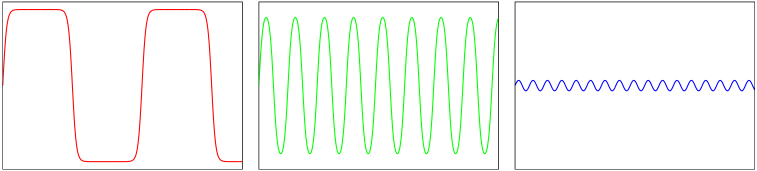
Inhomogeneous chiral symmetry breaking phases

Stefano Carignano
Michael Buballa



Phys.Rev. D **82**, 054009 (2010)
arXiv:1111.4400

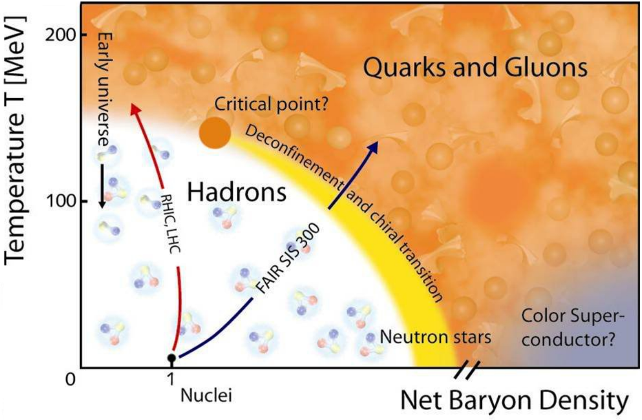
arXiv:1203.5343

H-QM Helmholtz Research School Quark Matter Studies

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

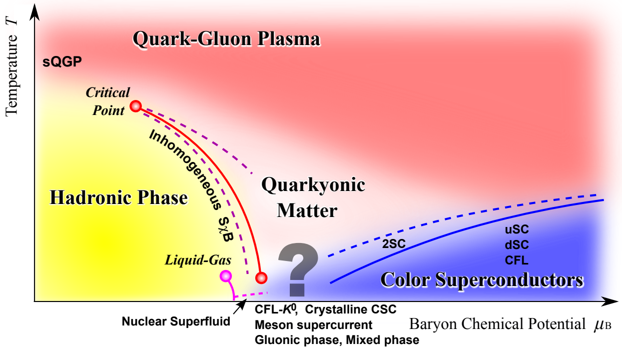
Motivation: the QCD phase diagram (so far ?)

TECHNISCHE UNIVERSITÄT DARMSTADT



Maybe it's not so simple...

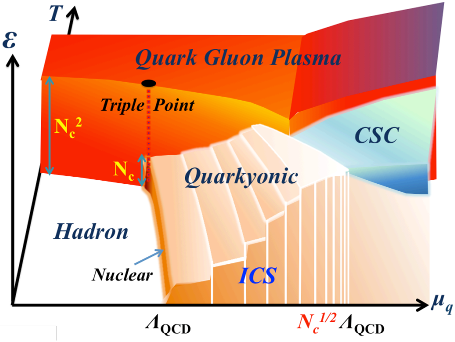
TECHNISCHE UNIVERSITÄT DARMSTADT



Fukushima and Hatsuda, arXiv:1005.4814

How about this one?

TECHNISCHE UNIVERSITÄT DARMSTADT



Kojo et al., arXiv:1107.2124

Why inhomogeneous phases ?

TECHNISCHE UNIVERSITÄT DARMSTADT

- Popular already for quite some time...
  - Overhauser pairing in nuclear matter
  - Pion condensation
  - (Color-) Superconductivity

- Recently rediscovered and revised
  - Studies of lower-dimensional models (GN_2, NJL_2, ...)
  - Quarkyonic chiral spirals
  - ...

- May be relevant for cold dense matter !
- This talk: inhomogeneous chiral symmetry breaking phases

Procedure

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Choose an effective model
- Allow for spatial modulations of the chiral condensate
- Determine the favored ground state of the system
- Study the properties of chiral crystalline phases
- Extend the model and see how crystalline phases are affected

The model

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Start from the two-flavor NJL Lagrangian

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi + G_s\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau^a\psi\right)^2\right)$$

- Mean-field approximation
- Retain spatial dependence of the condensates

$$\langle\bar{\psi}\psi\rangle = S(\vec{x})\,, \qquad \langle\bar{\psi}i\gamma^5\tau^a\psi\rangle = P_a(\vec{x})$$

- Mean-field Lagrangian:

$$\mathcal{L}_{MF} = \bar{\psi}(x)\mathcal{S}^{-1}(x)\psi(x) - G_s\left(S(\vec{x})^2 + P(\vec{x})^2\right)$$

$$\mathcal{S}^{-1} = i\gamma^\mu\partial_\mu - m + 2G_s\left(S(\vec{x}) + i\gamma^5\tau^a P_a(\vec{x})\right) \equiv \gamma^0(i\partial_0 - \mathcal{H}_{MF})$$

Thermodynamic potential

TECHNISCHE UNIVERSITÄT DARMSTADT

$$\Omega(T,\mu;S(\vec{x}),P(\vec{x})) = -\frac{T}{V}\text{Log}\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(\int_{x\in[0,\frac{1}{T}]\times V}(\mathcal{L}_{MF}+\mu\bar{\psi}\gamma^0\psi)\right)$$

$$= -\frac{TN_c}{V}\sum_n \text{Tr}_{D,f,V}\,\text{Log}\left(\frac{1}{T}(i\omega_n+\mathcal{H}_{MF}-\mu)\right)+\frac{G_s}{V}\int_V\left(S(\vec{x})^2+P(\vec{x})^2\right)$$

- If we can calculate the eigenvalues $\{E_n\}$ of $\mathcal{H}_{MF}$, it's

$$\Omega(T,\mu;M(\vec{x})) = -\frac{TN_fN_c}{V}\sum_{E_n}\text{Log}\left(2\cosh\left(\frac{E_n-\mu}{2T}\right)\right)+\frac{1}{V}\int_V\frac{|M(\vec{x})-m|^2}{4G_s}$$

Having defined $M(\vec{x}) = m - 2G_S\left(S(\vec{x})+iP(\vec{x})\right)$

Lower-dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- Restrict to lower-dimensional spatial modulations
- $[\mathcal{H}, P_\perp] = 0 \rightarrow$ Full spectrum from lower-dimensional eigenvalues λ
- Boost along the transverse directions:

$$(\lambda, \mathbf{0}) \rightarrow (\sqrt{p_\perp^2 + \lambda^2}, \mathbf{p}_\perp)$$

$$\Omega(T, \mu; M(\vec{x})) = -\frac{2TN_c}{V_{||}} \sum_\lambda \int \frac{d\vec{p}_\perp}{(2\pi)^{d_\perp}} \ln\left(2\cosh\left(\frac{\lambda\sqrt{1+\vec{p}_\perp^2/\lambda^2} - \mu}{2T}\right)\right) + \frac{1}{V}\int_V \frac{|M(\vec{x}) - m|^2}{4G_s} + \text{const.}$$

(D. Nickel, Phys.Rev.D80 074025, 2009)

One-dimensional modulations

$$M(\vec{x}) \to M(z)$$

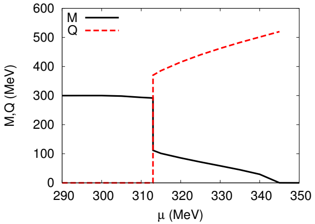
- Simplest case: one-dimensional modulations of the chiral condensate
- Different kinds of 1D modulations possible

One-dimensional modulations

$$M(\vec{x}) \rightarrow M(z)$$

- Simplest case: one-dimensional modulations of the chiral condensate
- Different kinds of 1D modulations possible

$$M(z) = M\,e^{iQz}$$

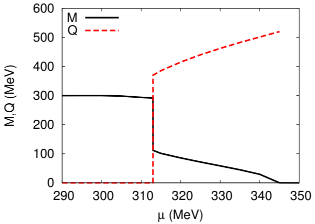


One-dimensional modulations

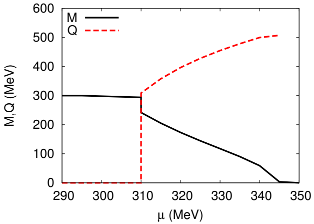
$$M(\vec{x}) \to M(z)$$

- Simplest case: one-dimensional modulations of the chiral condensate
- Different kinds of 1D modulations possible

$$M(z) = M\, e^{iQz}$$



$$M(z) = M \cos(Qz)$$

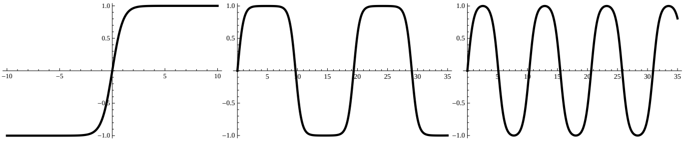


A more general 1D modulation

- Self-consistent real solutions known from studies of 1+1D Gross-Neveu model

  (M.Thies et al., Annals Phys. 314)

$$M(z) = \Delta \sqrt{\nu}\, \mathrm{sn}(\Delta z | \nu)$$



- Analytical expression for the eigenvalue spectrum of $\mathcal{H}_{MF}[M(z)]$
- Minimization of $\Omega[M(z)]$ w.r.t. two parameters (chiral limit): $\Omega(\Delta, \nu)$
- Away from chiral limit: add a third parameter δ

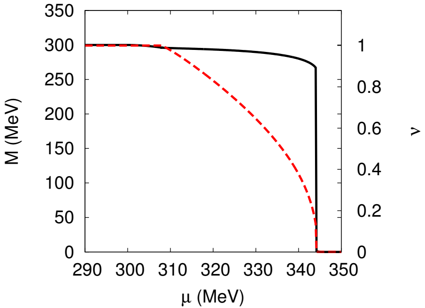
A more general 1D modulation

TECHNISCHE UNIVERSITÄT DARMSTADT

- Self-consistent real solutions known from studies of 1+1D Gross-Neveu model

  (M.Thies et al., Annals Phys. 314)

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu)$$



Two-dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- Brute-force diagonalization of $\mathcal{H} = \gamma^0 \left[i\vec{\gamma} \cdot \vec{\partial} + M(\vec{x}) \right]$
- Computationally demanding!
- Assume spatial periodicity of the order parameter $M(x, y)$
- Project onto the Brillouin zone $\rightarrow \mathcal{H}(k)$ becomes block-diagonal

$$\mathcal{H}_{\vec{p_{in}},\vec{p_{out}}} = \sum_{\vec{k} \in BZ} \mathcal{H}_{\vec{q_{in}},\vec{q_{out}}}(\vec{k}) \quad , \; \vec{q_{in}}, \vec{q_{out}} \in RL$$

- Diagonalize $\mathcal{H}$ numerically
- Minimize $\Omega\left[\{M_q\}, \{q\}\right]$

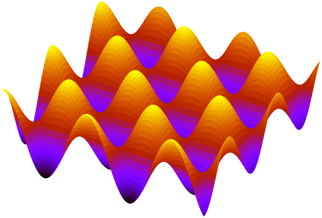
$$\mathcal{H}_{p_m,p_n} = \begin{pmatrix}
\square & & & \square & & & & & & \\
& \square & & & \square & & & & & \\
& & \square & & & \square & & & & \\
\square & & & \square & & & \square & & & \\
& \square & & & \square & & & \square & & \\
& & \square & & & \square & & & \square & \\
& & & \square & & & \square & & & \square \\
& & & & \square & & & \square & & \\
& & & & & \square & & & \square & \\
& & & & & & \square & & & \square
\end{pmatrix}$$

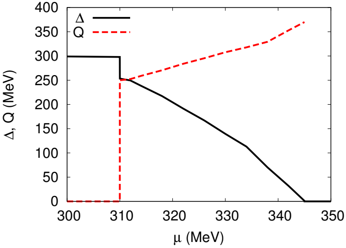
Egg carton

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Simple square crystal with LOFF-type modulation

$$M(x, y) = \Delta cos(Qx)cos(Qy)$$



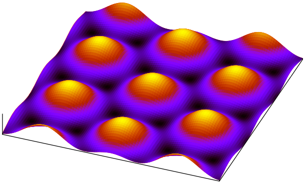


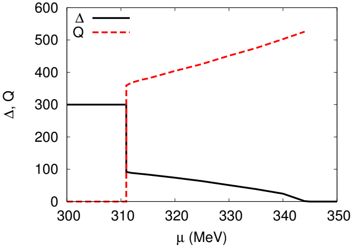
Hexagon-symmetric modulation

TECHNISCHE UNIVERSITÄT DARMSTADT

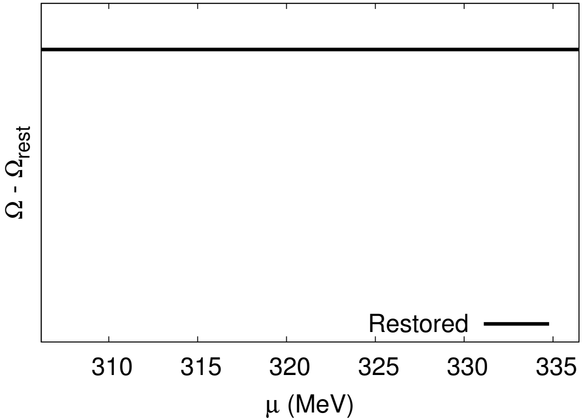
- Something more elaborate: hexagonal symmetry

$$M(x,y) = \Delta \left[\cos(Qy) + 2\cos\left(\frac{\sqrt{3}}{2}Qx\right)\cos\left(\frac{Q}{2}y\right) \right]$$



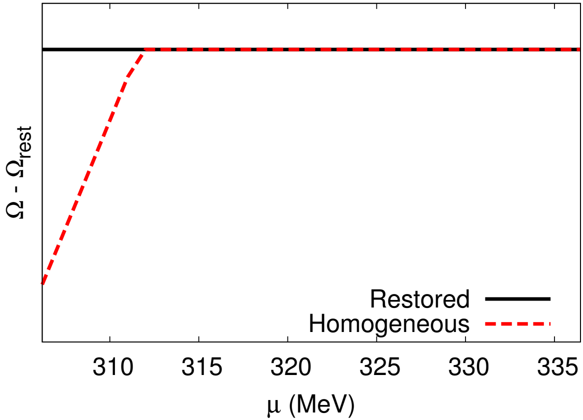


Comparison of free energies, $T = 0$



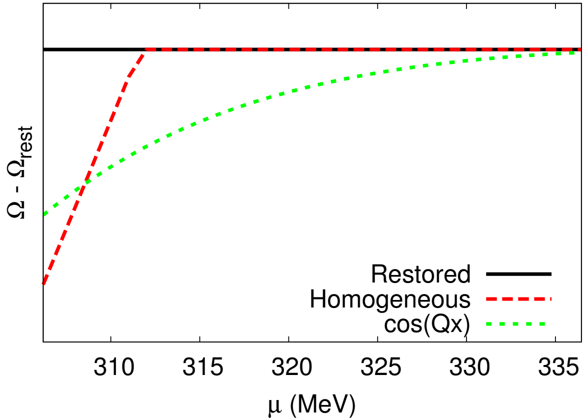
Comparison of free energies, $T = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT



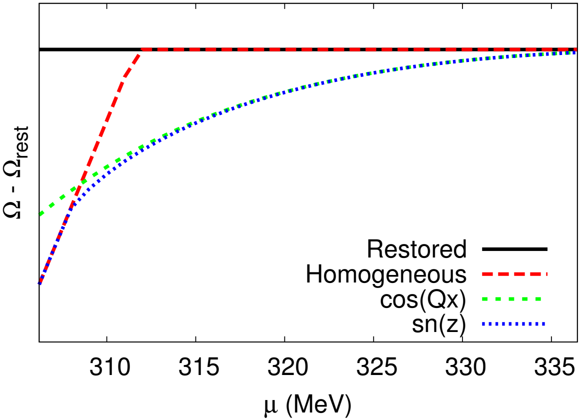
Comparison of free energies, $T = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT



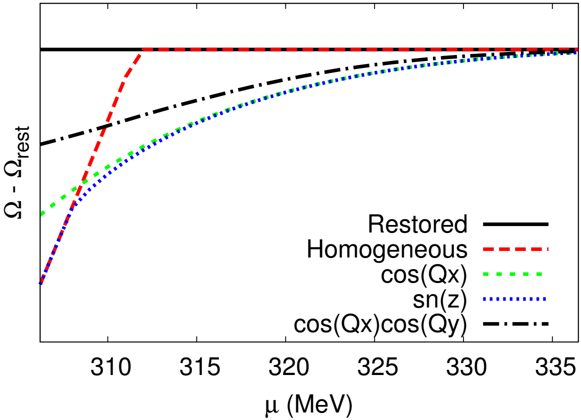
Comparison of free energies, $T = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT



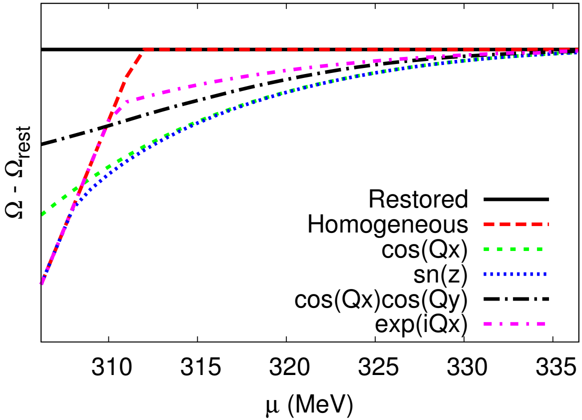
Comparison of free energies, $T = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT



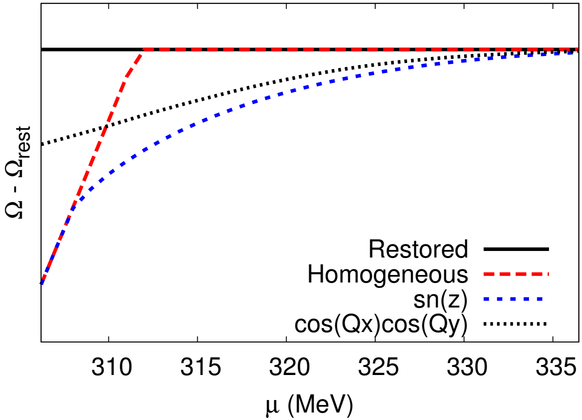
Comparison of free energies, $T = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT

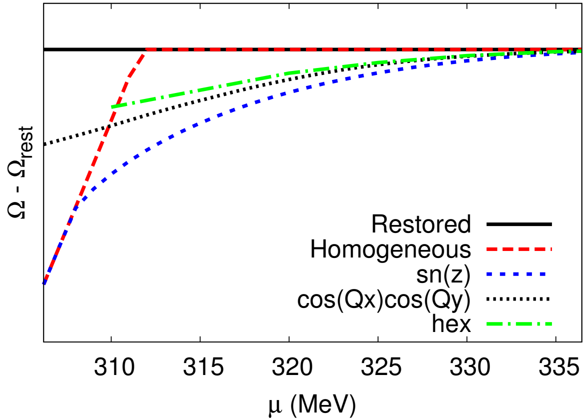


Comparison of free energies, $T = 0$

TECHNISCHE UNIVERSITÄT DARMSTADT



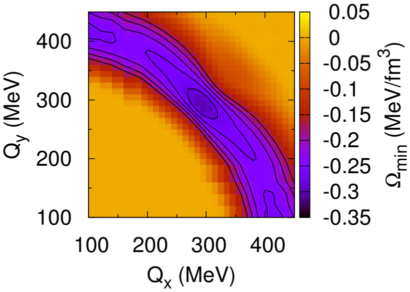
Comparison of free energies, $T = 0$



2D vs 1D

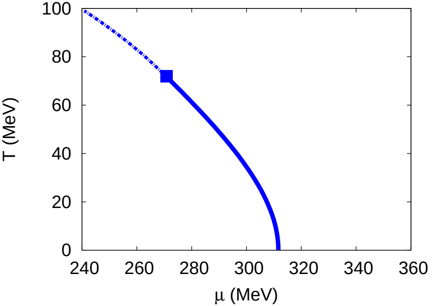
- One-dimensional solutions are the most favored!
- Extended “egg-carton” ansatz:

$$M(x, y) = \Delta \cos(Q_x x) \cos(Q_y y)$$



Phase diagram (chiral limit)

TECHNISCHE UNIVERSITÄT DARMSTADT

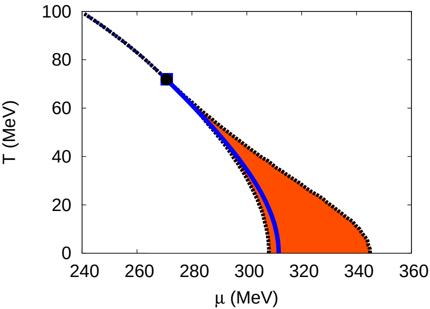


- Homogeneous only:

- First order phase transition
- ending at a critical point

Phase diagram (chiral limit)

TECHNISCHE UNIVERSITÄT DARMSTADT

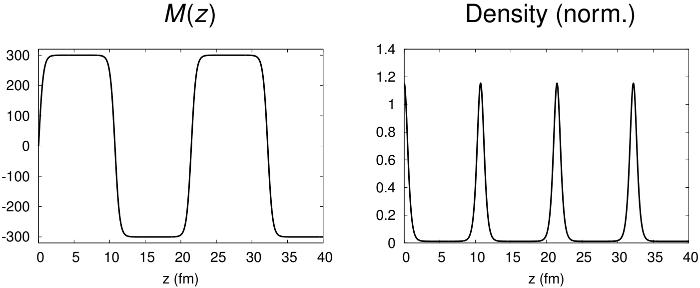


- Allow for 1D modulations like $M(z) = \Delta \sqrt{\nu}\, \mathrm{sn}(\Delta z|\nu)$
- First order transition line covered by inhomogeneous phase
- All phase transitions are 2nd order
- Critical point $\rightarrow$ Lifschitz point

Solitons: Mass and density profiles ($T = m = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta \tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$



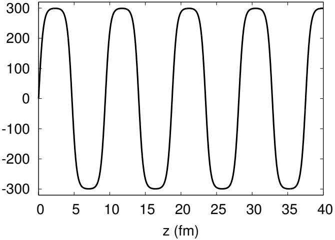
$\mu = 307.5$ MeV

Solitons: Mass and density profiles ($T = m = 0$)

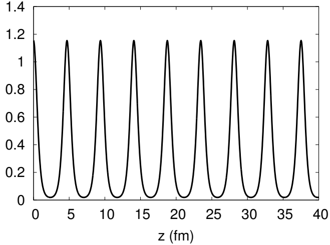
TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta\tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$

$M(z)$



Density (norm.)



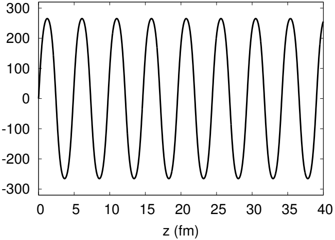
$\mu = 308$ MeV

Solitons: Mass and density profiles ($T = m = 0$)

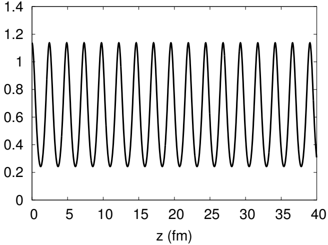
TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta \tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$

$M(z)$



Density (norm.)



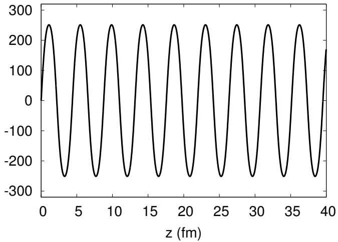
μ = 309 MeV

Solitons: Mass and density profiles ($T = m = 0$)

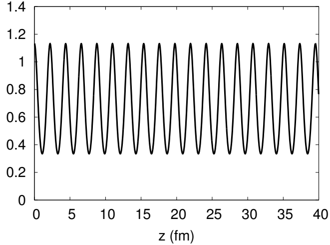
TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta\tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$

$M(z)$



Density (norm.)



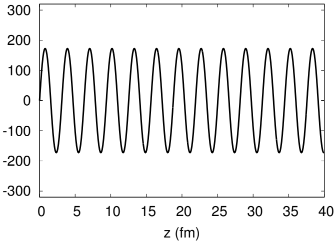
μ = 310 MeV

Solitons: Mass and density profiles ($T = m = 0$)

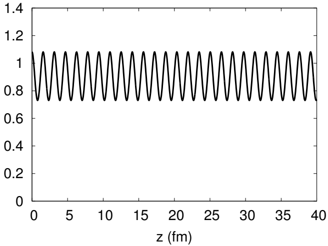
TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta \tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$

$M(z)$



Density (norm.)

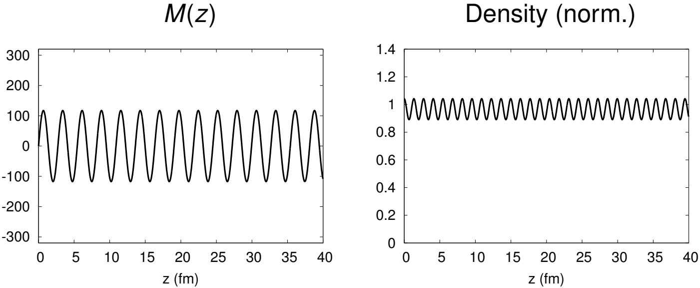


μ = 320 MeV

Solitons: Mass and density profiles ($T = m = 0$)

TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta \tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$



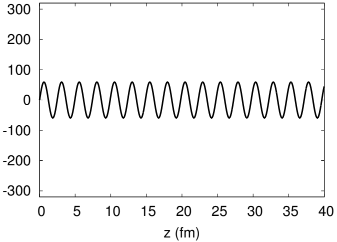
μ = 330 MeV

Solitons: Mass and density profiles ($T = m = 0$)

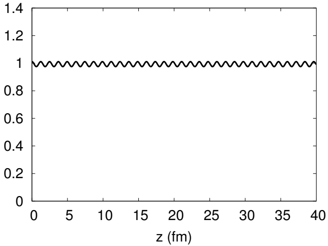
TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta \tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$

$M(z)$



Density (norm.)



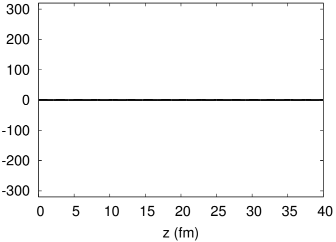
μ = 340 MeV

Solitons: Mass and density profiles ($T = m = 0$)

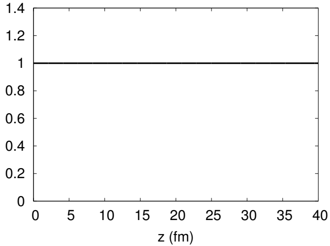
TECHNISCHE UNIVERSITÄT DARMSTADT

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu) = \begin{cases} \Delta \tanh(\Delta z) & \text{for } \nu \to 1 \\ \Delta\sqrt{\nu}\sin(\Delta z) & \text{for } \nu \to 0 \end{cases}$$

$M(z)$



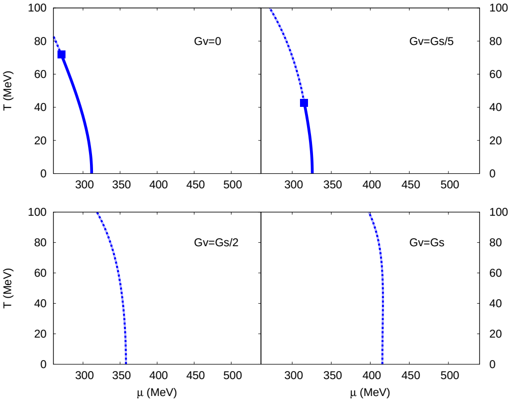
Density (norm.)



μ = 345 MeV

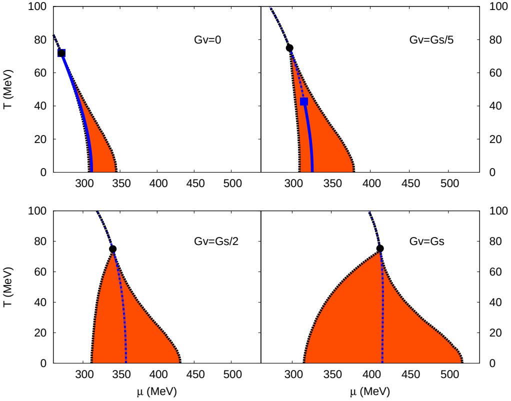
Vector interactions (Chiral limit)

TECHNISCHE UNIVERSITÄT DARMSTADT



- Homogeneous:
- Shift towards higher μ
- Strong G_V-dependence of the critical point

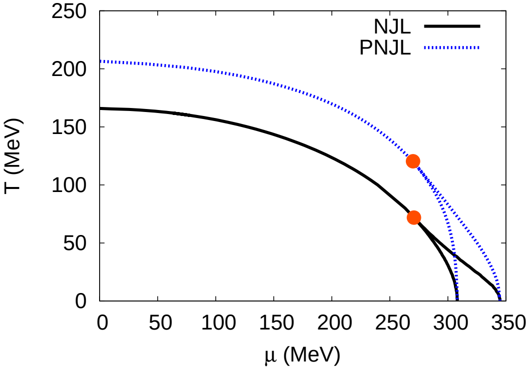
Vector interactions (Chiral limit)



- Inhomogeneous:
- Stretch towards higher μ
- Lifshitz point at constant T
- Lifshitz and critical points split

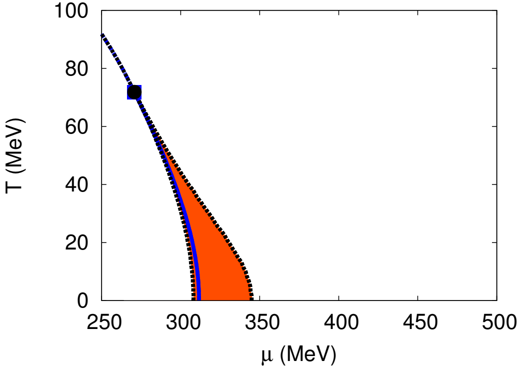
PNJL (Chiral limit)

TECHNISCHE UNIVERSITÄT DARMSTADT

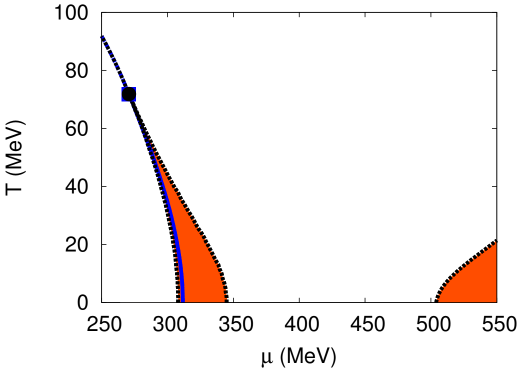


- Suppression of thermal effects
- Phase diagram stretched in T

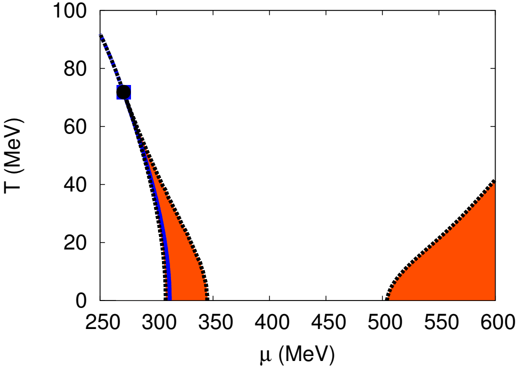
Islands and continents



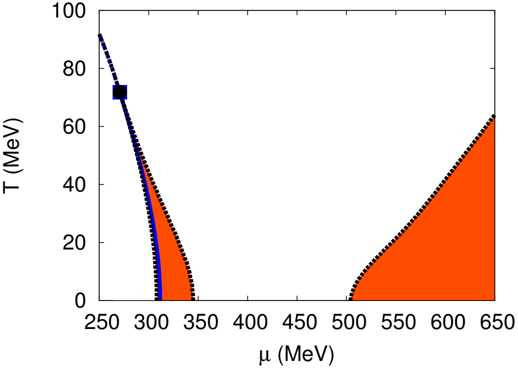
Islands and continents



Islands and continents



Islands and continents



Regularization artifact ?

TECHNISCHE UNIVERSITÄT DARMSTADT

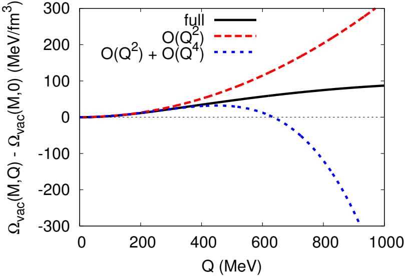
- Present for both Pauli-Villars and proper time regularizations

Regularization artifact ?

TECHNISCHE UNIVERSITÄT DARMSTADT

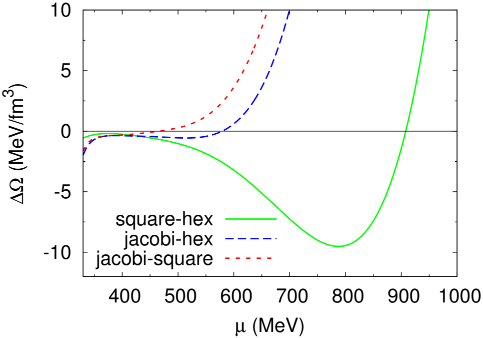
- Present for both Pauli-Villars and proper time regularizations
- Not present in Quark-Meson model !

- Difference NJL / QM ?
- Universal q^2 kinetic term
- Higher orders from NJL vacuum



2D crystals at high μ

- Pushing the model to its validity limits..



- 2D modulations become favored over 1D ones at high μ

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phase appears
  at intermediate densities and low temperatures

Conclusions

- Inhomogeneous phase appears
  at intermediate densities and low temperatures

- 2D modulations seem to be disfavored...

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phase appears
  at intermediate densities and low temperatures

- 2D modulations seem to be disfavored...
  (... except in the high μ region)

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phase appears at intermediate densities and low temperatures
- 2D modulations seem to be disfavored...
  (... except in the high μ region)
  (but it's kind of controversial)

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phase appears
  at intermediate densities and low temperatures

- 2D modulations seem to be disfavored...
  (... except in the high μ region)
  (but it's kind of controversial)

- Model extensions enlarge the inhomogeneous window

Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- Inhomogeneous phase appears
  at intermediate densities and low temperatures

- 2D modulations seem to be disfavored...
  (... except in the high μ region)
  (but it's kind of controversial)

- Model extensions enlarge the inhomogeneous window

- Many more things to try ..

backup

Lower-dimensional modulations

TECHNISCHE UNIVERSITÄT DARMSTADT

- Restrict to lower-dimensional spatial modulations
- $[\mathcal{H}, P_\perp] = 0 \rightarrow$ Full spectrum from lower-dimensional eigenvalues λ
- Boost along the transverse directions:

$$(\lambda, \mathbf{0}) \rightarrow (\sqrt{p_\perp^2 + \lambda^2}, \mathbf{p}_\perp)$$

$$\Omega(T, \mu; M(\vec{x})) = -\frac{2TN_c}{V_{||}} \sum_\lambda \int \frac{d\vec{p}_\perp}{(2\pi)^{d_\perp}} \ln\left(2\cosh\left(\frac{\lambda\sqrt{1 + \vec{p}_\perp^2/\lambda^2} - \mu}{2T}\right)\right) + \frac{1}{V}\int_V \frac{|M(\vec{x}) - m|^2}{4G_s} + \text{const.}$$

(D. Nickel, Phys.Rev.D80 074025, 2009 - arXiv:0906.5295)

One-dimensional modulations: $M(\vec{x}) \to M(z)$

- Restrict to one-dimensional modulations

$$M(\vec{x}) \to M(z)$$

- The Hamiltonian becomes

$$\mathcal{H} \to \mathcal{H}_{1D} = \begin{pmatrix} H_{1D}(M(z)) & \\ & H_{1D}(M(z)^*) \end{pmatrix}$$

$$\mathcal{H}_{1D}(M(z)) = \begin{pmatrix} -i\partial_z & M(z) \\ M(z)^* & i\partial_z \end{pmatrix}$$ Gross-Neveu Hamiltonian

Gross-Neveu

TECHNISCHE UNIVERSITÄT DARMSTADT

- One of the simplest interacting fermionic field theories
- Defined in 1+1 dimensions

$$\mathcal{L}_{GN} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \frac{1}{2} g^2 (\bar{\psi}\psi)^2$$

- Hartree-Fock $\rightarrow$ recover SUSY QM-like equation
- Real self-consistent solutions of the form

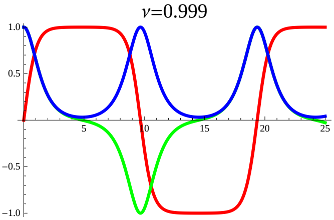
$$M(z) = \Delta \left(\nu \, \mathrm{sn}(b|\nu)\mathrm{sn}(\Delta z|\nu)\mathrm{sn}(\Delta z + b|\nu) + \frac{\mathrm{cn}(b|\nu)\mathrm{dn}(b|\nu)}{\mathrm{sn}(b|\nu)} \right)$$

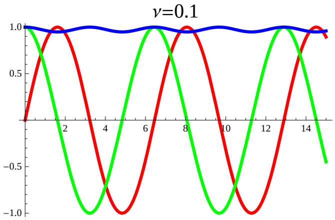
- Eigenvalue spectrum well known

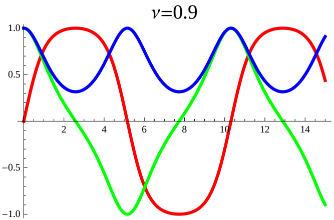
(M.Thies et al., Annals Phys. 314 (2004) 425-447, arXiv:hep-th/0402014)

Elliptic functions: sn$(z|\nu)$, cn$(z|\nu)$, dn$(z|\nu)$
$\nu \in [0, 1]$

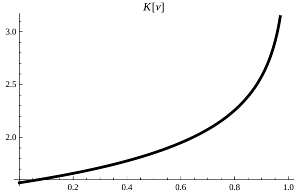
TECHNISCHE UNIVERSITÄT DARMSTADT







Period $\propto$ **K**(ν):



Some technicalities...

TECHNISCHE UNIVERSITÄT DARMSTADT

$$\Omega_{MF}^{NJL}(T,\mu;\Delta,\nu,\delta) = -2N_c \int_0^\infty dE\, \tilde{\rho}(E;\nu,\Delta)\tilde{f}_{\rm bare}\left(\sqrt{E^2+\delta\Delta^2}\right) + \frac{1}{4G_sP}\int_0^P dz\, |M(z)-m|^2 + C$$

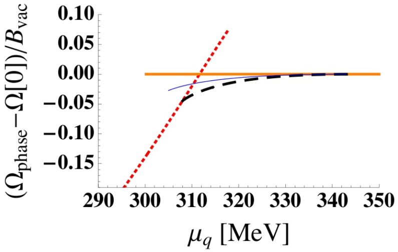
$$\begin{aligned}
\tilde{f}_{\rm bare}(x) &= \tilde{f}_{\rm vac}(x) + \tilde{f}_{\rm medium}(x)\\
\tilde{f}_{\rm vac}(x) &= x\\
\tilde{f}_{\rm medium}(x) &= T\ln\left(1+\exp\left(-\frac{x-\mu}{T}\right)\right) + T\ln\left(1+\exp\left(-\frac{x+\mu}{T}\right)\right)
\end{aligned}$$

$$\tilde{f}_{\rm UV}(x) \to \tilde{f}_{\rm PV}(x) = \sum_{j=0}^{3} c_j\sqrt{x^2+j\Lambda^2} \qquad (c_0=1,\, c_1=-3,\, c_2=3,\, c_3=-1)$$

Real VS complex modulations

- Real modulations $\rightarrow P(x) = 0$

- Solitons: $M(z) \sim \Delta\sqrt{\nu}\, sn(z|\nu)$
- Chiral density wave: $M(z) = \Delta e^{iqz}$
- Homogeneous broken: $M(z) = \Delta$

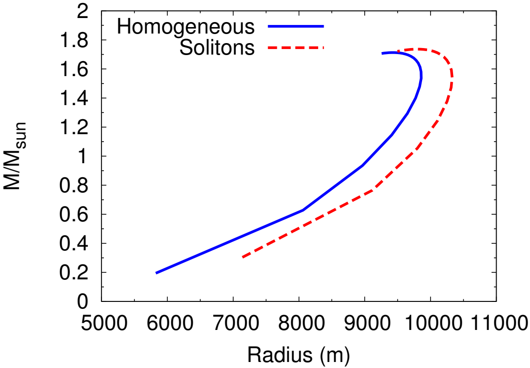


(D. Nickel, PRD 80)

- (Real) solitons are always favored over chiral density wave!

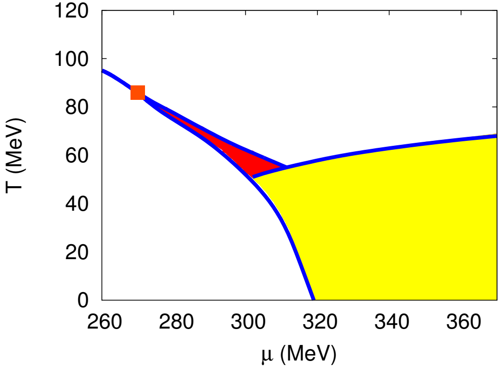
Mass-radius plot
(quark matter only!!)

TECHNISCHE UNIVERSITÄT DARMSTADT



preliminary!

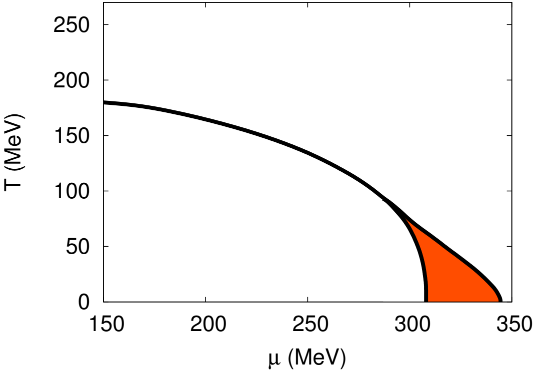
Comparison with (homogeneous) 2SC phase (with D. Nowakowski)



Preliminary!!

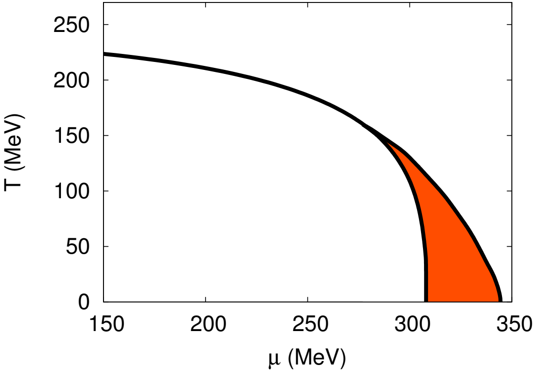
PNJL - Large N_C results

TECHNISCHE UNIVERSITÄT DARMSTADT



$N_C = 3$

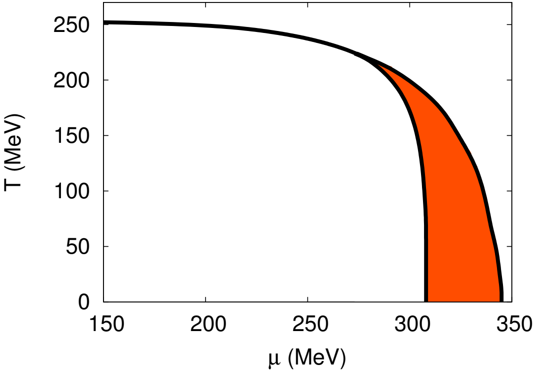
PNJL - Large N_C results



$N_C = 10$

PNJL - Large N_C results

TECHNISCHE UNIVERSITÄT DARMSTADT



$N_C = 50$

Vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

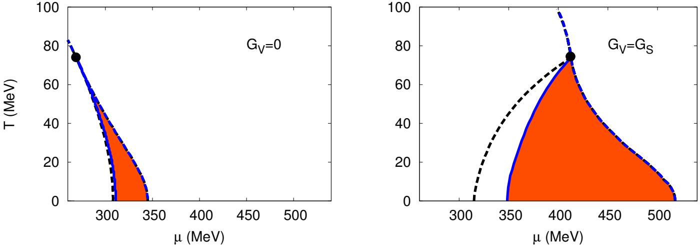
- Additional vector term: $\mathcal{L} = \mathcal{L}_{NJL} - G_V(\bar{\psi}\gamma^\mu\psi)^2$
- New mean field: $\bar{\psi}\gamma^\mu\psi \to \langle\bar{\psi}\gamma^\mu\psi\rangle \equiv n(\vec{x})\delta^{\mu 0}$ (density!)
- Introduce shifted chemical potential $\tilde{\mu}(\vec{x}) = \mu - 2G_V n(\vec{x})$
- Determine $\tilde{\mu}$ via $\frac{\delta\Omega}{\delta\tilde{\mu}} = 0$
- Sacrifice complete self-consistency: pick $\tilde{\mu} \equiv \langle\tilde{\mu}\rangle_z$ instead of $\tilde{\mu}(z)$
  - Most questionable in the inhomogeneous phase at low μ and T
  - More reliable close to the restored phase and the Lifshitz point

$$\Omega(T,\mu) \to \Omega(T,\tilde{\mu}) - \frac{(\mu-\tilde{\mu})^2}{4G_V}$$

Chiral density wave and vector interactions

TECHNISCHE UNIVERSITÄT DARMSTADT

- How good is our $\tilde{\mu}(z) \to \langle\tilde{\mu}(z)\rangle$ approximation ?
- Cross-check: Chiral density wave $\to M(z) = \Delta e^{iqz} \;\to\; n(z) = \text{const.}$



- Same qualitative behaviour as the solitonic solutions
- Lifshitz point at the same position
- Different (1st order) homogeneous $\to$ inhomogeneous transition line

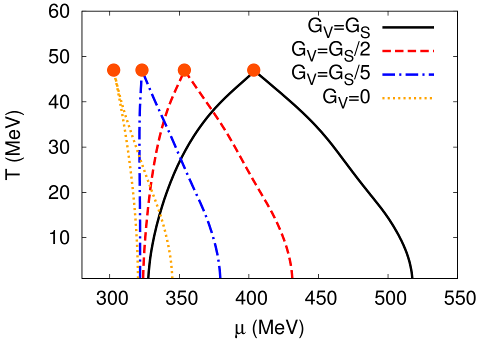
More phase diagrams: massive quarks

TECHNISCHE UNIVERSITÄT DARMSTADT

- Self-consistent solutions take the form

$$M(z) = \Delta \left(\sqrt{\nu}\, \mathrm{sn}(b|\nu)\mathrm{sn}(\Delta z|\nu)\mathrm{sn}(\Delta z + b|\nu) + \frac{\mathrm{cn}(b|\nu)\mathrm{dn}(b|\nu)}{\mathrm{sn}(b|\nu)} \right)$$

- Additional parameter: *b*
- Same qualitative features as $m = 0$
- Results for $m = 5$ MeV



Coupling with Polyakov Loop

TECHNISCHE UNIVERSITÄT DARMSTADT

- PNJL model:

$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - \hat{m}\right)\psi + G_s\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau^a\psi\right)^2\right) - \mathcal{U}(L, \bar{L})$$

  - Covariant derivative: $D_{\mu} = \partial_{\mu} + iA_0\delta_{\mu 0}$
  - Polyakov loop: $L(\vec{x}) = \mathcal{P}exp[i\int_0^{1/T} d\tau A_4(\tau, \vec{x})]\,,\quad A_4(\tau, \vec{x}) = iA_0(t = -i\tau, \vec{x})$
  - Expectation values: $\ell = \frac{1}{N_c}\langle Tr_c L\rangle\,,\quad \bar{\ell} = \frac{1}{N_c}\langle Tr_c L^{\dagger}\rangle$
- Assumption: $\ell\,,\bar{\ell}$ space-time independent
- Main effect:

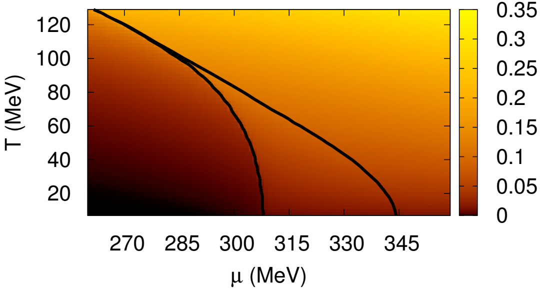
$$N_c T \log\left(1 + e^{-\frac{E-\mu}{T}}\right) \to T\log\left(1 + e^{-3(E-\mu)/T} + 3\,\ell\, e^{-(E-\mu)/T} + 3\,\bar{\ell}\, e^{-2(E-\mu)/T}\right)$$

- Thermally excited quarks are suppressed at small $\ell\,,\bar{\ell}$

Polyakov loop expectation value

TECHNISCHE UNIVERSITÄT DARMSTADT

- How good is our approximation of constant $\ell, \bar{\ell}$?

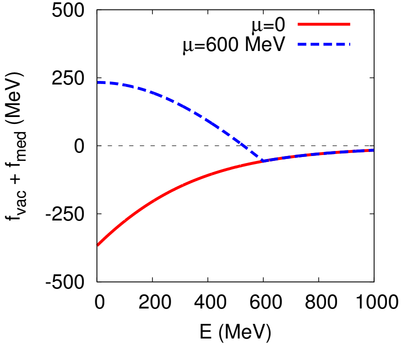


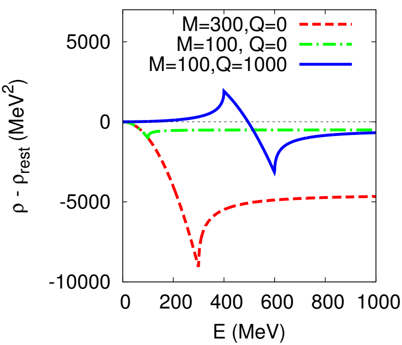
- Inhomogeneous regime: $\ell, \bar{\ell} \leq 0.2$
- Effects of neglecting spatial variations of $\ell, \bar{\ell}$ presumably small

Continent - Origin?
(Chiral density wave, T = 0)

TECHNISCHE UNIVERSITÄT DARMSTADT

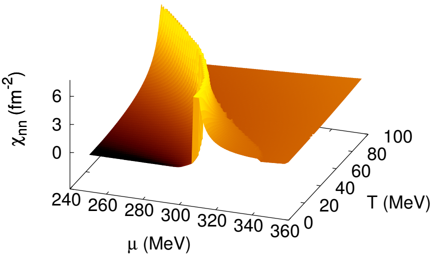
$$\Omega - \Omega_{rest} = -2N_c \int_0^\infty dE\,[\rho(E;M,Q) - \rho_{rest}(E)][f_{vac}(E) + f_{med}(E,\mu)] + \frac{M^2}{4G}$$



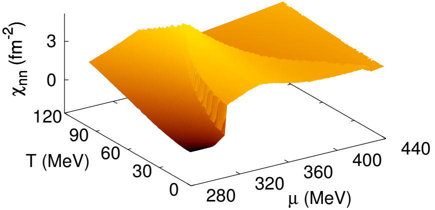


Quark number susceptibilities

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial \bar{n}}{\partial \mu}$$



$G_V = 0$



$G_V = G_S/2$

Susceptibilities