Collisional energy loss of heavy quarks

by Alex Meistrenko
with A. Peshier (Cape Town), J. Uphoff (Frankfurt) and C. Greiner (Frankfurt)
reference: arXiv:1204.2397v1 (04.2012)

NeD/TURIC 28.06.2012

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

H-QM Helmholtz Research School Quark Matter Studies

HIC for FAIR Helmholtz International Center

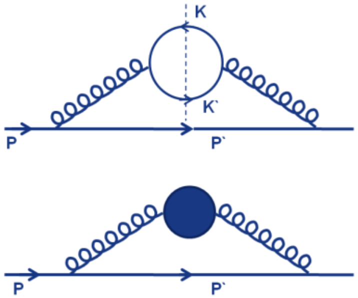
Elastic energy loss model

- elastic dE/dx of heavy quarks in the case of a non-static thermalized QGP
- pQCD transition matrix approach with
    - quantum distribution functions
    - running coupling
    - effective screening mass adjusted to HTL calculations
    - Peterson fragmentation and meson decay with Pythia
- comparison with RHIC data for electorn yields from heavy flavor decays

Mean energy loss in the QGP

interaction rate: Σ is evaluated at the energy $p_0 = E + i\epsilon$:

$$\Gamma_i(E) = -\frac{1}{2E}\left(1 - n_F(E)\right)\mathrm{tr}\left[\left(\gamma^\mu P_\mu + M\right)\mathrm{Im}\Sigma(P)\right], \quad \frac{dE_i}{dx} = \frac{1}{v}\int d\omega\, \frac{\partial \Gamma_i}{\partial \omega}\omega$$



contribution from $|t| < |t^*|$ with $m_D^2 \ll |t^*| \ll T^2$:

$$\frac{dE_i}{dx} = \frac{K(C_F,\alpha)}{v^2}\int_{t^*}^{0} dt\,(-t)\int_{-v}^{v} dx \frac{x}{\left(1-x^2\right)^2}\left[\rho_L + \left(v^2 - x^2\right)\rho_T\right], \quad t = \omega^2 - q^2, \quad x = \omega/q$$

spectral functions and HTL propagators:

$$\rho_{L,T}(\omega, q) := -\frac{1}{\pi}\mathrm{Im}\left[\Delta_{L,T}(\omega + i\epsilon, q)\right], \quad \Delta_L(\omega, q) = \frac{1}{q^2 + \Pi_L(x)}, \quad \Delta_T(\omega, q) = \frac{1}{\omega^2 - q^2 - \Pi_T(x)}$$

with the self-energies:

$$\Pi_L(x) = m_D^2\left[1 - Q(x)\right], \quad \Pi_T(x) = \frac{m_D^2}{2}x\left(1 - x^2\right)Q'(x), \quad Q(x) := \frac{x}{2}\ln\frac{x+1}{x-1}$$

Mean energy loss in the QGP

contribution from $|t| > |t^*|$ with $\int_k := \int d^3k/(2\pi)^3$:

$$\frac{dE_i}{dx} = \frac{1}{2Ev}\int_k \frac{n_i(k)}{2k}\int_{k'}\frac{\overline{n}_i}{2k'}\int_{p'}\frac{1}{2E'}(2\pi)^4\delta^{(4)}\left(P+K-P'-K'\right)\frac{1}{d}\sum_{spin,color}\left|\mathcal{M}_i\right|^2\omega$$

in the limit $E \to \infty$ and $E >> T$:

$$\frac{dE_i}{dx} = d_i\int_k \frac{n_i(k)}{2k}\int_{t_{min}}^{t^*} dt(-t)\frac{d\sigma_i}{dt}$$

both contributions from $|t| < |t^*|$ and $|t| > |t^*|$ lead to the NLL formula:

$$\frac{dE}{dx} = \frac{4\pi\alpha_s^2T^2}{3}\left[\left(1+\frac{n_f}{6}\right)\ln\frac{ET}{m_D^2}+\frac{2}{9}\ln\frac{ET}{M^2}+c(n_f)\right] \quad \text{with} \quad c\left(n_f\right)\approx 0.146\cdot n_f+0.050$$

and for running coupling:

$$\frac{dE}{dx} = \frac{4\pi T^2}{3}\alpha_s(m_D^2)\alpha_s(ET)\left[\left(1+\frac{n_f}{6}\right)\ln\frac{ET}{m_D^2}+\frac{2}{9}\frac{\alpha_s(M^2)}{\alpha_s(m_D^2)}\ln\frac{ET}{M^2}+c(n_f)+\mathcal{O}\left(\alpha_s(m_D^2)\ln\frac{ET}{m_D^2}\right)\right]$$

Mean energy loss with QGP flow

replace Bose and Fermi by the Jüttner distribution functions:

$$n_{BJ} = \frac{1}{e^{\gamma\left(E_k - \vec{\beta}\cdot\vec{k}\right)/T} - 1}\,, \qquad n_{FJ} = \frac{1}{e^{\gamma\left(E_k - \vec{\beta}\cdot\vec{k}\right)/T} + 1}\,.$$

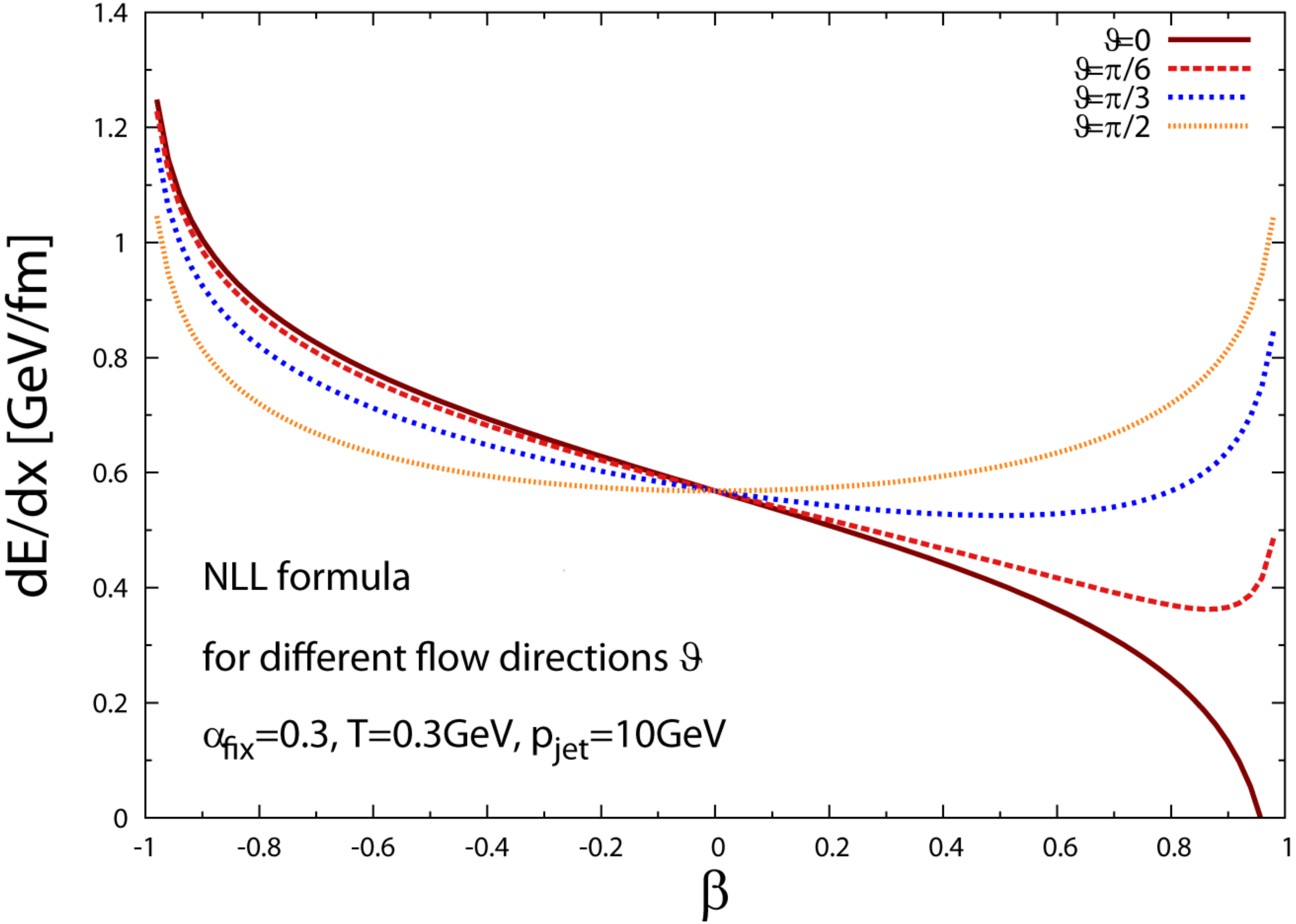
in the parallel case ($\vec{\beta}\|\vec{p}$):

$$\left.\frac{dE}{dx}\right|_{\|}^{fix} = \frac{4\pi\alpha_s^2 T^2}{3}\left[\left(1+\frac{n_f}{6}\right)\ln\frac{E\gamma\left(1-\beta\right)T}{m_D^2} + \frac{2}{9}\ln\frac{E\gamma\left(1-\beta\right)T}{M^2} + c\left(n_f\right)\right]$$

in general:

$$\frac{dE}{dx} = \frac{4\pi\alpha_s^2 T^2}{3}\left[\left(1+\frac{n_f}{6}\right)\ln\frac{p_\mu\beta^\mu T}{m_D^2} + \frac{2}{9}\ln\frac{p_\mu\beta^\mu T}{M^2} + c\left(n_f\right)\right]$$

Mean energy loss with QGP flow

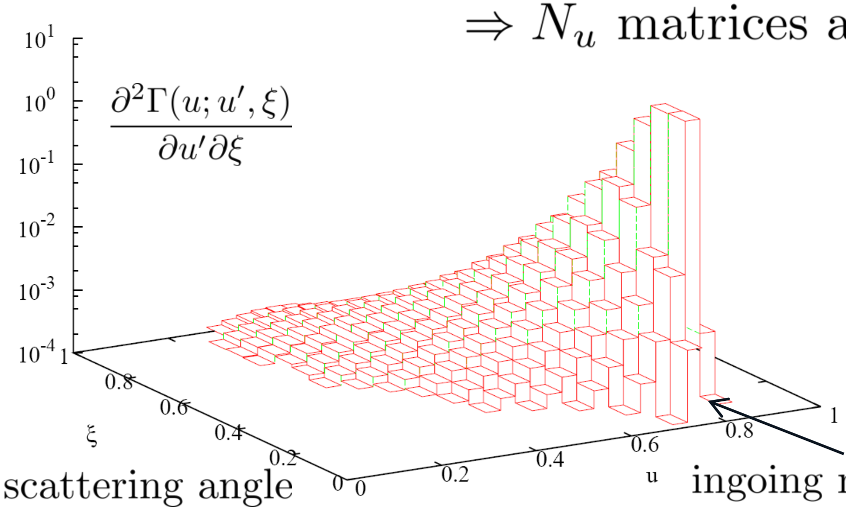


MC-simulation: transition matrix

thermalized medium with quarks and gluons: $\tau = 0.6 - 0.8\, fm/c$

$$P_{ij} \approx \Delta t \sum_{k=1}^{n} \frac{\Delta u' \Delta \xi}{n} \cdot \frac{\partial^2 \Gamma(u; u'_{i,k}, \xi_{j,k})}{\partial u' \partial \xi} \,, \quad 0 \leq i < N_u \,,\, 0 \leq j < N_\xi$$

$\Rightarrow N_u$ matrices are needed (ingoing u)



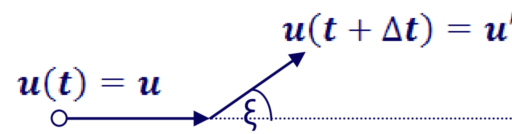
scattering angle

ingoing momentum u(t)

mappings:

$$u : \mathbb{R}_0^+ \longrightarrow [0,1),\ p' \longmapsto u(p') = \frac{e^{p'/p_0} - 1}{e^{p'/p_0} + 1}$$

$$\xi : [0,\pi] \longrightarrow [0,1],\ \vartheta \longmapsto \xi = \left(\frac{1-\cos\vartheta}{2}\right)^{\frac{1}{4}}$$



as input: available partonic or hydro models for the background medium, $T_{cell}(\vec{x},t)$, $\vec{v}_{cell}(\vec{x},t)$

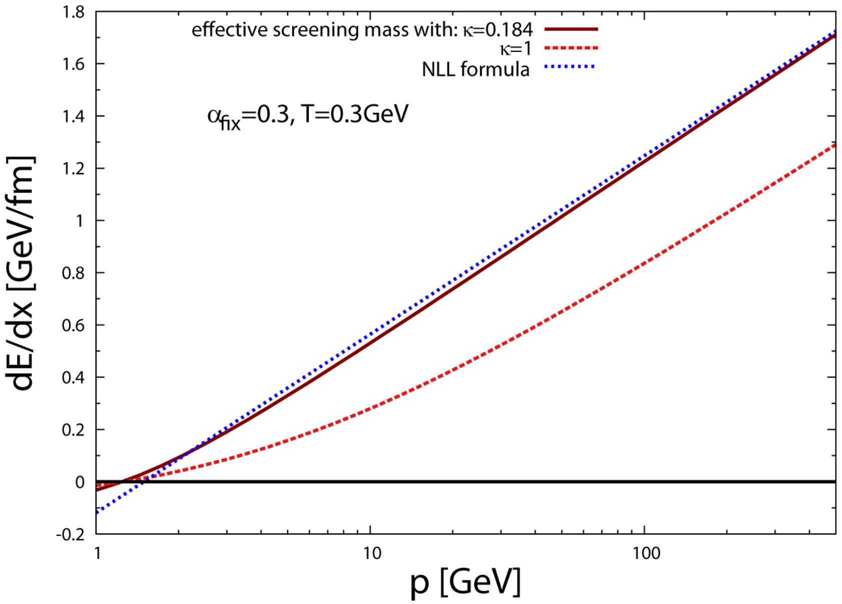
Effective screening mass and numerics

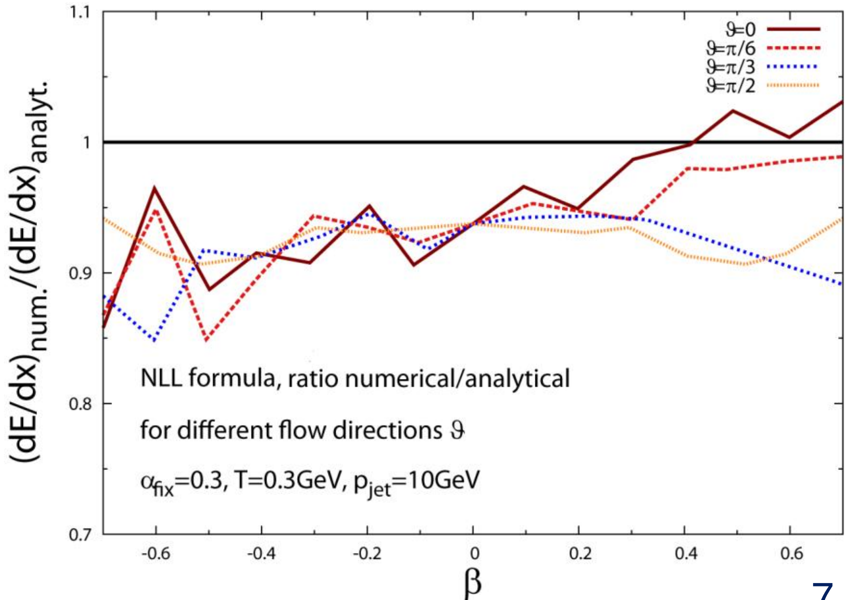
no convergence for the Debye screening mass (soft part of $\partial\Gamma/\partial\omega$ has to be screened): $\left.\frac{dE}{dx}\right|_{num.} \neq \left.\frac{dE}{dx}\right|_{analyt.}$

effective screening mass for the Born cross sections: $\mu^2(t) = \kappa\cdot 4\pi\left(1+\frac{n_f}{6}\right)\alpha(t)\,T^2 \Rightarrow \frac{\alpha_s}{t-\Pi_T(\omega,q)} \rightarrow \frac{\alpha_s}{t-\mu^2(t)}$

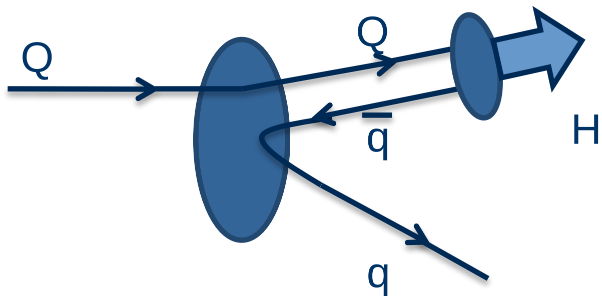
analytical evalution of $\partial\Gamma/\partial\omega$ with $\mu^2(t)$ and comparing with HTL calculation of dE/dx leads to: $\kappa = \frac{1}{2e} \simeq 0.2$

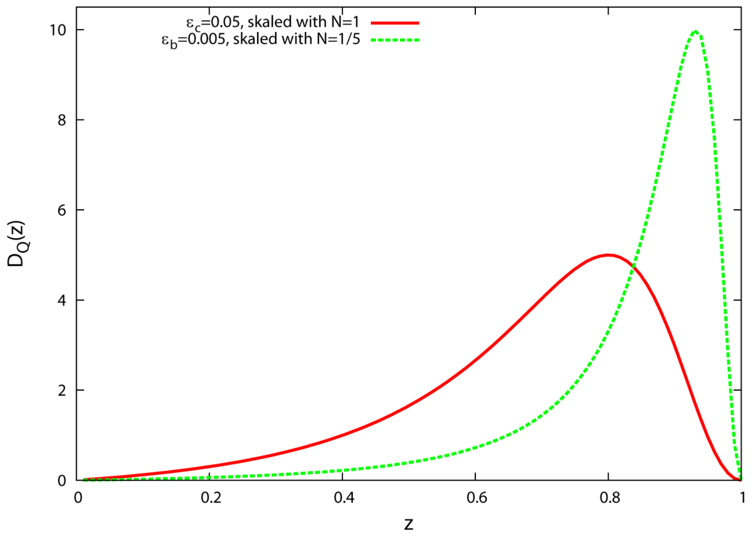
$\Rightarrow$ convergence of the numerical results against the analytic NLL formula





Peterson fragmentation





$$\mathcal{M}(Q \to H + q) \sim \frac{1}{E_H + E_q - E_Q}$$

$$\Rightarrow D_Q^H(z) = \frac{N}{z\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^2}$$

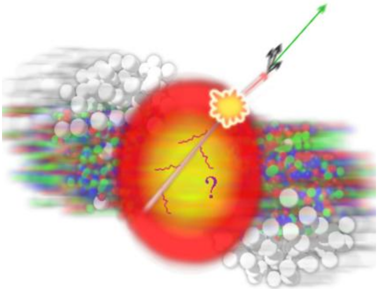
$$z = \frac{p_H}{p_Q}, \quad N = \left(\sum_H \int dz\, D_Q^H(z)\right)^{-1}$$

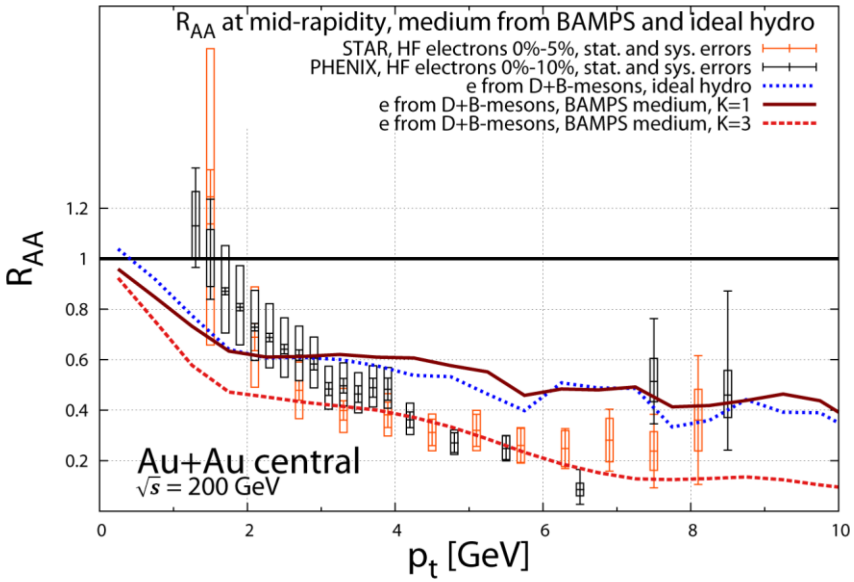
$$\epsilon_Q \sim (m_q/m_Q)^2 \simeq \begin{cases} 0.05\,, & \text{for c-quarks,} \\ 0.005\,, & \text{for b-quarks.} \end{cases}$$

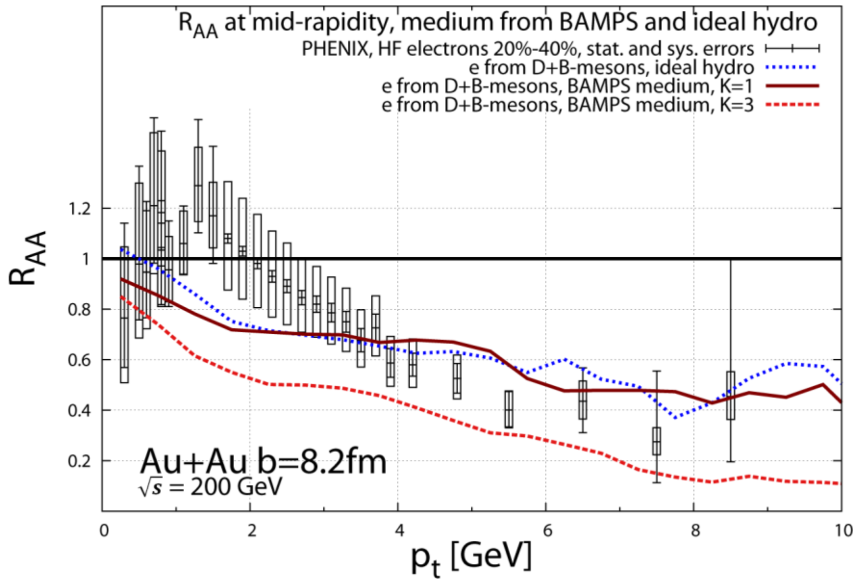
the following stage of heavy meson decays to electrons is calculated with PYTHIA 8.1

Results: nuclear modification factor

$$R_{AA} = \frac{d^2N_{AA}/dp_T dy}{N_{b.coll.} d^2N_{NN}/dp_T dy} \simeq \frac{d^2N_{AA}^{final}/dp_T dy}{d^2N_{AA}^{initial}/dp_T dy}$$



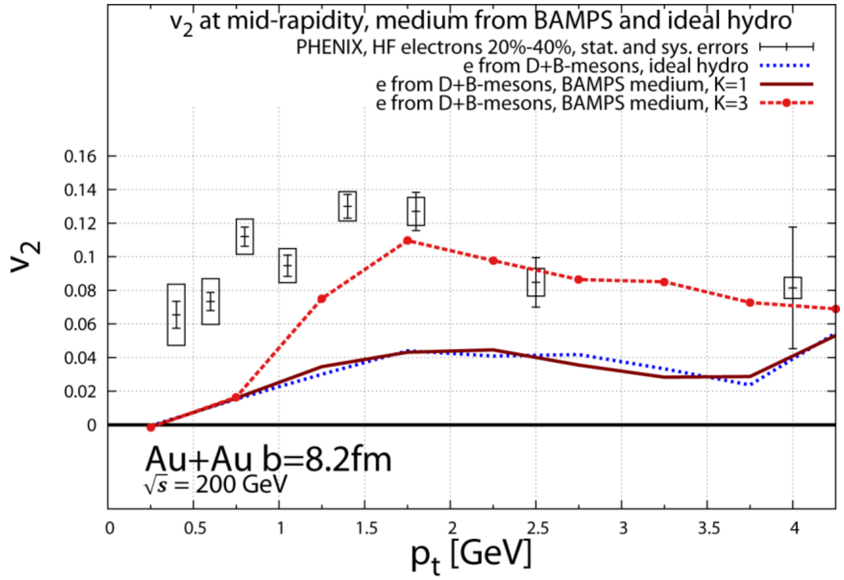


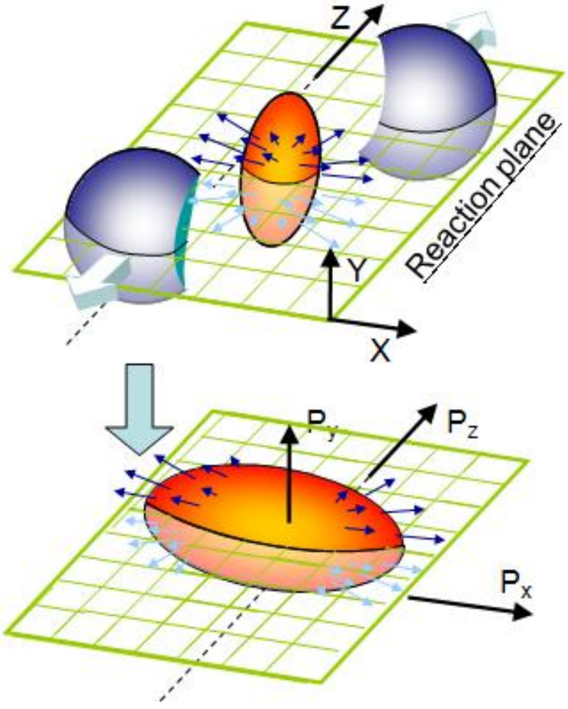


Results: elliptic flow

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle = \langle \cos(2\varphi) \rangle = \frac{1}{\frac{d^2N}{p_T dp_T dy}} \int d\varphi \frac{d^3N}{p_T dp_T d\varphi dy} \cos(2\varphi)$$

$$\text{from} \quad E\frac{d^3N}{d^3p} = \frac{d^2N}{2\pi p_T dp_T dy}\left(1 + \sum_{n=1}^{\infty} 2v_n \cos(n\varphi)\right)$$





Conclusion

- more realistic scenario
  - quantum statistics, running coupling, effective screening mass, light quarks and gluons, fragmentation, decay of mesons, different background media
- significant contribution of elastic processes
  - R_{AA} can be reproduced up to a factor of 2
  - v_2 can be reproduced up to a factor of 3
  - discrepancy could be an effect of radiative energy loss and/or uncertain initial conditions
- possible modifications:
  - different initial conditions
  - hadronization of the medium particles
  - implementation of bremsstrahlung

Thank you
for
your attention!