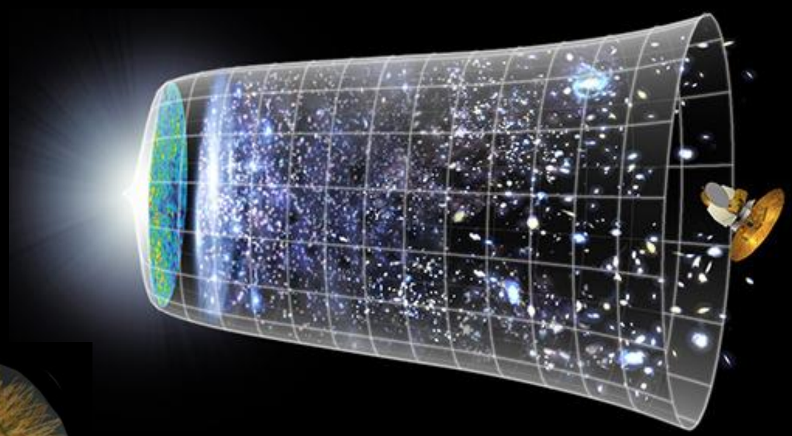
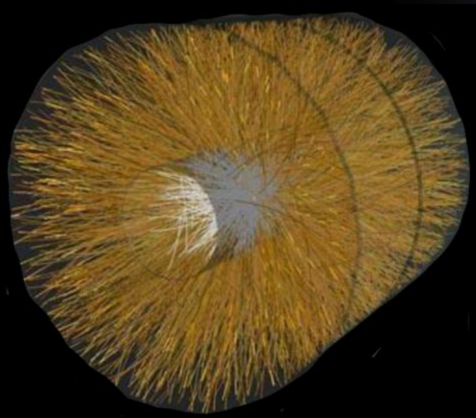
Turbulence and Bose Condensation: From Heavy Ions to Cold Atoms



WMAP Science Team



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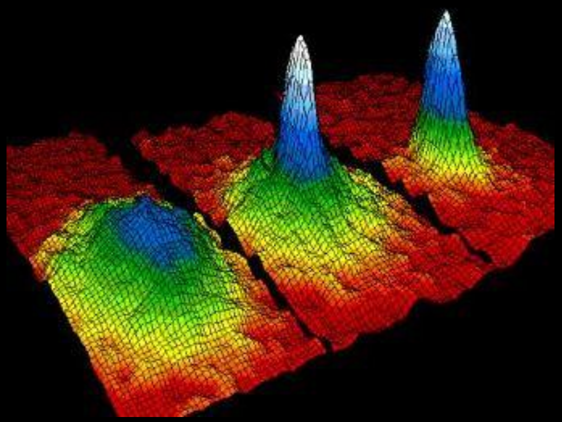

JILA/NIST

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NeD-2012, Crete

Content

I. Nonthermal fixed points

II. Turbulence, Bose condensation

III. Applications

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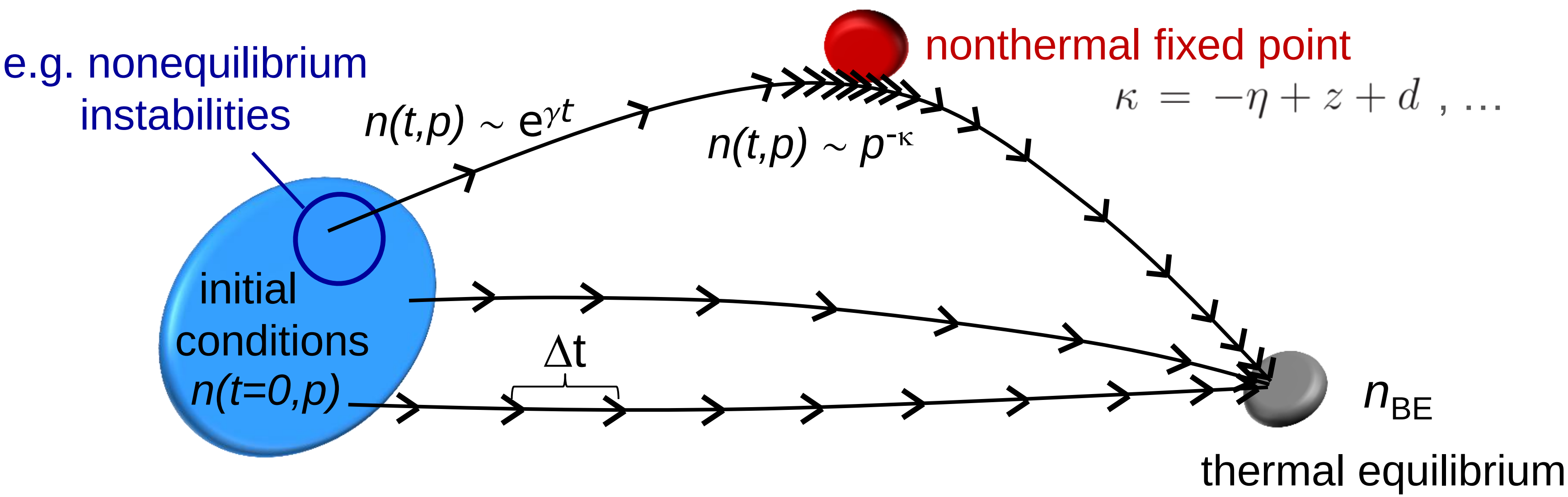
Nonequilibrium initial value problems

Thermalization process in quantum many-body systems?

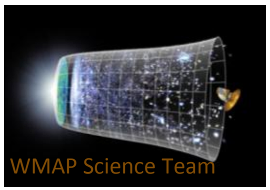
Schematically:



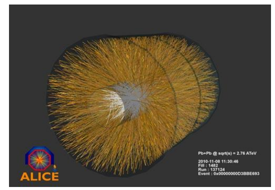
- **Characteristic nonequilibrium time scales? Relaxation? Instabilities?**
- **Diverging time scales far from equilibrium? Nonthermal fixed points?**

Universality far from equilibrium

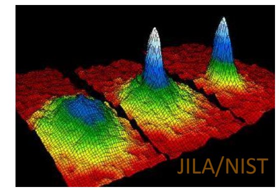
Early-universe preheating
(~10^{16} GeV)



Heavy-ion collisions
(~100 MeV)



Cold quantum gas dynamics
(~10^{-13} eV)



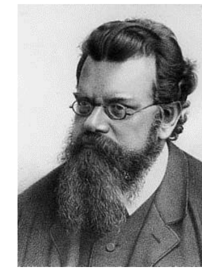
Instabilities, \`overpopulation´, …

Nonthermal fixed points

Very different microscopic dynamics can lead to
same *macroscopic scaling phenomena*

Digression: weak wave turbulence

Boltzmann equation for *relativistic* 2↔2 scattering, $n_1 \equiv n(t,p_1)$:



http://www.aeiou.at/aeiou.encyclop.b/b638771.htm

$$\frac{\mathrm{d}n_1}{\mathrm{d}t} = \int \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} \int \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \int \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4}$$

$$\times \quad \delta^3(p_1 + p_2 - p_3 - p_4)\,\delta(E_1 + E_2 - E_3 - E_4)\,(2\pi)^4 |M|^2$$

momentum conservation — energy conservation — scattering

$$\times \Big(n_3 n_4 (1 + n_1)(1 + n_2) - n_1 n_2 (1 + n_3)(1 + n_4) \Big)$$

"gain" term — "loss" term

Different stationary solutions, $\mathrm{d}n_1/\mathrm{d}t=0$, in the (classical) regime $n(p) \gg 1$:

1. $n(p) = 1/(e^{\beta\omega(p)} - 1)$ thermal equilibrium
2. $n(p) \sim 1/p^{4/3}$ turbulent *particle* cascade
3. $n(p) \sim 1/p^{5/3}$ *energy* cascade

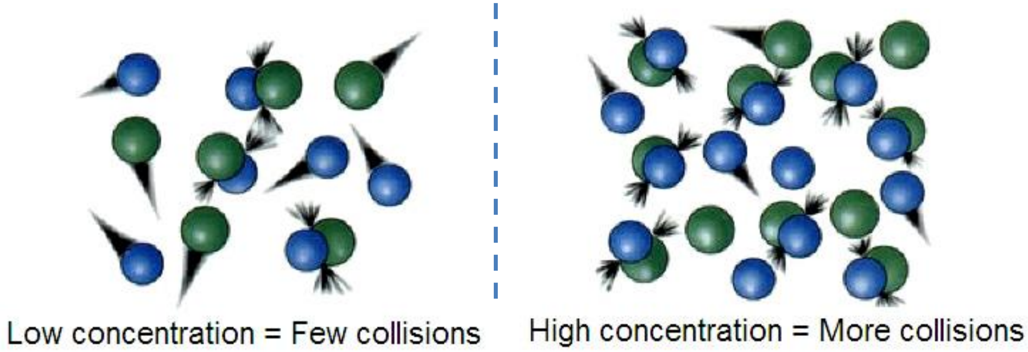
(2. and 3.): *Kolmogorov -Zakharov spectrum*

…associated to stationary transport of conserved quantities

Range of validity of Kolmogorov-Zakharov

E.g. self-interacting scalars with quartic coupling: $|M|^2 \sim \lambda^2 \ll 1$

$n(p) \lesssim 1$ | $1 \ll n(p) \ll 1/\lambda$ | $n(p) \sim 1/\lambda$



`overpopulation´

(non-perturbative)

analytically well described by QFT (2PI effective action resummation)!

Very high concentration = ?

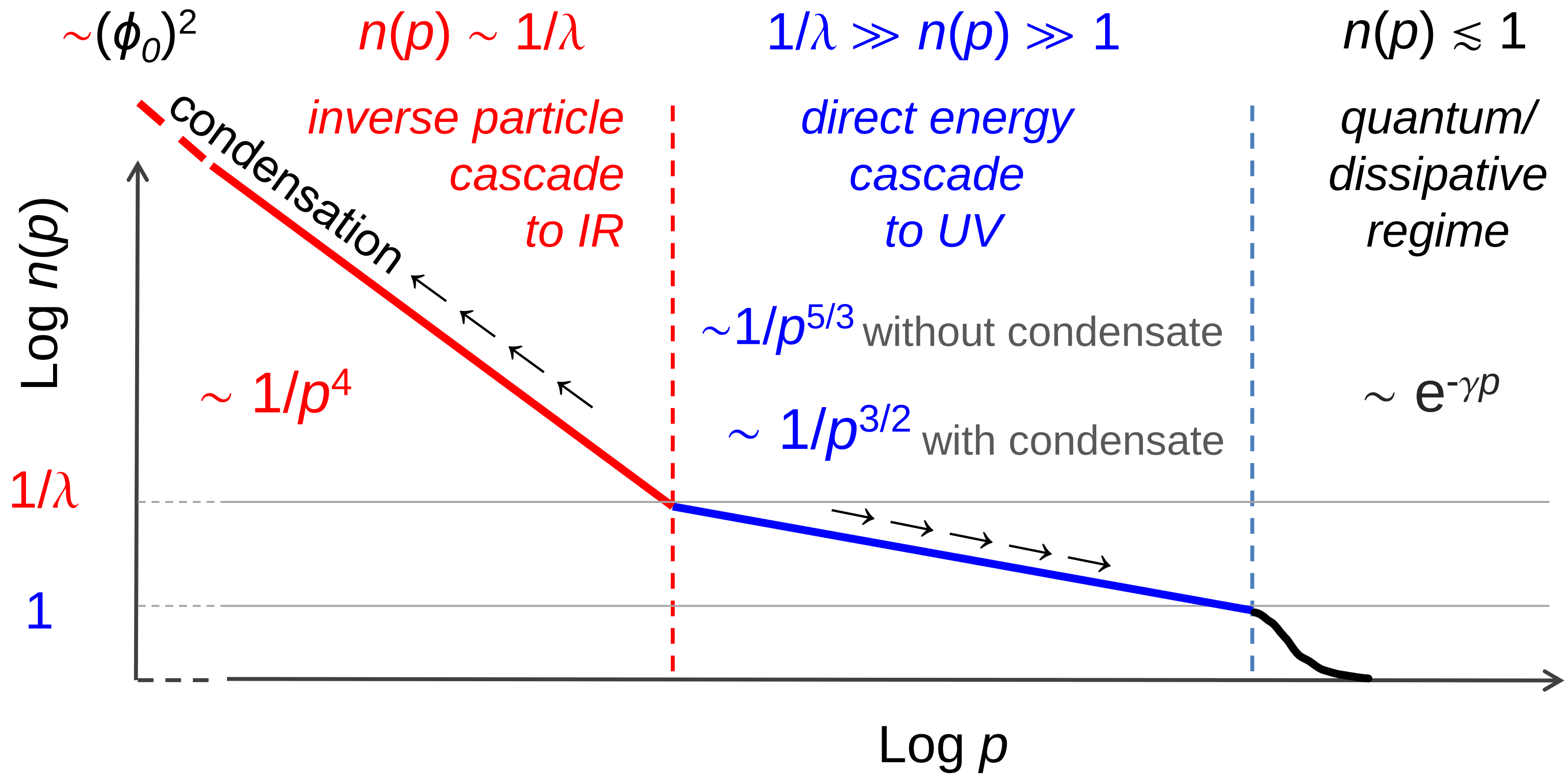
http://upload.wikimedia.org/wikipedia/commons/4/41/Molecular-collisions.jpg

Weak wave turbulence solutions are limited to the "window"

$1 \ll n(p) \ll 1/\lambda$, since for

$n(p) \sim 1/\lambda$ the n$\leftrightarrow$m scatterings for n,m=1,..,∞ are as important as 2$\leftrightarrow$2 **!**

Beyond weak wave turbulence: *here relativistic, d=3*



Non-thermal fixed point: $n(p) \sim 1/p^{d+z-\eta}$

Berges, Rothkopf, Schmidt
PRL 101 (2008) 041603

Bose-Einstein condensation from inverse particle cascade:

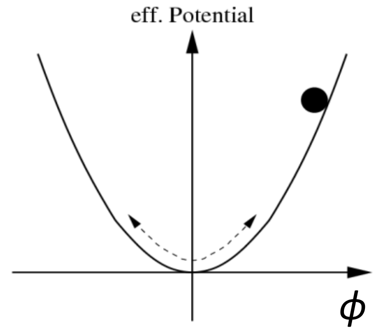
$$\sim (2\pi)^d \delta^{(d)}(\vec{p})\, \phi_0^2(t)$$

Berges, Sexty, PRL 108 (2012) 161601

Berges, Hoffmeister NPB813 (2009) 383; Nowak et al. PRA85 (2012) 043627; Nowak, Gasenzer arXiv:1206.3181

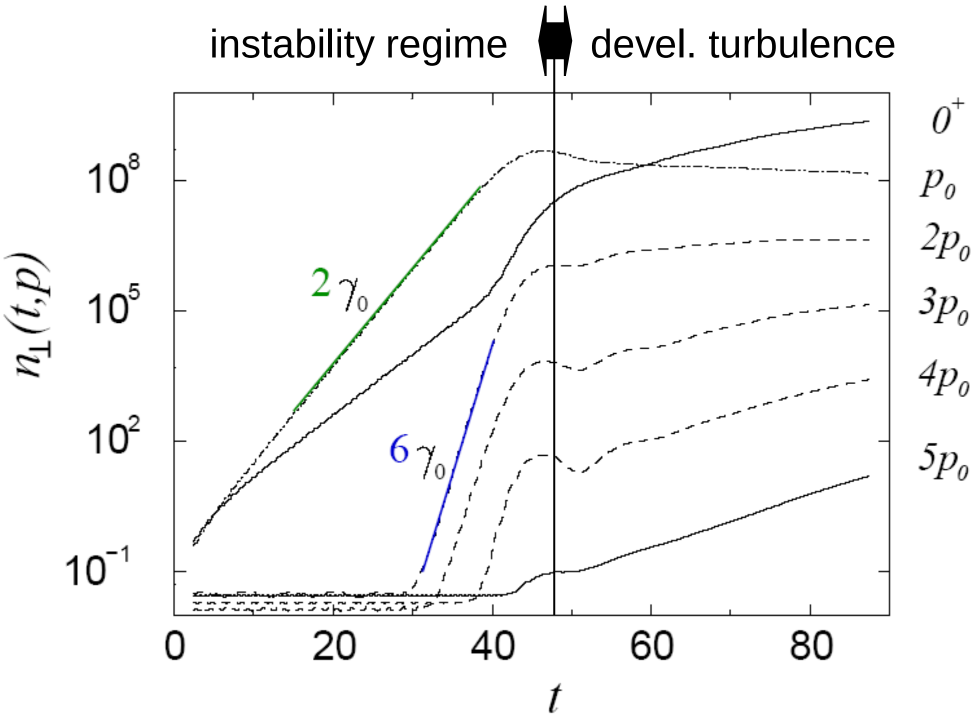
Dual cascade for linear sigma model in QFT

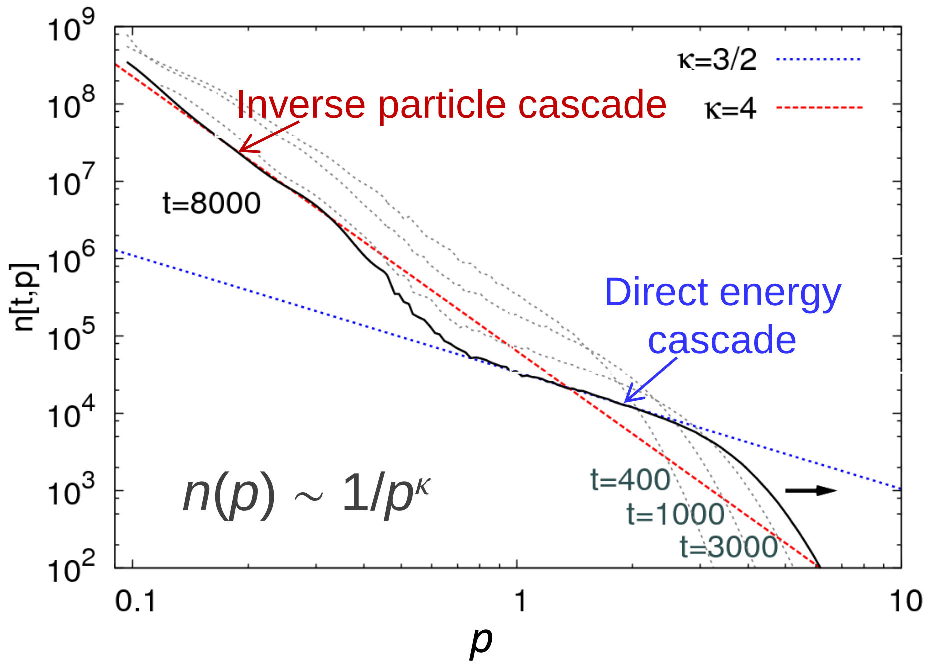
Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603



Parametric resonance instability (2PI 1/N to NLO):

$$\Phi(t,k) = (\ \phi(t,k),\ \pi(t,k)\ )$$





O(N=4) symmetric , here $\lambda \sim 10^{-4}$, $\phi\ (t) = \sigma(t)\sqrt{6N/\lambda}$ in units of $\sigma(t$=0)

Similar results also for other instabilities auch as spinodal dynamics!

Bose condensation from infrared particle cascade

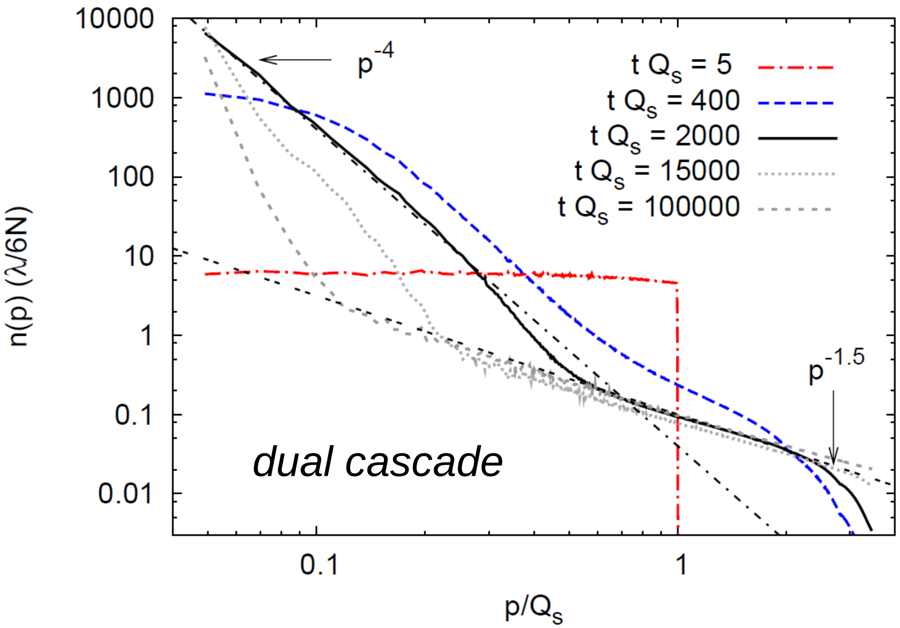
$$F(t,t';\vec{x}-\vec{y}) = \left\langle \left\{ \hat{\phi}(t,\vec{x}), \hat{\phi}(t',\vec{y}) \right\} \right\rangle$$

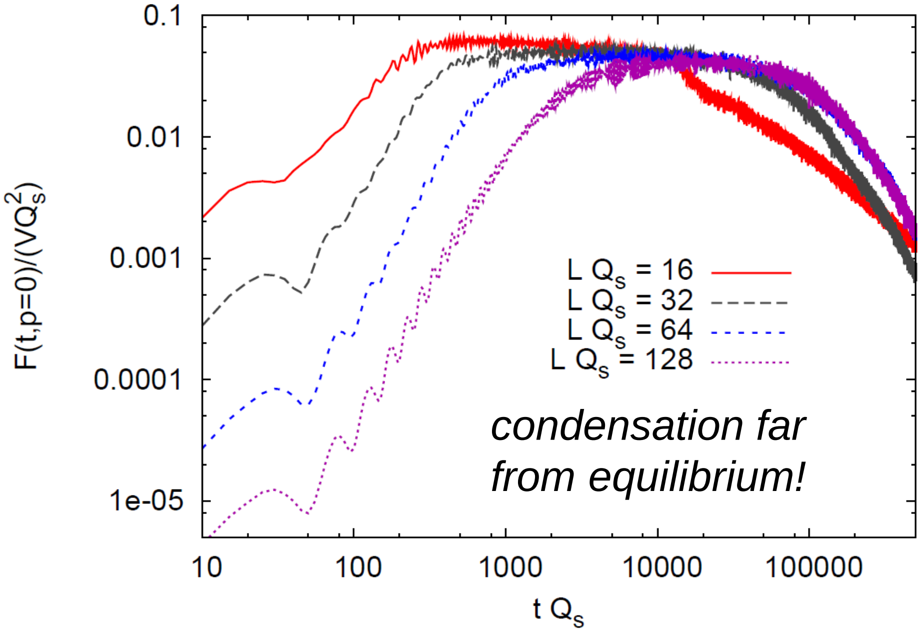
time-dependent condensate

$$F(t,t;p) = \frac{1}{\omega_p(t)} \left(n_p(t) + \frac{1}{2} \right) + (2\pi)^d \delta^{(d)}(\vec{p})\, \phi_0^2(t)$$

starting from `overpopulation´ ($\phi_0(0)=0$):

finite volume: $(2\pi)^d \delta^{(d)}(0) \rightarrow V$



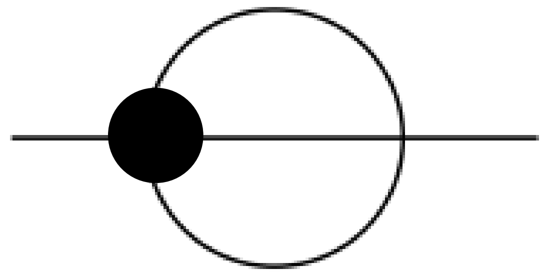


Berges, Sexty, PRL 108 (2012) 161601

From complexity to simplicity

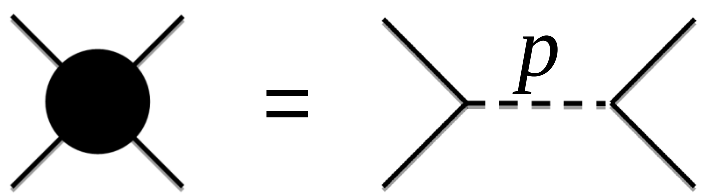
Complexity: many-body $n \leftrightarrow m$ processes for $n,m = 1,..,\infty$
as important as $2 \leftrightarrow 2$ scattering (\`overpopulation´)!

Simplicity: Resummation of the infinitely many processes leads to *effective kinetic theory* (2PI 1/*N* to NLO) dominated in the IR by



approximately number conserving! → particle cascade

describing $2 \leftrightarrow 2$ scattering with an *effective coupling:*

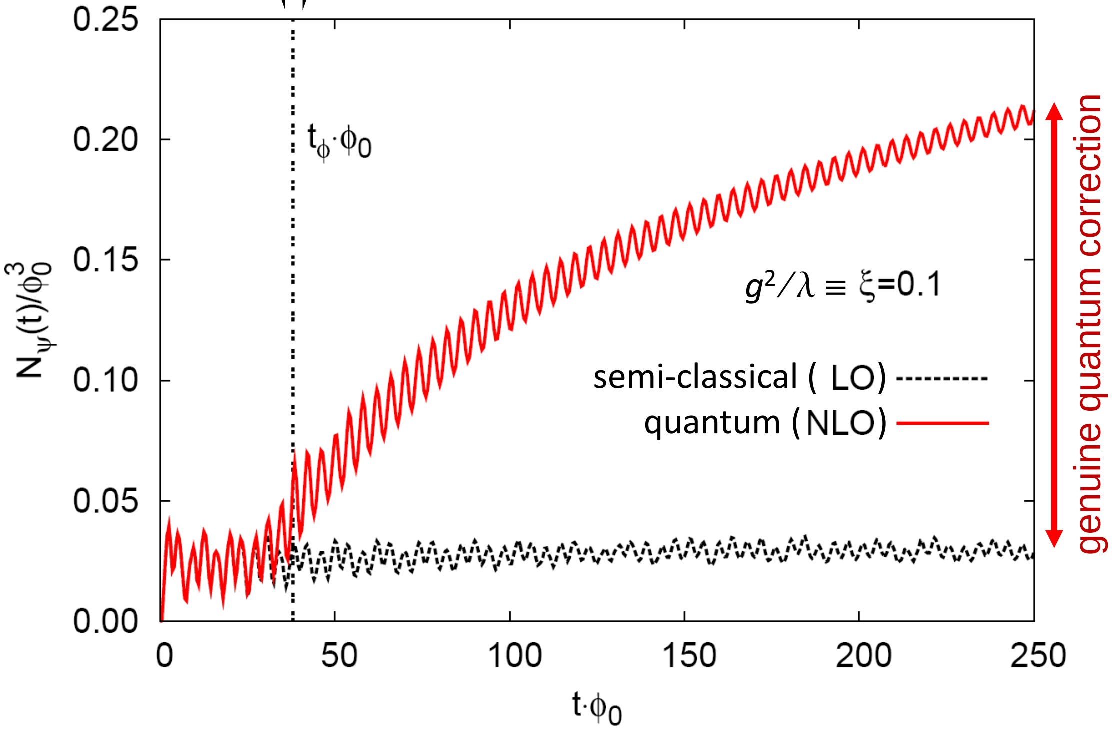


$\sim \; p^{8-4\eta}$

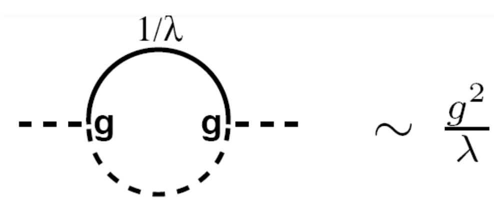
Overpopulation as a quantum amplifier

$SU_L(2) \times SU_R(2)$ quark meson model:

instability regime ⟷ *overpopulation, turbulent regime*



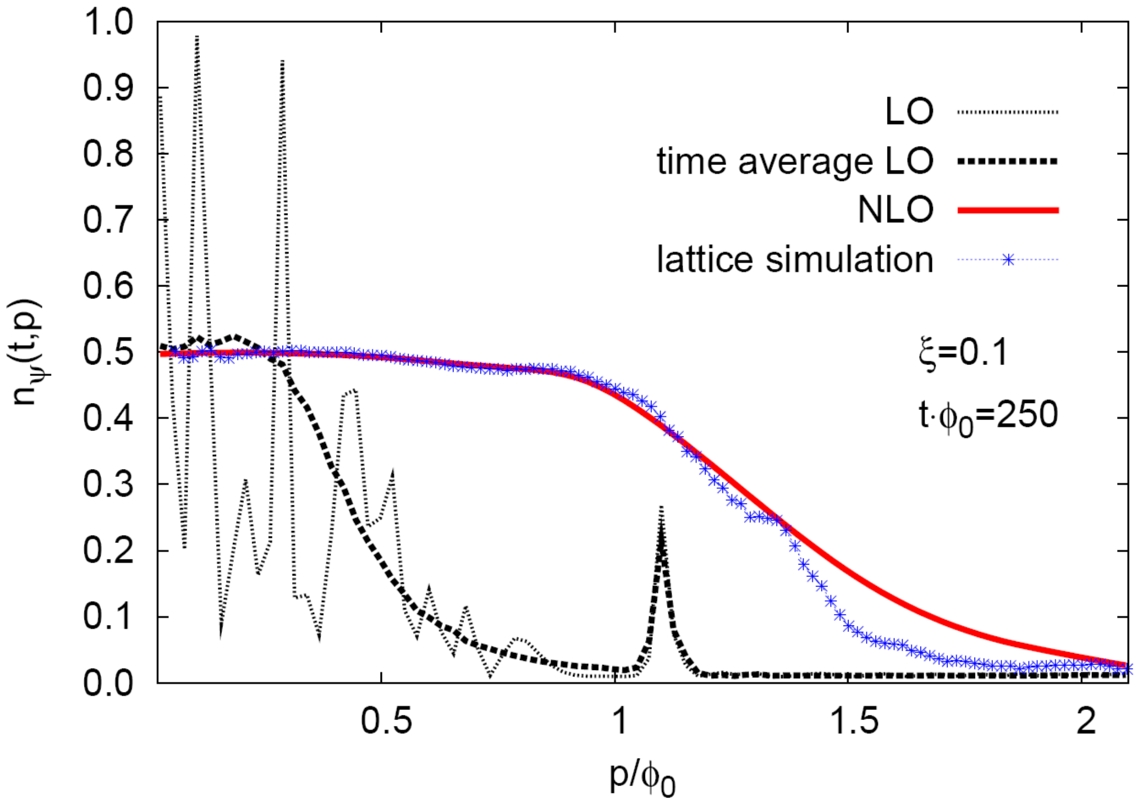
2PI-NLO:



Berges, Gelfand, Pruschke
PRL 107 (2011) 061301

Strongly enhanced quark production rate $\sim (g^2/\lambda)\ \phi_0$ **!**

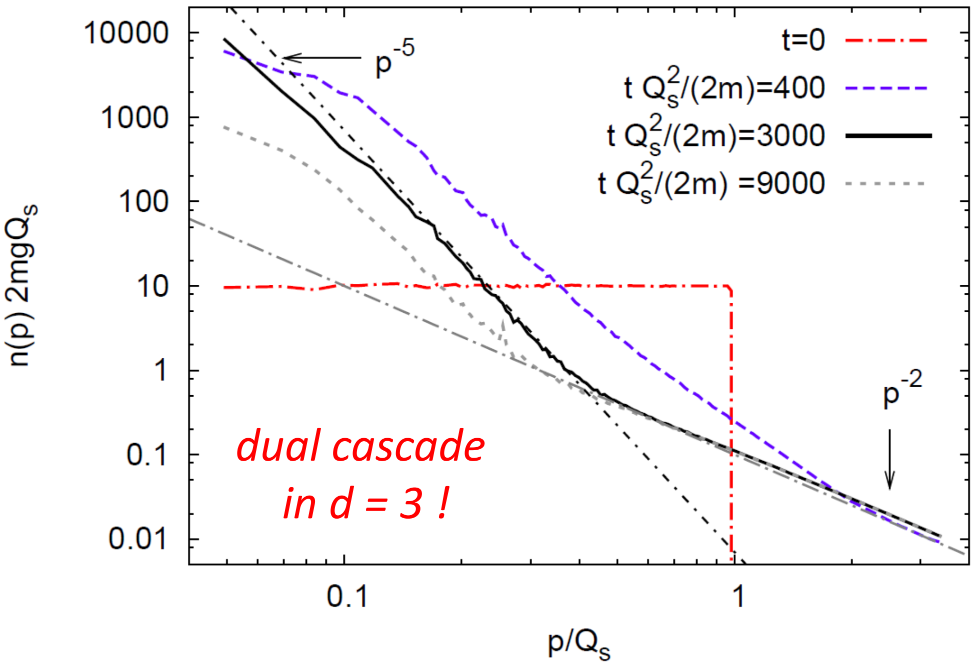
Lattice field theory simulations: quark meson model



Berges, Gelfand, Pruschke, PRL 107 (2011) 061301

- Real-time Wilson fermions on a (64^3) lattice in $d = 3 + 1$ for the first time!
- Very good agreement with NLO quantum result (2PI) for $\xi \ll 1$
  (differences at larger *p* depend on Wilson term → larger lattices)
- Lattice simulation can be applied to $\xi \sim 1$ relevant for *QCD*

Comparison to cold Bose gas (Gross-Pitaevskii)



Berges, Sexty, PRL 108 (2012) 161601

Infrared particle cascade leads to Bose condensation without subsequent decay

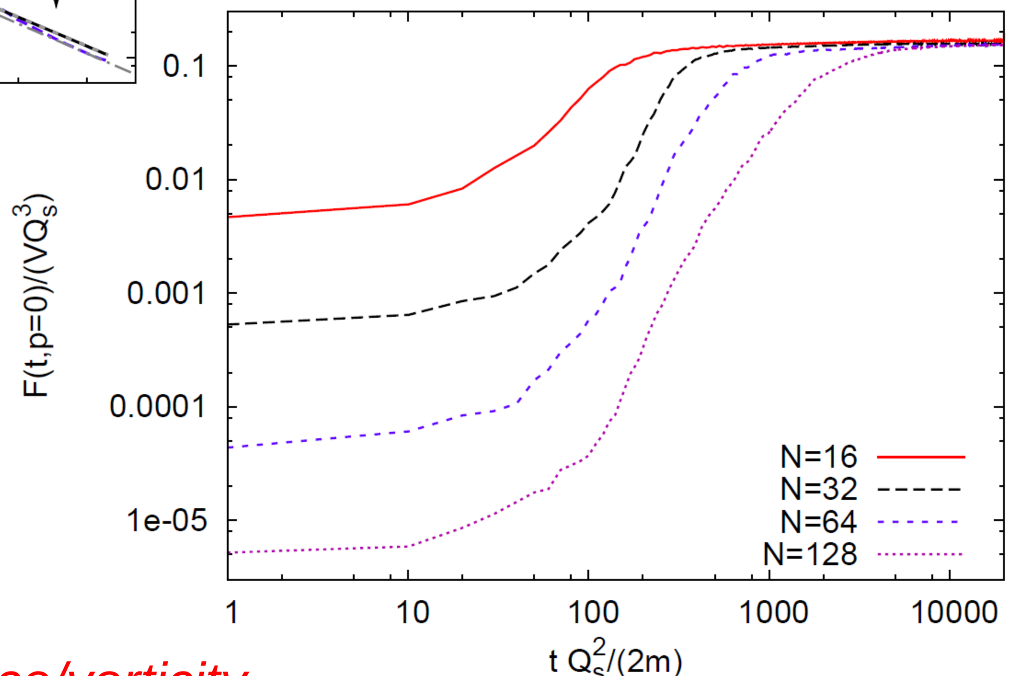
(no number changing processes)

Expected infrared cascade:

$$n(p) \sim 1/p^{d+2-\eta}$$

for non-relativistic dynamics

Scheppach, Berges, Gasenzer, PRA 81 (2010) 033611; Nowak, Sexty, Gasenzer, PRB 84 (2011) 020506(R); Nowak, Gasenzer arXiv:1206.3181



See also talk by T. Gasenzer !

→ connection to *quantum turbulence/vorticity*

Turbulence/Bose condensation for gluons?

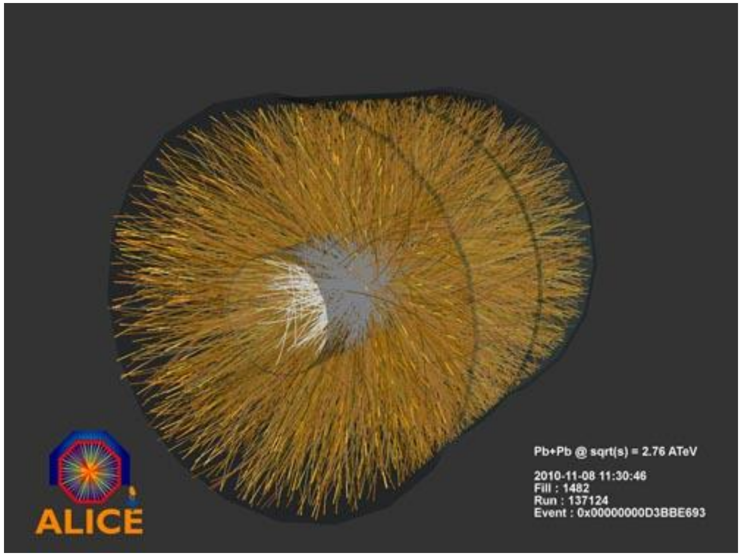
Field strength tensor, here for *SU*(2):

$$F^a_{\mu\nu}[A] = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

Equation of motion:

$$(D_\mu[A] F^{\mu\nu}[A])^a = 0$$

$$D^{ab}_\mu[A] = \partial_\mu \delta^{ab} + g\epsilon^{acb} A^c_\mu$$



Sampling introduces classical-statistical fluctuations (‘collision terms‘)

→ with quantum initial conditions accurate description for sufficiently ’large fields/high occupation’ numbers if

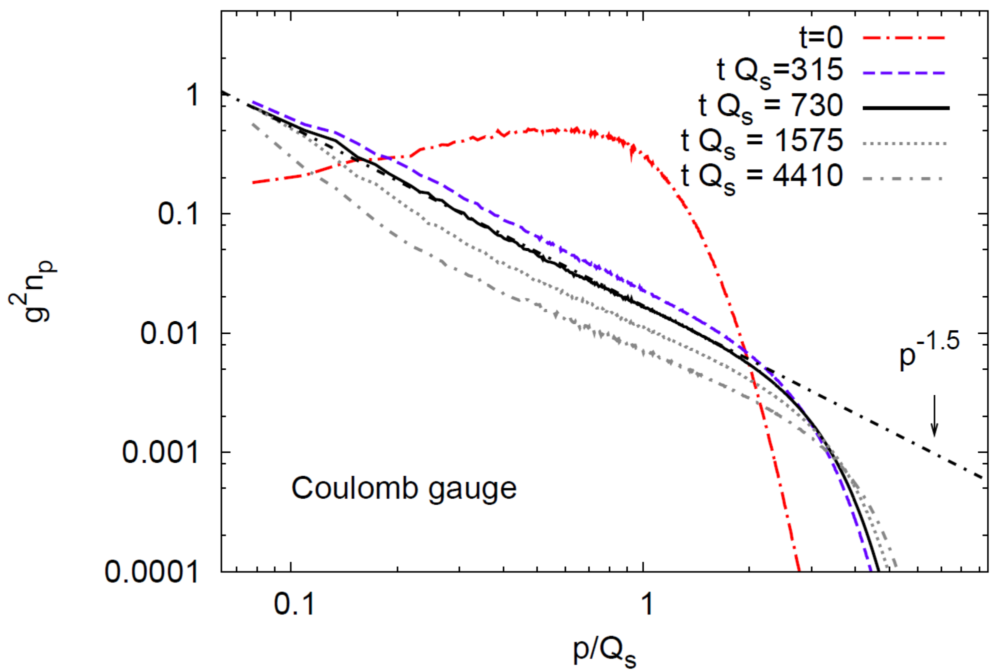
anti-commutators $\langle\{A, A\}\rangle \gg \langle[A, A]\rangle$ commutators

i.e. “n(p)“ $\gg$ 1

Classical-statistical lattice gauge theory

Occupancy: $\sim \sqrt{\langle |A^2(p)| \rangle \langle |E^2(p)| \rangle}$

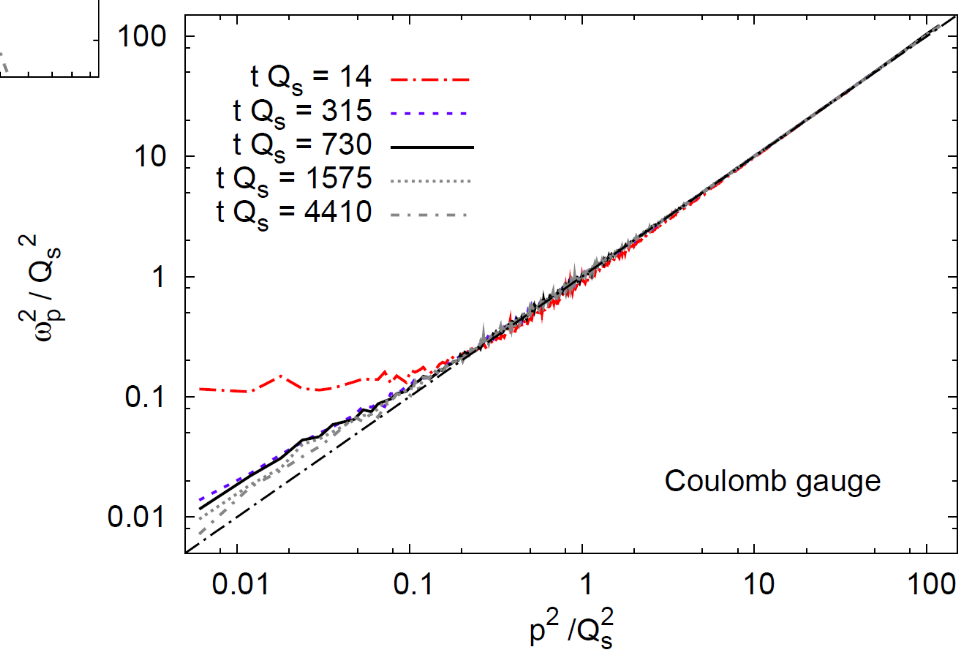
Berges, Schlichting, Sexty, arXiv:1203.4646



Initial overpopulation (fixed box):

$$\epsilon \sim \frac{Q_s^4}{g^2} \quad \text{i.e.} \quad n(p \simeq Q_s) \sim \frac{1}{g^2}$$

Dispersion: $\sim \sqrt{\langle |E^2(p)| \rangle / \langle |A^2(p)| \rangle}$



- Wave turbulence exponent 3/2 (as for scalars with condensate)!?

- No stable occupation numbers exceeding $g^2 n_p \sim 1$ observed yet

Scaling analysis

Leading (2PI) resummed perturbative contribution ($O(g^2)$):

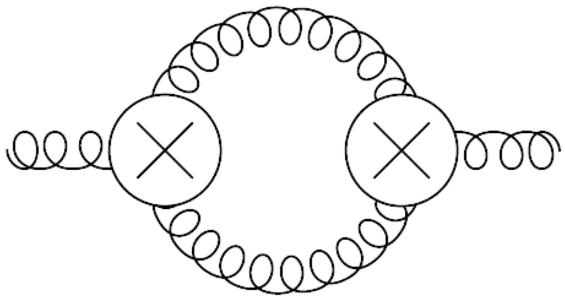


Figure 4: Gluon part of the one-loop contribution to the self-energy with (2PI) resummed propagator lines. The crossed circles indicate an effective three-vertex in the presence of a background gauge field potential.

Standard scaling analysis gives *for slowly varying background field:*

$n(p) \sim 1/p^{\kappa}$ $\quad$ $\kappa = \frac{3}{2}$, or $\kappa = 1$

energy cascade $\quad$ particle cascade

Lattice results explained in terms of intermediate `Bose condensation´ !?

Berges, Schlichting, Sexty, arXiv:1203.4646, Berges, Scheffler, Sexty, PLB 681 (2009) 362

Conclusions

Nonthermal fixed points:

- crucial for thermalization process from instabilities/overpopulation!

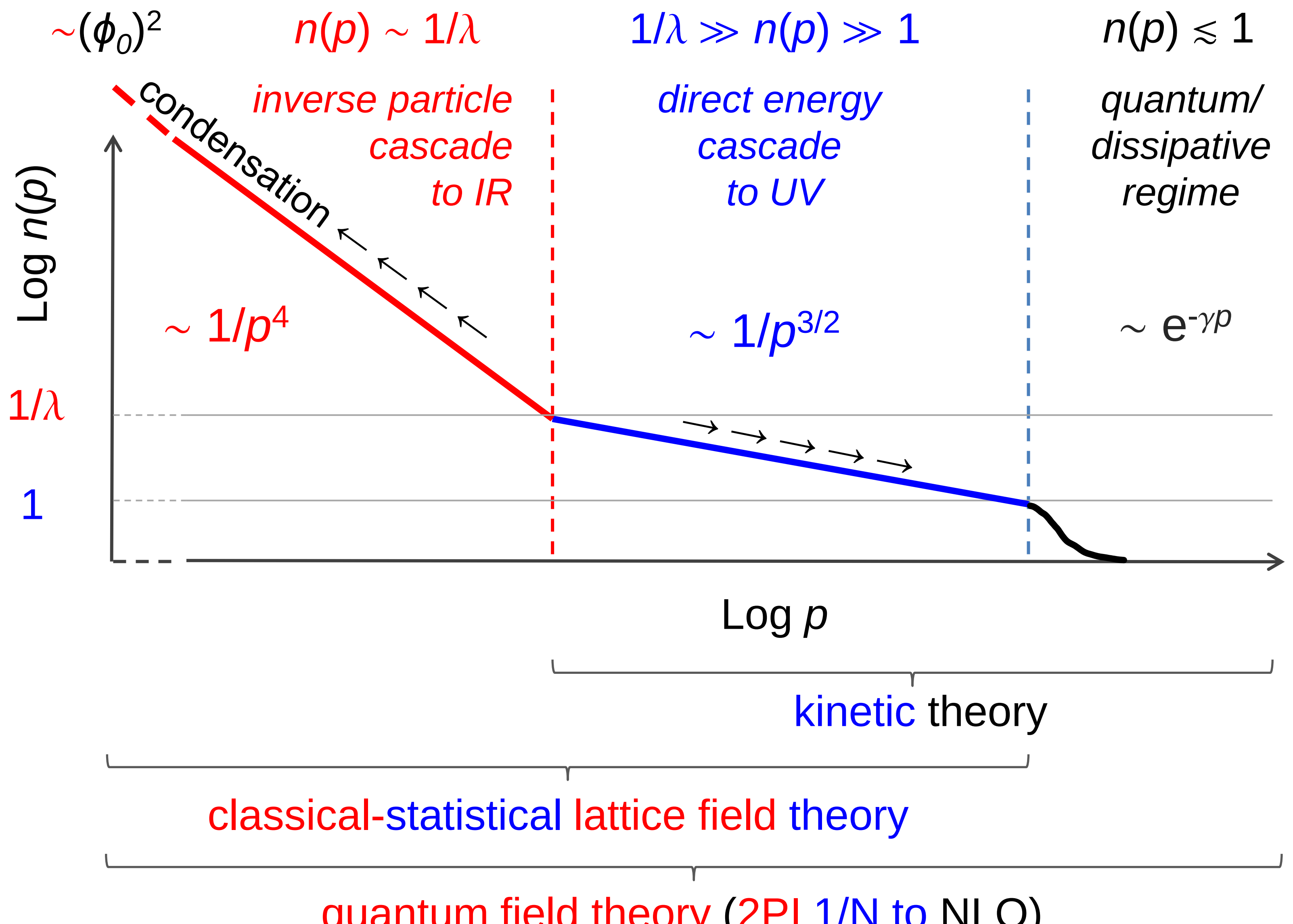
- strongly nonlinear regime of stationary transport (*dual* cascade)!

- Bose condensation for scalars from inverse particle cascade!

- large amplification of quark production!

- gauge theory results indicate the same weak wave turbulence exponents as for scalars!

Methods

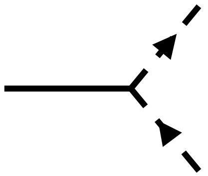


Berges, NPA 699 (2002) 847; Aarts, Ahrensmeier, Baier, Berges, Serreau PRD 66 (2002) 045008

Quark Meson Model

- generic interaction of Yukawa type for N_f (massless) Dirac fermions:

$$-\frac{g}{N_f}\bar{\psi}_i\left(\frac{1-\gamma^5}{2}\Phi^\dagger_{ij}+\frac{1+\gamma^5}{2}\Phi_{ij}\right)\psi_j$$



→ couples left- and right-handed components

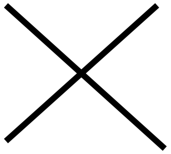
$$\psi_L=\frac{1-\gamma^5}{2}\psi \quad , \quad \psi_R=\frac{1+\gamma^5}{2}\psi$$

i.e. acting like a mass term for $\langle \Phi \rangle \neq 0$

- we consider $N_f = 2$ with symmetry group $SU_L(2)\times SU_R(2)\sim O(4)$

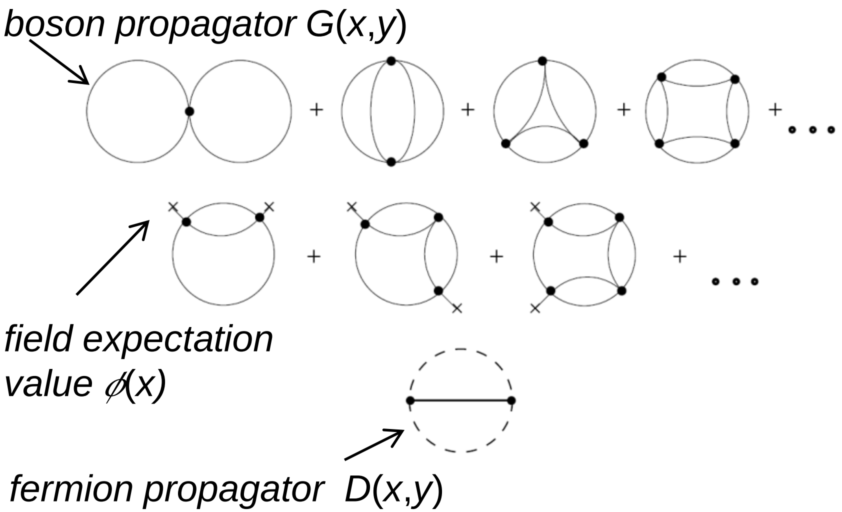
$$\Phi=\frac{1}{2}\left(\sigma+i\vec{\pi}\vec{\tau}\right)$$

→ N=4 component linear σ-model with quartic self-interaction



Approximation I

- 2PI effective action Γ:



NLO in the number N_s = 4 of inflaton field components $(\sigma, \vec{\pi})$

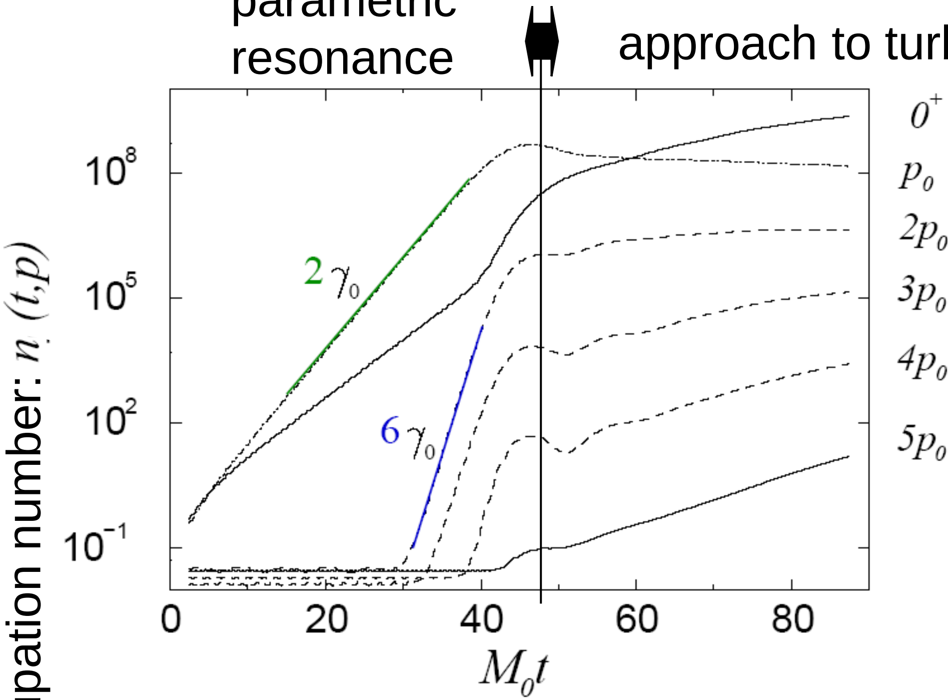
$\mathcal{O}(g^2)$

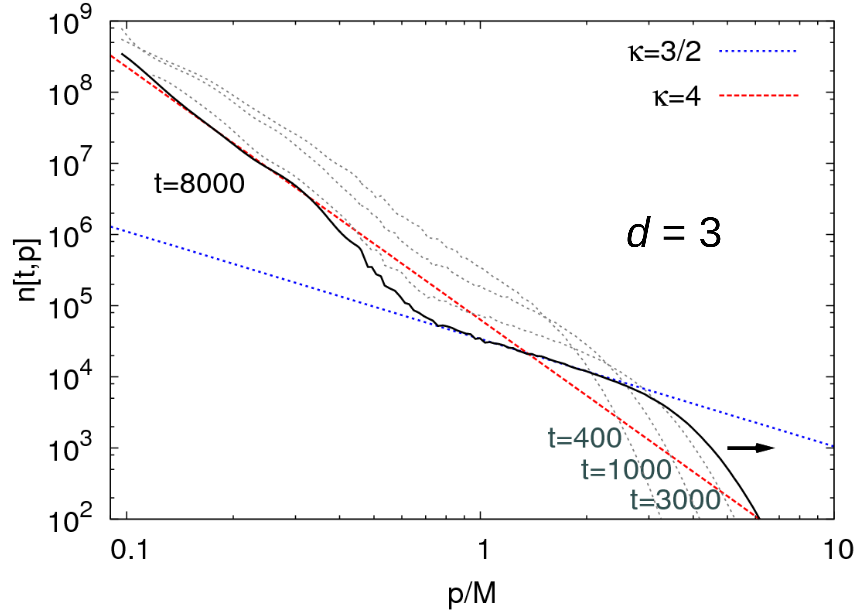
corresponding to self-consistently dressed self-energies Σ:

etc.

Bosonic sector ($g = 0$)

parametric resonance | approach to turbulence:

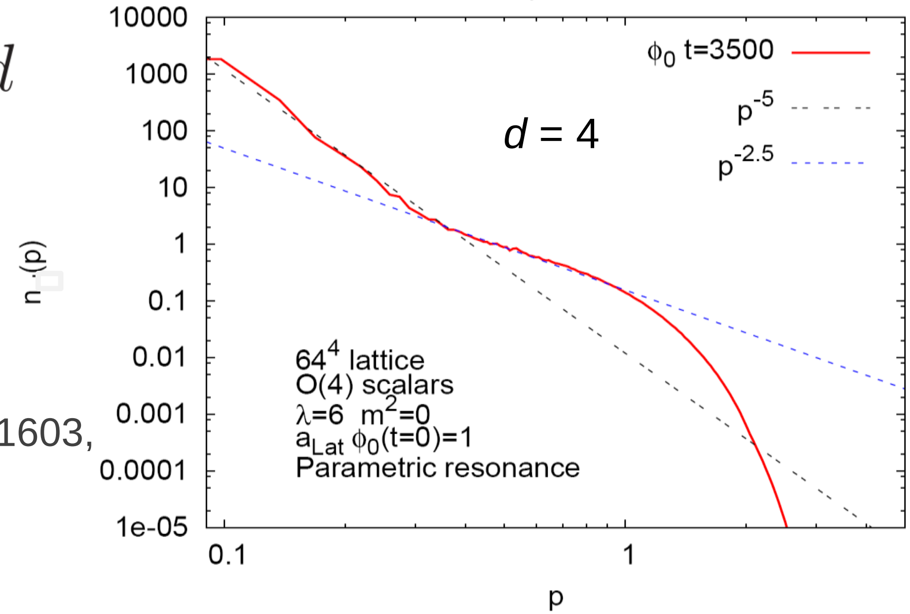




$n(t,p) \sim p^{-\kappa}$ with $\kappa = -\eta + z + d$

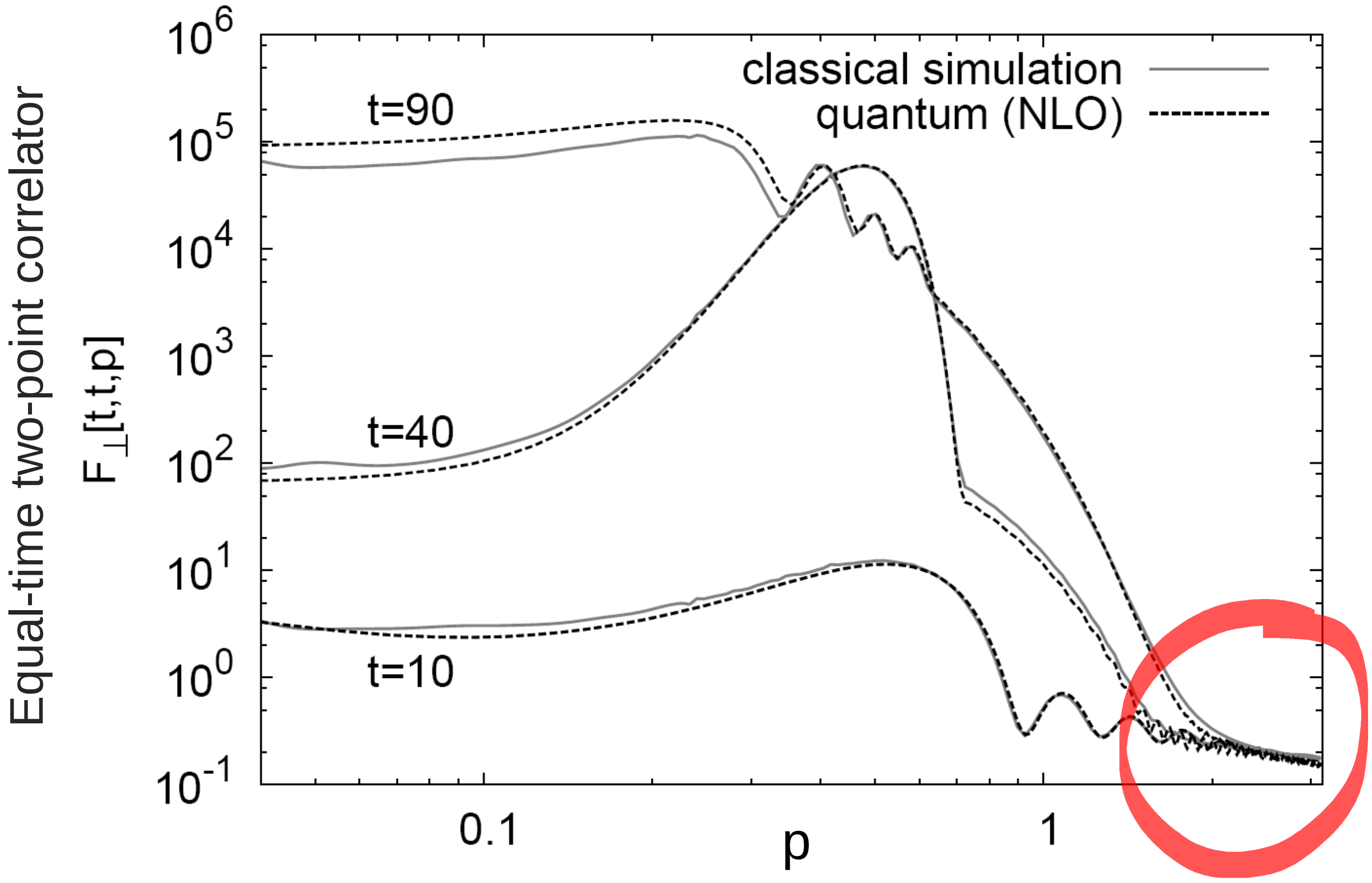
→ $\kappa = 4$ for $d = 3$,
$\kappa = 5$ for $d = 4$ ✓ IR

for $z = 1$ (relativistic), $\eta = 0$



Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603,
Berges, Hoffmeister, NPB 813 (2009) 383,
Berges, Sexty, PRD 83 (2011) 085004

Comparing classical to quantum (g=0)



Practically no *bosonic* quantum corrections at the end of instability

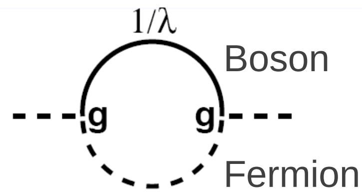
Accurate nonperturbative description by quantum 2PI-1/*N* to NLO

Fermions: failure of semi-classical approach

LO: $$iD^{-1}_{0,ij}(x,y) = \left[i\gamma^\mu \partial_\mu - m_\psi - \frac{g}{N_f}\phi(t) \right] \delta^{(4)}(x-y)\,\delta_{ij}$$

Baacke, Heitmann, Pätzold, *PRD* 58 (1998) 125013; Greene, Kofman, *PLB* 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, *JHEP* 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP *0002* (2000) 034; …

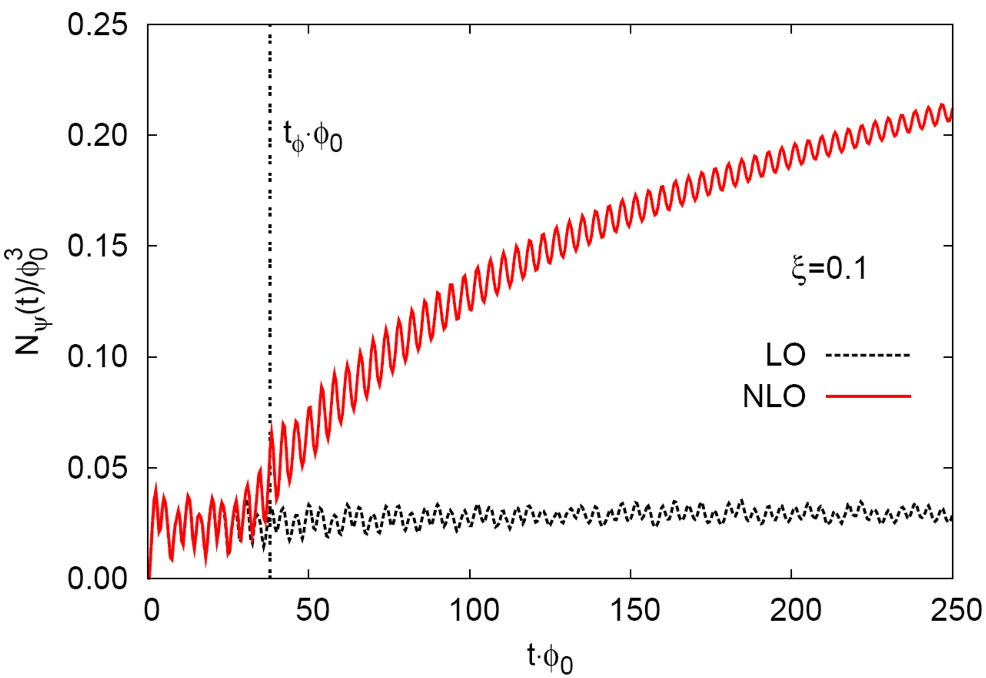
2PI-NLO: +



$\sim \frac{g^2}{\lambda}$

small self-coupling λ leads to *large* corrections!

Berges, Gelfand, Pruschke, PRL 107 (2011) 061301



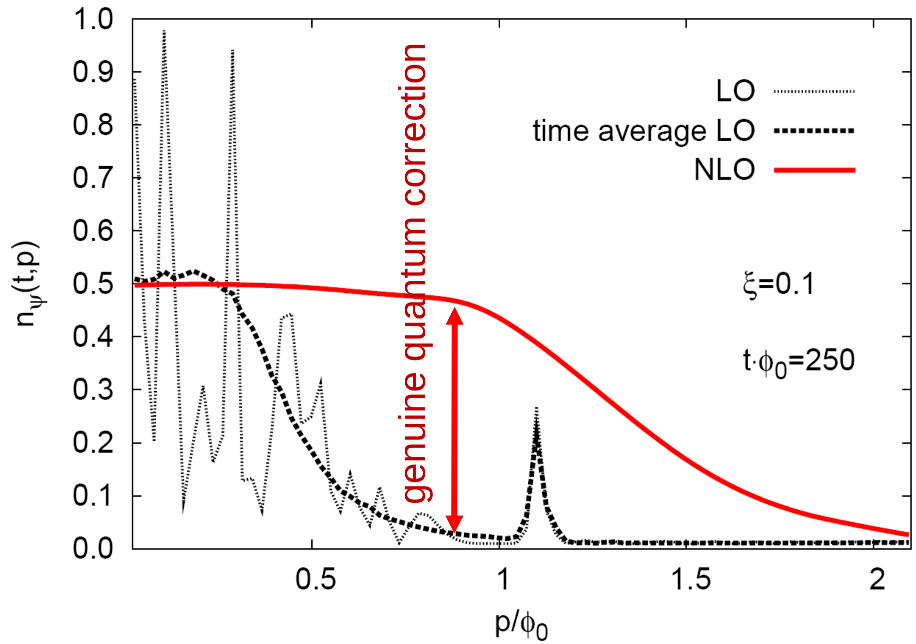
Parametric resonance preheating

$m_\psi = 0$

$\xi \equiv g^2/\lambda$

$$\phi = \phi_0 \sqrt{6N_s/\lambda}$$

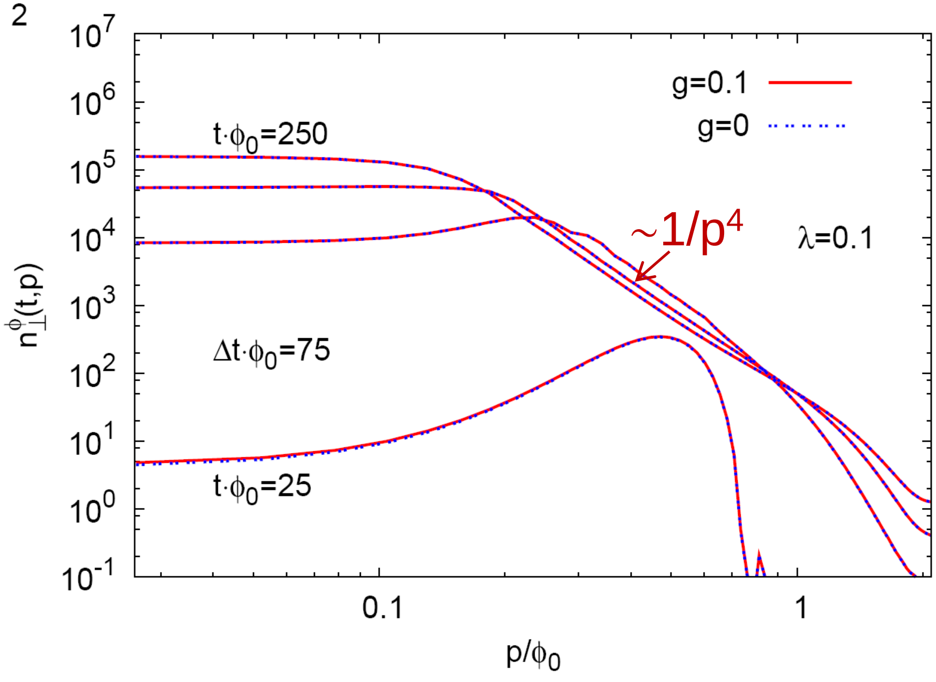
Occupation number distributions



Fermions

IR fermions thermally occupied

Bosons



Bosons still far from equilibrium

Nonequilibrium fermion spectral function

$$\rho(x,y) = i\,\langle\{\psi(x),\bar{\psi}(y)\}\rangle$$

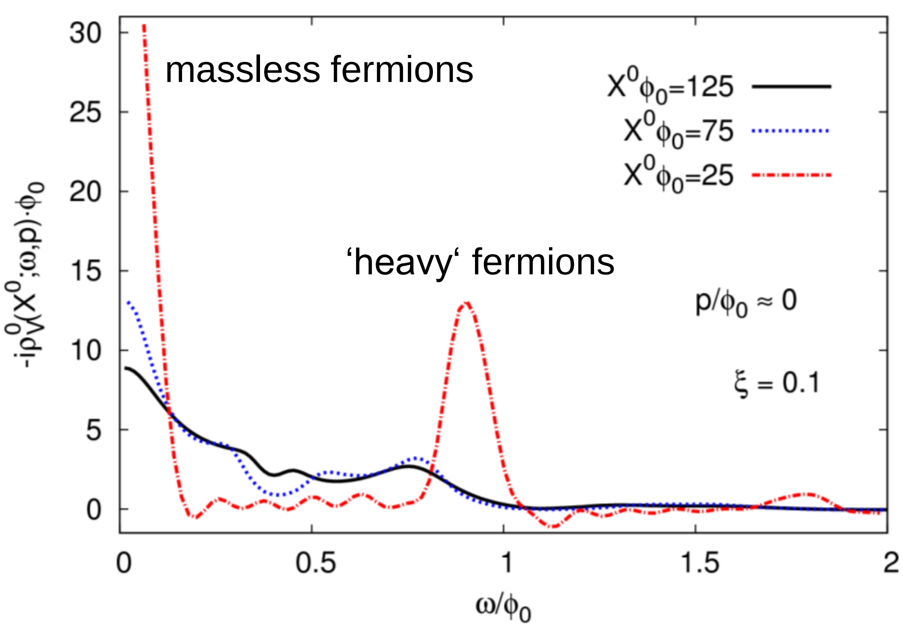
$$\rho_V^\mu = \frac{1}{4}\mathrm{tr}\,(\gamma^\mu\rho)$$ vector components

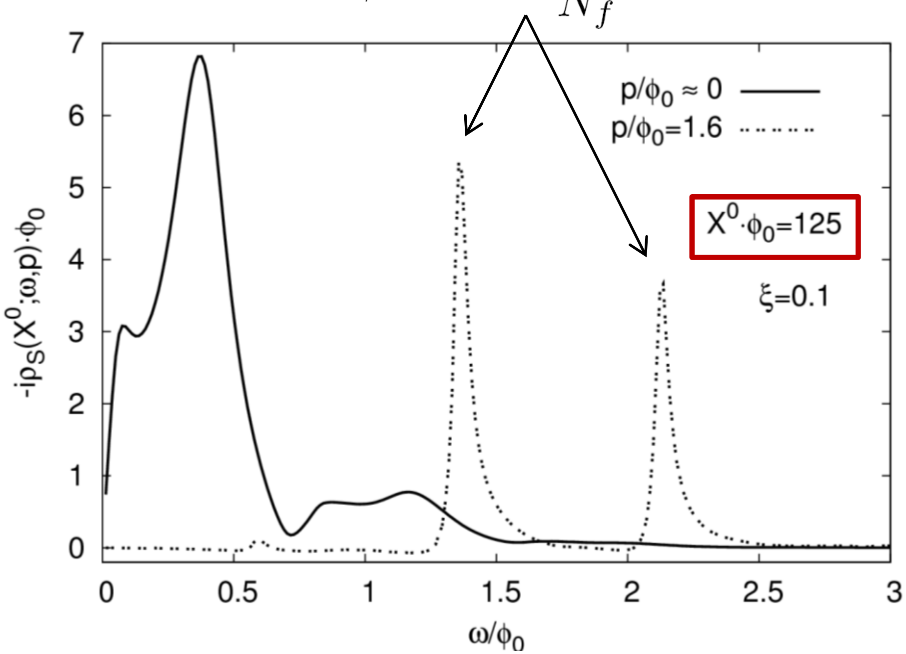
$$\rho_S = \frac{1}{4}\mathrm{tr}\,(\rho)$$ scalar component

quantum field anti-commutation relation: $-i\rho_V^0(t,t;\mathbf{p}) = 1$

Wigner transform: ($X^0 = (t + t')/2$)

$$M_\psi^{\mathrm{eff}}(t) \simeq \pm\frac{g}{N_f}|\phi(t)|$$





Lattice simulations with dynamical fermions

Consider general class of models including lattice gauge theories with covariant coupling to fermions:

$$\mathcal{L} = \frac{1}{2}\partial\Phi^*\partial\Phi - V(\Phi) + \sum_k^{N_f} \left[i\bar{\Psi}_k\gamma^\mu\partial_\mu\Psi_k - \bar{\Psi}_k(\underset{\nearrow\, \frac{1}{2}(1-\gamma^5)}{\overset{m - g\Phi(x)\,\downarrow}{M} P_L} + M^* \underset{\nwarrow\, \frac{1}{2}(1+\gamma^5)}{P_R})\Psi_k \right]$$

$$\int \prod_k D\Psi_k^+ D\Psi_k e^{i\int \mathcal{L}(\Phi,\Psi^+,\Psi)} \implies \boxed{\partial_x^2\Phi(x) + V'(\Phi(x)) + N_f J(x) = 0}$$

$$J(x) = J^{\mathrm{S}}(x) + J^{\mathrm{PS}}(x)$$

$$J^{\mathrm{S}}(x) = -g\left\langle \bar{\Psi}(x)\Psi(x) \right\rangle = g\mathrm{Tr}\, D(x,x)\,,$$
$$J^{\mathrm{PS}}(x) = -g\left\langle \bar{\Psi}(x)\gamma^5\Psi(x) \right\rangle = g\mathrm{Tr}\, D(x,x)\gamma^5$$

For classical $\Phi(x)$ the exact equation for the fermion $D(x,y)$ reads:

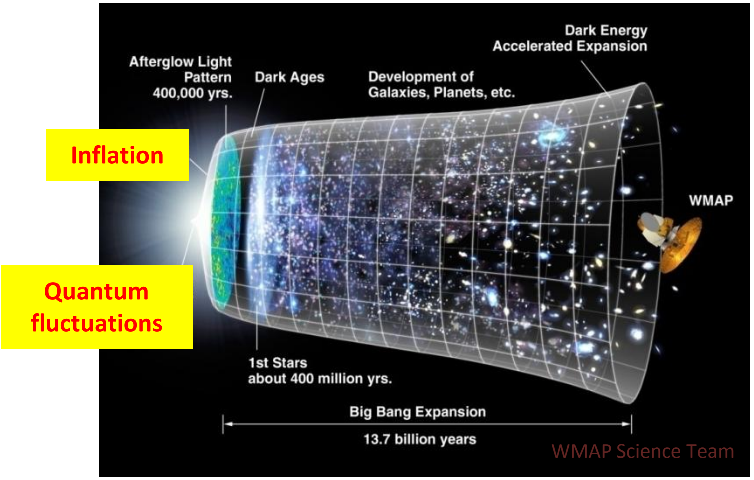
$$\boxed{(i\gamma^\mu\partial_{x,\mu} - m + g\mathrm{Re}\,\Phi(x) - ig\mathrm{Im}\,\Phi(x)\gamma^5)D(x,y) = 0}$$

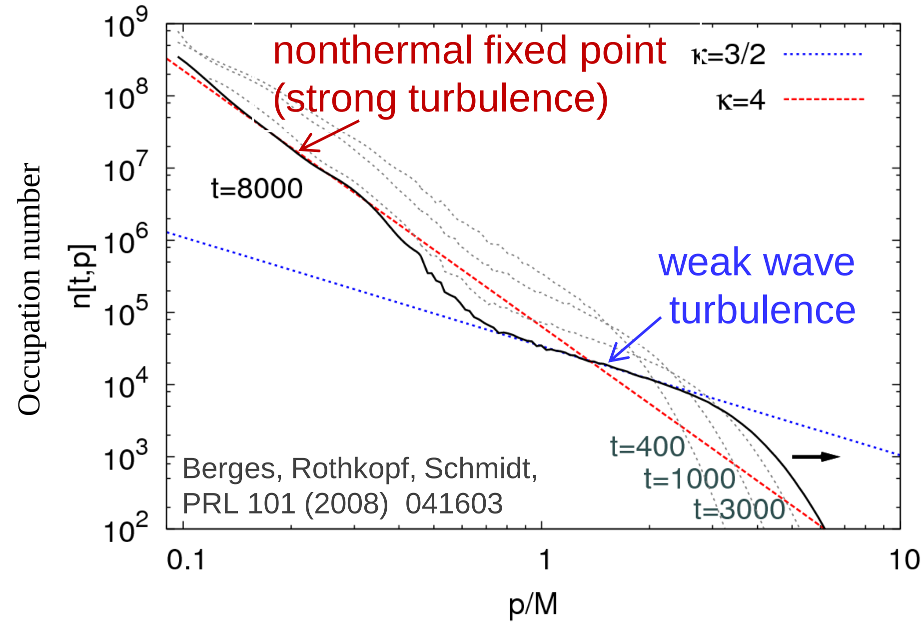
Very costly (4 4 N^3 N^3)! Use low-cost fermions of Borsanyi & Hindmarsh!

Aarts, Smit, NPB 555 (1999) 355 PRD 79 (2009) 065010

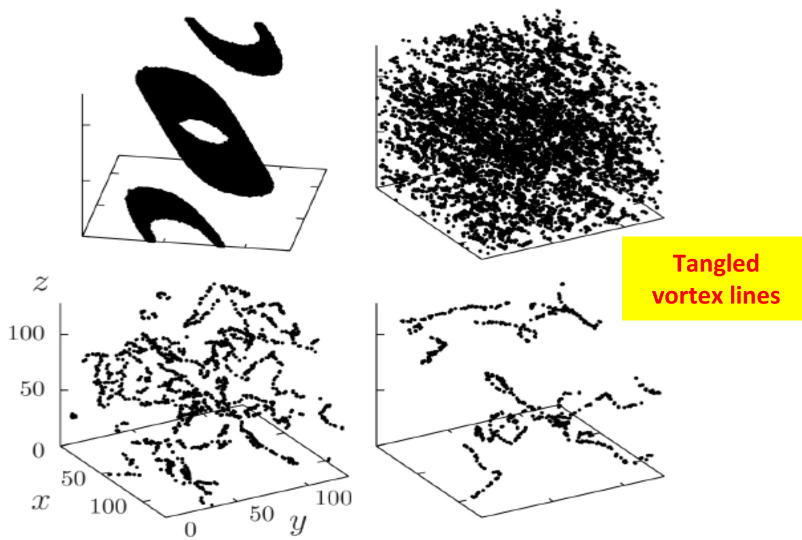
Strong turbulence: $\lim_{p\to 0} n(p) \sim \frac{1}{p^4}$ ← *universal scaling exponent*

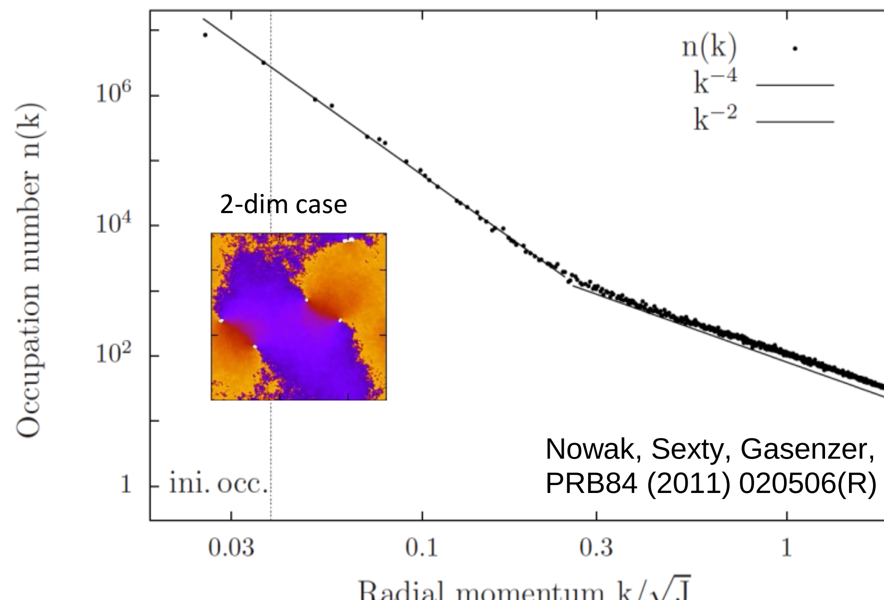
- Reheating dynamics after chaotic inflation



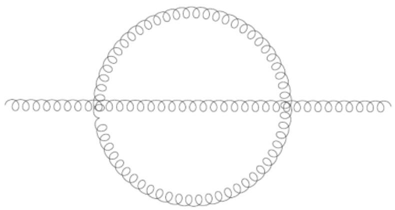


- Superfluid turbulence in a cold Bose gas





SU(2) gauge theory



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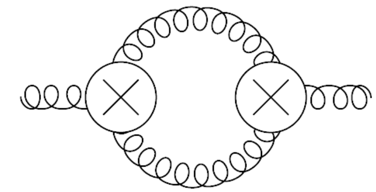

$$\kappa = \frac{5}{3}, \quad \text{or} \quad \kappa = \frac{4}{3} \qquad\qquad \kappa = \frac{3}{2}, \quad \text{or} \quad \kappa = 1$$

‘condensate‘