Transition from ideal to viscous Mach Cones in a partonic transport model

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I. Bouras et al., Phys. Rev. Lett. 103:032301 (2009)

I. Bouras et al., PRC 82, 024910 (2010)

I. Bouras et al., Phys.Lett. B710 (2012)

HGS-HIRe for FAIR
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Bundesministerium
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und Forschung

GOETHE
UNIVERSITÄT
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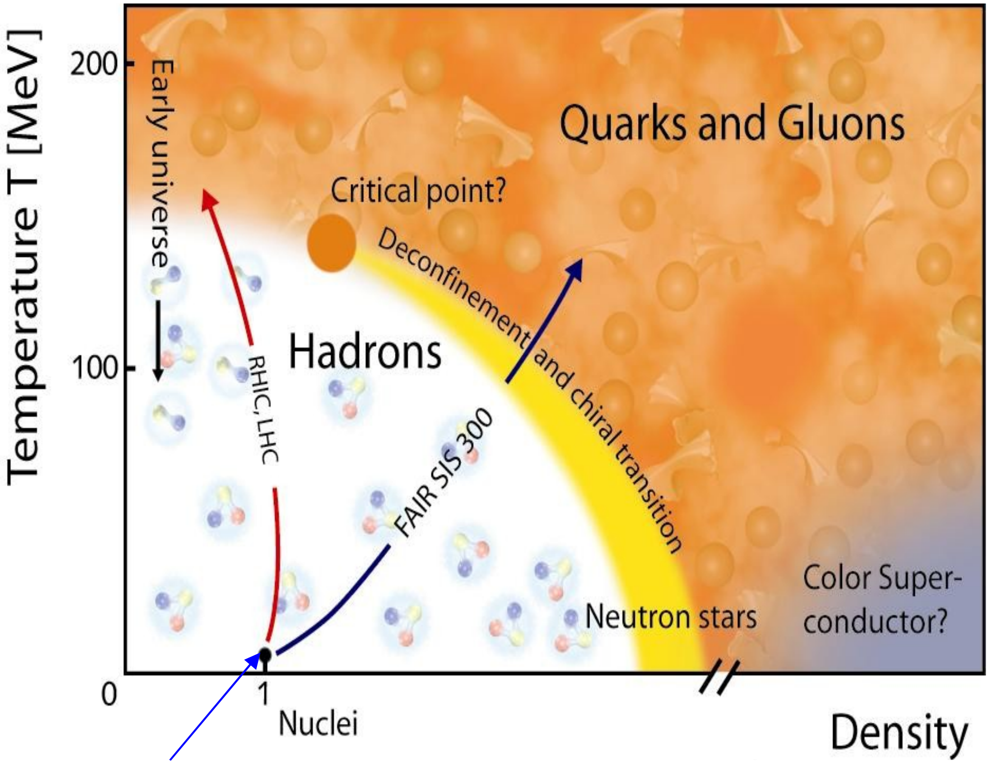
Helmholtz International Center

NED/Turic
Crete, Greece

May, 2012

Motivation

QCD Phase diagram

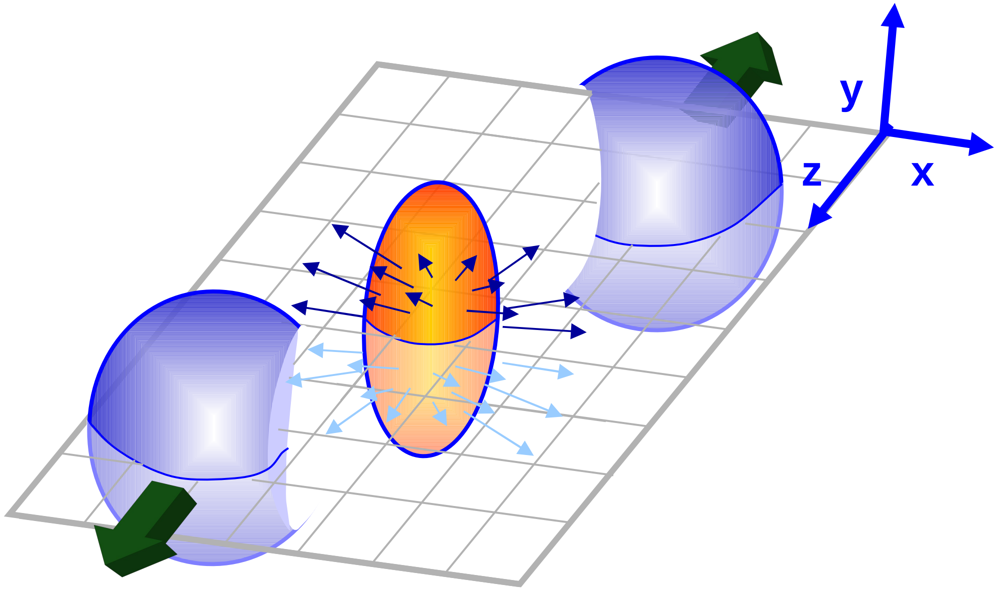


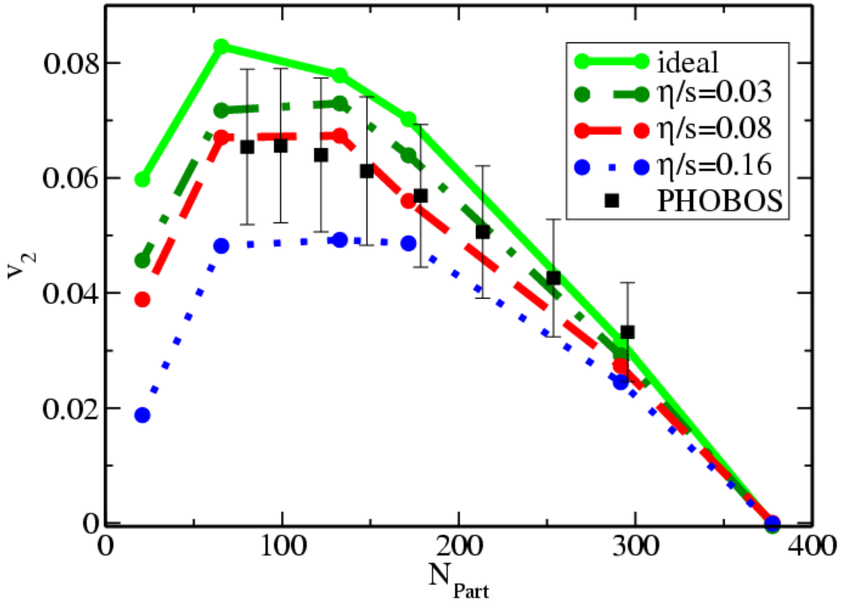
ordinary world

QCD is most probably the theory we have to describe

Motivation

Elliptic Flow

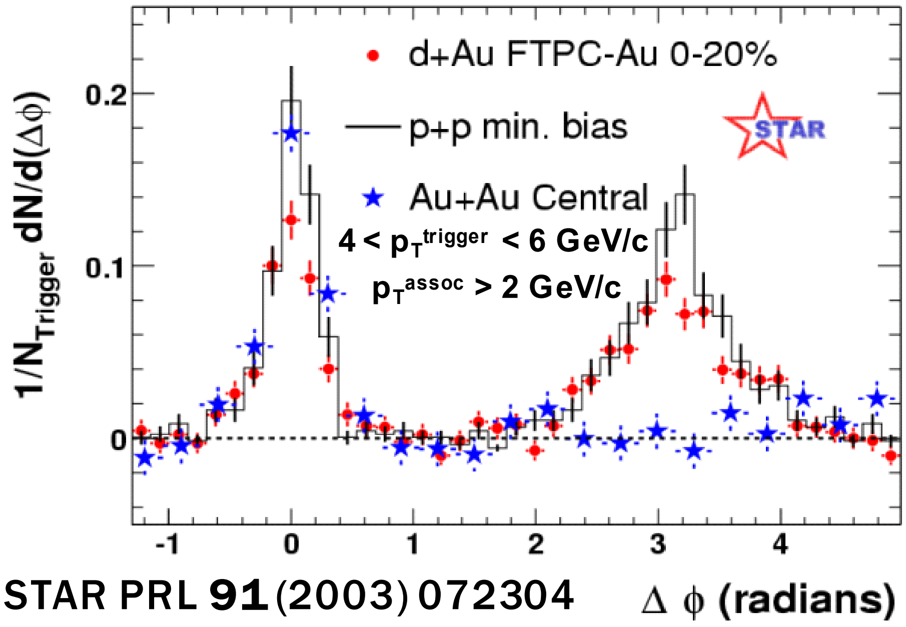




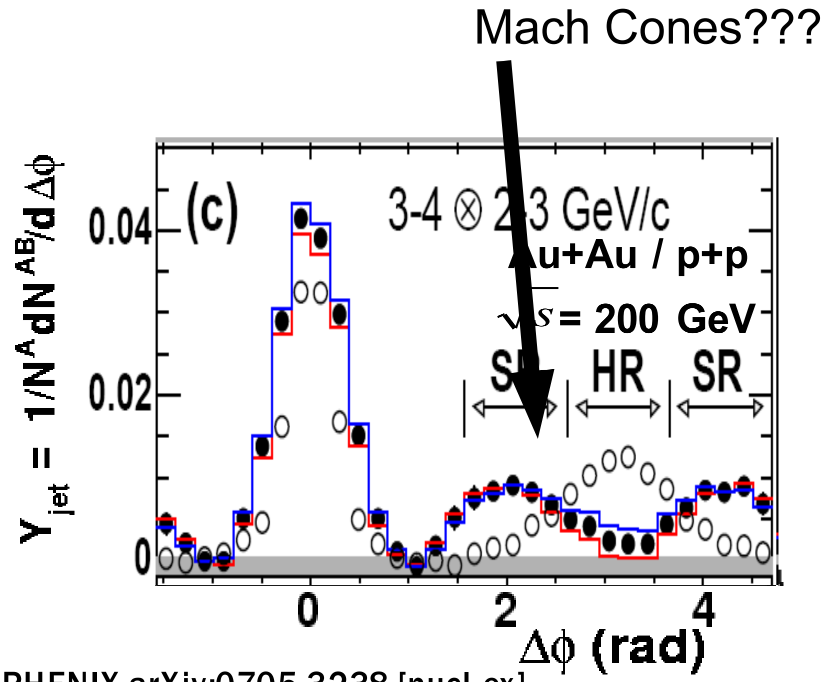
- Matter behaves like a nearly perfect fluid
- Early thermalization

Motivation

Jet-Quenching and Two-particle correlations



STAR PRL **91** (2003) 072304


Mach Cones???

PHENIX arXiv:0705.3238 [nucl-ex]

- Jet-physics is another good observable ot understand the Porperties of the matter

Do Mach Cones have something to do with double peaks?
→ Then answer is given in the end of the talk

The Parton Cascade BAMPS

- Transport algorithm solving the **Boltzmann equation** using Monte Carlo techniques

$$p^{\mu} \partial_{\mu} f(x,p) = C_{22} + C_{23} + \dots$$

Boltzmann
Approach for
Multi-
Parton
Scatterings

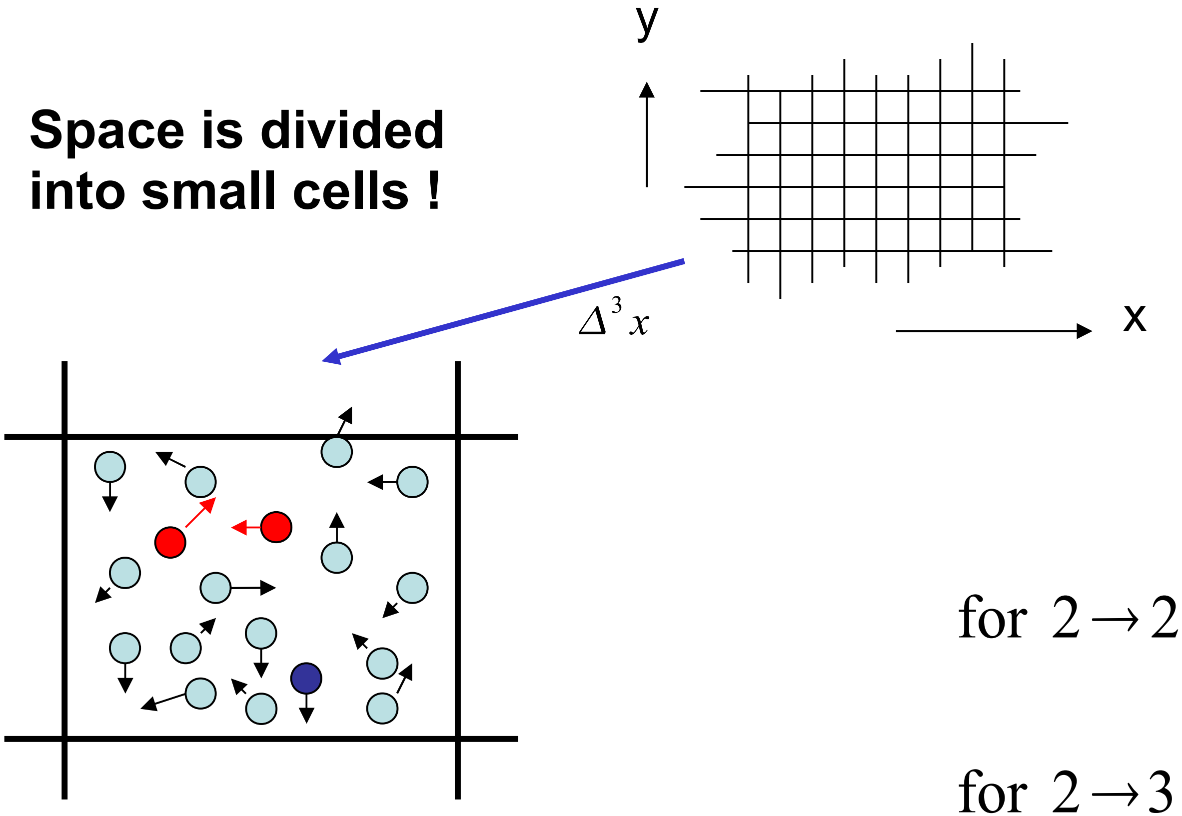
- Stochastic interpretation of collision rates

$$P_{2i} = v_{rel} \frac{\sigma_{2i}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

- In general:
  pQCD interactions, 2 ↔ 3 processes,
  quarks and gluons

The Parton Cascade BAMPS



Space is divided into small cells !

Boltzmann
Approach for
Multi-
Parton
Scatterings

for $2 \to 2$ $\quad P_{22} = v_{rel} \dfrac{\sigma_{22}}{N_{test}} \dfrac{\Delta t}{\Delta^3 x}$

for $2 \to 3$ $\quad P_{23} = v_{rel} \dfrac{\sigma_{23}}{N_{test}} \dfrac{\Delta t}{\Delta^3 x}$

for $3 \to 2$ $\quad P_{32} = \dfrac{1}{8 E_1 E_2 E_3} \dfrac{I_{32}}{N_{test}^2} \dfrac{\Delta t}{(\Delta^3 x)^2}$

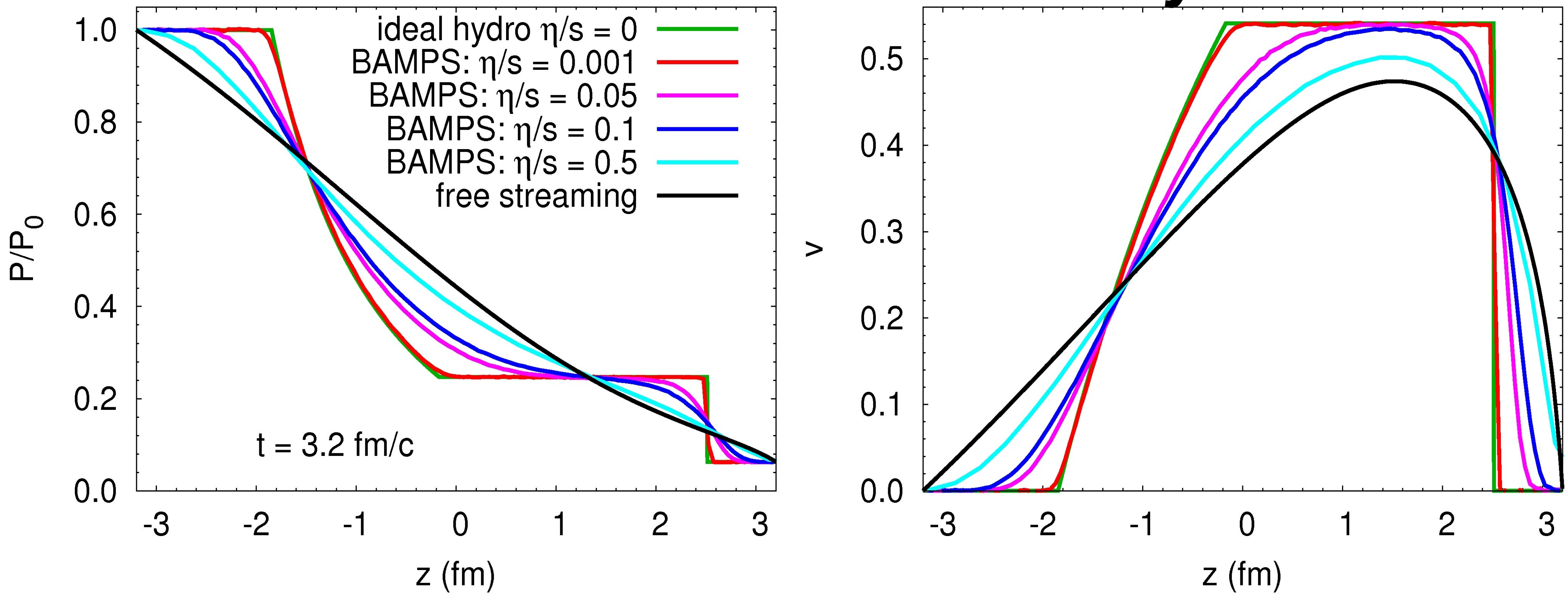
Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

$$I_{32} = \frac{1}{2} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} |M_{123 \to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p'_1 - p'_2)$$

The Relativistic Riemann Problem

Investigation of Shock Waves in one dimension

Boltzmann solution of the relativistic Riemann problem
->what effects have viscosity?



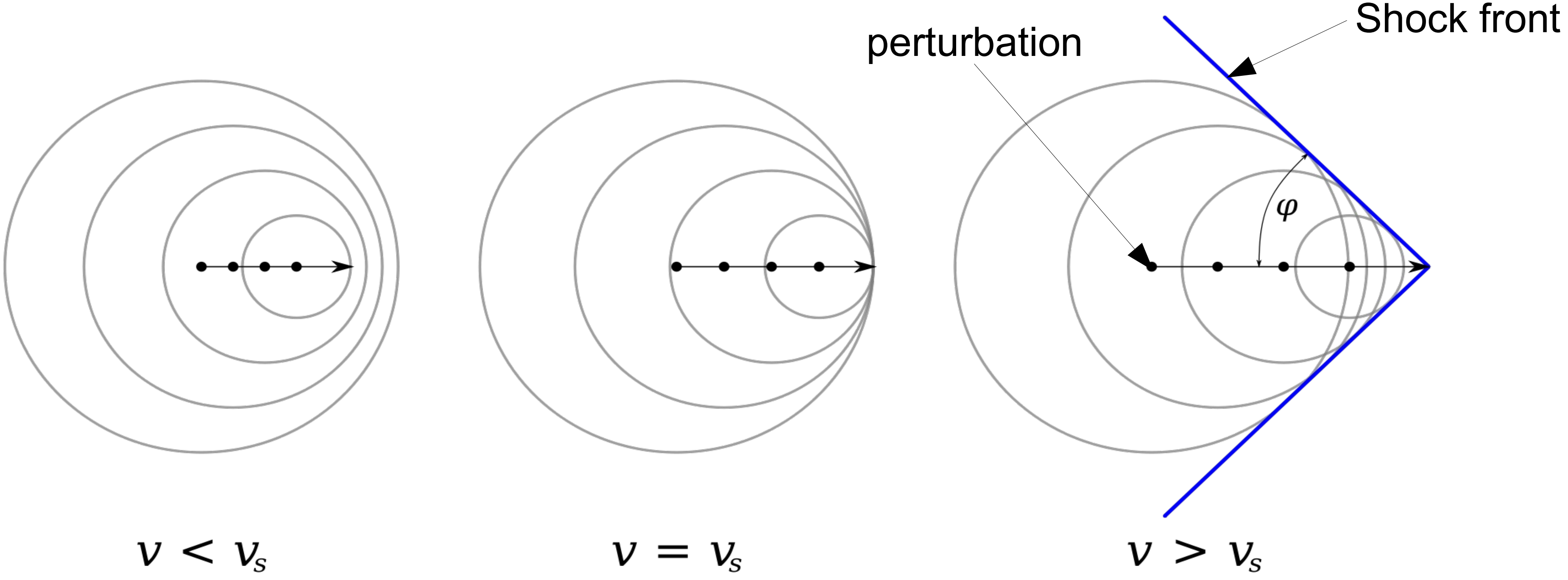
Transition from ideal hydro to free streaming

I. Bouras et al., Phys. Rev. Lett. 103:032301 (2009)

I. Bouras et al., PRC 82, 024910 (2010)

Mach Cones

- If source (perturbation) is propagating faster than the speed of sound, then a Mach Cone structure is observed



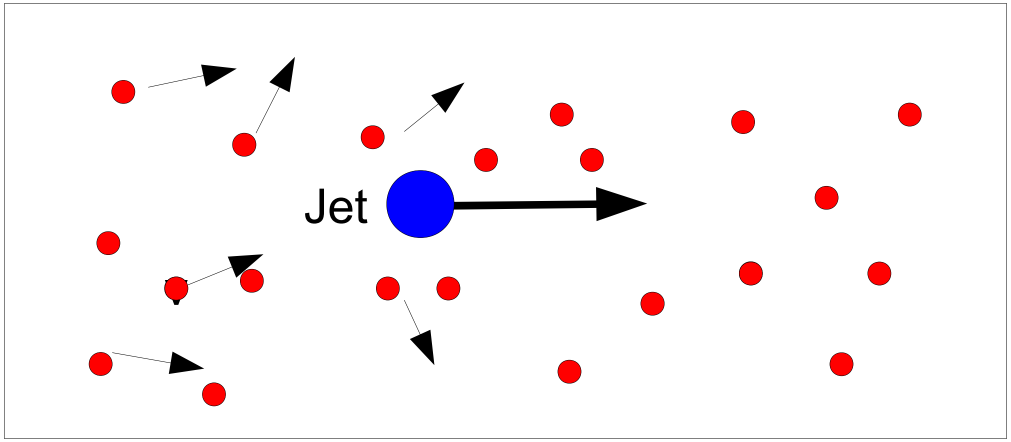
”Source” Terms in BAMPS

1) Punch Through Scenario

2) Pure energy deposition scenario

Punch Through Scenario

A scenario usefull to investigate the shape and development of ideal Mach Cones

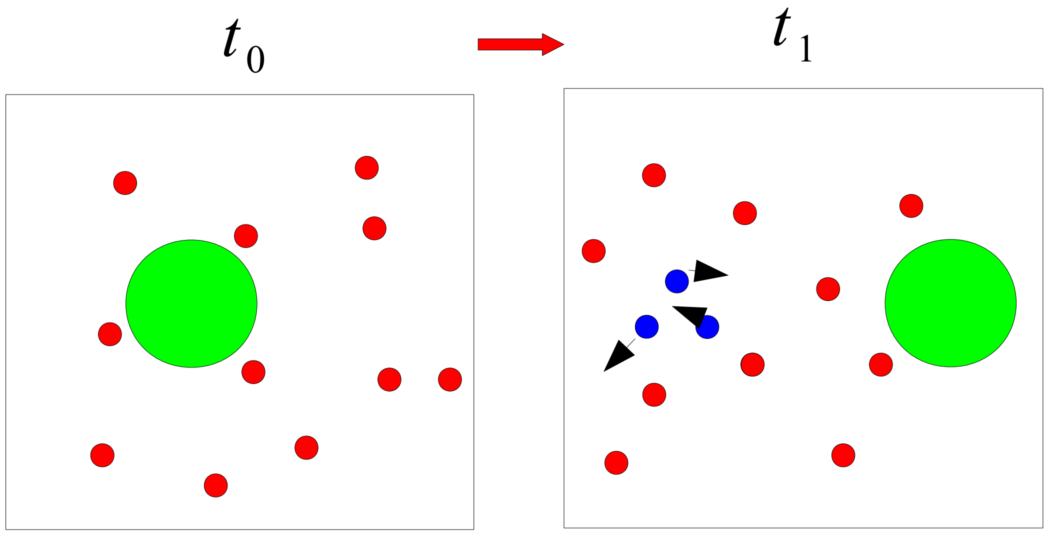


- Jet has finite initial energy and momentum E = pz and is massless; no transverse momentum → px = py = 0
- The Jet deposits energy to the medium due to binary collisions with particles
- After every collision with a thermal particle of the medium the energy of the jet gets recharged to its inital value

Movie:
Evolution of Mach Cones
in BAMPS
For the *Punch Through Scenario*

Pure energy deposition Scenario

Energy deposition via the creation of thermal distributed particles

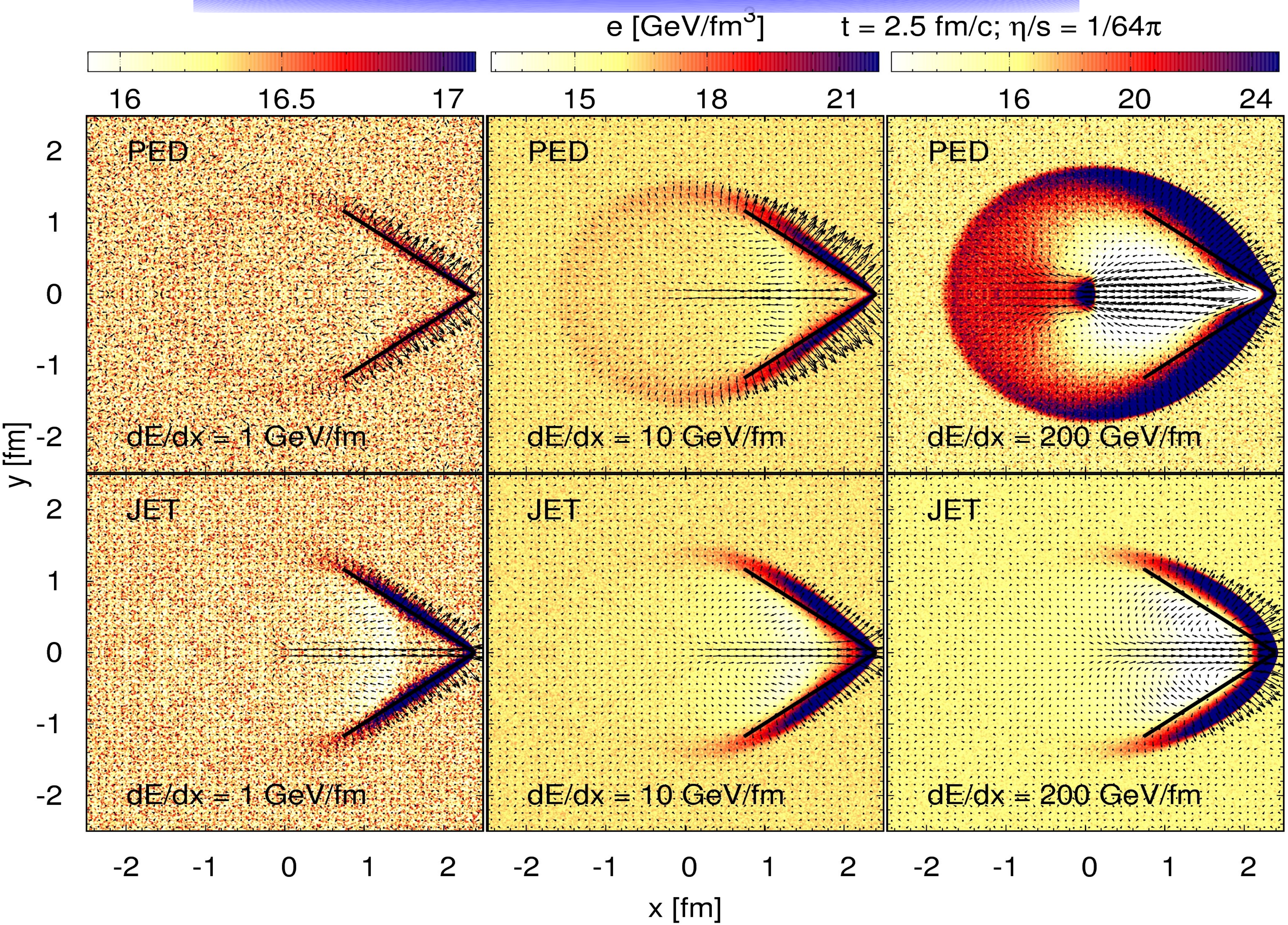


- The source (green) propagates with the speed of light and generates new particles (blue) at different timesteps
- The advantage of that method: a constant energy deposition but no momentum deposition, because new particles are thermal distributed

$$\longrightarrow f_{ped}(x,p)=e^{-E/T}$$

Movie:
Evolution of Mach Cones
in BAMPS
For the *Pure energy deposition scenario*

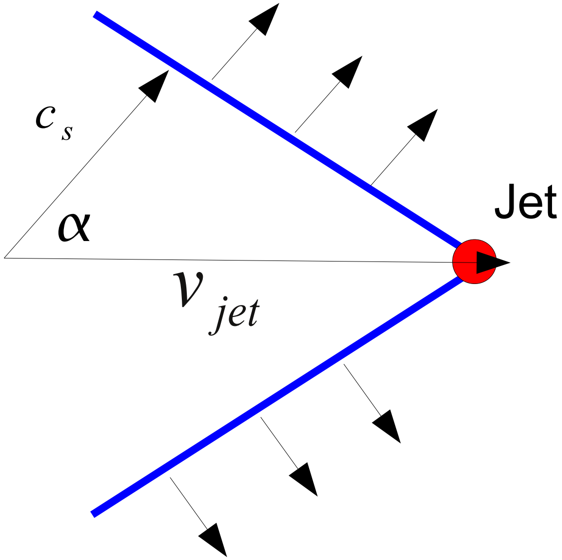
Ideal Solutions of Mach Cones



Mach Cones

Mach angle dependence

Scenario for a very weak perturbation



Mach Cones

Mach angle dependence

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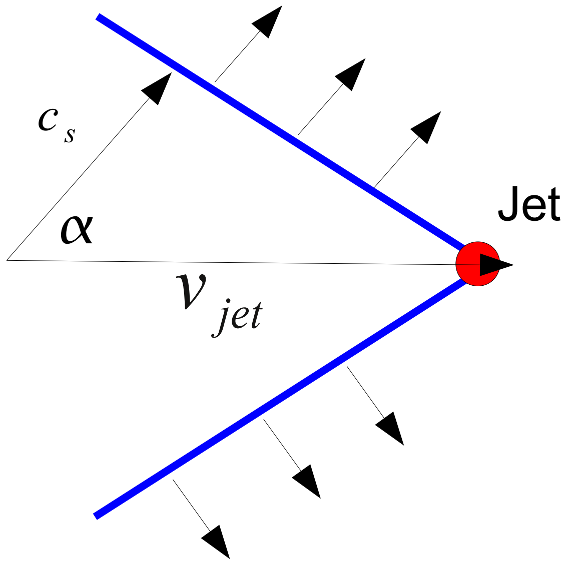


- In the case of a perfect fluid, i.e. $\eta=0$, the Mach angle is

$$\alpha=\arccos\frac{c_s}{v_{jet}}\approx 54.7^o$$

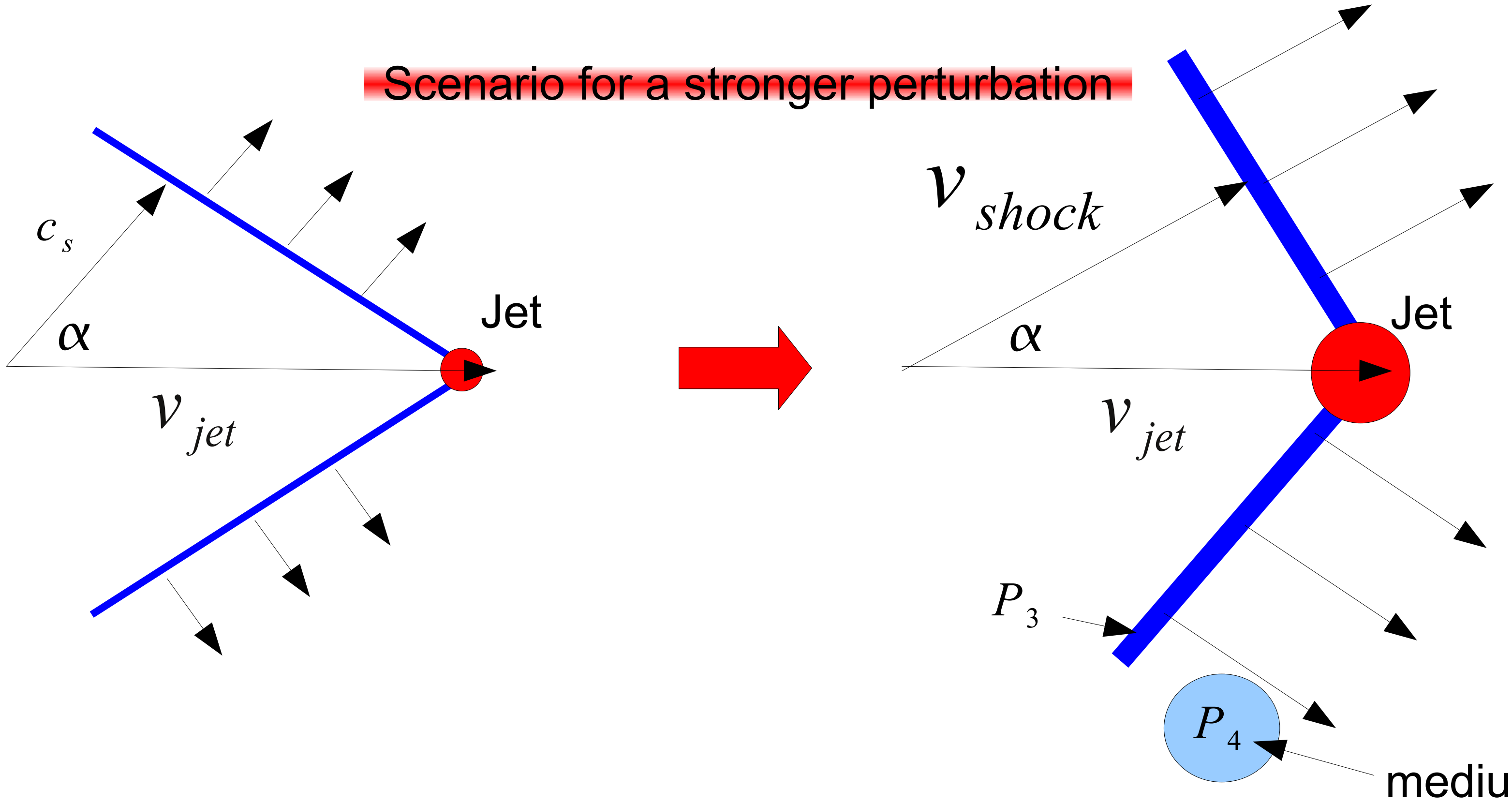
  for a massless Boltzmann gas, i.e. $e=3\mathrm{P}$, with $c_s=1/\sqrt{3}$ and $v_{jet}=1$

- This is only valid for small perturbation, i.e. energy of the jet is infinite small

Mach Cones

Mach angle dependence

Scenario for a stronger perturbation

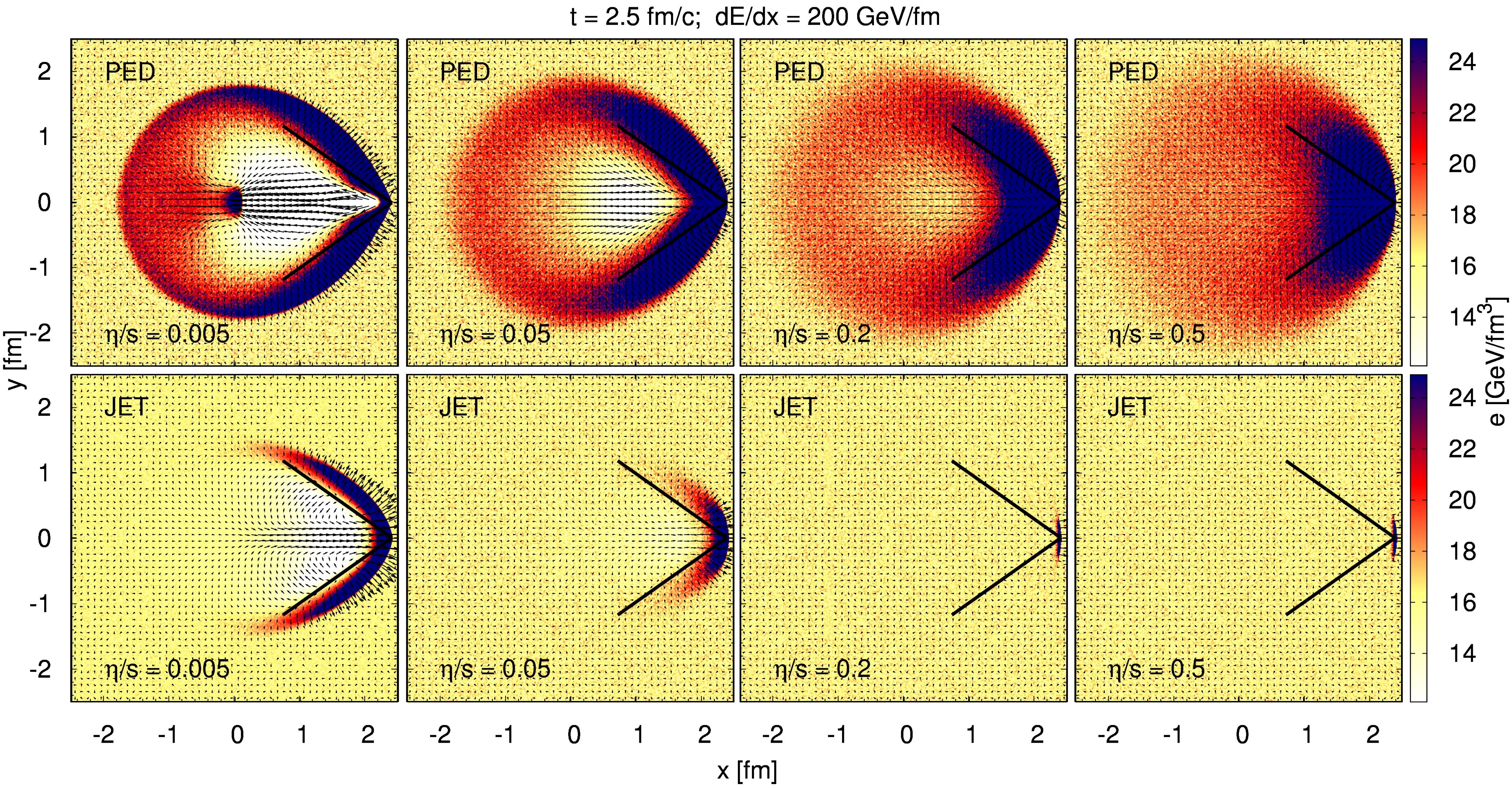


- In the case of a stronger perturbation the energy deposition is larger and therefore shock waves develop which exceed the speed of sound. Therefore the angle is approximately given by

$$\alpha = \arccos \frac{v_{shock}}{v_{jet}} \qquad v_{shock} = \left[\frac{(P_4 - P_3)(e_3 + P_4)}{(e_4 - e_3)(e_4 + P_3)} \right]^{\frac{1}{2}}$$

- The emission angle α changes to smaller values than in the weak perturbation case

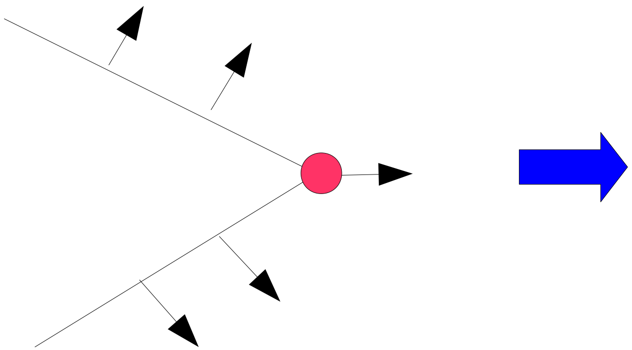
Viscous Solutions of Mach Cones

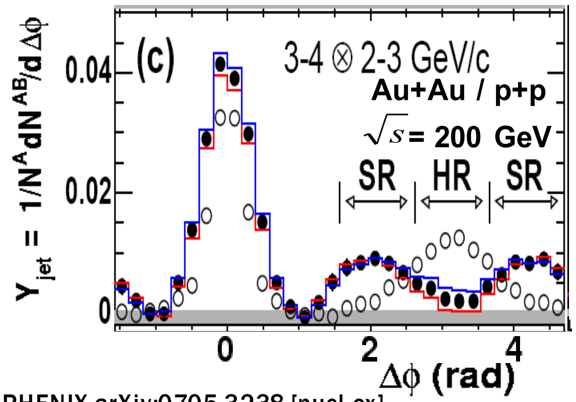


Mach Cones in BAMPS

Two Particle Correlations

- First, we (have) expect(ed) that the double peak observed in experimental data is a hint for a conical structure...because of the naive picture





PHENIX arXiv:0705.3238 [nucl-ex]

Mach Cones in BAMPS

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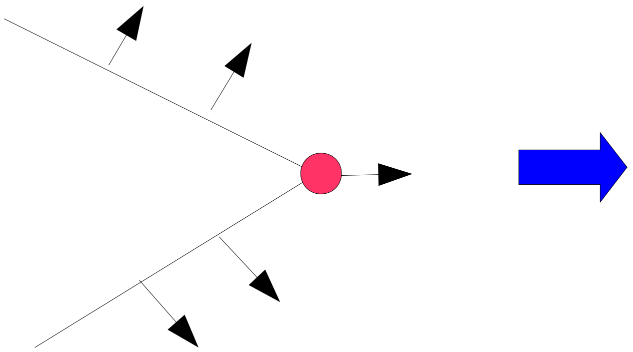

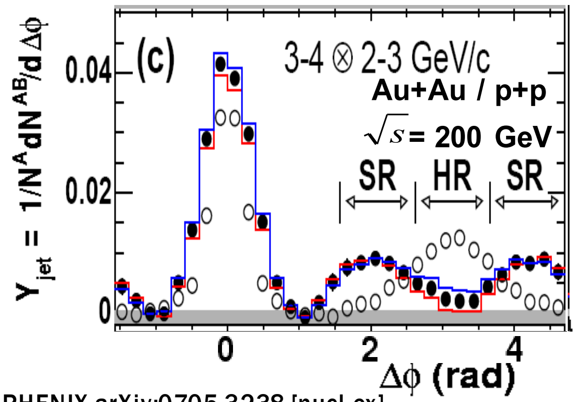

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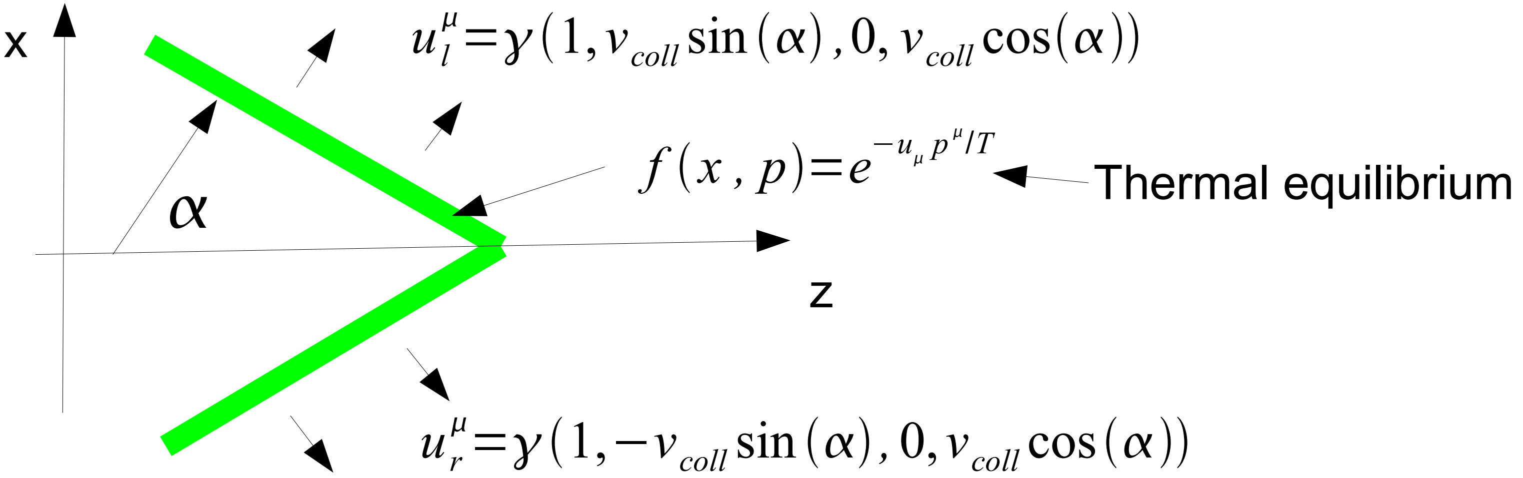
- But....
  1) viscosity is not zero in heavy-ion collisions (HIC)...and as we have already seen, viscosity in order expected in HIC destroys the conical structure to a very weak signal
  2) The jet in reality has not infinite energy....and the formation-time is finite
  3) The angle changes of the Mach Cone changes depending on the energy deposition
  4) The diffusion wake and head shock will have a big contribution...as we will see..

- However, one can can find an analytical expression for the two-particle correlations of Mach Cones....

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution

Assume two wings in thermal equilibrium



$$u_l^{\mu}=\gamma\left(1, v_{coll}\sin(\alpha), 0, v_{coll}\cos(\alpha)\right)$$

$$f(x, p)=e^{-u_{\mu}p^{\mu}/T}$$

Thermal equilibrium

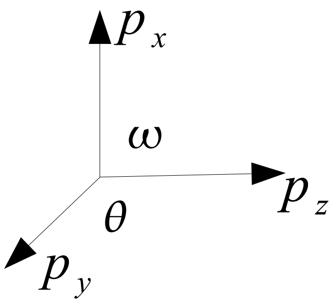
$$u_r^{\mu}=\gamma\left(1, -v_{coll}\sin(\alpha), 0, v_{coll}\cos(\alpha)\right)$$

alpha is a const and corresponds to the Mach angle, where v_coll is the collective velocity of matter velocity in the wings

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution

- We are looking for the angle ω, which is the angle in the p_x and p_z plane

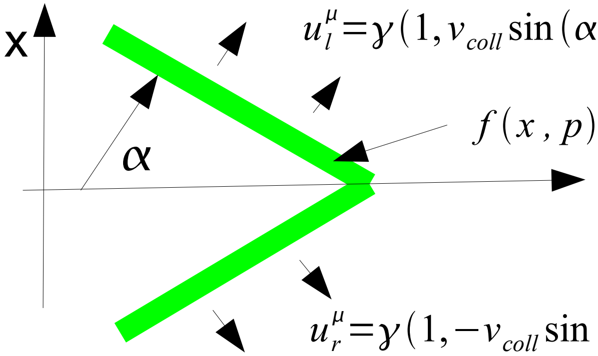


$$p_z = p\cos(\omega)\sin(\theta)$$
$$p_x = p\sin(\omega)\sin(\theta)$$
$$p_y = p\cos(\theta)$$

One calculate for each wing the particle distribution

$$\frac{dN}{d\omega} = \frac{V}{(2\pi)^3} \iint p^2 \sin(\theta) e^{-u_\mu p^\mu / T} \, dp \, d\theta$$

In the end one has to add both contributions!



$$u_l^\mu = \gamma(1, v_{coll}\sin(\alpha), 0, v_{coll}\cos(\alpha))$$

$$f(x, p) = e^{-u_\mu p^\mu / T}$$

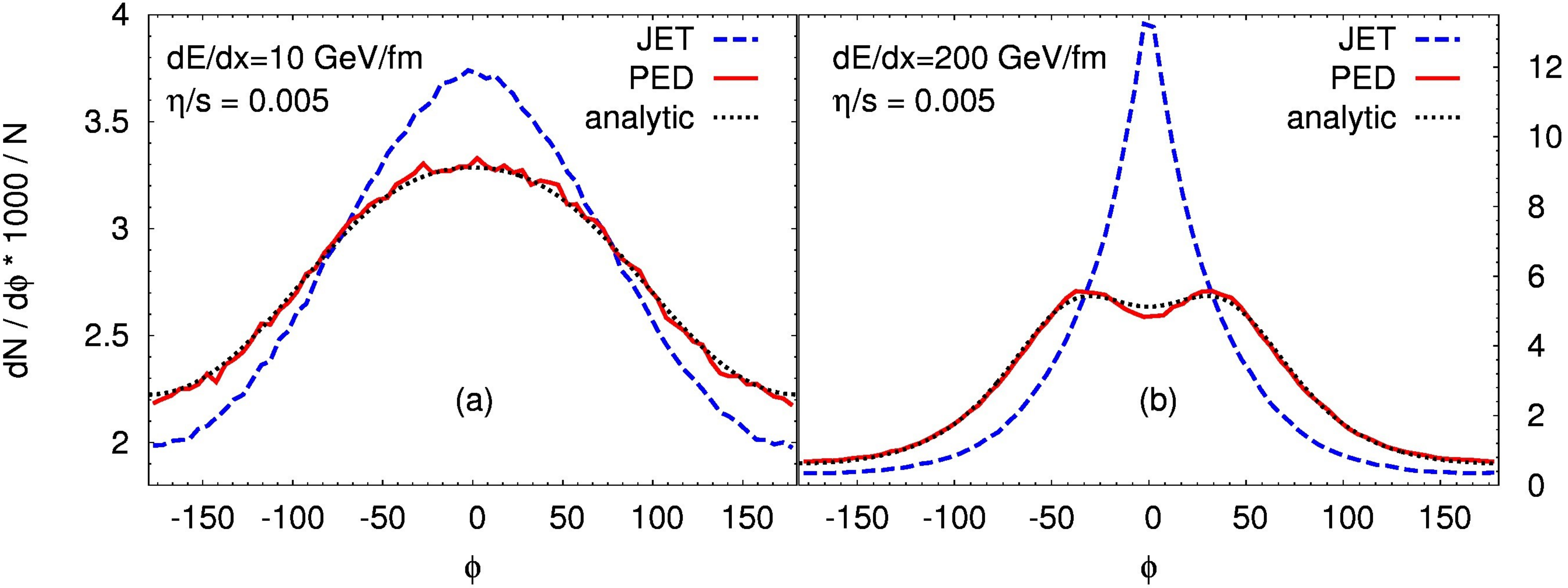
$$u_r^\mu = \gamma(1, -v_{coll}\sin(\alpha), 0, v_{coll}\cos(\alpha))$$

Mach Cones in BAMPS

Two Particle Correlations for ideal solution
Numerical Results

10 GeV/fm

200 GeV/fm

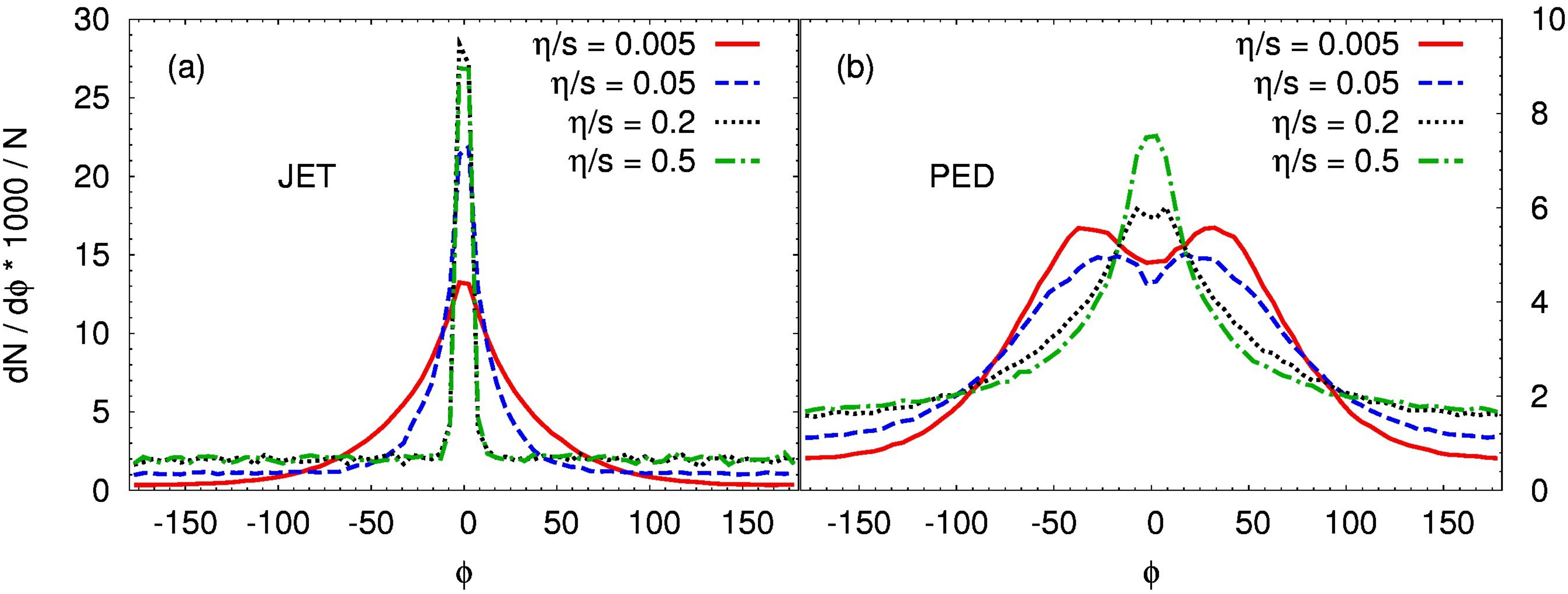


The source term plays a big role for observation a double peak structure

Mach Cones in BAMPS

Two Particle Correlations for viscous solution

Numerical Results



Viscosity does not help for the development fo the double peak structure

Conclusion ....

- BAMPS is an excellent benchmark to investigate phenomena like shock waves and Mach Cones in the ideal and viscous region

- Mach Cones might exist in heavy-ion collisions...

...but have **NOT** to be the origin of the famous ”double peak structure”....

... and Outlook

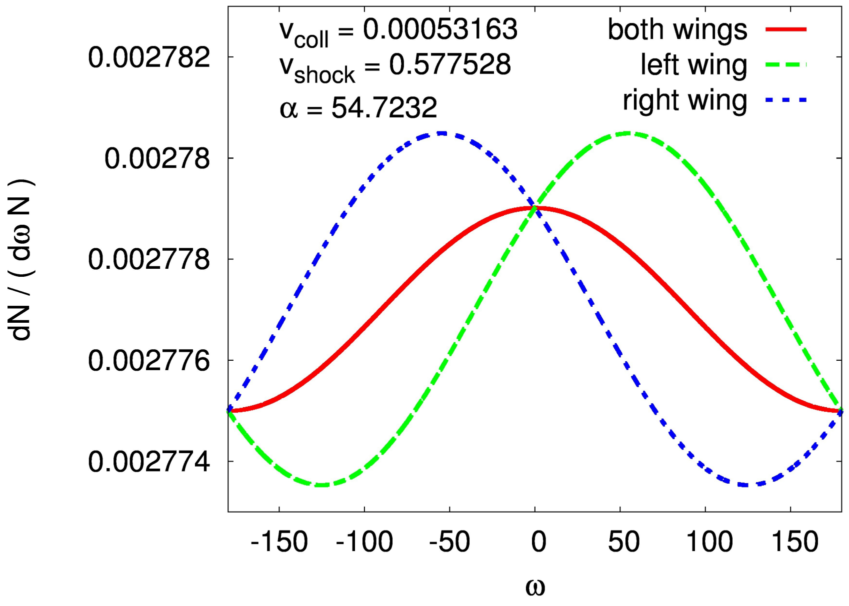
- **Investigation of Mach cones in a full 3D simulation of HIC**
- **3-particle correlations**

Thank you

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution - Results

Taking the very weak perturbation case in account, we do not observe a double peak structure as we expected.

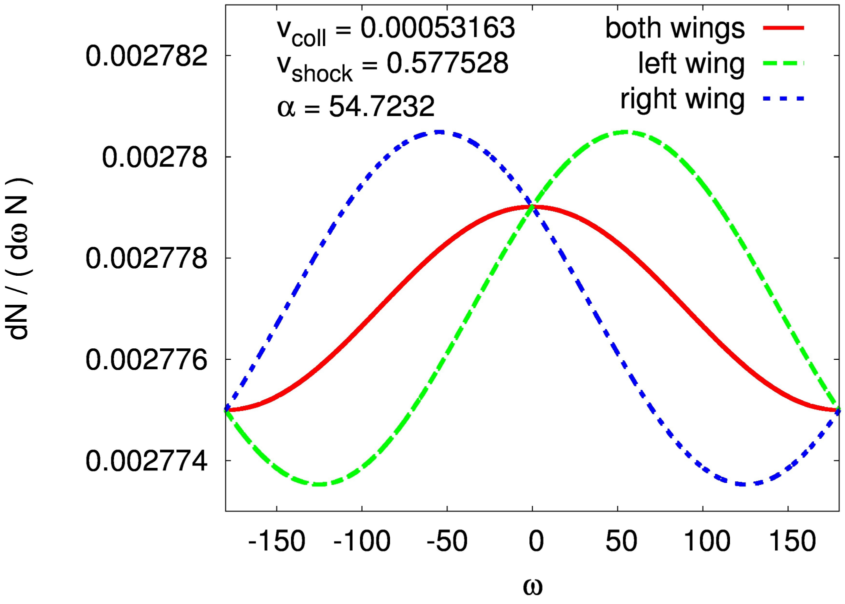


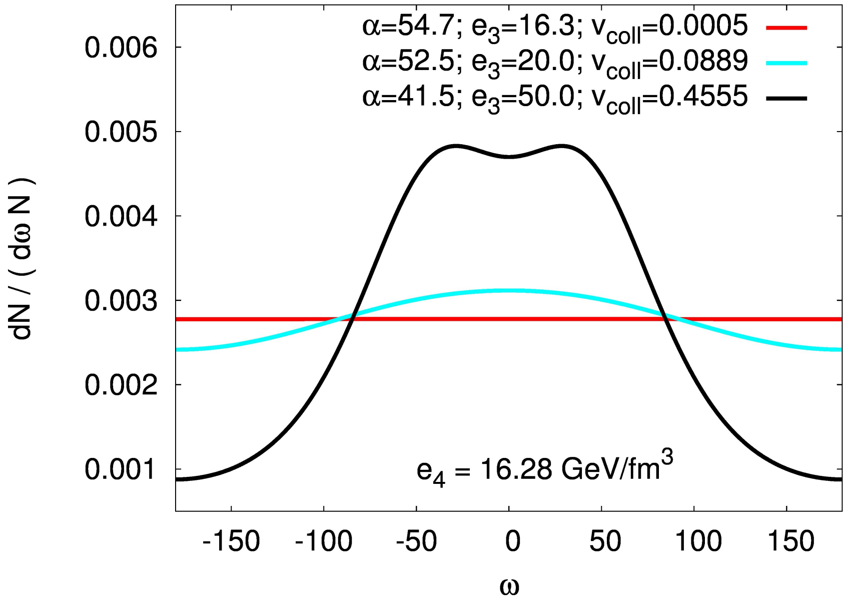
alpha and v_coll depends on the ratio of density in the wing and medium in rest

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution - Results

Taking the very weak perturbation case in account, we do not observe a double peak structure as we expected.
→ Only if the shock gets stronger a double peak is observed
→ If the shock gets stronger, also v_coll gets larger and therefore the double peak is clearer





alpha and v_coll depends on the ratio of density in the wing and medium in rest

The Parton Cascade BAMPS

For this setup :

- **Boltzmann gas, isotropic cross sections, elastic processes only**
- **Implementing a constant η/s, we locally get the cross section σ_{22}:**

$$\eta=\frac{4}{15}\frac{\epsilon}{R^{tr}}$$

Transport collision rate R^{tr}

For isotropic elastic collisions:

$$R_{22}^{tr}=n\frac{2}{3}\sigma_{22}$$

$$\epsilon=3\mathrm{nT}$$

$$s=4\mathrm{n}-n\ \ln(\lambda_{fug})$$

$$\lambda_{fug}=\frac{n}{n_{eq}} \qquad n_{eq}=\frac{g}{\pi^2}T^3$$

g = 16 for gluons

Z. Xu & C. Greiner,
Phys.Rev.Lett.100:172301,2008

$$\sigma_{22}=\frac{6}{5}\frac{T}{s}\left(\frac{\eta}{s}\right)^{-1}$$