On transport coefficients in Yang-Mills theory

Jan M. Pawlowski
Universität Heidelberg & ExtreMe Matter Institute

Crete, June 27th 2012

based on

Transport coefficients in QCD; **Michael Haas, JMP, in prep.**

using

thermal YM/QCD-correlation functions from

FunMethods

L. Fister, JMP, ’11 & in prep.

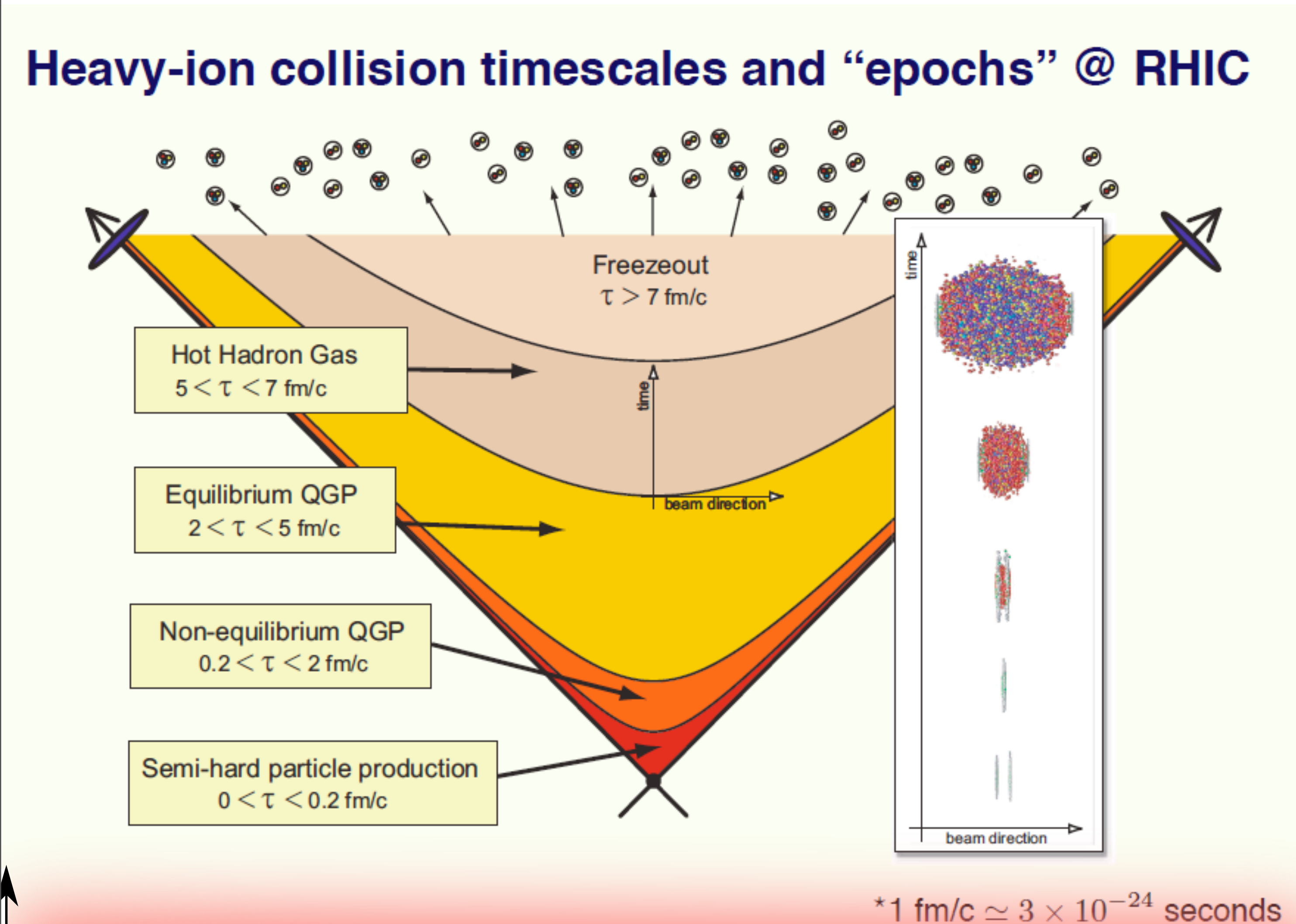
J. Braun, L.M. Haas, F. Marhauser, JMP, ’09

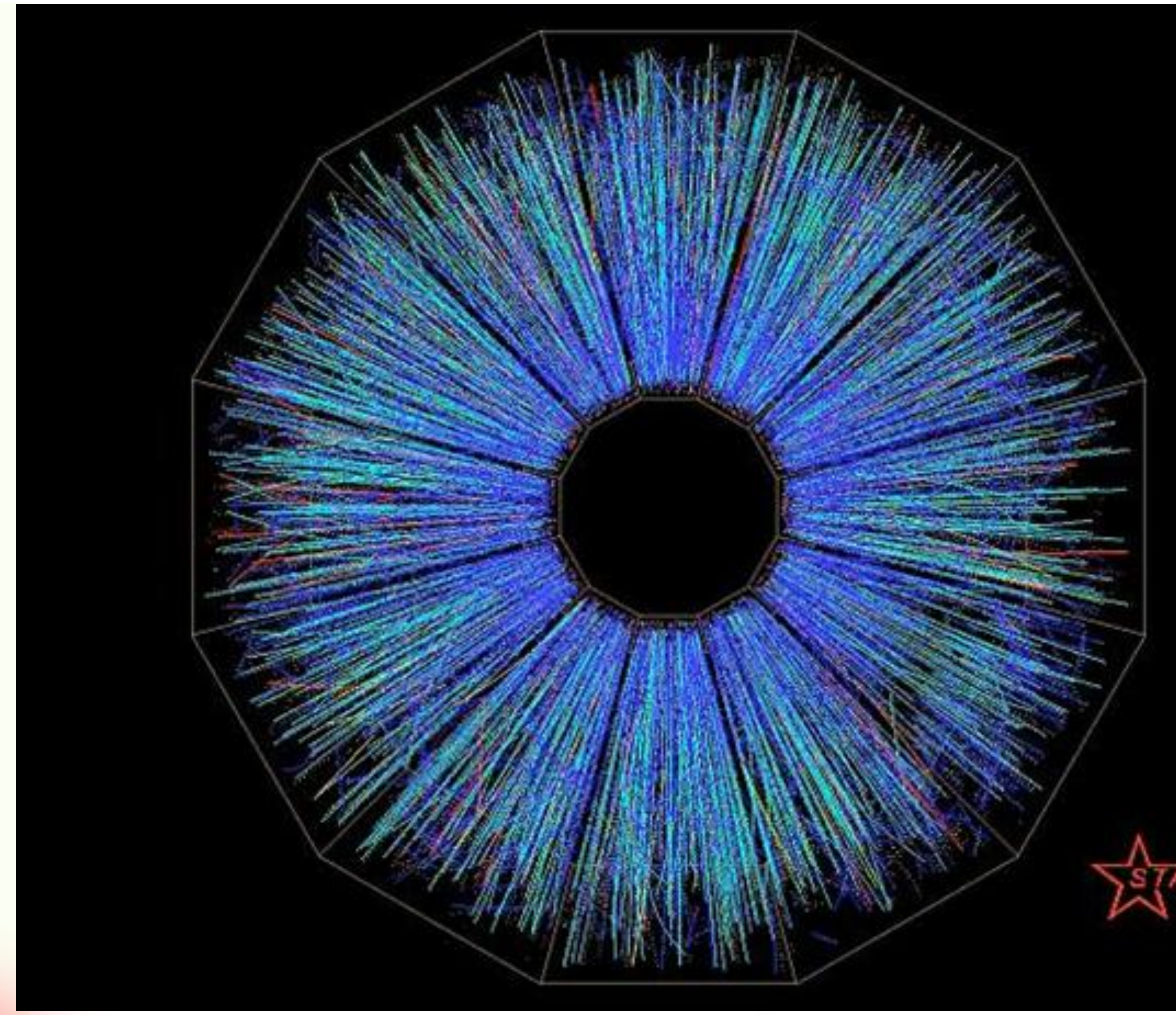
J. Braun, L. Fister, L.M. Haas, JMP, in prep.

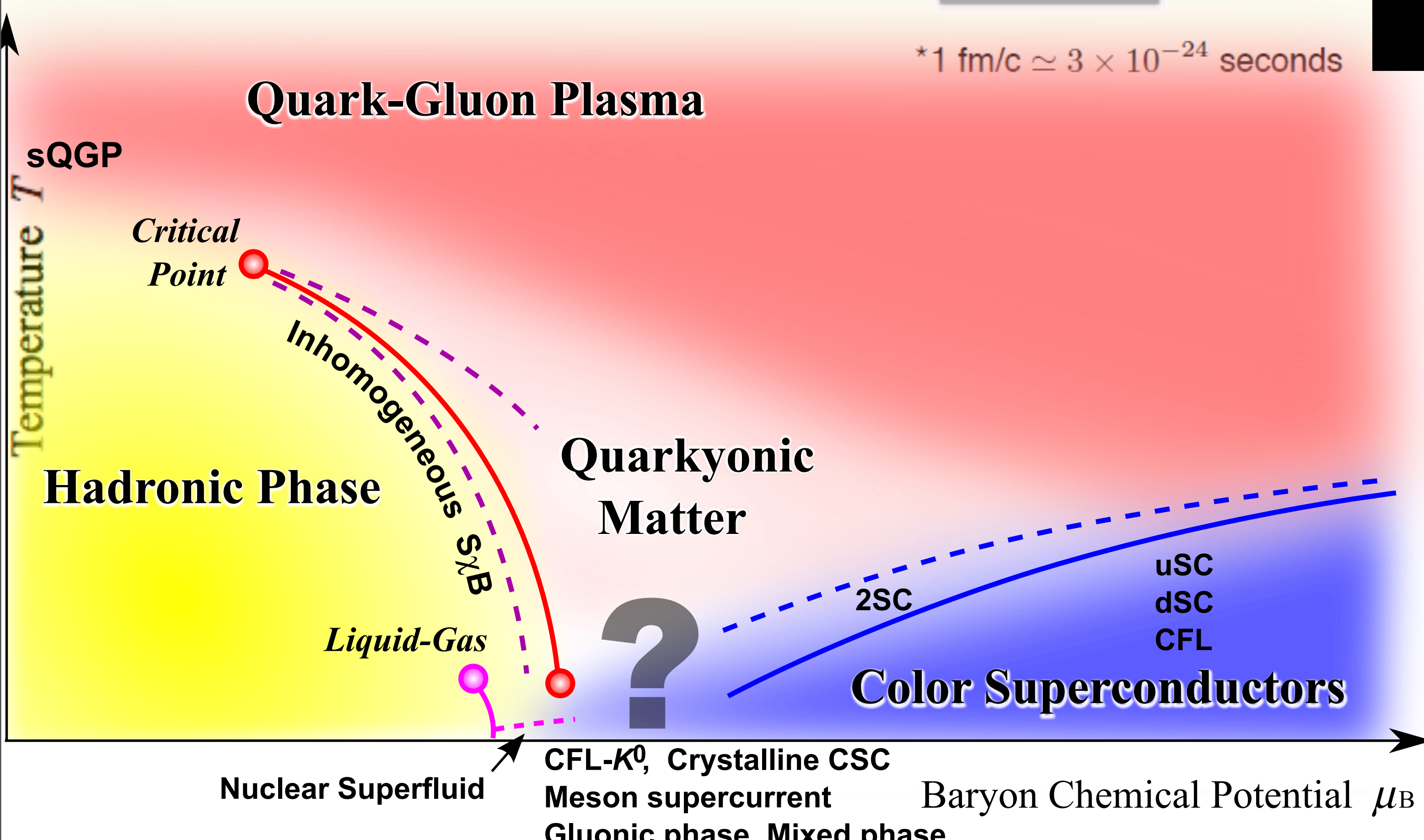
Lattice

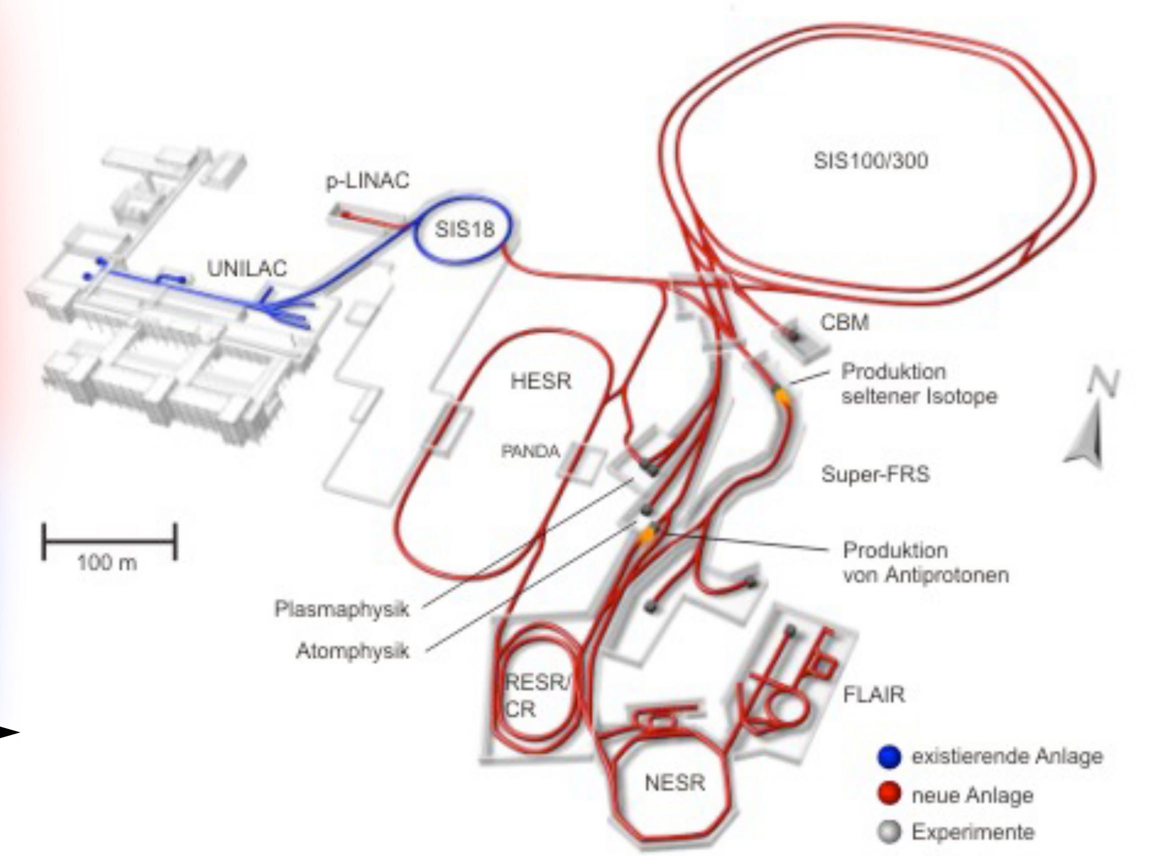
A. Maas, JMP, D. Spielmann, L. von Smekal, ’11

Heavy ion collisions





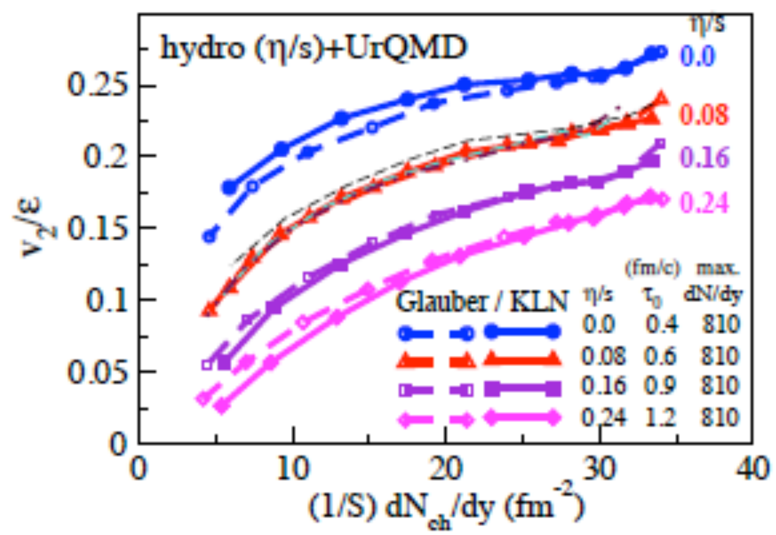


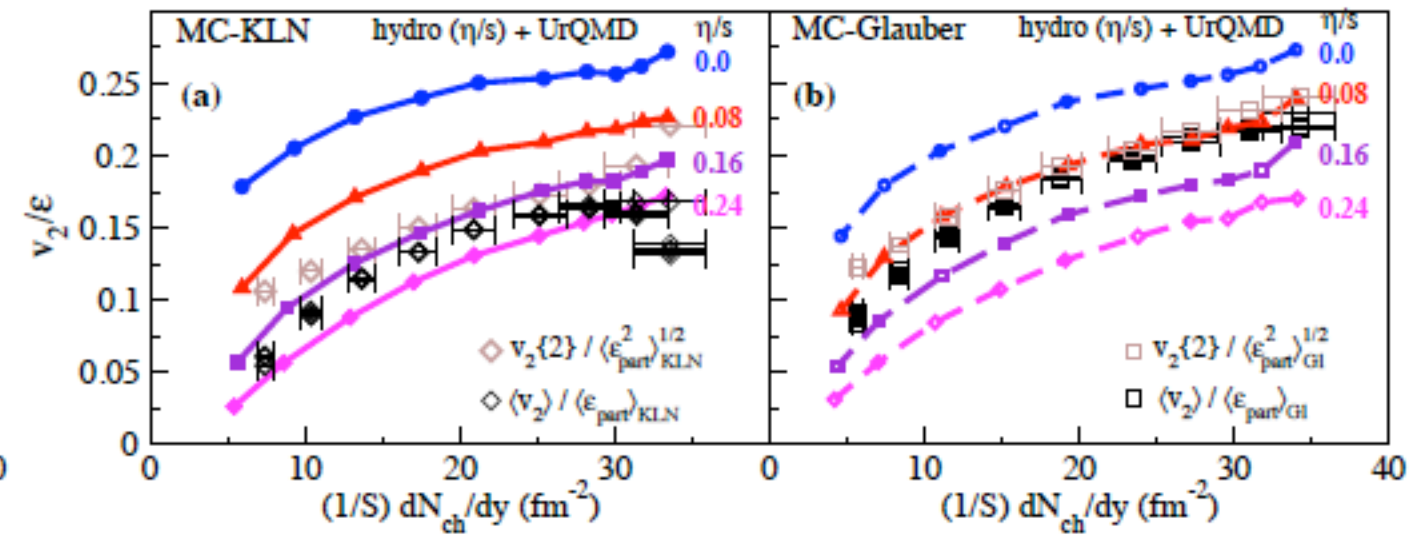


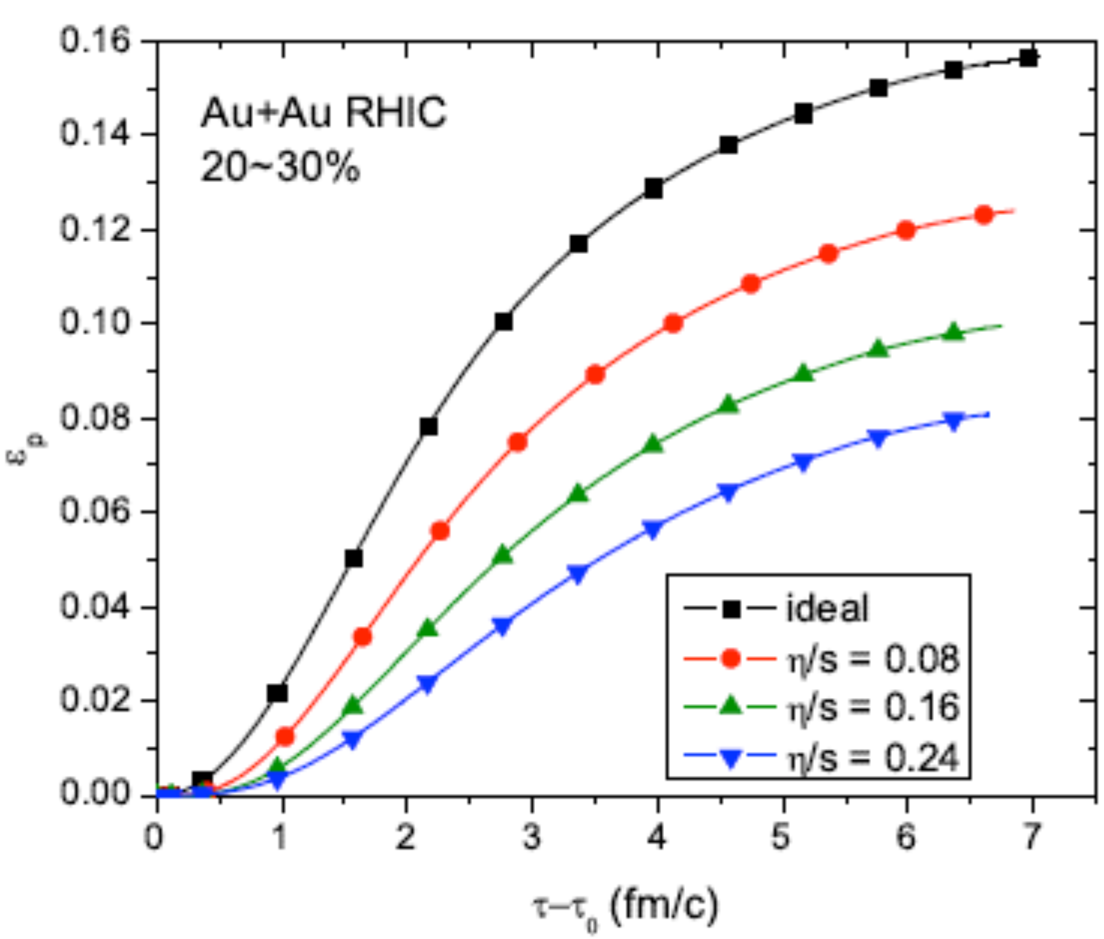
Heavy ion collisions

Extraction of $(\eta/s)_{QGP}$ from AuAu@RHIC

H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRL106 (2011) 192301







$$1 < 4\pi(\eta/s)_{QGP} < 2.5$$

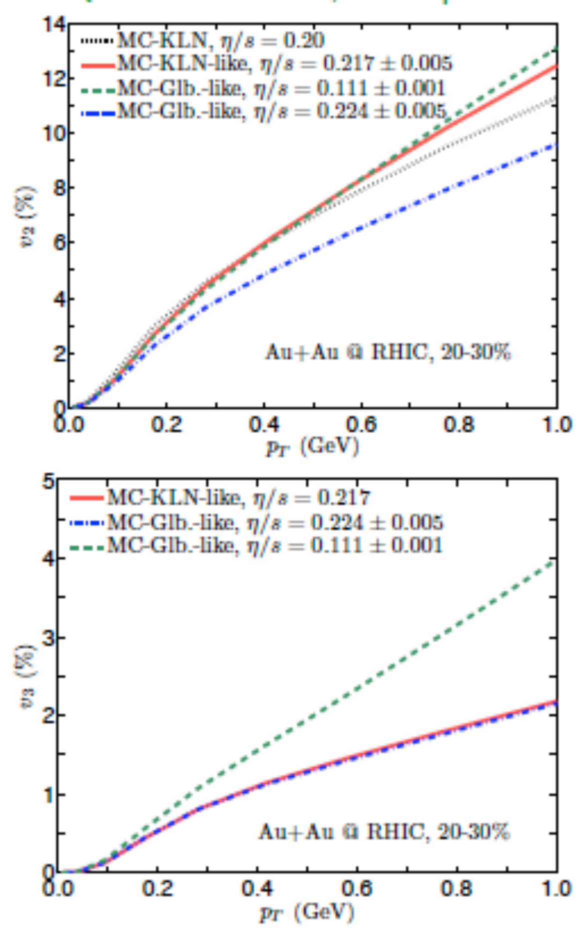
U. Heinz, talk at RETUNE '12

Heavy ion collisions

Shooting the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



- Take ensemble of sum of deformed Gaussian profiles, $s(\boldsymbol{r}_\perp) = s_2(\boldsymbol{r}_\perp; \tilde{\varepsilon}_2, \psi_2) + s_3(\boldsymbol{r}_\perp; \tilde{\varepsilon}_3, \psi_3)$, with
  1. equal Gaussian radii $R_2^2 = R_3^2 = 8\,\mathrm{fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
  2. $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_3$ adjusted such that
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{KLN}}^{20-30\%}$ ("MC-KLN-like")
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{Gl}}^{20-30\%}$ ("MC-Glauber-like")
  3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{Gl}}^{v3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

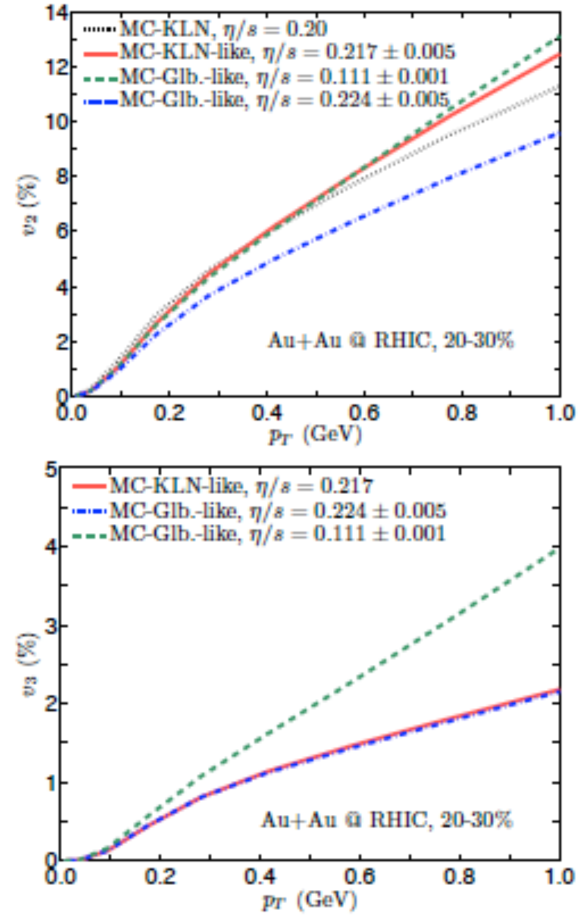
U. Heinz, talk at RETUNE '12

Heavy ion collisions

Computing the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



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Heavy ion collisions

Goal: microscopic transport description of the partonic and hadronic phase

Problems:

- How to model a QGP phase in line with lQCD data?
- How to solve the hadronization problem?

Ways to go:

pQCD based models:

- QGP phase: pQCD cascade
- hadronization: quark coalescence

→ AMPT, HIJING, BAMPS

‚Hybrid' models:

- QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

→ Hybrid-UrQMD

- microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

→ PHSD

Off-shell dynamical approach for relativistic heavy-ion collisions

Elena Bratkovskaya

RETUNE '12

Heavy ion collisions

DQPM thermodynamics (N_f=3) and lQCD

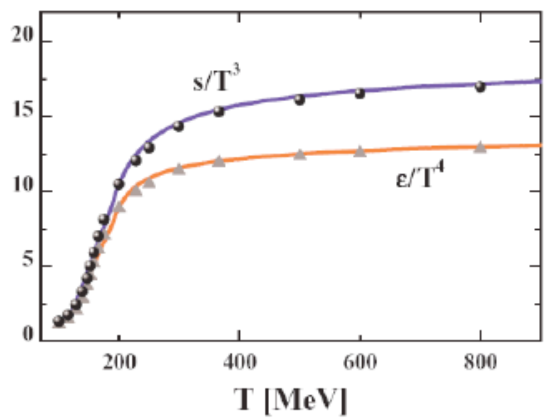
entropy $s = \frac{\partial P}{dT}$ → pressure P

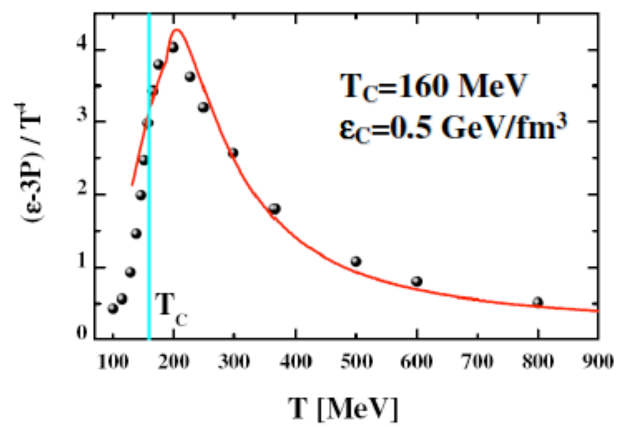
energy density: $\epsilon = Ts - P$

interaction measure:

$W(T) := \epsilon(T) - 3P(T) = Ts - 4P$

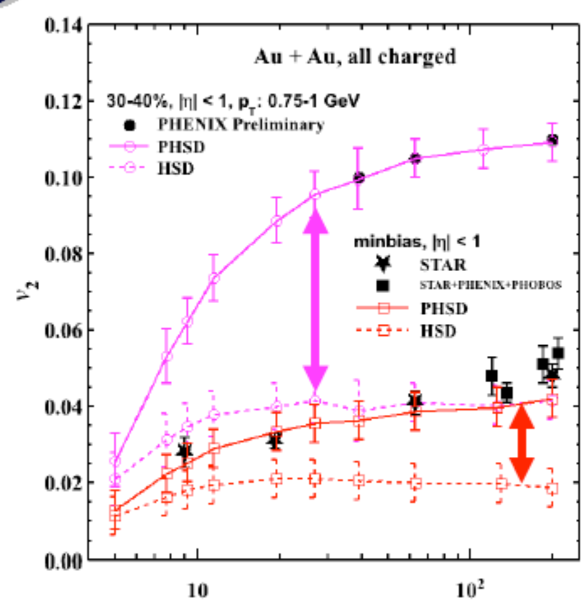
lQCD: Wuppertal-Budapest group
Y. Aoki et al., JHEP 0906 (2009) 088.

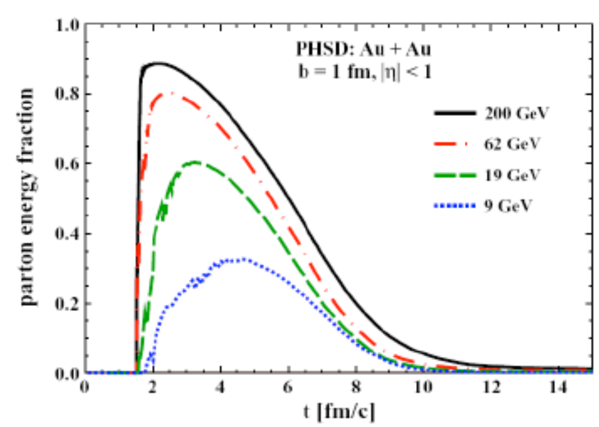




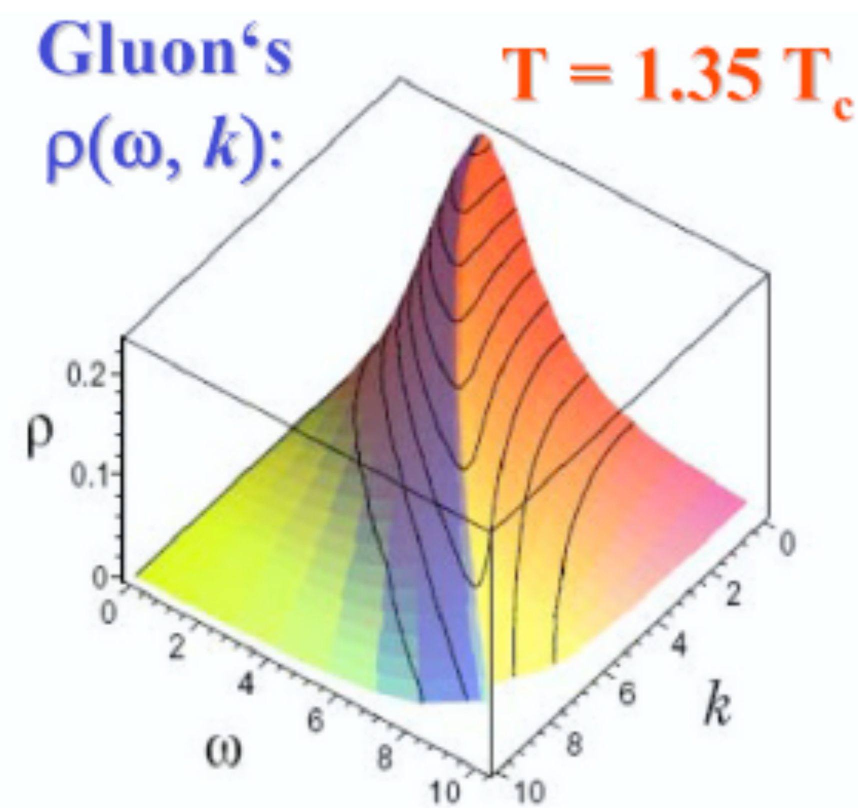
DQPM gives a good description of lQCD results !

Elliptic flow v_2 vs. collision energy for Au+Au





→ Broad spectral function :



Off-shell dynamical approach for relativistic heavy-ion collisions

Elena Bratkovskaya

RETUNE '12

Outline

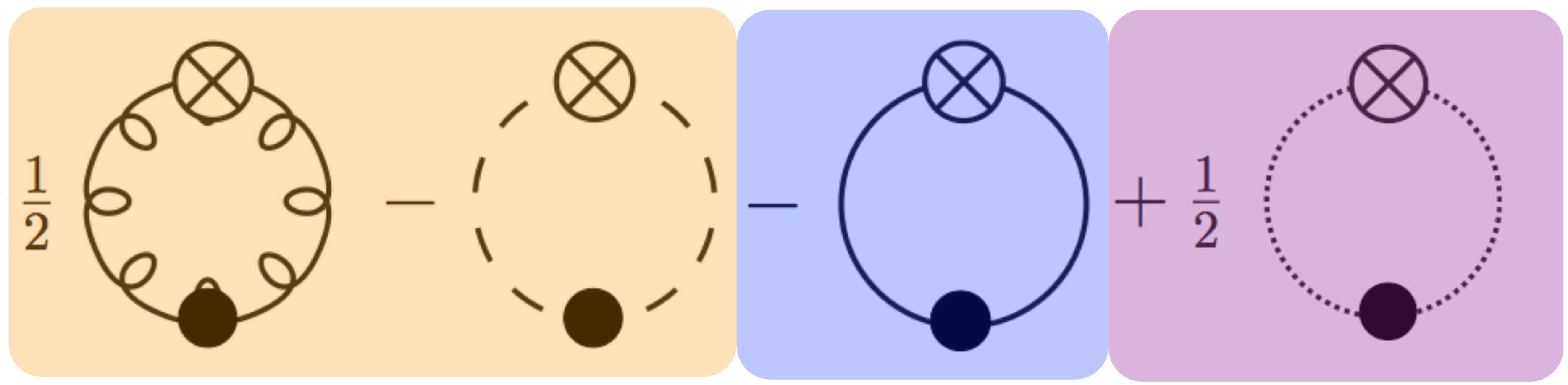
- Functional Methods in QCD
- Confinement & thermodynamics
- Transport in QCD
- Viscosity in YM
- Outlook

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

glue quantum fluctuations | **hadronic quantum fluctuations**

$$\partial_t \Gamma_k[\phi] = $$



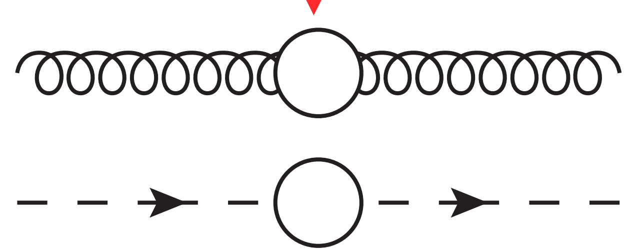
free energy | **quark quantum fluctuations** | RG-scale k: $t = \ln k$

Yang-Mills:

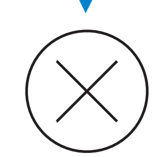
$$\partial_t \Gamma_k[A, \bar{c}, c] = \frac{1}{2} \mathrm{Tr} \left\{ \frac{1}{\Gamma^{(2)}[A, \bar{c}, c] + R_k} \, \partial_t R_k \right\} - \partial_t C_k$$

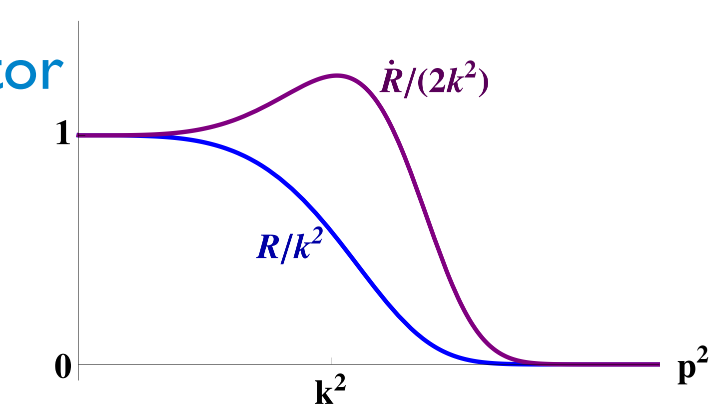
$$\partial_t = k \, \partial_k$$

full propagator



regulator





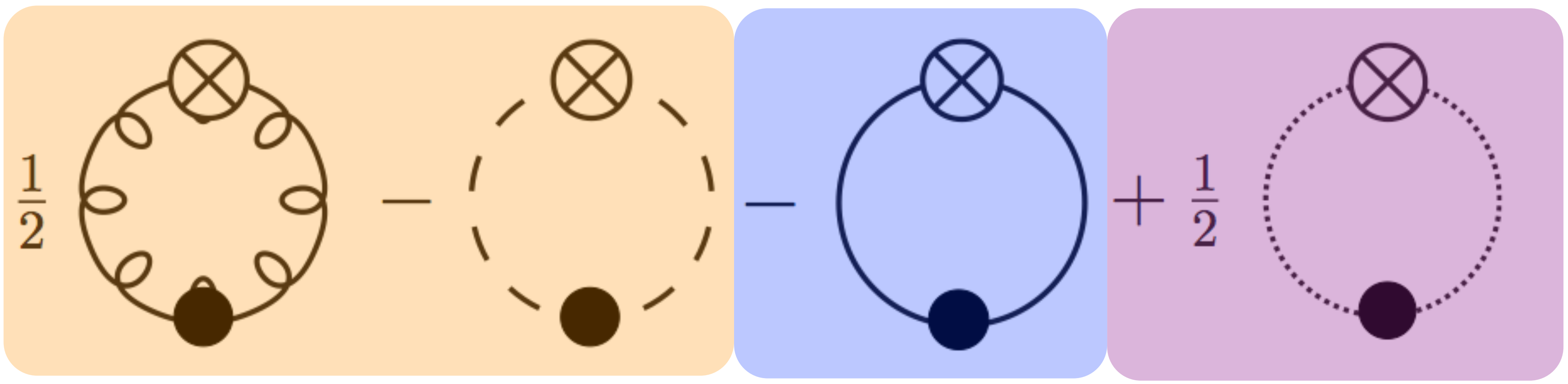
DSE: see lectures of C.F. Fischer

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

glue quantum fluctuations | **hadronic quantum fluctuations**

$\partial_t \Gamma_k[\phi] =$



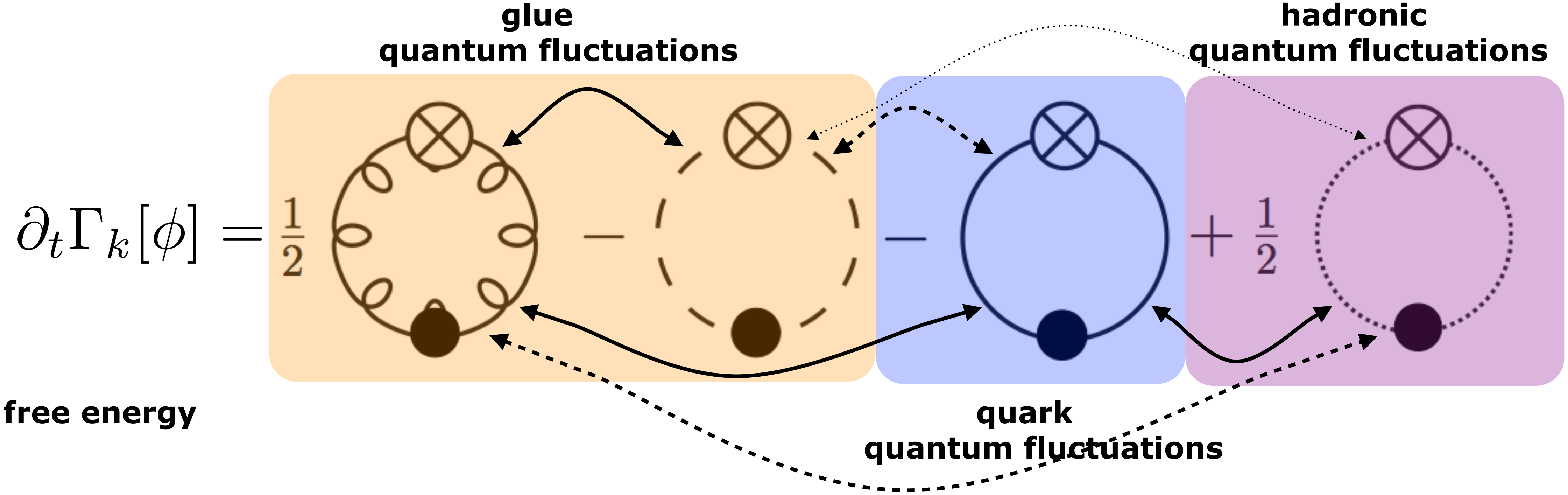
free energy | **quark quantum fluctuations** | RG-scale k: $t = \ln k$

- **Gluons have cost us decades**
- **Fermions are straightforward** though ‘physically’ complicated
  - **no sign problem**
  - **chiral fermions**
- **bound states via dynamical hadronisation**

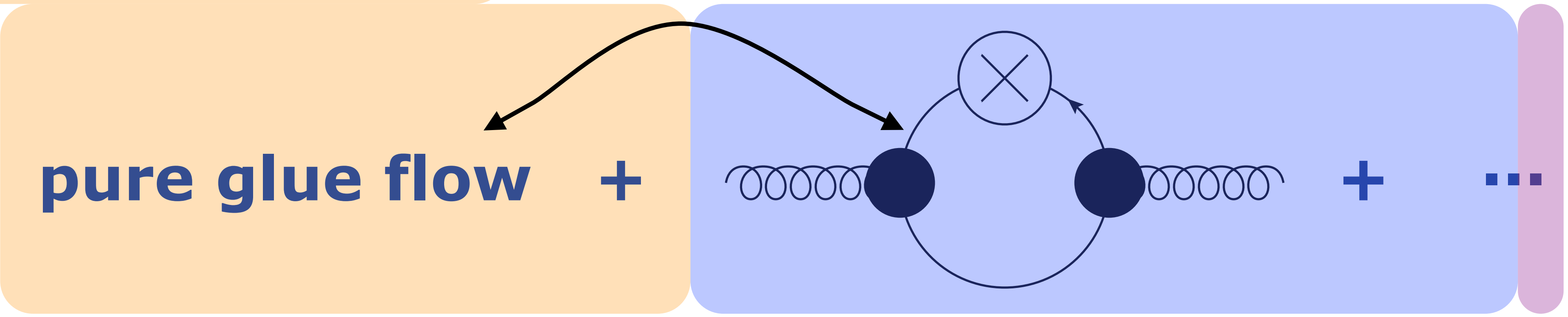
Complementary to lattice!

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

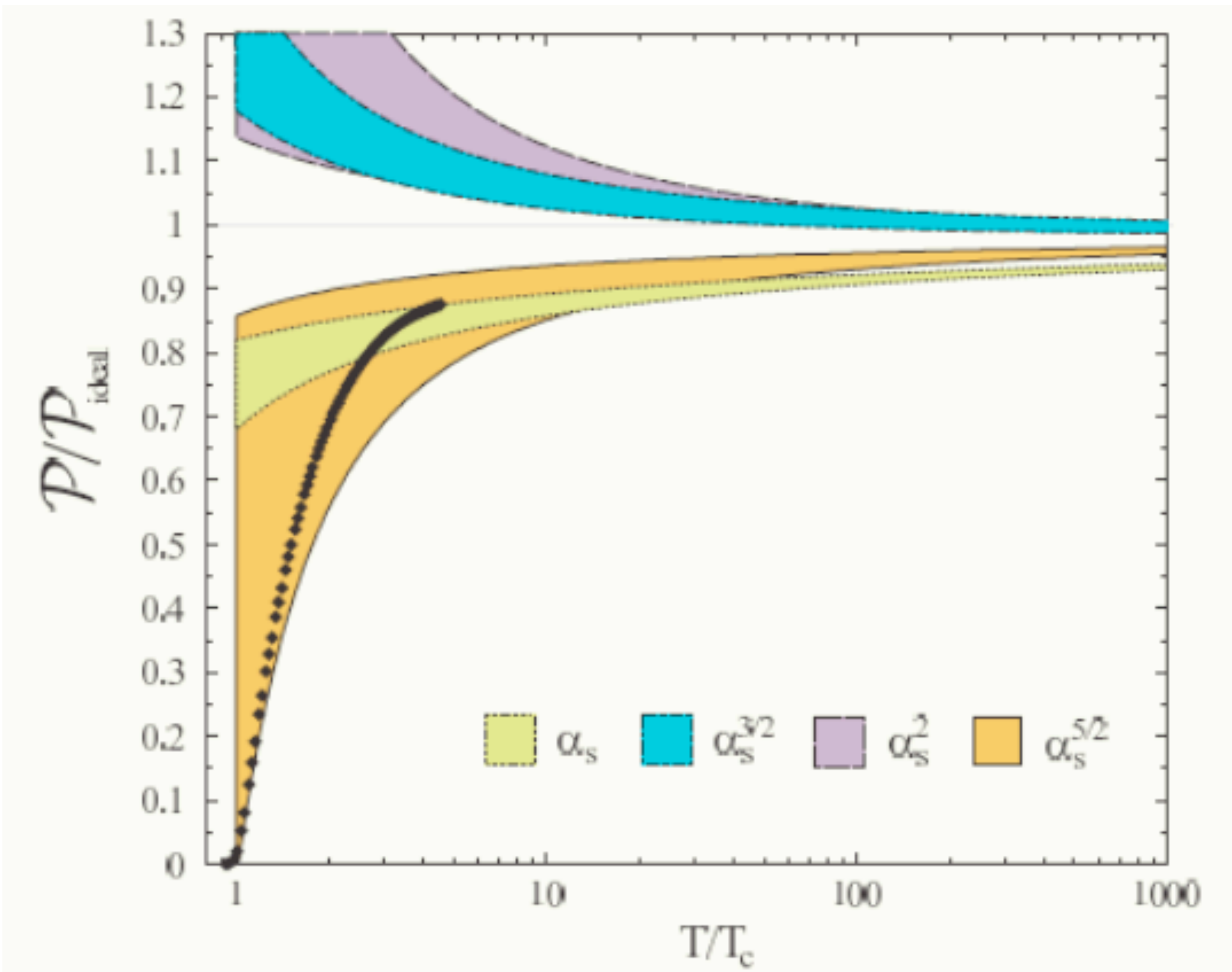


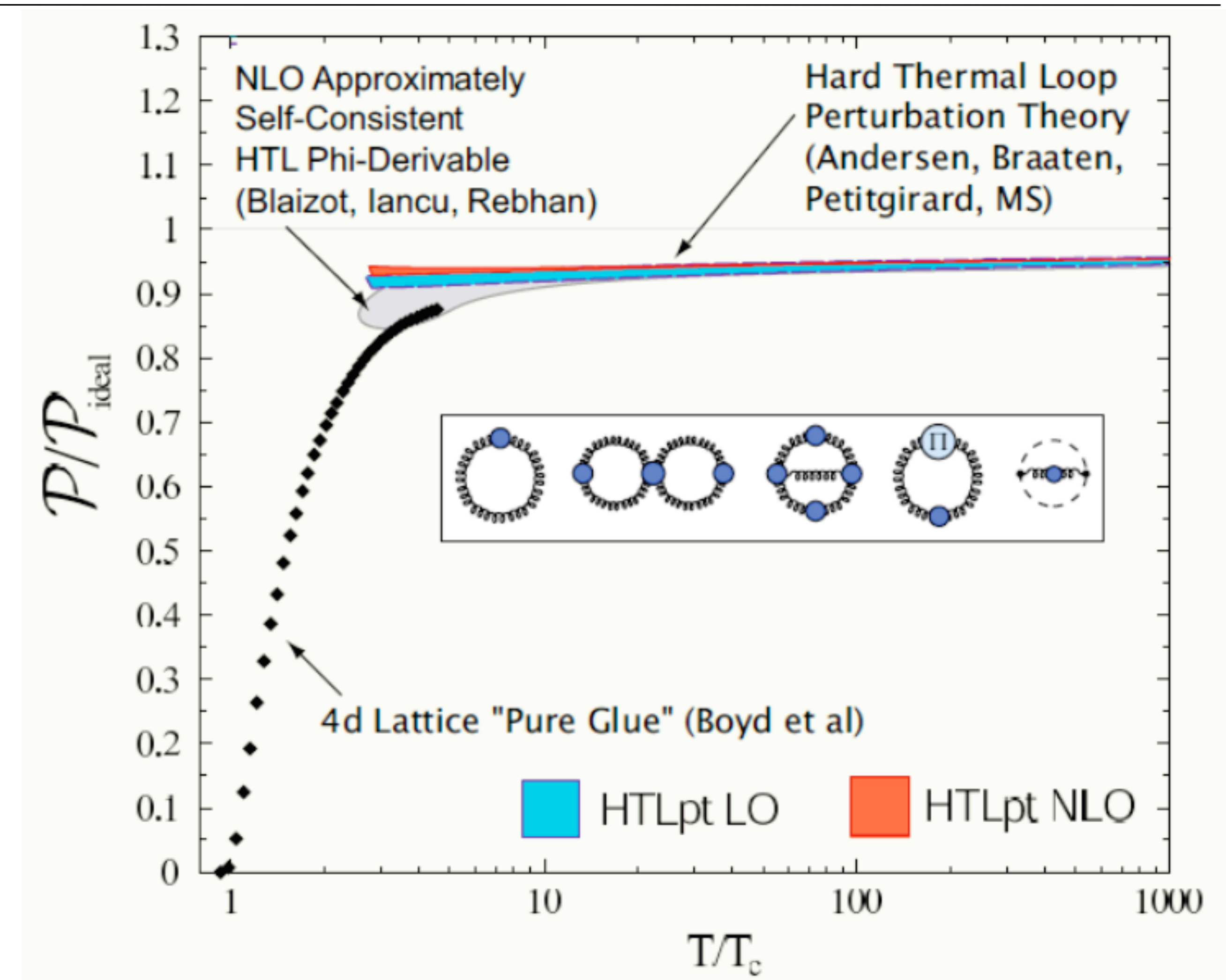
flow of gluon propagator



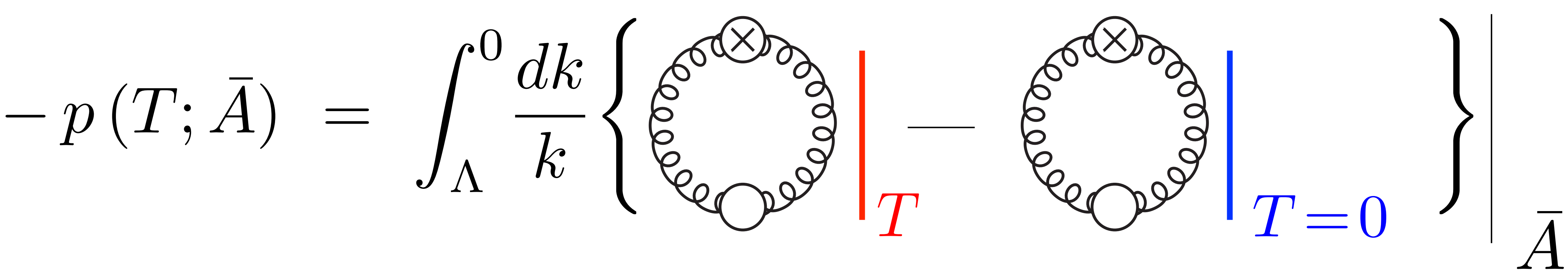
Naturally encorporates PQM/PNJL models as specific low order trunations

Confinement & Thermodynamics





Strickland



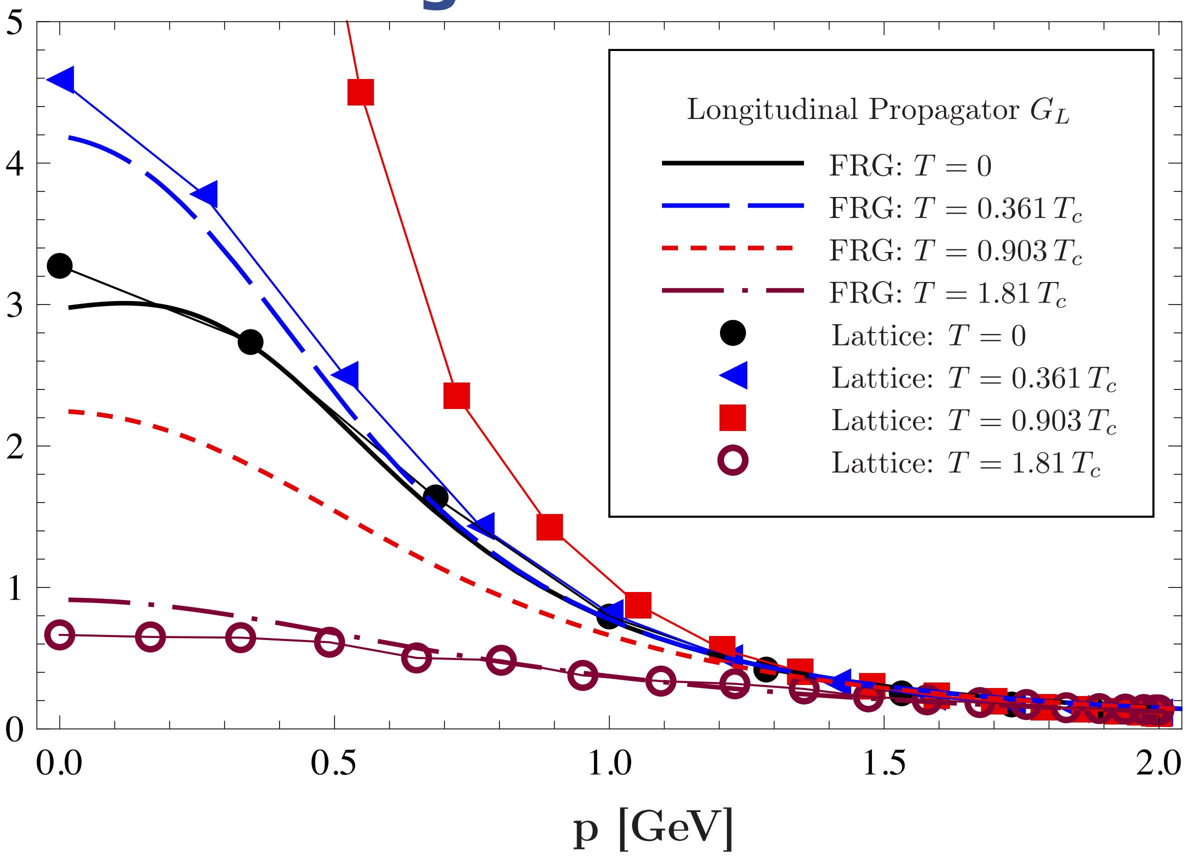
Fister, JMP

Confinement

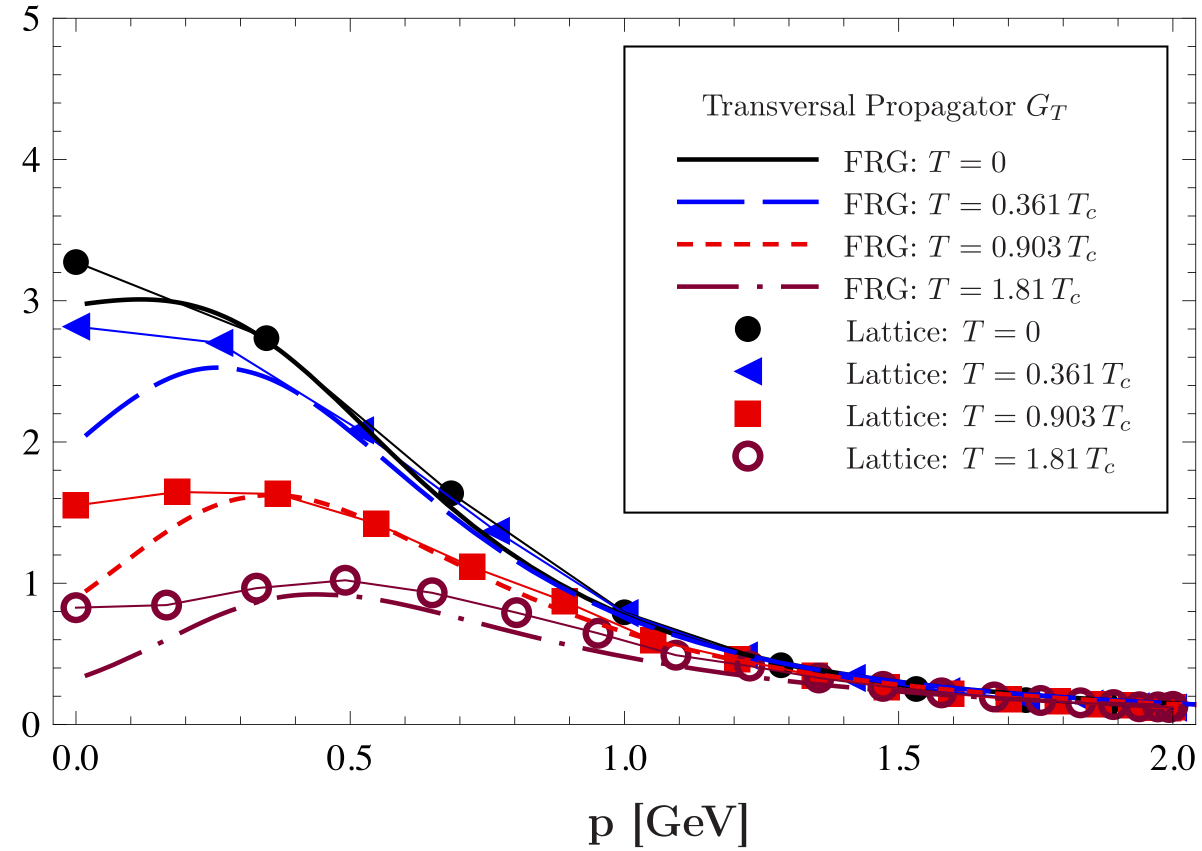
thermal gluon propagators

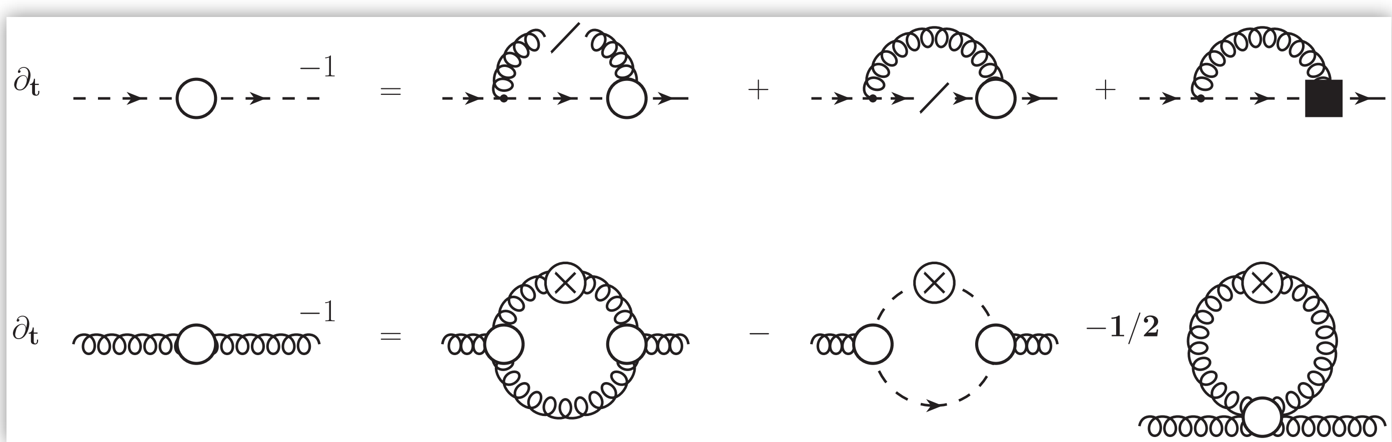
Fister, JMP '11

longitudinal

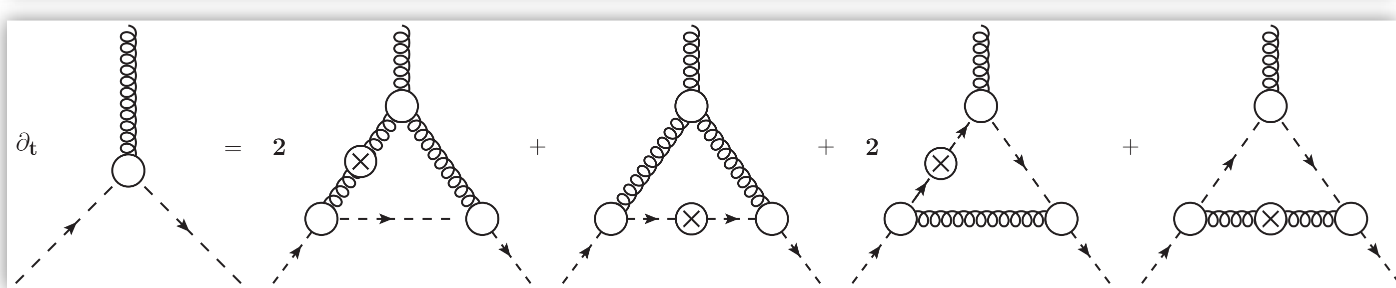


transversal





Lattice: Maas, JMP, Spielmann, von Smekal '11

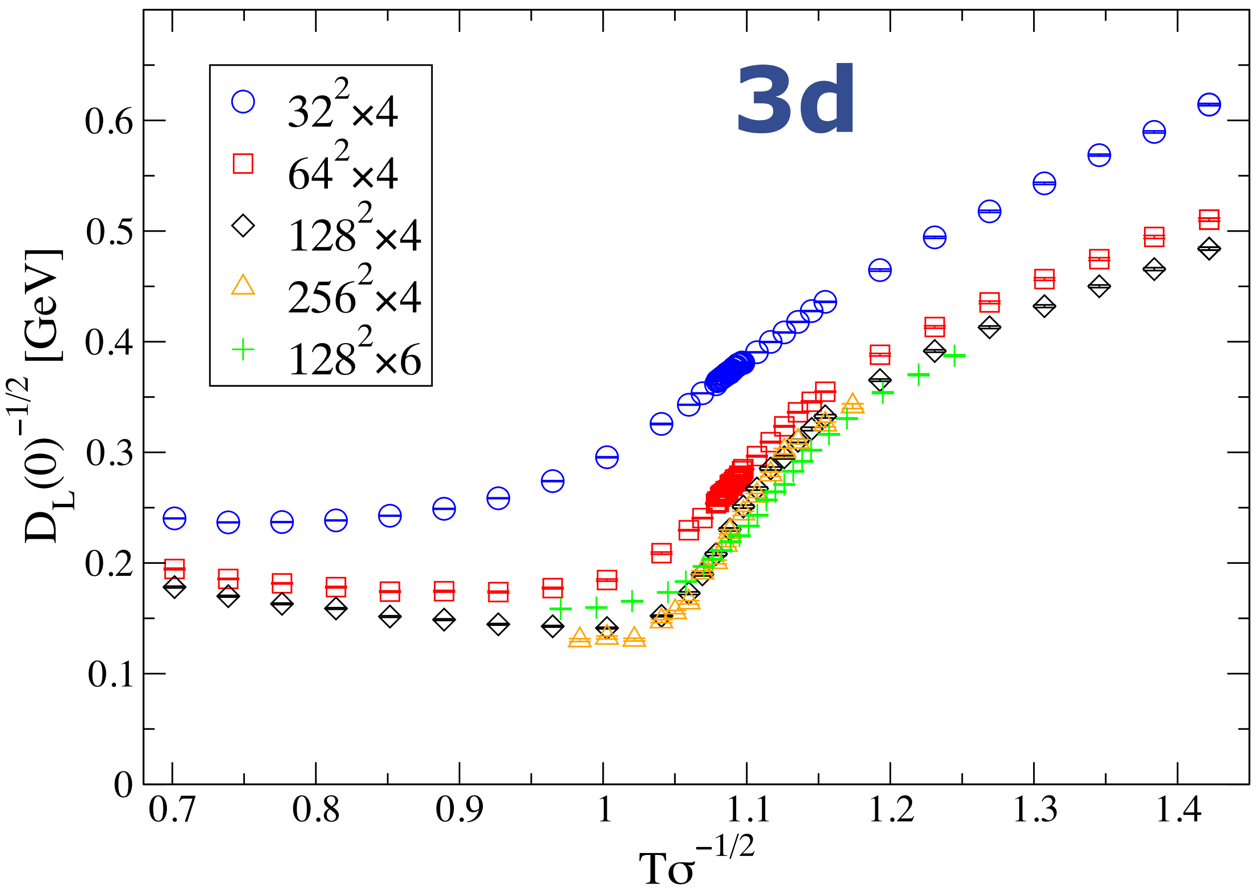


+ dressed gluonic vertices

Confinement

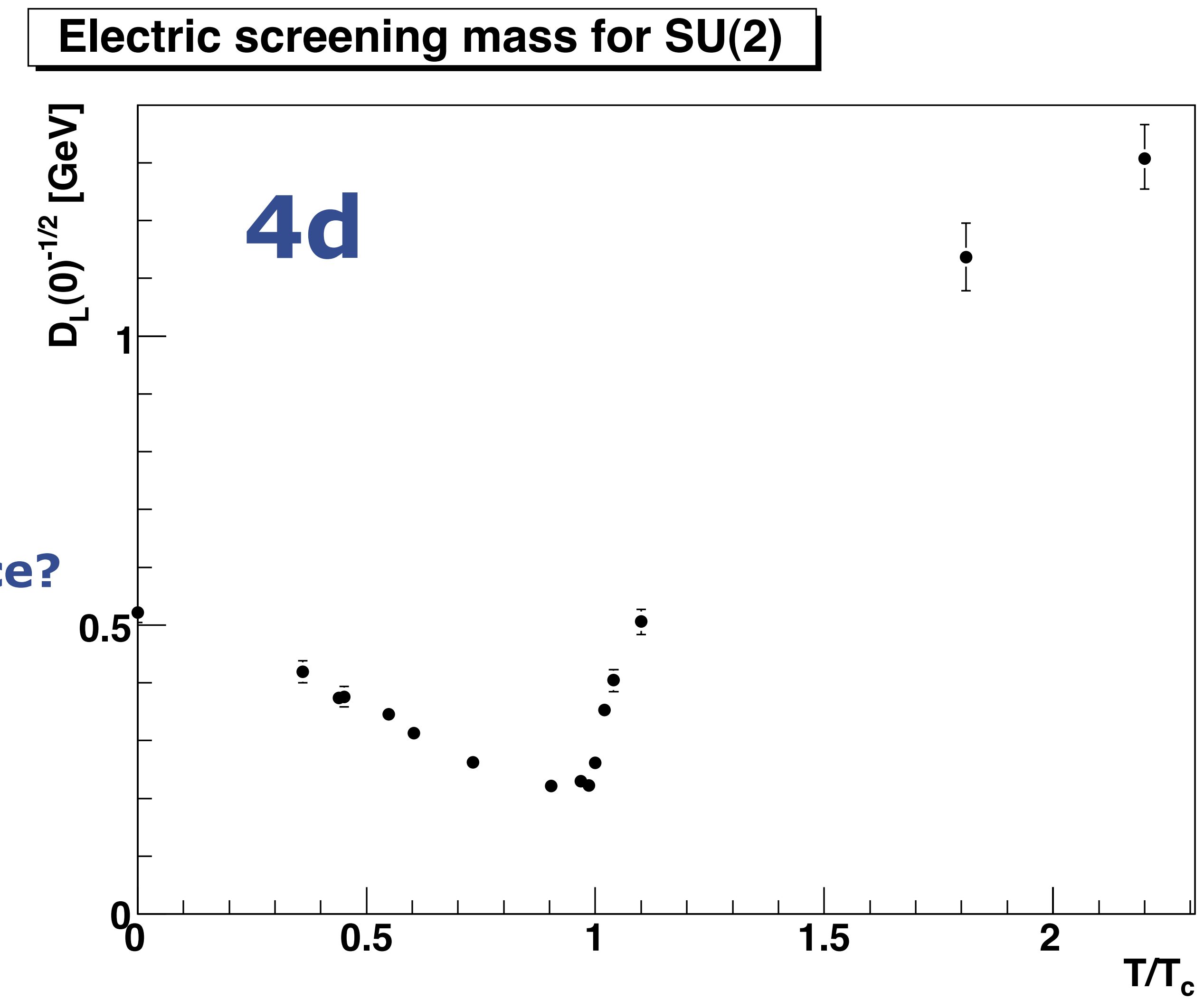
chromo-electric propagator

Maas, JMP, Spielmann, von Smekal ’11



$$\mathbf{D_L(0) = \langle A\,A \rangle_T(0)}$$

$\nu \approx 1$



critical scaling in Landau gauge props on the lattice?

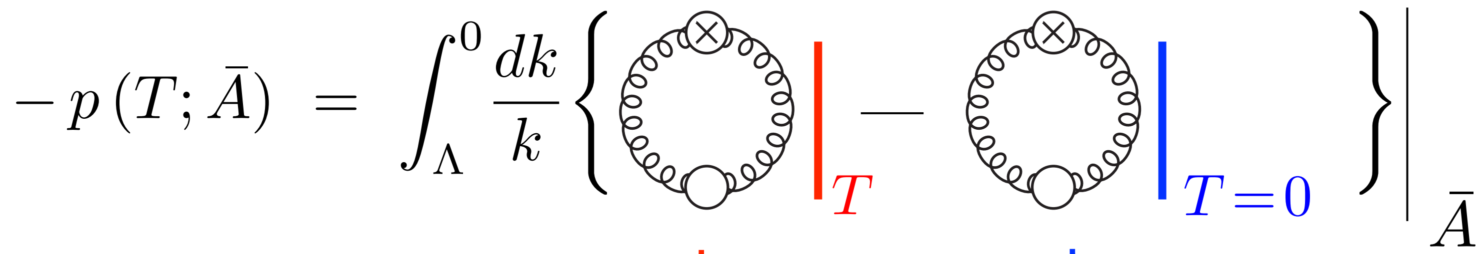
$$\mathbf{D_L(0)^{-1/2} \propto |T - T_c|^{\nu} + \cdots}$$

critical scaling in Landau gauge props in FunMethods!

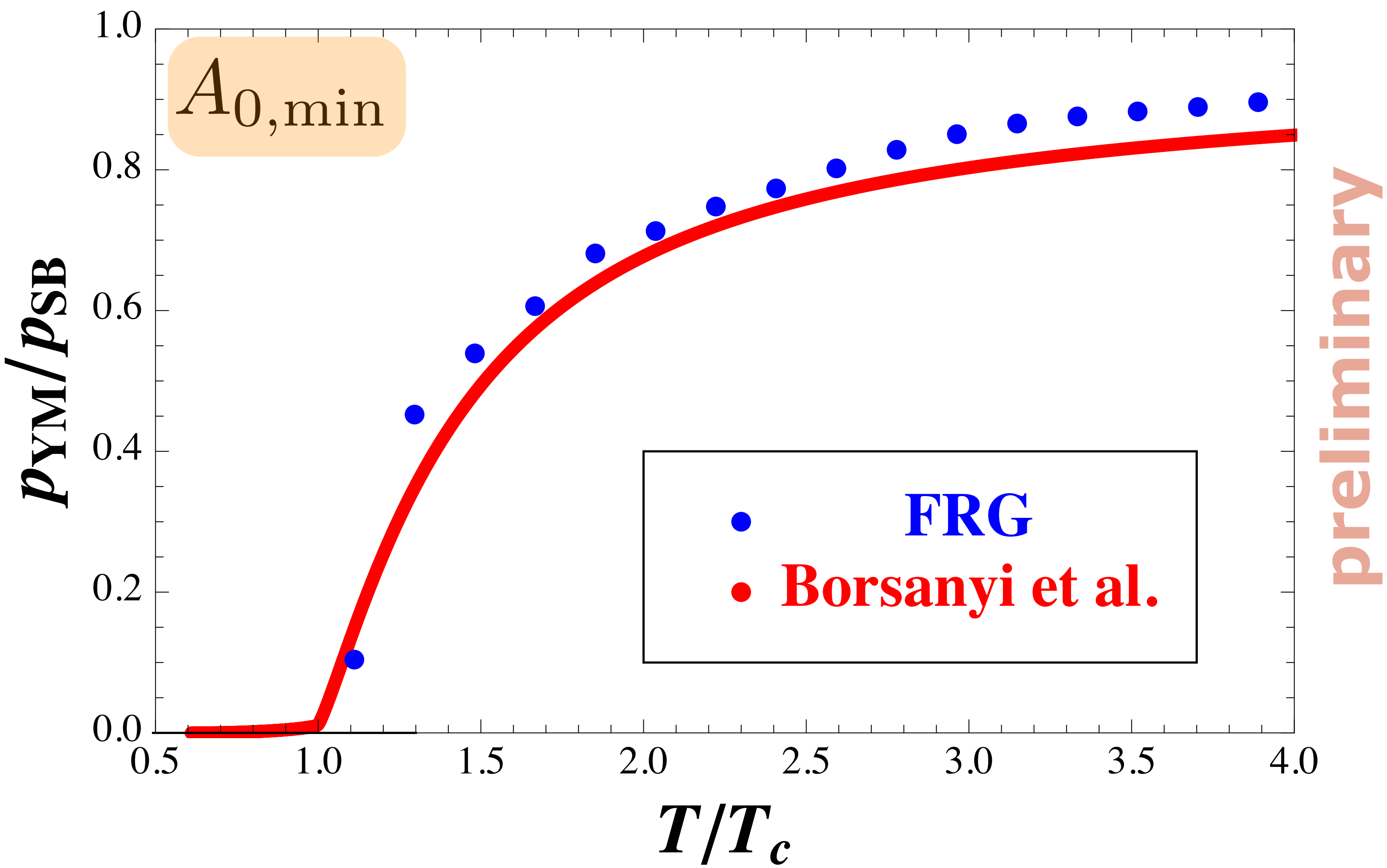
$\nu \approx 0.68$

Confinement & Thermodynamics

Fister, JMP '11



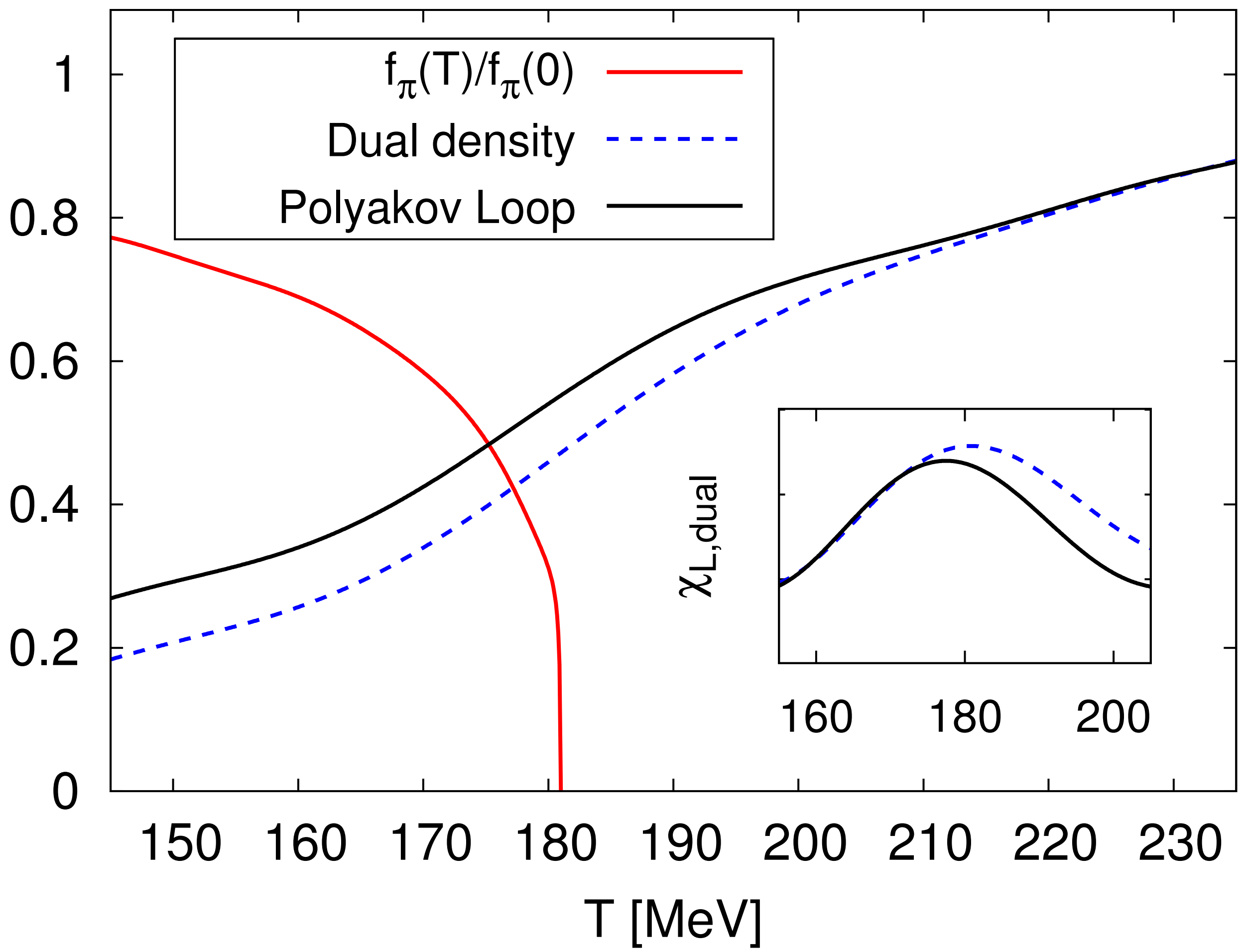
$$\sum\!\!\!\!\!\!\int_p G_{T,k}\ \partial_t R_k \qquad\qquad \int_p G_{T=0,k}\ \partial_t R_k$$



Full dynamical QCD: $N_f = 2$ & chiral limit

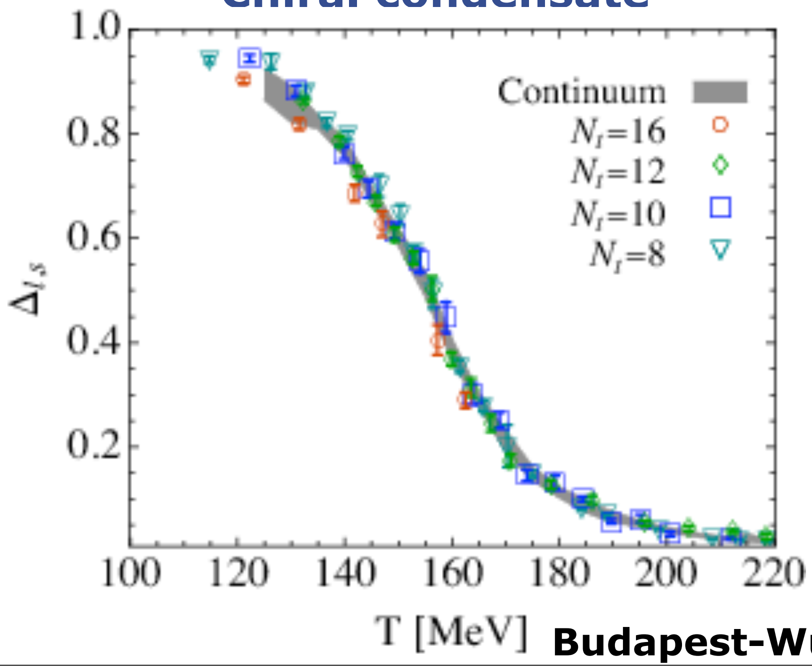
Phase structure

Braun, Haas, Marhauser, JMP '09



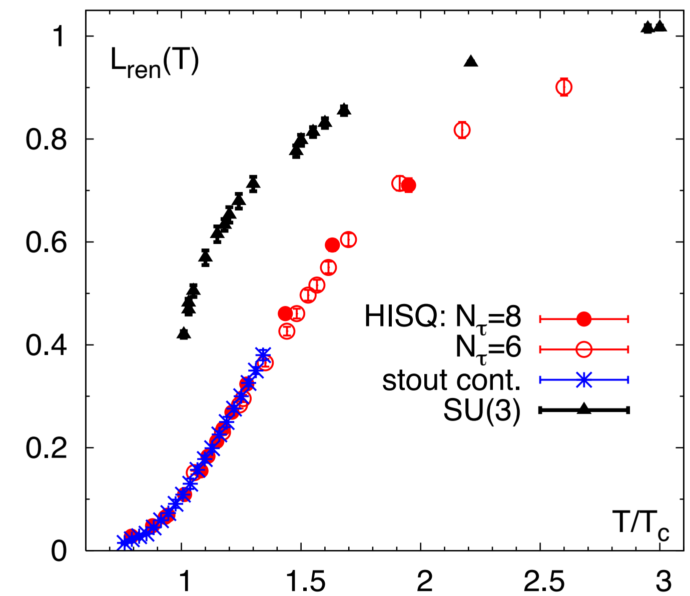
- $T_\chi \simeq T_{\rm conf} \simeq 180\mathrm{MeV}$
- **Width** $\Delta T_{\rm conf} \simeq \pm 20\mathrm{MeV}$

Chiral condensate



Budapest-Wuppertal '10

Polyakov loop



hotQCD '10

Transport in QCD

Shear viscosity & Kubo relations

Energy-momentum tensor

$$\pi_{ij} = F^a{}_i{}^\mu \, F^a{}_{j\mu} - \frac{1}{3}\delta_{ij} F^{a\,\mu k} \, F^a{}_{\mu k}$$ **+ matter**

Shear viscosity

$$\eta = \frac{1}{20} \, \frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$ **Kubo relation**

$$\rho_{\pi\pi}(\omega, \vec{k}) = -2 \, \mathrm{Im} \, G_{\pi\pi,R}(\omega, \vec{k})$$

Transport in QCD

Shear viscosity & Kubo relations

Energy-momentum tensor

$$\pi_{ij} = F^a{}_i{}^\mu \, F^a{}_{j\mu} - \frac{1}{3}\delta_{ij} F^{a\,\mu k}\, F^a{}_{\mu k} \quad \textbf{+ matter}$$

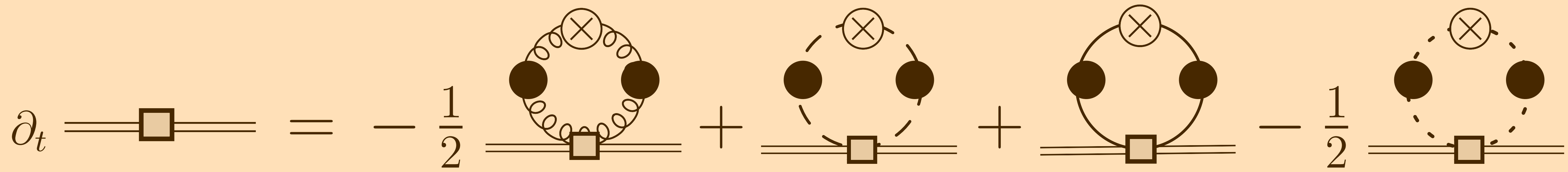
Shear viscosity

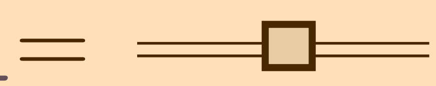
$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$ **Kubo relation**

$$G_{\pi\pi,R}(\omega,\vec{k}) = -i \int dt \int d^3x \, e^{i(\omega t - \vec{k}\vec{x})} \theta(t) \langle [\pi_{ij}(x)\,,\, \pi_{ij}(0)] \rangle$$

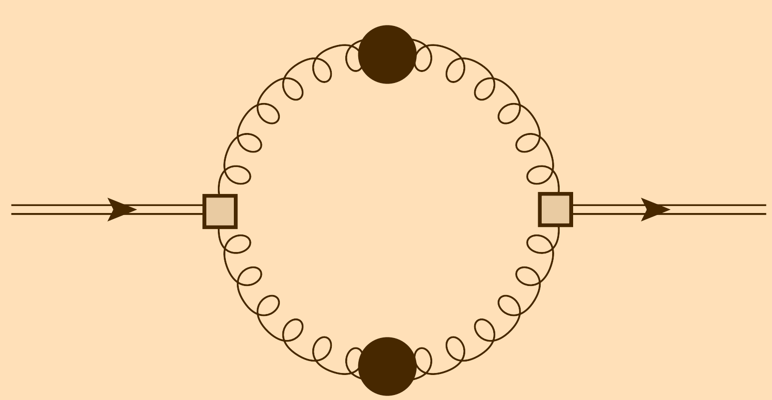
Transport in QCD

flow of $\rho_{\pi\pi}$





current approximation

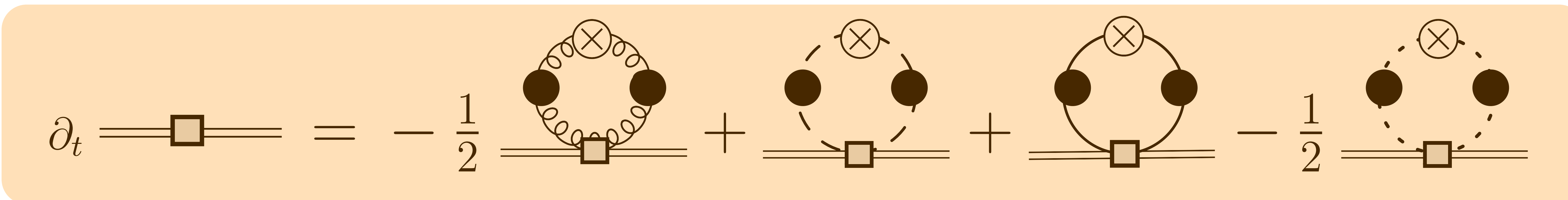


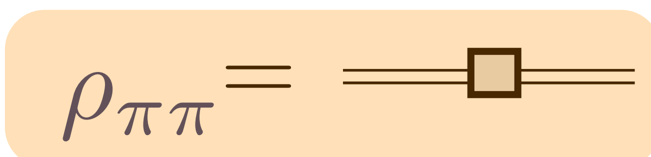
$\rho_{T/L}$ **with MEM**

$$\rho_{\pi\pi}(p) = \frac{2}{3}(N_c^2 - 1)\int \frac{d^4k}{(2\pi)^4}\,[n(k_0) - n(k_0+p_0)]\,(V_{TT}(k)\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L$$

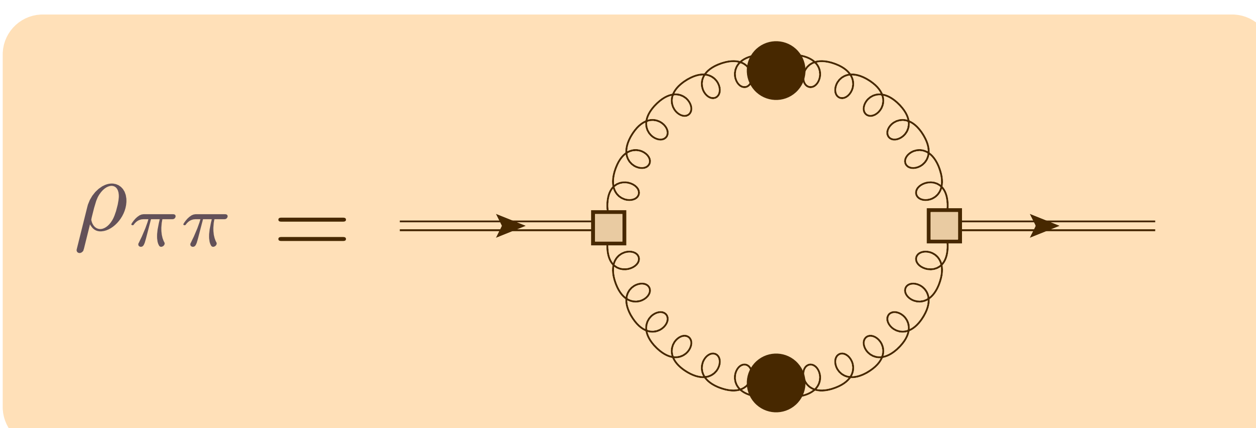
Transport in QCD

flow of $\rho_{\pi\pi}$





current approximation



'Those are my methods (principles), and if you don't like them...well, I have others'

Groucho Marx

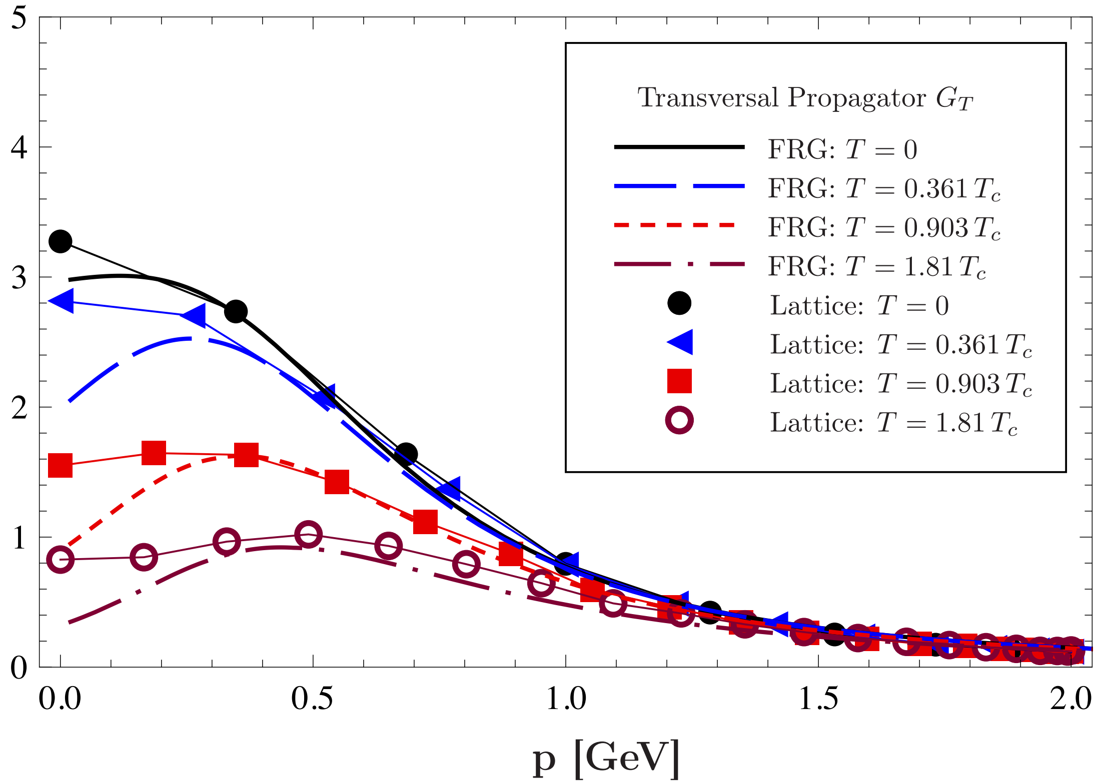
$\rho_{T/L}$ **with MEM**

$$\rho_{\pi\pi}(p) = \frac{2}{3}(N_c^2 - 1)\int \frac{d^4k}{(2\pi)^4}\,\left[n(k_0) - n(k_0+p_0)\right]\left(V_{TT}(k)\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L\right.$$

Viscosity in YM

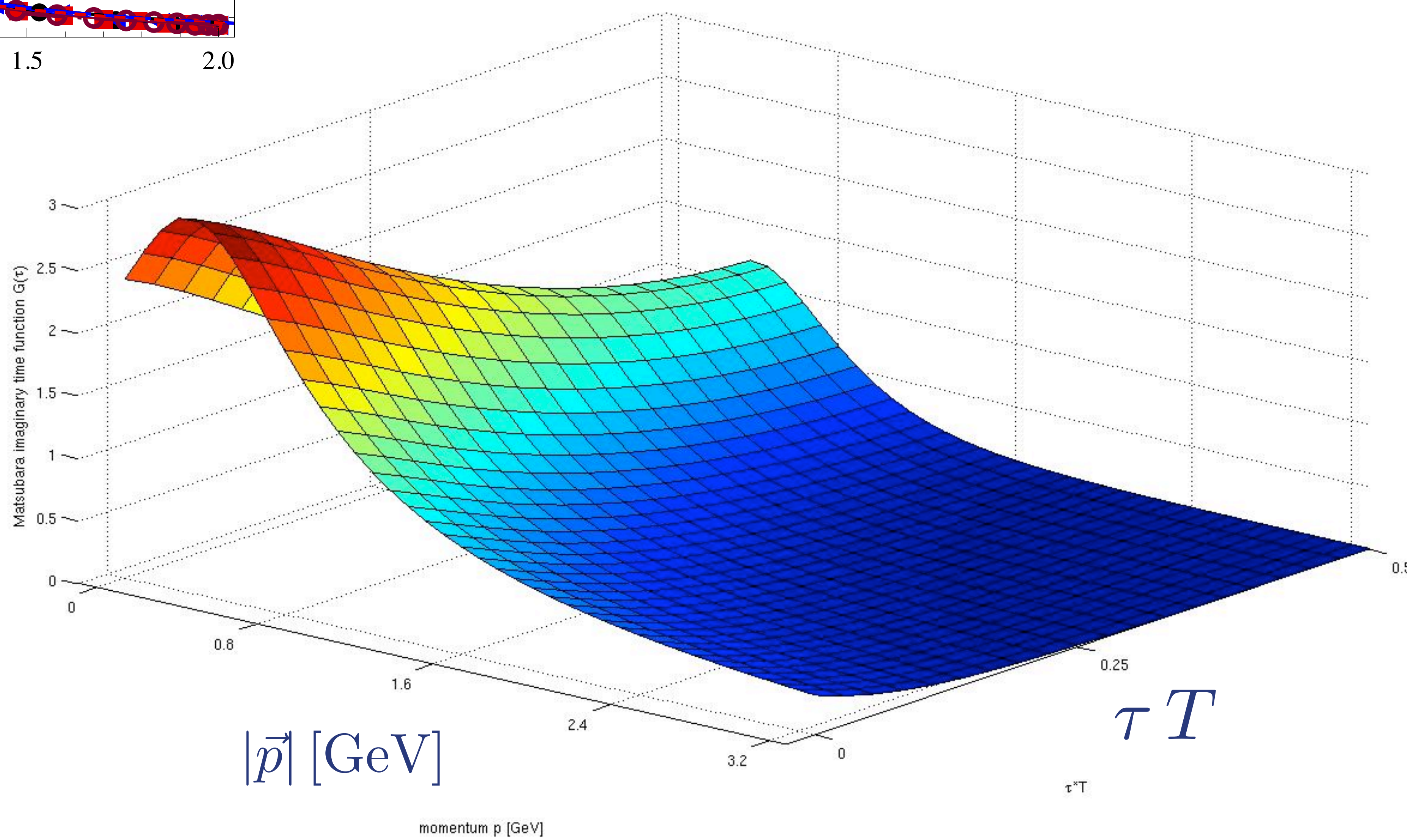
imaginary time correlations

M. Haas, JMP, in prep.



transversal gluon propagator

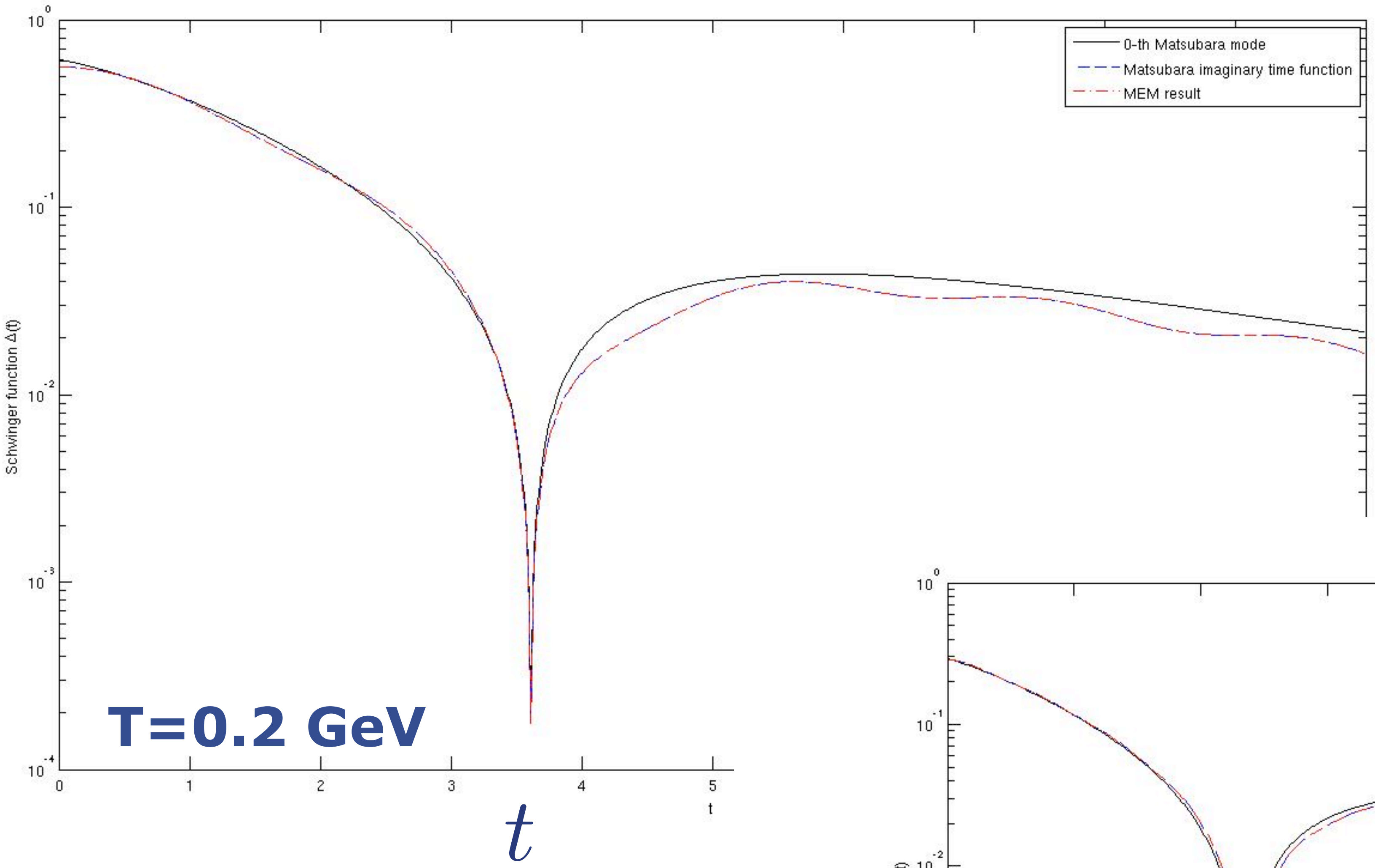
$G_T(\tau, \vec{p})$

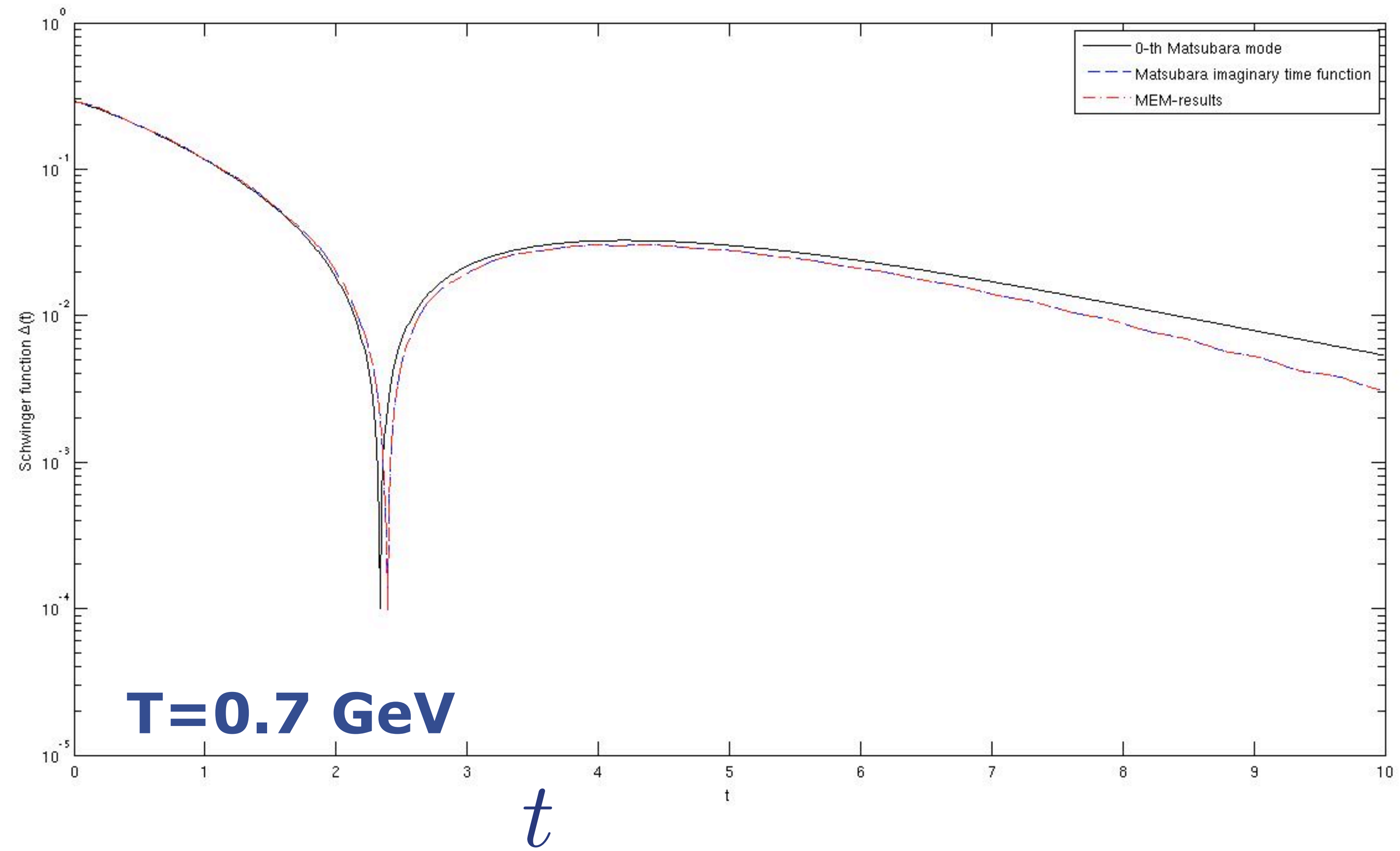


Viscosity in YM

tranversal Schwinger functions

M. Haas, JMP, in prep.





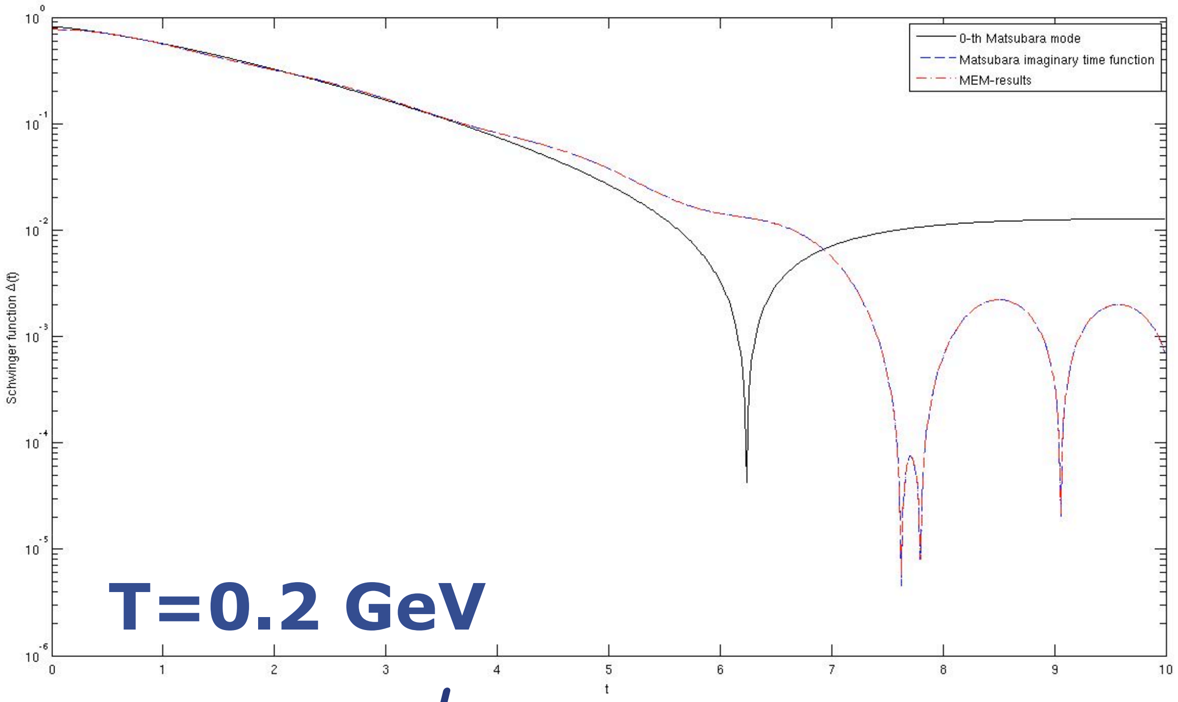
$$\Delta(t) = \frac{1}{\pi}\int_0^\infty d\omega\ \cos(t\omega) G(\omega, \vec{p}=0)$$

Schwinger function

Viscosity in YM

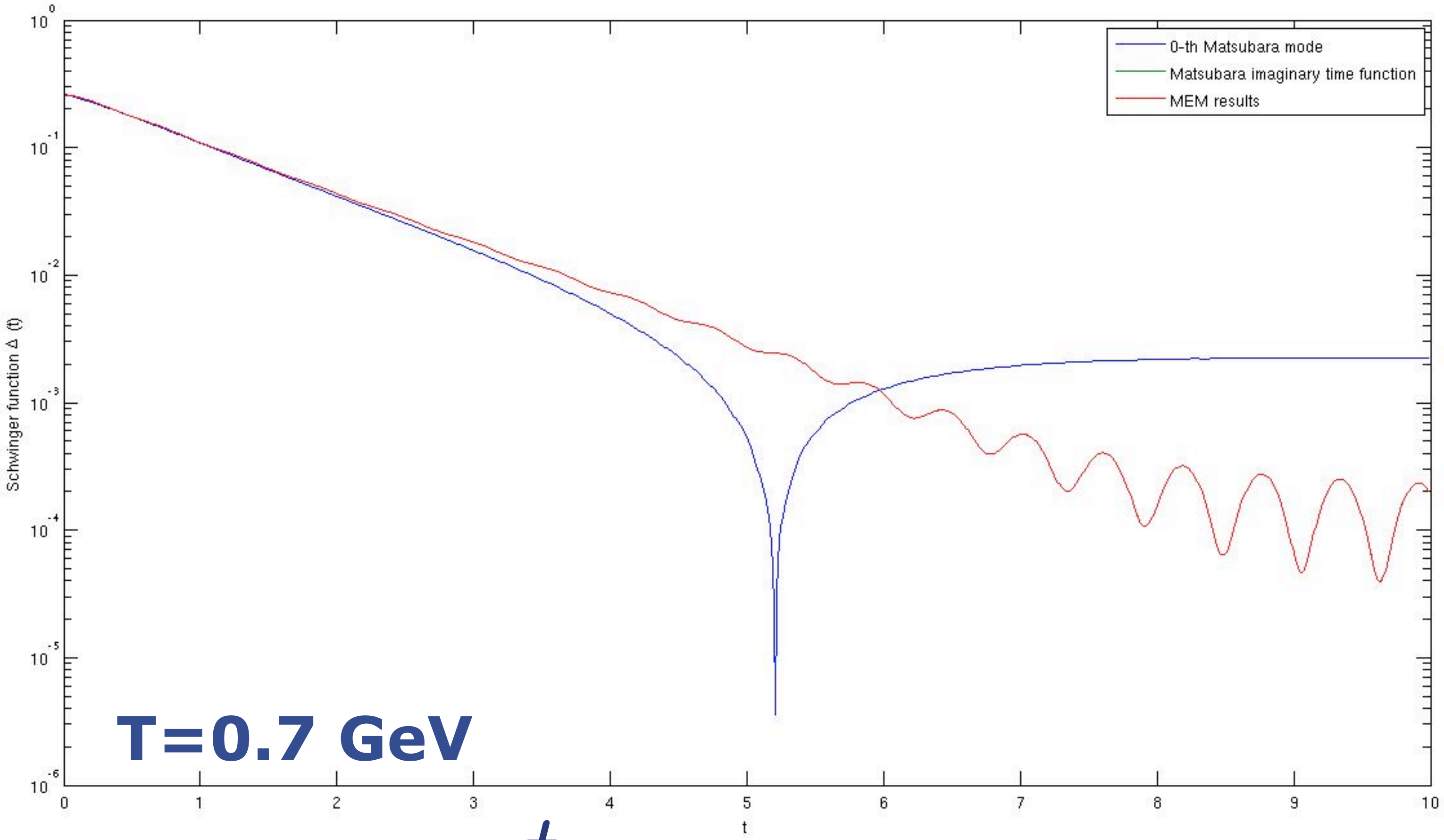
longitudinal Schwinger functions

M. Haas, JMP, in prep.



T=0.2 GeV

t



T=0.7 GeV

t

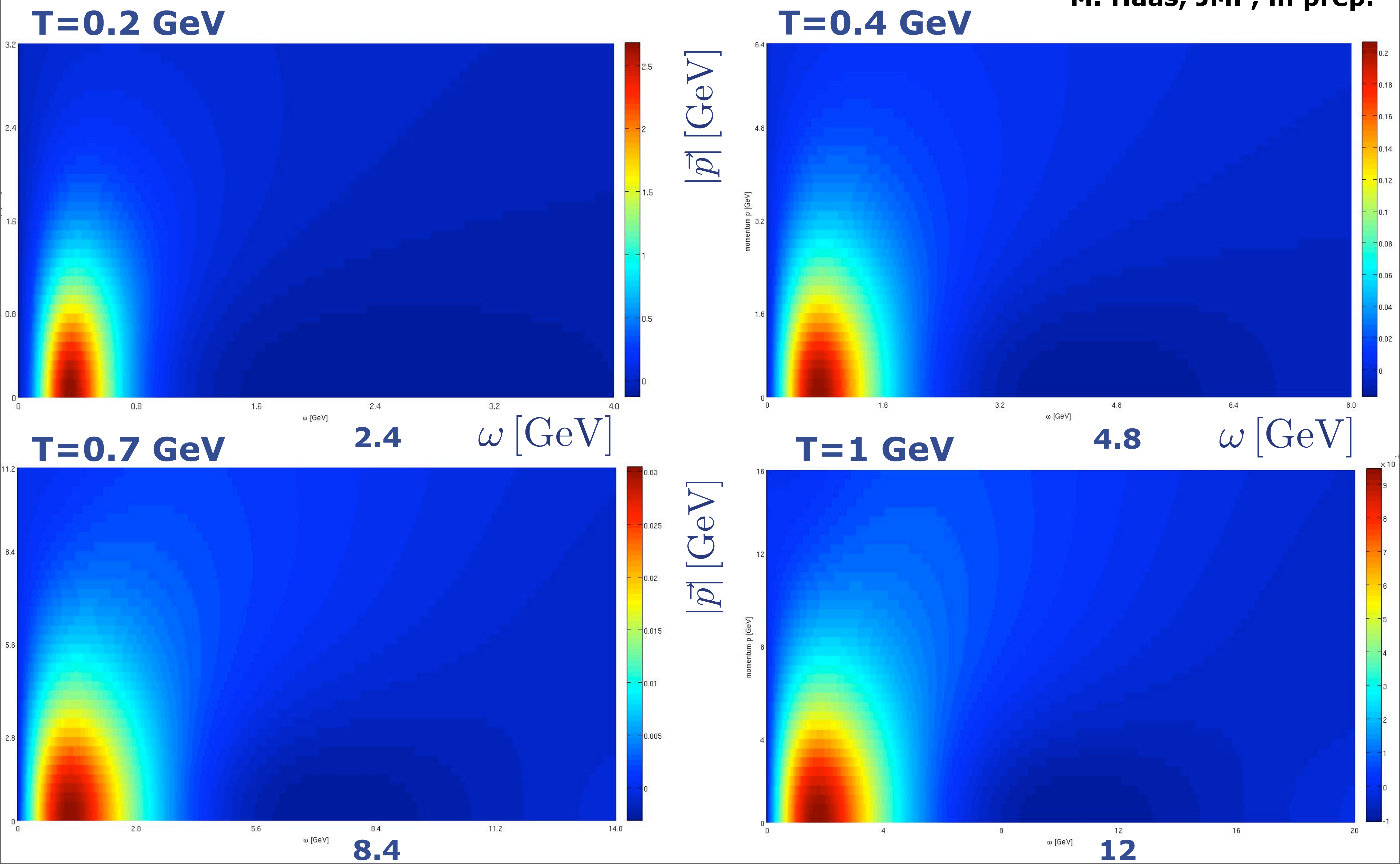
high temperature limit DSE

Maas, Wambach, Grüter, Alkofer ’04

Viscosity in YM

transversal spectral functions

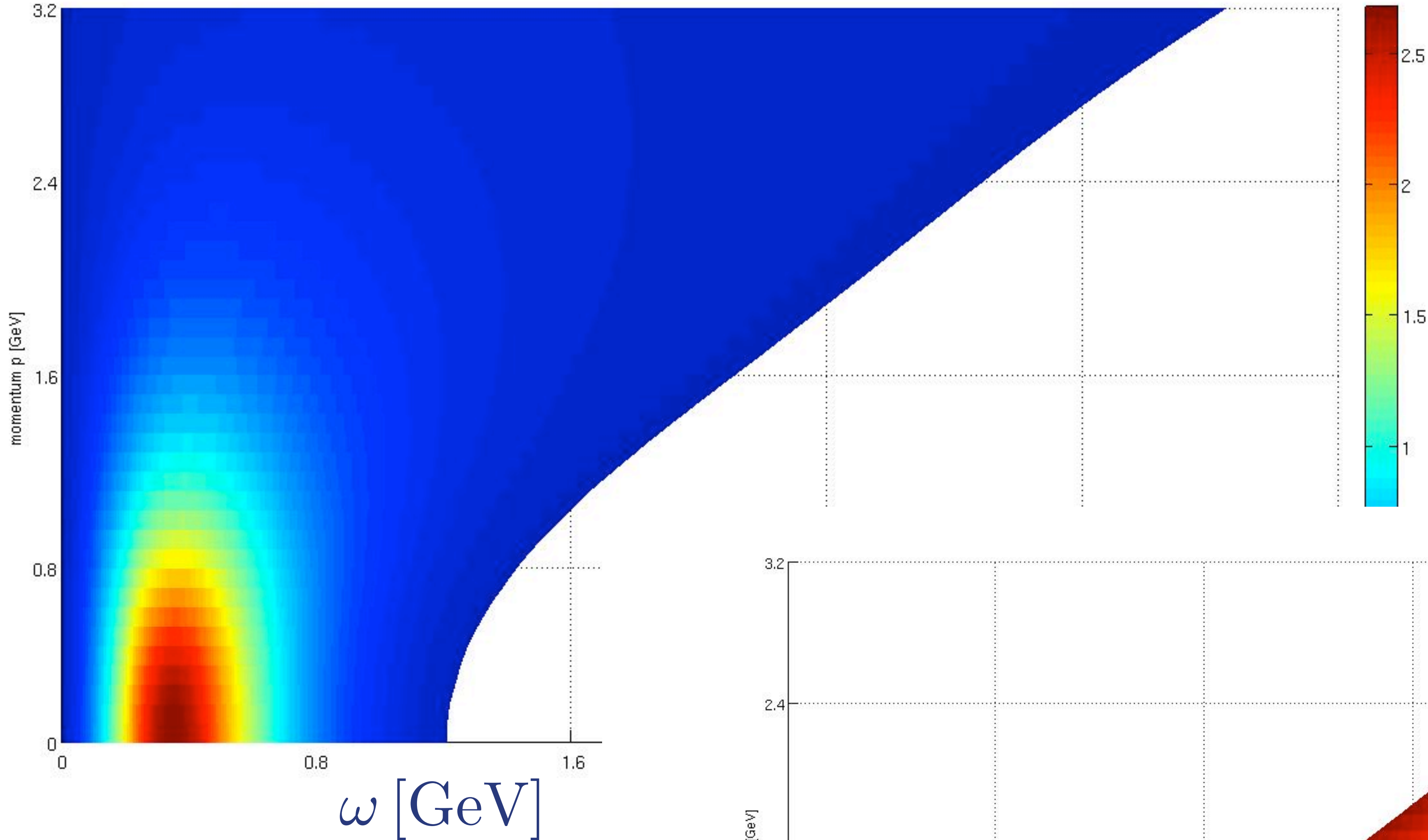
M. Haas, JMP, in prep.



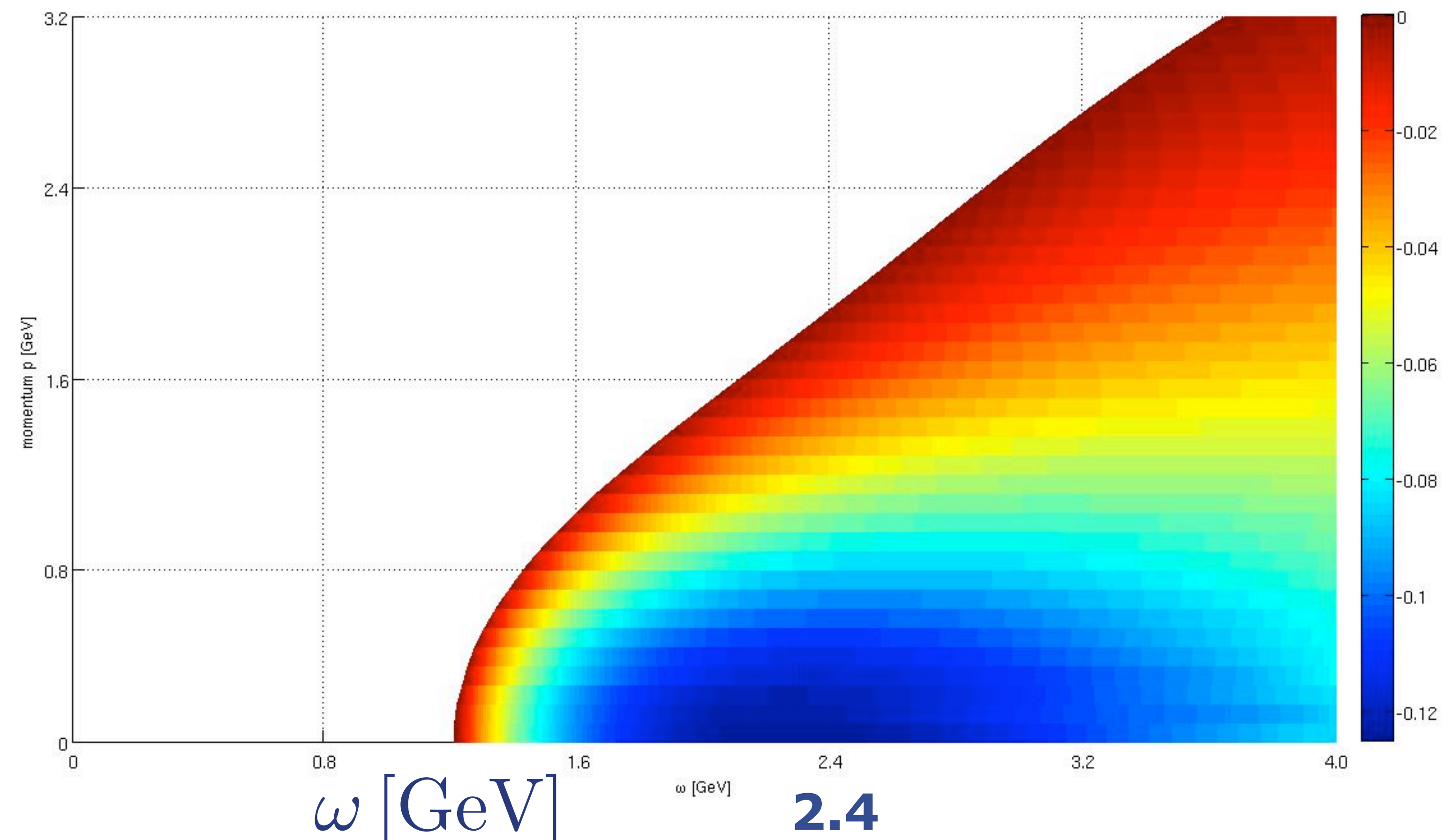
Viscosity in YM

transversal spectral functions

M. Haas, JMP, in prep.



T=0.2 GeV



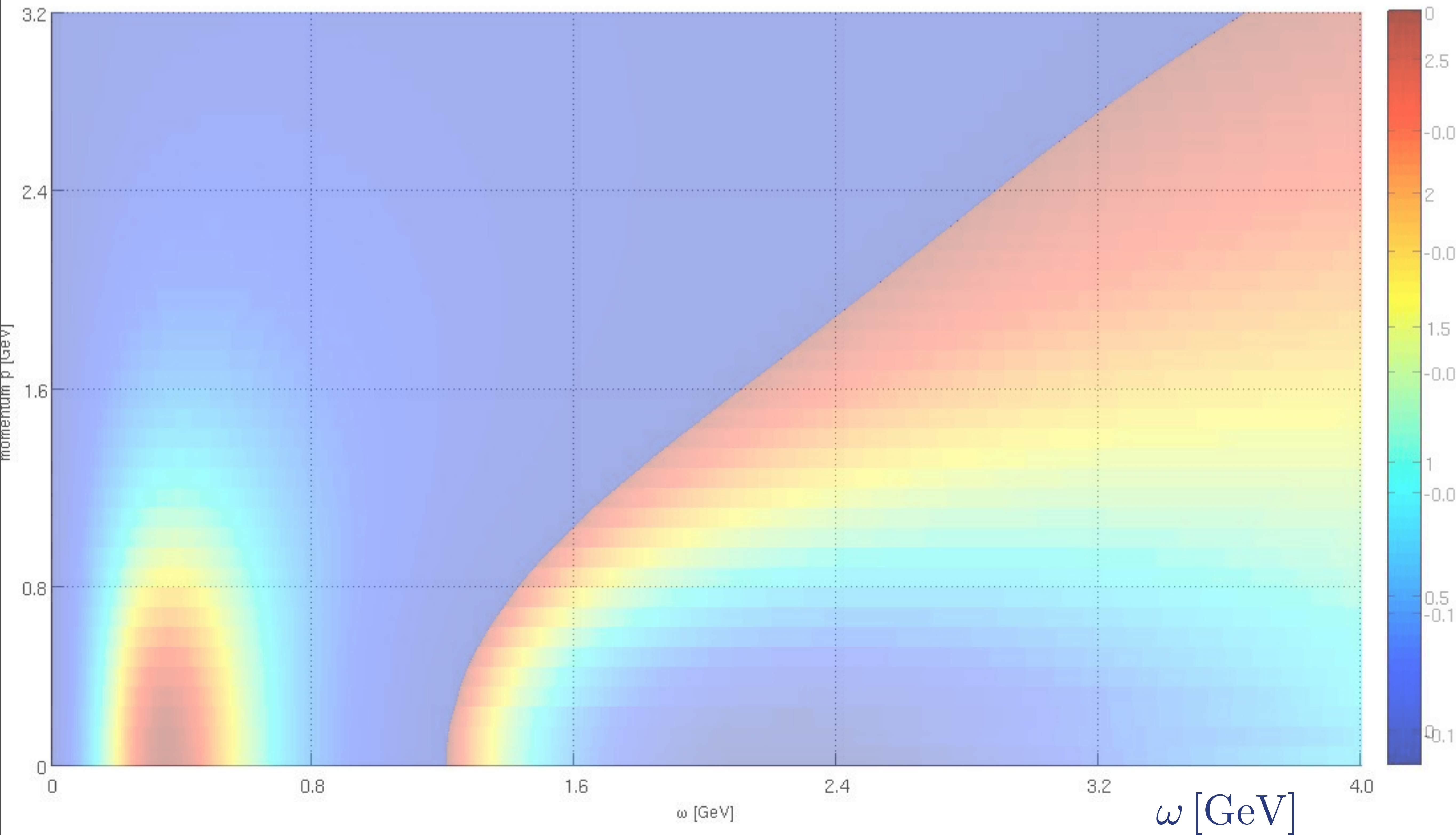
2.4

Viscosity in YM

transversal spectral functions

M. Haas, JMP, in prep.

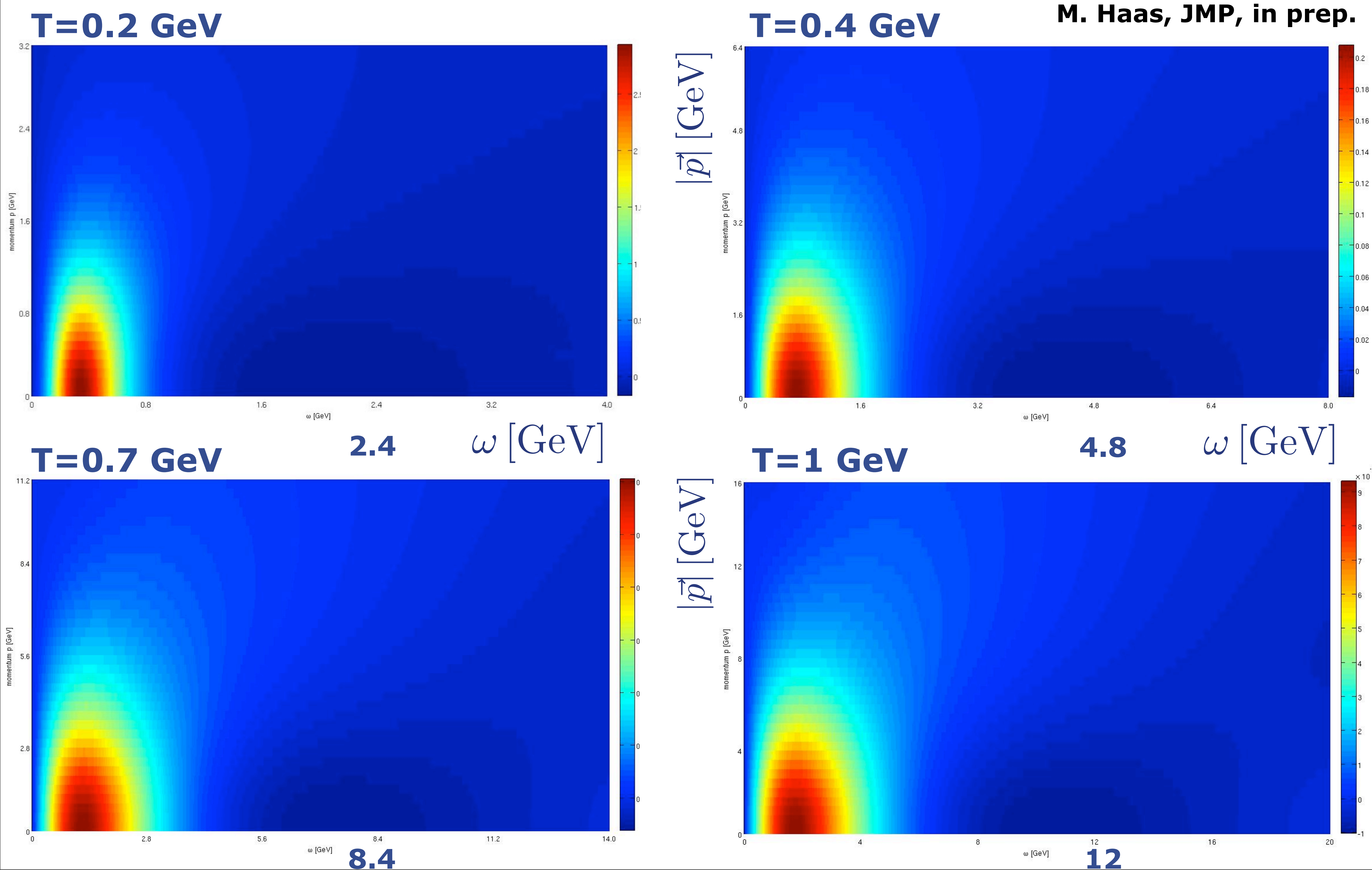
T=0.2 GeV



Viscosity in YM

longitudinal spectral functions

M. Haas, JMP, in prep.

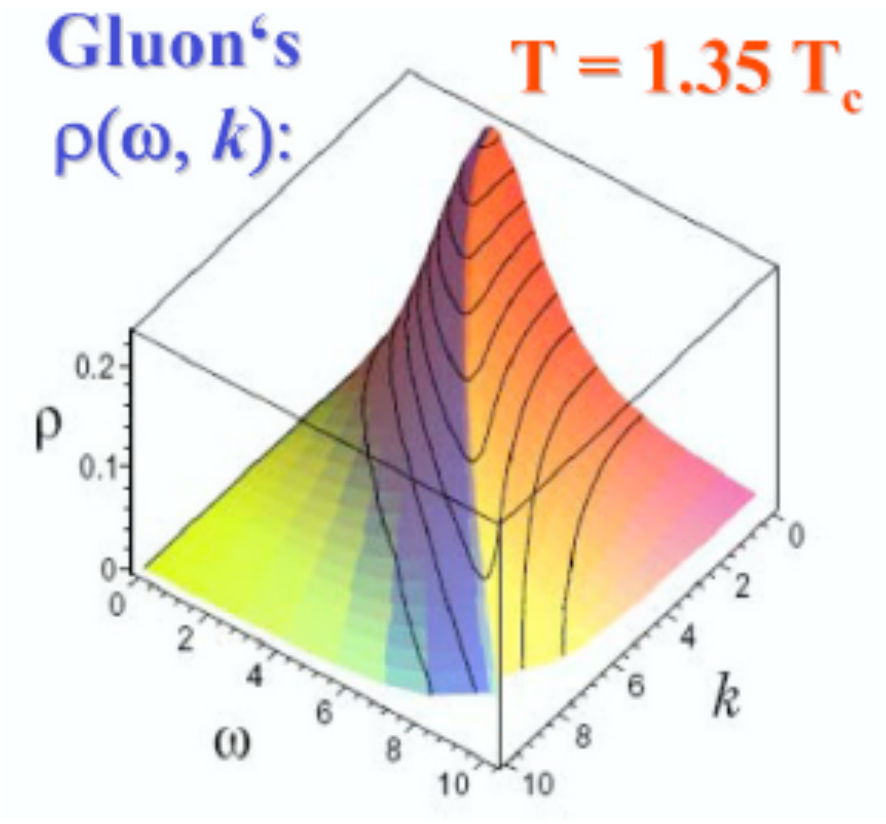


Viscosity in YM

spectral functions

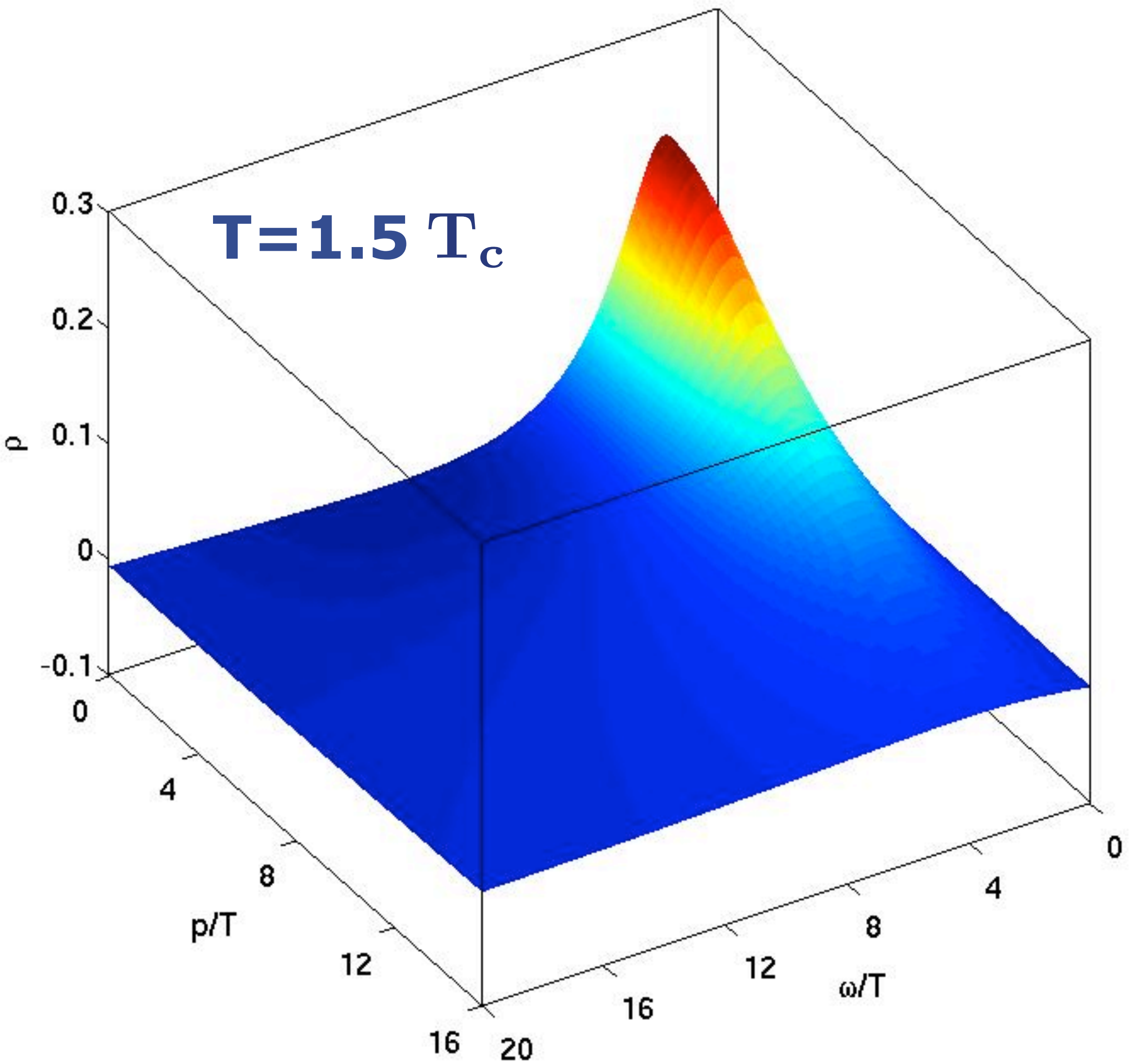
M. Haas, JMP, in prep.

➔ Broad spectral function :



E. Bratkovskaya, talk at RETUNE '12

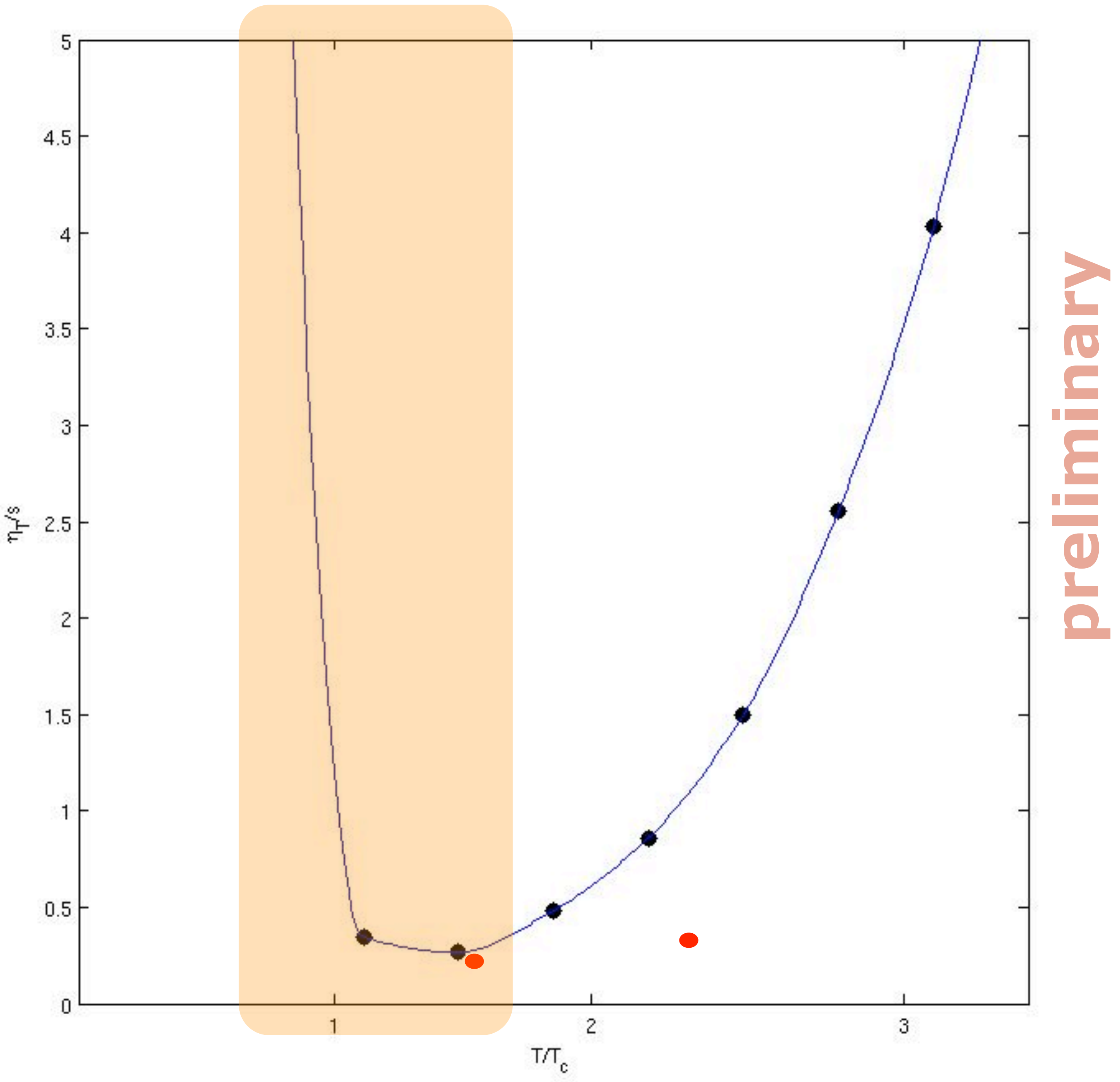
transversal spectral function



Viscosity in YM

shear viscosity

M. Haas, JMP, in prep.

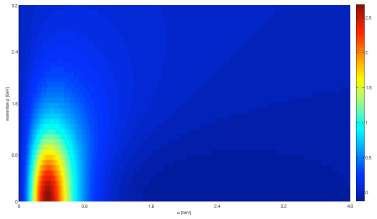


● **lattice: H.B. Meyer '09**

	$1.58T_c$	$2.32T_c$	free gluons	$\lambda = \infty$ SYM
$(\eta + \frac{4}{3}\zeta)/s$	0.20(3)	0.26(3)	∞	$\frac{1}{4\pi} \approx 0.080$
$2\pi T \tau_\Pi$	3.1(3)	3.2(3)	∞	$2 - \log 2 \approx 1.31$
$(\eta + \frac{4}{3}\zeta)/(T\tau_\Pi s)$	0.40(5)	0.51(5)	0.17	0.38

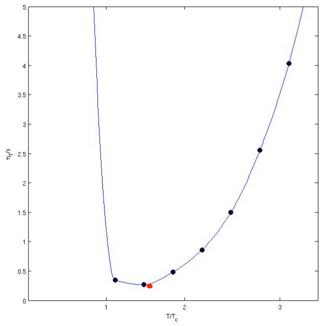
Summary & outlook

- **Real time correlation functions**
  - **spectral functions in YM**



  - **spectral functions in QCD**

- **Transport coefficients in QCD**



  - **viscosity over entropy in YM**
  - **viscosity over entropy in QCD**

- **Towards quantitative reliability**

Additional material

Transport in QCD

Conversation laws

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu \left(n\, u^\mu\right) = 0 \qquad \partial_\mu \left(s\, u^\mu\right) = 0$$

Equation of state

$$P = P(\epsilon, n)$$

Dissipation

$$\delta T^{\mu\nu} = -\eta\, \sigma^{\mu\nu} - \zeta \Delta^{\mu\nu} \partial u$$

$$\sigma^{\mu\nu} = \Delta^{\mu\alpha} \delta^{\nu\beta} \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} \eta_{\alpha\beta} \partial u \right)$$

MEM

- Simultaneous optimisation of the spectral function to preknowledge about its shape and to the Matsubara correlator.
- Likelihood given by:

$$L = \int d\tau \, (G_E(\tau) - G_\rho(\tau))^2$$

- Extensions of Maximum Likelihood Method by adding an entropy term of Shannon-Jaynes type:

$$S = \int d\omega \, [\rho(\omega) - m(\omega) - \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)}]$$

- Minimisation of $Q = L - \alpha S$ with a weight α

MEM Input

- Pure Yang-Mills gauge theory at finite temperature T

- Gluon propagator
  - Transversal part: $D_L(\omega_n = 0, q^2)$
  - Longitudinal part: $D_T(\omega_n = 0, q^2)$

- FRG results for the 0-th Matsubara mode of $D_L(0, q^2)$ and $D_T(0, q^2)$.