\mathbf{k}}$$

From an explicit calculation we found:

$$\mathrm{tr}[\widehat{\rho} :\Psi^\dagger(0,\mathbf{x})\Sigma_z\Psi(0,\mathbf{x}):] = D(r)$$

$$= \sum_M \sum_{\xi=\pm 1}\sum_{l=1}^{\infty}\int_{-\infty}^{\infty}{\rm d}p_z \left[\frac{1}{{\rm e}^{(\varepsilon - M\omega+\mu)/T}+1} + \frac{1}{{\rm e}^{(\varepsilon - M\omega-\mu)/T}+1}\right]\frac{p_{Tl}^2\left[J^2_{|M-\frac{1}{2}|}(p_{Tl}r) - {b_\xi^{(+)}}^2 J^2_{|M+\frac{1}{2}|}(p_{Tl}r)\right]}{4\pi R J^2_{|M-\frac{1}{2}|}(p_{Tl}R)(2Rm_{Tl}^2 + 2\xi M m_{Tl} + m)}$$

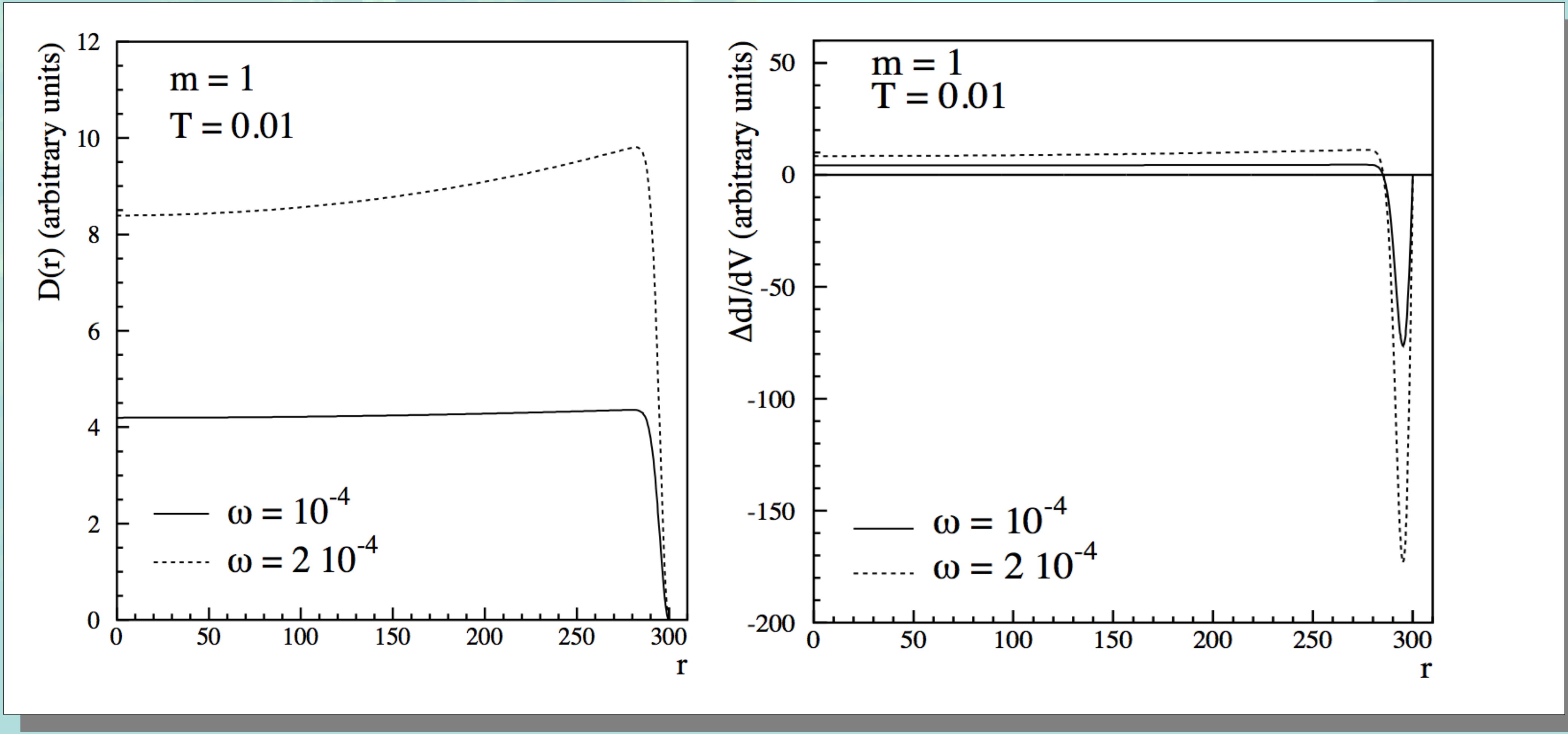
We proved that this function is not vanishing if there is a non vanishing angular velocity

D(r) in the non-relativistic limit

It is the sum of a particle and antiparticle term:

$$D(r)^{\pm} = \hbar \mathrm{tr}\left[\widehat{\rho}\left(:\Psi^{\dagger}\Sigma_z\Psi:\right)^{\pm}\right] \simeq \frac{1}{2}\frac{\hbar\omega}{KT}\hbar \mathrm{tr}\left[\widehat{\rho}\left(:\Psi^{\dagger}\Psi:\right)^{\pm}\right] = \hbar\frac{1}{2}\frac{\hbar\omega}{KT}\left(\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{x}}\right)^{\pm}$$

we can make a numerical computation of the D(r) function:



Linear response and kinetic coefficients

for an isotropic fluid:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu} + \Pi_0 \Delta^{\mu\nu} \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

where:

$$q^\mu = k(\Delta^{\mu\alpha}\partial_\alpha T - T A^\mu) \qquad \Pi^{\mu\nu} = \eta\left(\Delta^{\mu\alpha}\partial_\alpha u^\nu + \Delta^{\nu\alpha}\partial_\alpha u^\mu - \frac{2}{3}\partial\cdot u \Delta^{\mu\nu}\right) \qquad \Pi_0 = \zeta\partial\cdot u$$

$$T^{\mu\nu}_{\text{eq.}} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \qquad u^\mu_{\text{eq.}} = (1, \mathbf{0})$$

the variation from the equilibrium

linear in the small change:

$$\delta u^y \Rightarrow u^0 = \sqrt{1 + (\delta u^y)^2} \simeq 1 + \frac{1}{2}(\delta u^y)^2 = 1$$

$$\partial_x \delta u^y \neq 0$$

is

$$T_{xy} = \delta T_{xy} = q_x u_y + \Pi_{xy} = \eta\partial_x u_y$$

Zubarev method for relativistic Kubo formulas

$$\widehat{\rho} = \frac{1}{Z} \mathrm{e}^{-\widehat{\Upsilon}}$$

$$Z = \mathrm{tr}\left(\mathrm{e}^{-\widehat{\Upsilon}}\right)$$

We use the non equilibrium stationary operator to obtain the macroscopic energy momentum tensor

for the Belinfante symmetrized case:

$$\widehat{\Upsilon} = \lim_{\epsilon \longrightarrow 0} \epsilon \int_{-\infty}^{t} \mathrm{d}t_1 \, \mathrm{e}^{\epsilon(t_1 - t)} \int \mathrm{d}^3\mathbf{x} \, \widehat{T}^{0\mu}(t_1, \mathbf{x}) \beta_\mu(t_1, \mathbf{x})$$

where $\beta^\mu = (1/T) u^\mu$ *is a relativistic generalization of the temperature of a system*

In the specific case of the canonical equilibrium where $\beta^\mu_{\mathrm{eq.}} = \overline{\beta} \delta^\mu_0$ **it reduces to the known canonical density matrix**

$$\widehat{\rho} = \frac{1}{Z} \mathrm{e}^{-\overline{\beta} \widehat{H}}$$

Zubarev D N 1974 *Nonequilibrium Statistical Thermodynamics* (New York: Plenum Press)

To get the average values of the stress-energy tensor we divide the exponent in

$$-\widehat{\Upsilon} = \widehat{A} + \widehat{B}$$

where $\widehat{A} = -\int \mathrm{d}^3\mathbf{x}\, \widehat{T}^{0\mu}\beta_\mu$ *corresponds to a local equilibrium distribution* $\widehat{\rho}_{\mathrm{L.E.}} = \frac{1}{Z_{\mathrm{L.E.}}}\mathrm{e}^{\widehat{A}}$

and $\widehat{B}$ *linearly depends on the four temperature gradients.*

As long as the deviation from equilibrium is small we can make the approximations:

$$Z = \mathrm{tr}\left(\mathrm{e}^{\widehat{A}+\widehat{B}}\right) \simeq \mathrm{tr}\left(\mathrm{e}^{\widehat{A}}\left[1+\widehat{B}\right]\right) = Z_{\mathrm{L.E.}}\left(1+\langle\widehat{B}\rangle_{\mathrm{L.E.}}\right) \Rightarrow \frac{1}{Z} \simeq \frac{1}{Z_{\mathrm{L.E.}}}\left(1-\langle\widehat{B}\rangle_{\mathrm{L.E.}}\right)$$

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \left[1+\int_0^1 \mathrm{d}z\, \mathrm{e}^{z(\widehat{A}+\widehat{B})}\widehat{B}\mathrm{e}^{-z\widehat{A}}\right]\mathrm{e}^{\widehat{A}} \simeq \left[1+\int_0^1 \mathrm{d}z\, \mathrm{e}^{z\widehat{A}}\widehat{B}\mathrm{e}^{-z\widehat{A}}\right]\mathrm{e}^{\widehat{A}}$$

to finally get the relativistic Kubo formula:

$$\eta = -i \lim_{\epsilon,\kappa\to 0}\int_{-\infty}^{0}\mathrm{d}\tau\,\frac{1-\mathrm{e}^{\epsilon\tau}}{\epsilon}\int \mathrm{d}^3\mathbf{x}\langle\left[\widehat{T}^{xy}(\tau,\mathbf{x}),\widehat{T}^{xy}(0,\mathbf{0})\right]\rangle_{\mathrm{eq.}}\cos(\kappa x)$$

When there is a spin tensor we have to add another contibution

$$\widehat{\rho} = \frac{1}{Z} \mathrm{P}_V \exp \left\{ - \left[\lim_{\epsilon \to 0} \epsilon \int_{-\infty}^{t} \mathrm{d}t_1 \mathrm{d}^3\mathbf{y} \; \mathrm{e}^{\epsilon(t_1 - t)} \left(\widehat{T}^{0\nu}(t_1, \mathbf{y}) \; \beta_\nu(t_1, \mathbf{y}) - \frac{1}{2} \widehat{\mathcal{S}}^{0,\mu\nu} \omega_{\mu\nu} \right) \right] \right\}$$

otherwise we wouldn't get the rotating density matrix

$$\widehat{\rho} = \frac{1}{Z} \mathrm{P}_V \mathrm{e}^{-\overline{\beta} \widehat{H} + \omega \widehat{J}_z}$$

corresponding to the equilibrium four temperature:

$$\beta_{\mathrm{eq.}} = \overline{\beta}(1, \boldsymbol{\omega} \times \mathbf{x})$$

There is a constraint at equilibrium over the two rank tensor

$$\omega^{\mu\nu}_{\mathrm{eq.}} = -\frac{1}{2} (\partial^\mu \beta^\nu_{\mathrm{eq.}} - \partial^\nu \beta^\mu_{\mathrm{eq.}})$$

The density matrix changes if we change the quantum tensors

$$\widehat{\Upsilon}' - \widehat{\Upsilon} = \frac{1}{2}\lim_{\epsilon\to 0} \epsilon \int_{-\infty}^{t} \mathrm{d}t_1 \mathrm{e}^{\epsilon(t_1 - t)} \left\{ \int \mathrm{d}S n_i \left[\widehat{\phi}^{i,0\nu} - \widehat{\phi}^{0,i\nu} - \widehat{\phi}^{\nu,i0} \right] \delta\beta_\nu + \right.$$
$$- \int \mathrm{d}^3\mathbf{x}\, \widehat{\phi}^{\lambda,0\nu} \left(\partial_\lambda \delta\beta_\nu + \partial_\nu \delta\beta_\lambda \right) +$$
$$\left. + \int \mathrm{d}^3\mathbf{x}\, \widehat{\phi}^{0,\mu\nu} \left[\frac{1}{2} \left(\partial_\mu \delta\beta_\nu - \partial_\nu \delta\beta_\mu \right) + \delta\omega_{\mu\nu} \right] \right\}$$

The las term vanishes if we assume that the relation

$$\omega_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu \right)$$

holds out of equilibrium as at equilibrium

It will still remain

$$-\frac{1}{2}\lim_{\epsilon\to 0} \epsilon \int_{-\infty}^{t} \mathrm{d}t_1 \mathrm{e}^{\epsilon(t_1 - t)} \int \mathrm{d}^3\mathbf{x}\, \widehat{\phi}^{\lambda,0\nu} \left(\partial_\lambda \delta\beta_\nu + \partial_\nu \delta\beta_\lambda \right)$$

For a generic “pseudo-gauge” transformation

$$\widehat{T'}^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha\left(\widehat{\phi}^{\alpha,\mu\nu} - \widehat{\phi}^{\mu,\alpha\nu} - \widehat{\phi}^{\nu,\alpha\mu}\right)$$

$$\widehat{\mathcal{S}'}^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\phi}^{\lambda,\mu\nu}$$

If we call $\widehat{\Xi}^{\alpha\mu\nu} = \widehat{\phi}^{\mu,\alpha\nu} + \widehat{\phi}^{\nu,\alpha\mu}$

the transport coefficients will be different in general, in particular for the shear viscosity:

$$\eta' - \eta = i \lim_{\epsilon,\kappa\to 0}\left\{\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\epsilon\tau} \int_V \mathrm{d}\mathbf{x} \langle\left[\widehat{\Xi}^{0xy}(\tau,\mathbf{x}), \widehat{T}^{xy}_{\mathrm{s.}}(0,\mathbf{0})\right]\rangle_{\mathrm{eq.}} \cos(\kappa x) + \right.$$
$$\left. + \frac{1}{4}\int_{-\infty}^{0} \mathrm{d}\tau\left(\delta(\tau) - \epsilon\mathrm{e}^{\epsilon\tau}\right)\int_V \mathrm{d}\mathbf{x}\langle\left[\widehat{\Xi}^{0xy}(\tau,\mathbf{x}), \widehat{\Xi}^{0xy}(0,\mathbf{0})\right]\rangle_{\mathrm{eq.}} \cos(\kappa x)\right\}$$

This difference in transport coefficients directly depends on the microscopic spin tensor

$$\widehat{\mathcal{S}'}^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\phi}^{\lambda,\mu\nu} \Rightarrow \widehat{\phi}^{\lambda,\mu\nu} = -\Delta\widehat{\mathcal{S}}^{\lambda,\mu\nu} = -\left(\widehat{\mathcal{S}'}^{\lambda,\mu\nu} - \widehat{\mathcal{S}}^{\lambda,\mu\nu}\right)$$

so, if the starting couple is the Belinfante one $\widehat{\phi}^{\lambda,\mu\nu} = -\widehat{\mathcal{S}}^{\lambda,\mu\nu}$

$$\eta - \eta_{\mathrm{B.}} = -i \lim_{\epsilon,\kappa\to 0} \left\{ \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\epsilon\tau} \int_V \mathrm{d}\mathbf{x} \langle \left[\widehat{\mathcal{S}}^{x,0y}(\tau,\mathbf{x}) + \widehat{\mathcal{S}}^{y,0x}(\tau,\mathbf{x}), \widehat{T}^{xy}_{\mathrm{B.}}(0,\mathbf{0}) \right] \rangle_{\mathrm{eq.}} \cos(\kappa x) + \right.$$

$$\left. -\frac{1}{4} \int_{-\infty}^{0} \mathrm{d}\tau \left(\delta(\tau) - \epsilon\mathrm{e}^{\epsilon\tau} \right) \int_V \mathrm{d}\mathbf{x} \langle \left[\widehat{\mathcal{S}}^{x,0y}(\tau,\mathbf{x}) + \widehat{\mathcal{S}}^{y,0x}(\tau,\mathbf{x}), \widehat{\mathcal{S}}^{y,0x}(0,\mathbf{0}) + \widehat{\mathcal{S}}^{x,0y}(0,\mathbf{0}) \right] \rangle_{\mathrm{eq.}} \cos(\kappa x) \right\}$$

Can we measure it?

What's the magnitude of this difference in relevant cases?

Summary & outlook

- Couples of stress-energy and spin tensor, previously thought equivalent are actually thermodynamically inequivalent.
- This inequivalence persists in the non-relativistic limit and can be measured, at least in principle.
- Different couples of microscopic tensors give different transport coefficients.
- How can this difference in transport coefficients prediction be measured?