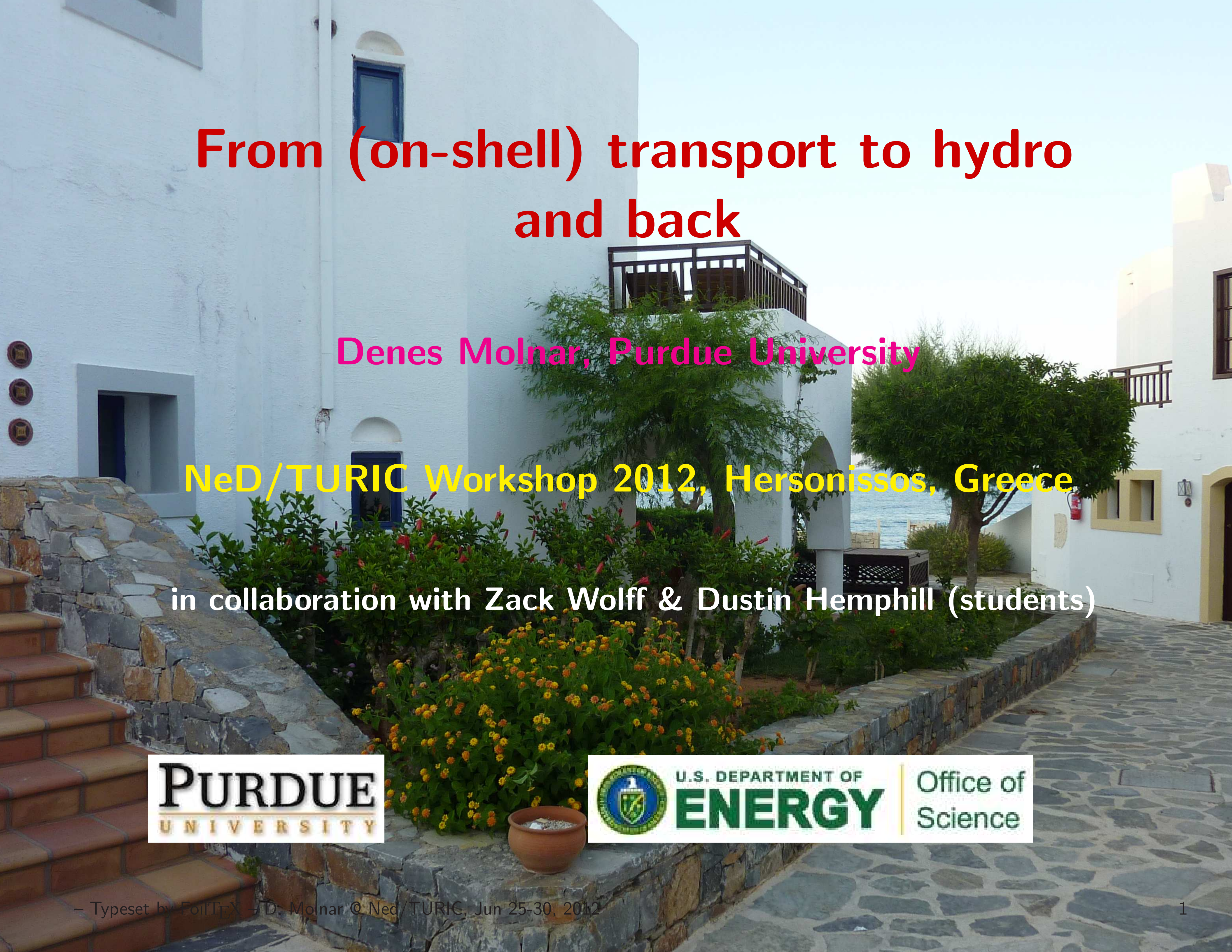

From (on-shell) transport to hydro and back

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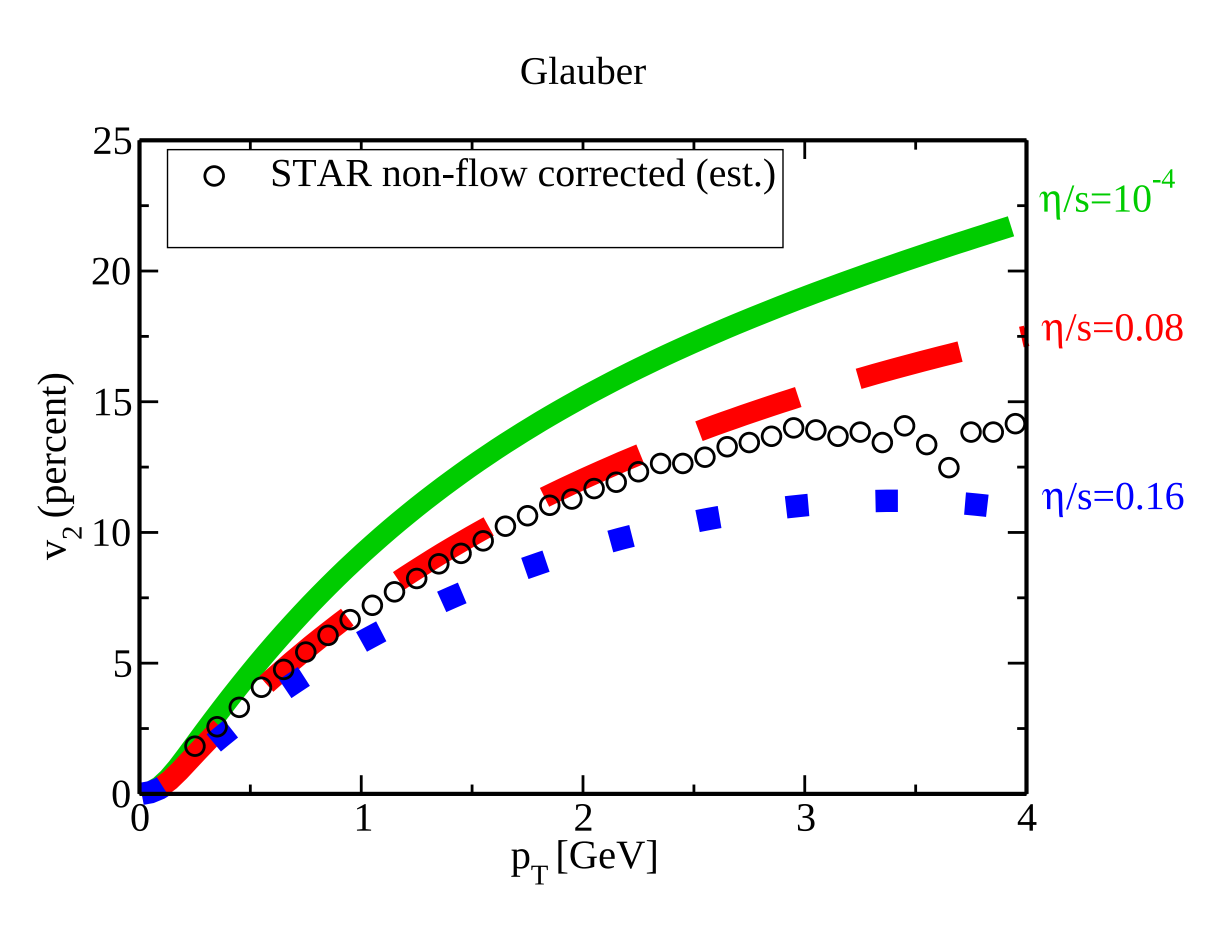
NeD/TURIC Workshop 2012, Hersonissos, Greece

in collaboration with Zack Wolff & Dustin Hemphill (students)

Motivation / Outline

Romatschke & Luzum, PRC78 ('08): **hydro - nice & simple**



- **but thermalization puzzle is complicated - e.g., radiative transport**
- **moreover, cannot escape understanding hadronic transport**

Our MPC/Grid radiative $3 \leftrightarrow 2$ transport (on-shell)

Covariant transport

(on-shell) phase-space density $f(x, \vec{p}) \equiv \frac{dN(\vec{x},\vec{p},t)}{d^3x d^3p}$

transport equation:

$$p^\mu \partial_\mu f_i(x,p) = C^i_{2\to 2}[\{f_j\}](x,p) + C^i_{2\leftrightarrow 3}[\{f_j\}](x,p) + \cdots$$

with, e.g.,

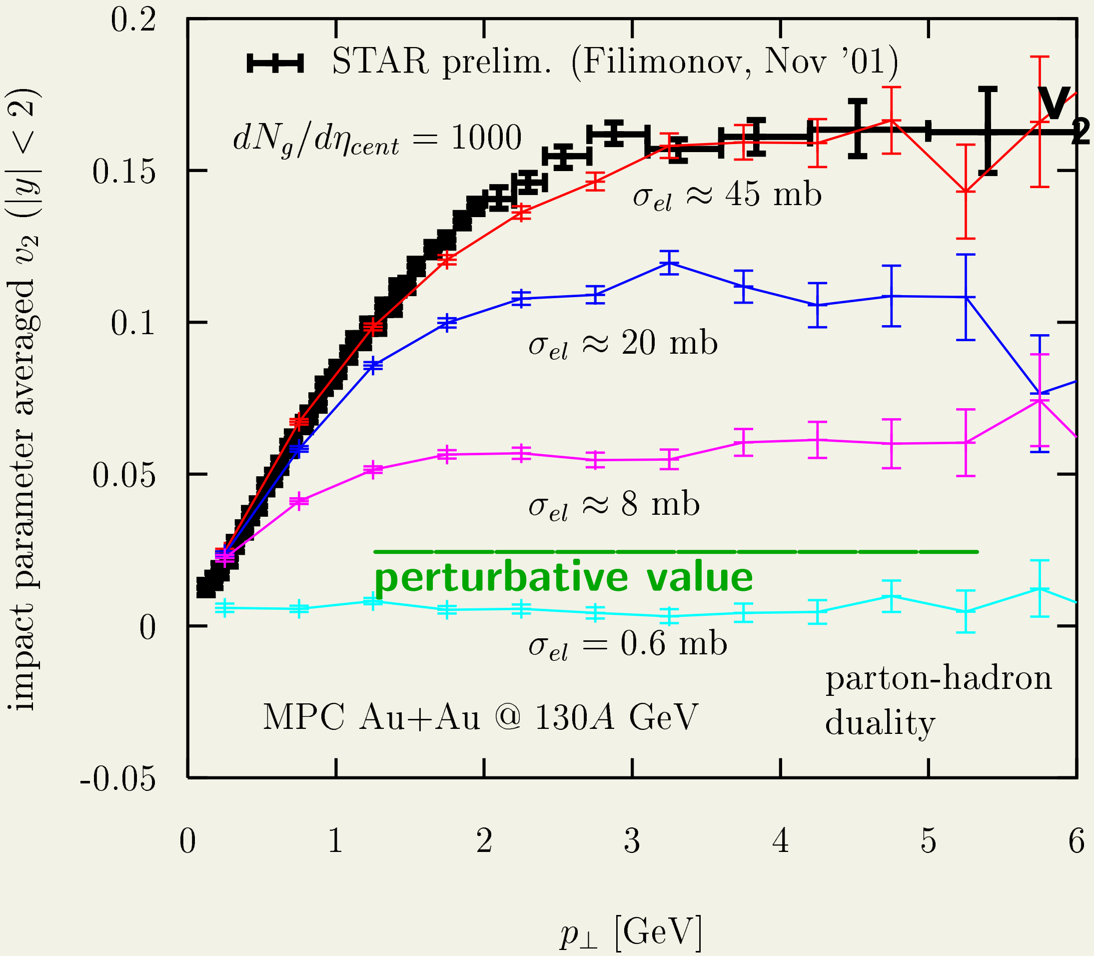
$$C^i_{2\to 2} = \frac{1}{2} \sum_{jkl} \int_{234} (f_3^k f_4^l - f_1^i f_2^j) W^{ij\to kl}_{12\to 34} \qquad \left(\int_j \equiv \int \frac{d^3p_j}{2E_j}\ , \quad f_a^k \equiv f^k(x,p_a) \right)$$

fully causal and stable, can equilibrate

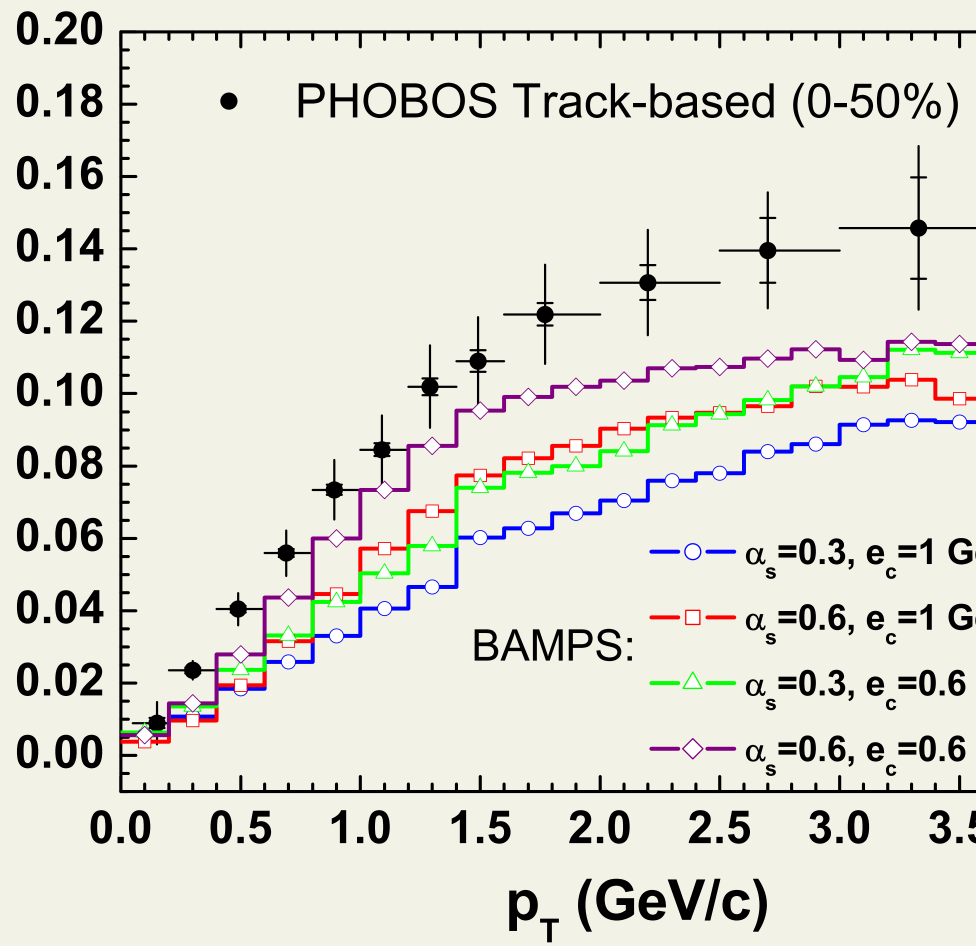
near hydrodynamic limit, transport coefficients and relaxation times:

$\eta \approx 1.2T/\sigma_{tr}$, $\qquad \tau_\pi \approx 1.2\lambda_{tr}$

$2 \rightarrow 2$ **transport** DM & Gyulassy, NPA 697 ('02)



radiative $3 \leftrightarrow 2$ Xu & Greiner, ('08)



perturbative $2 \rightarrow 2$ **rates not enough**, **need** $\sim 15\times$ **higher to get enough** v_2

but radiative $3 \leftrightarrow 2$ **seems to help**

MPC/Grid

our transport equation solver using test particles on a rectangular grid

parameters: **time step Δt, cell sizes d_x, d_y, d_z, subdivision ℓ**

- **collision probability during one time step for pair/triplet**

$$P_{2\to X} = \frac{\sigma_{2\to X} v_{rel} \Delta t}{V_{cell}}$$

$$P_{3\to 2} = \frac{K_{3\to 2} \Delta t}{V_{cell}^2}$$

- **outgoing momenta generated according to matrix elements**

- **we can use subdivision to control number of particles per cell**

Main advantage: **5 adjustable knobs instead of just 1 for cascade algorithm**

$\Rightarrow$ **more flexibility**

Main question: how much faster is equilibration with $3 \leftrightarrow 2$?

pQCD-motivated matrix elements coming soon...

but for now, massless particles with energy-independent, isotropic scattering

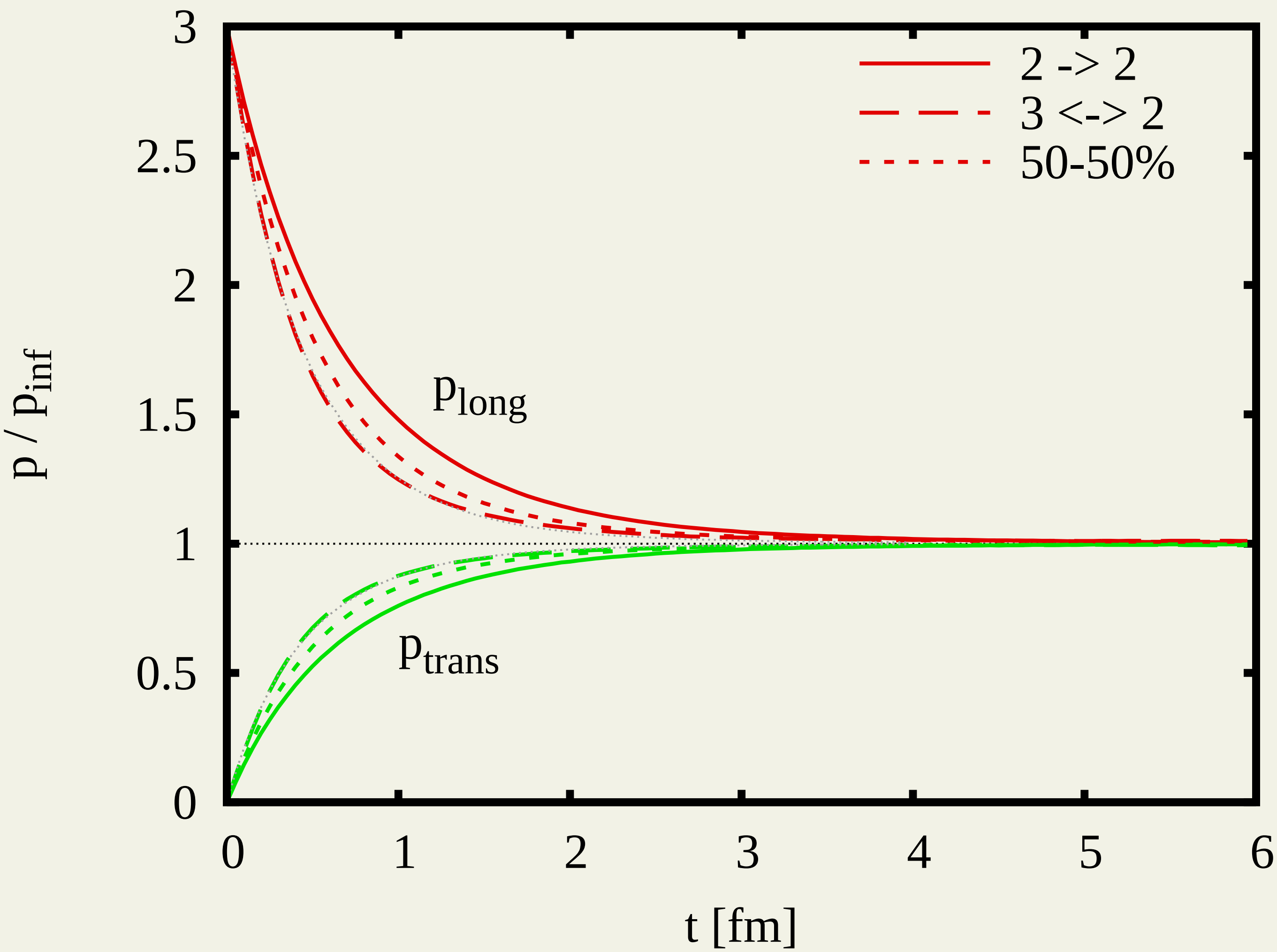
i) $d\sigma_{2\to 2}/d\Omega = const \qquad \Rightarrow \qquad |\bar{M}_{2\to 2}|^2 = 16\pi s \sigma_{2\to 2}$

ii) $\sigma_{2\to 3} = const$, $d\sigma_{2\to 3} = const \times d^3p_3 d^3p_4 d^3p_5 \qquad \Rightarrow \qquad |\bar{M}_{2\leftrightarrow 3}|^2 = 3072\pi^3 \sigma_{2\to 3}$

$$\Rightarrow K_{3\to 2} = \frac{24\pi^2 \sigma_{2\to 3}}{g E_1 E_2 E_3}$$

compare i) pure $2 \to 2$ **with** $\sigma_{22} = \sigma_0$
ii) pure $3 \leftrightarrow 2$ **with the same** $\sigma_{23} = \sigma_0$
iii) 50-50% split, $\sigma_{22} = \sigma_{23} = \sigma_0/2$

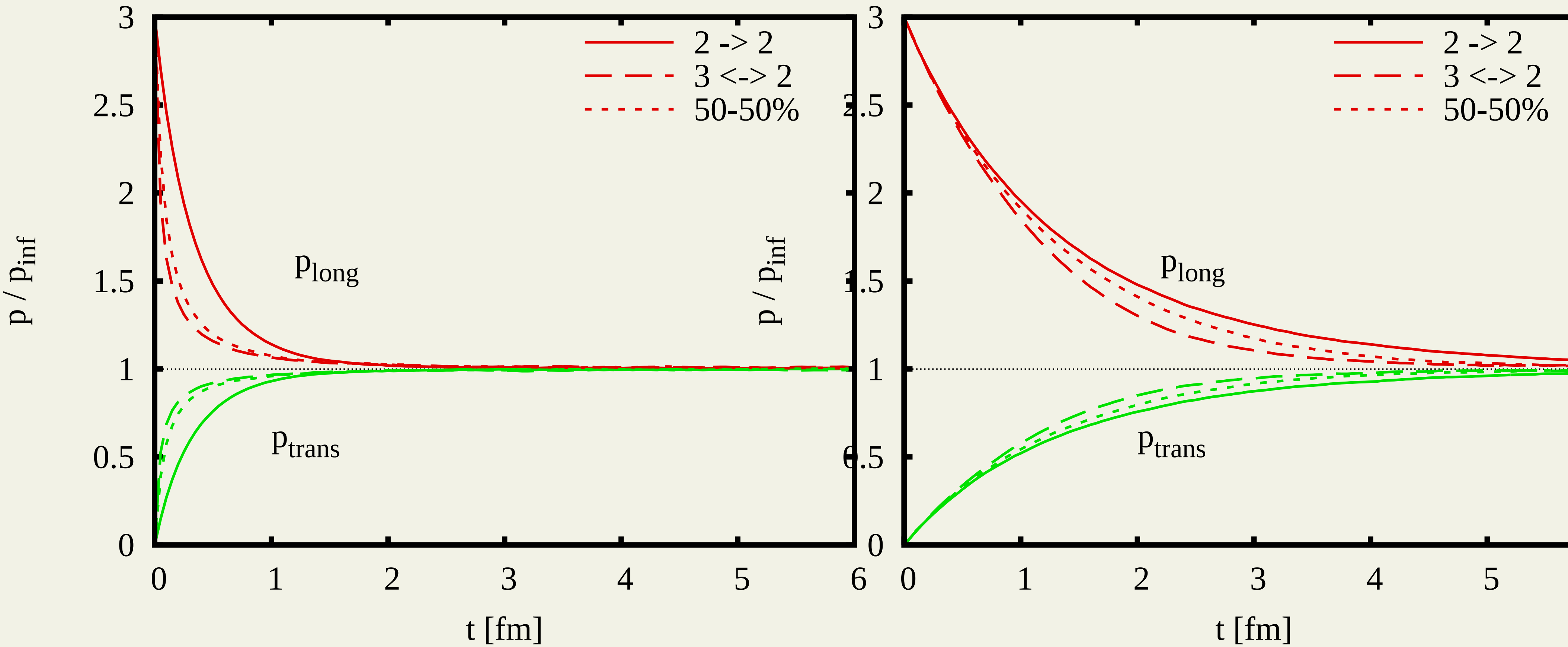
equilibration in uniform box with initial $f(\vec{p}) \propto \delta(p_z - p_0) + \delta(p_z + p_0)$



with $n|_{t=0} = n_{equil}$

$3 \leftrightarrow 2$ **is $\sim$ 50% more efficient (isotropic case)**

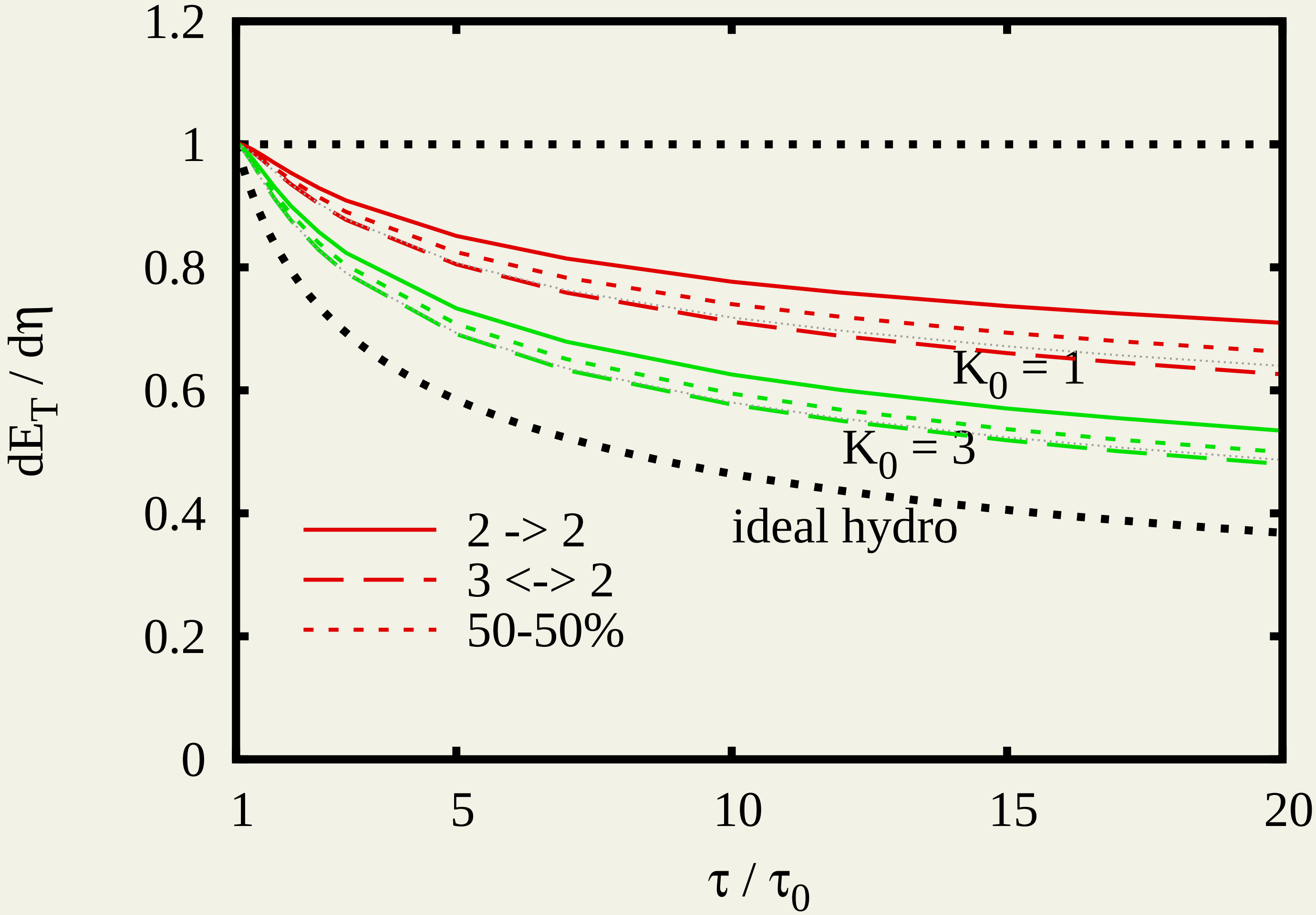
$n|_{t=0} = 2n_{equil}$ $\qquad$ $n|_{t=0} = n_{equil}/2$



higher/lower efficiency in over/undersaturated case

cooling in longitudinal Bjorken scenario (pdV work)

extends Zhang, Pang, Gyulassy ('97); and DM & Gyulassy ('99)



$$K \equiv \frac{\tau}{\lambda_{tr}}$$

same ~ 50% enhancement

conclusion for QCD will likely be better

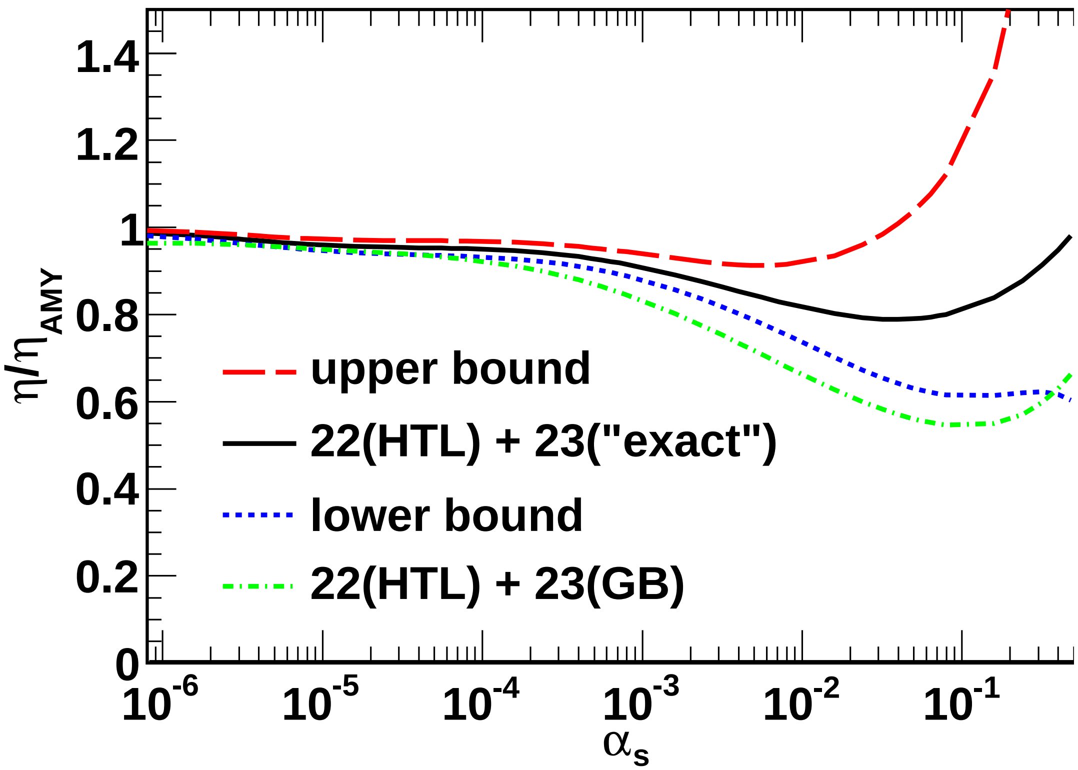
$2 \to 2$ **strongly forward-peaked**

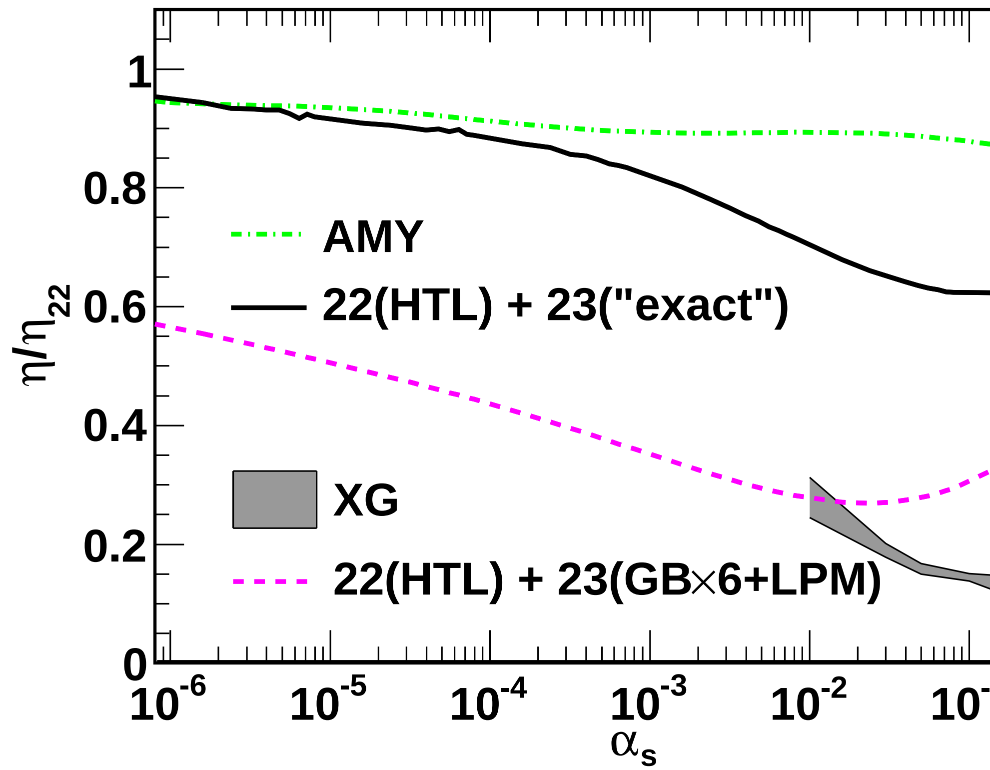
$3 \leftrightarrow 2$ **opens up more phase space**

but importance of $3 \leftrightarrow 2$ **is debated, e.g.,** Arnold, Moore, Yaffe, JHEP 0011, and Chen, Deng, Dong & Wang, arXiv:1107.0522v4 **claim only modest effect**

we will check this soon...

Chen, Deng, Dong & Wang, arXiv:1107.0522v4





Why viscous hydro calculations need to know about transport

Hydrodynamics

Equations of motion

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad , \quad \partial_\mu N_B^\mu(x) = 0$$

Ideal hydro:

$$T_{id}^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu} \quad , \quad N_B^\mu = n_B u^\mu$$

Viscous hydro:

$$T^{\mu\nu}(x) = T_{id}^{\mu\nu}(x) + \pi^{\mu\nu}(x) - \Pi(x)\Delta^{\mu\nu}(x)$$

$$\dot{\pi}^{\mu\nu} = F^{\mu\nu}(e, u, \pi, \Pi) \quad , \quad \dot{\Pi} = G(e, u, \pi, \Pi)$$

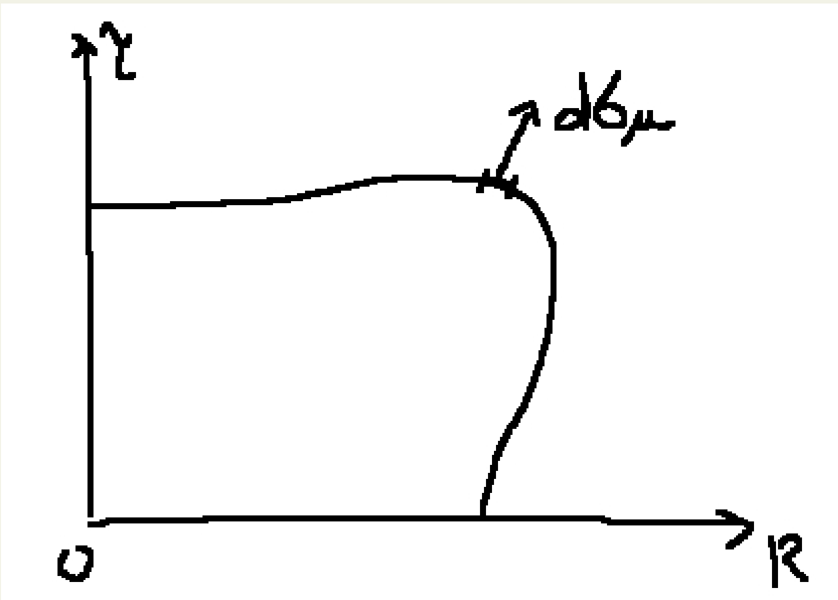
(e.g. Israel-Stewart theory)

Needs:

- **equation of state** $p(e, n_B)$, $T(e, n_B)$ **and transport properties** η, ζ, τ_π, ...
- **initial conditions**
- **decoupling (freezeout) prescription** $*$

Cooper-Frye freezeout

Assume sudden transition to a gas on a 3D hypersurface (typically $T = const$ or $\varepsilon = const$)



$$E\, dN = p^\mu d\sigma_\mu(x)\, d^3p\, f_{gas}(x, \vec{p})$$

(covariant analog of $t = const$ freezeout $dN/d^3xd^3p = f(\vec{x}, \vec{p}, t_{fo})$)

Good: - conserves energy-momentum and charges locally

Bad: - negative contributions possible $p \cdot d\sigma < 0$
- arbitrariness in choice of HS & self-consistency problem

exist alternative approaches, e.g., Kodama, Grassi et al; Csernai et al

Hydro $\longrightarrow$ particles

In hydro and hydro+transport studies one must convert fluid to particles.

two effects: - **dissipative corrections to hydro fields** u^μ, T, n
- **dissipative corrections to thermal distributions** $f \to f_0 + \delta f$

$$T^{\mu\nu}(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(p,x)$$

- **in local equilibrium (ideal hydro) - "one to one"**

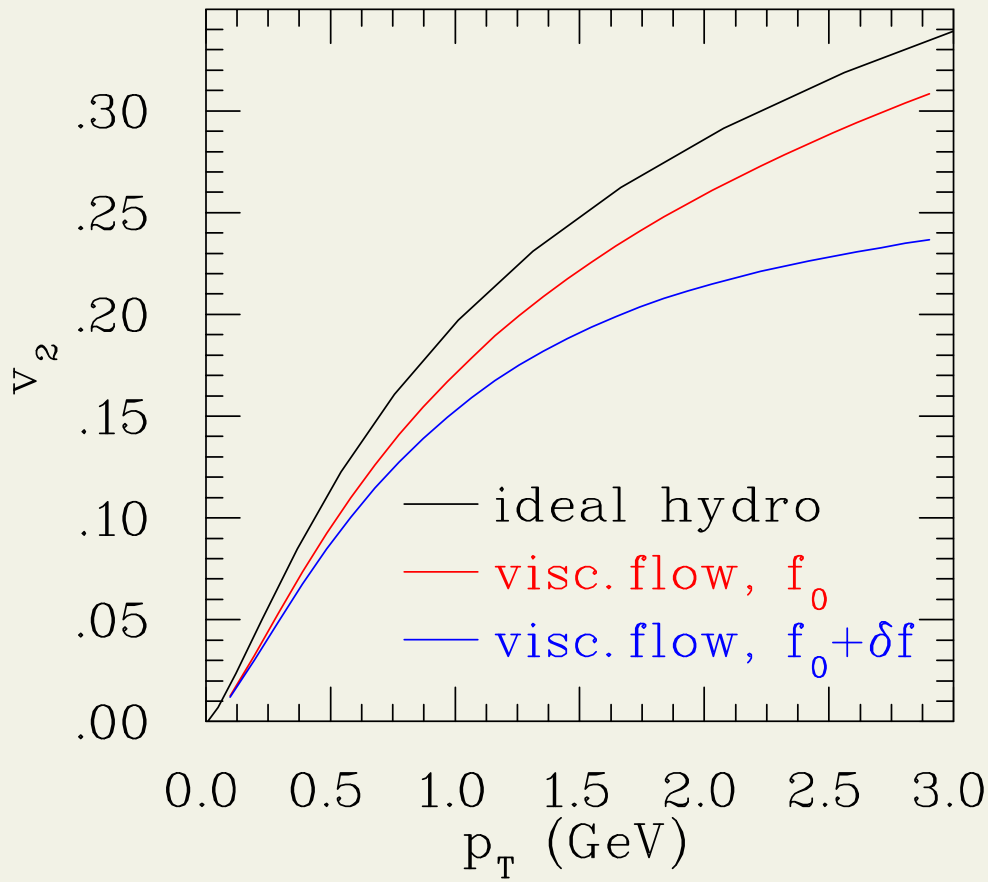
$$T^{\mu\nu}_{LR}(x) = diag(e,p,p,p) \qquad \Leftrightarrow \qquad f_{eq,i}(x,p) = \frac{g_i}{(2\pi)^3} e^{-p^\mu u_\mu / T}$$

- **near local equilibrium (viscous hydro) - "few to many"**

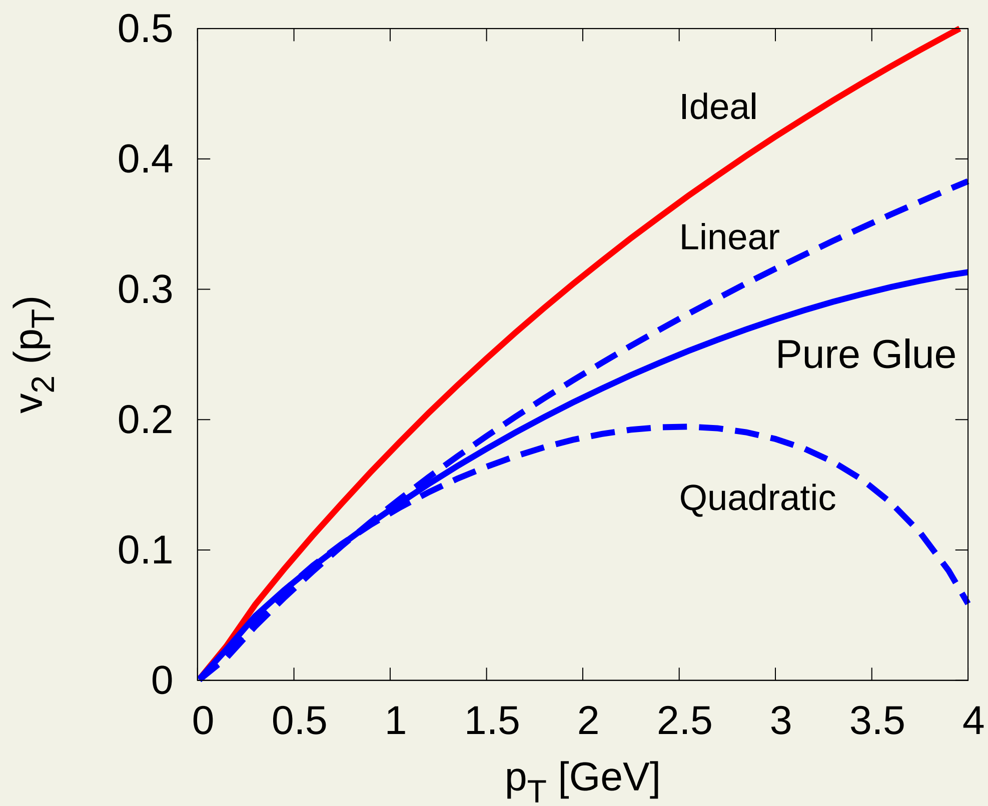
$$T^{\mu\nu}(x) = T^{\mu\nu}_{ideal})(x) + \pi^{\mu\nu}(x) \qquad \Leftarrow \qquad f(x,p) = f_{eq,i}(x,p) + \delta f_i(x,p)$$

common choice - "democratic" Grad ansatz: $\delta f_i \equiv f_i^{eq} \times \frac{\pi^{\mu\nu}}{2(e+p)} \frac{p_{\mu,i} p_{\nu,i}}{T^2}$

Huovinen & DM ('08) **Grad $\delta f \propto p^2$**



Dusling et al, ('09) - **linear response $\delta f_{q,g} \sim p^{1.5}$**



large effects at higher momenta (δf blows up, can even lead to $f < 0$)

Problem: “democratic Grad” ignores microscopic dynamics

$$\delta f_i \equiv f_i^{eq} \times \frac{\pi^{\mu\nu}}{2(e+p)} \frac{p_{\mu,i} p_{\nu,i}}{T^2}$$

answer CANNOT be universal

$\rightarrow$ **investigate this in a nonequilibrium framework**

Setup - 1D Bjorken $\quad \rightarrow \quad f_i = f_i(p_T, \xi, \tau)$, where $\xi \equiv \eta - y$

i) compute f_i from full nonequilibrium transport $p\partial f_i = \sum_j C_{ij}^{2\to 2}[f_i, f_j]$

using MPC code

ii) from f_i, determine $T^{\mu\nu}$ and $\pi_i^{\mu\nu}$

iii) study partial shear stresses $\pi_{L,i}(\tau)/p(\tau)$, and relative magnitude of δf_i

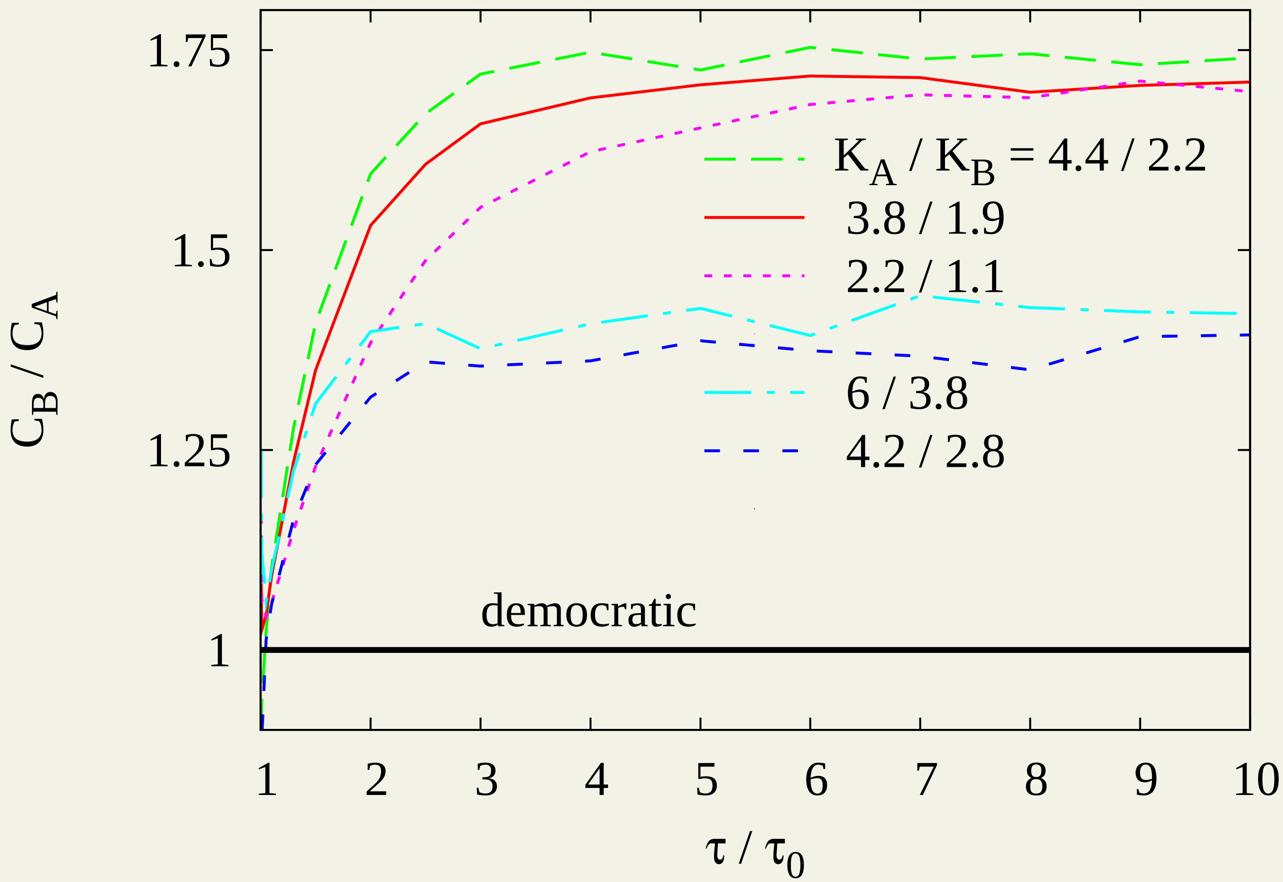
———

expect dynamics to be governed by inverse Knudsen numbers:

$$K_i \equiv \frac{\tau}{\lambda_i} = \tau \sum_j n_j \sigma_{ij} = \sum_j K_{i(j)}$$

Two-component, massless system. A set to equilibrate faster than B.

$$\text{assume } \delta f_i = C_i \, (p_T/T)^2(\mathrm{sh}^2 y - 1/2) f_i^{eq} \quad \Rightarrow \quad \pi_{L,i}/p_i = 8C_i$$



$n_A : n_B$	$\sigma_{AA} : \sigma_{AB} : \sigma_{BB}$
3 : 1	20 : 10 : 5
2 : 2	20 : 10 : 5
2 : 2	12 : 6 : 3
3 : 1	24 : 24 : 12
2 : 2	20 : 13.3 : 8.89

viscous corrections are <u>not</u> proportional to K_i but shear stress sharing seems universal at late times

δf from linear response

standard linear response to flow shear $\sigma^{\mu\nu} \equiv \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial u)$, same as computation of shear viscosity de Groot, et al ('70s)... Arnold, Moore, Jaffe, JHEP 0011...

$$p\partial f_i = \sum_j C_{ij}^{2\to 2}[f_i, f_j]$$

small deviations from local equilibrium $f_i = f_{0i} + \delta f_i$, 2-component case:

$$\begin{aligned}
p\partial f_{0A} &= C_{AA}[f_{0A}, \delta f_A] + C_{AA}[\delta f_A, f_{0A}] + C_{AB}[\delta f_A, f_{0B}] + C_{AB}[f_{0A}, \delta f_B] \\
p\partial f_{0B} &= C_{BB}[f_{0B}, \delta f_B] + C_{BB}[\delta f_B, f_{0B}] + C_{BA}[\delta f_A, f_{0B}] + C_{BA}[f_{0A}, \delta f_B]
\end{aligned}$$

No δf on LHS - relaxation implicitly assumed, moments beyond $T^{\mu\nu}$ ignored.

Can be recast as a variational problem:

$$\delta Q[\delta f_A, \delta f_B] = 0$$

where Q_{max} is proportional to the shear viscosity.

<u>One</u> unknown function per particle species

$$\delta f_i(x,p) = \chi_i(p/T)\, \hat{p}_\mu \hat{p}_\nu \frac{\sigma^{\mu\nu}}{T}$$

one-component case, $2 \to 2$

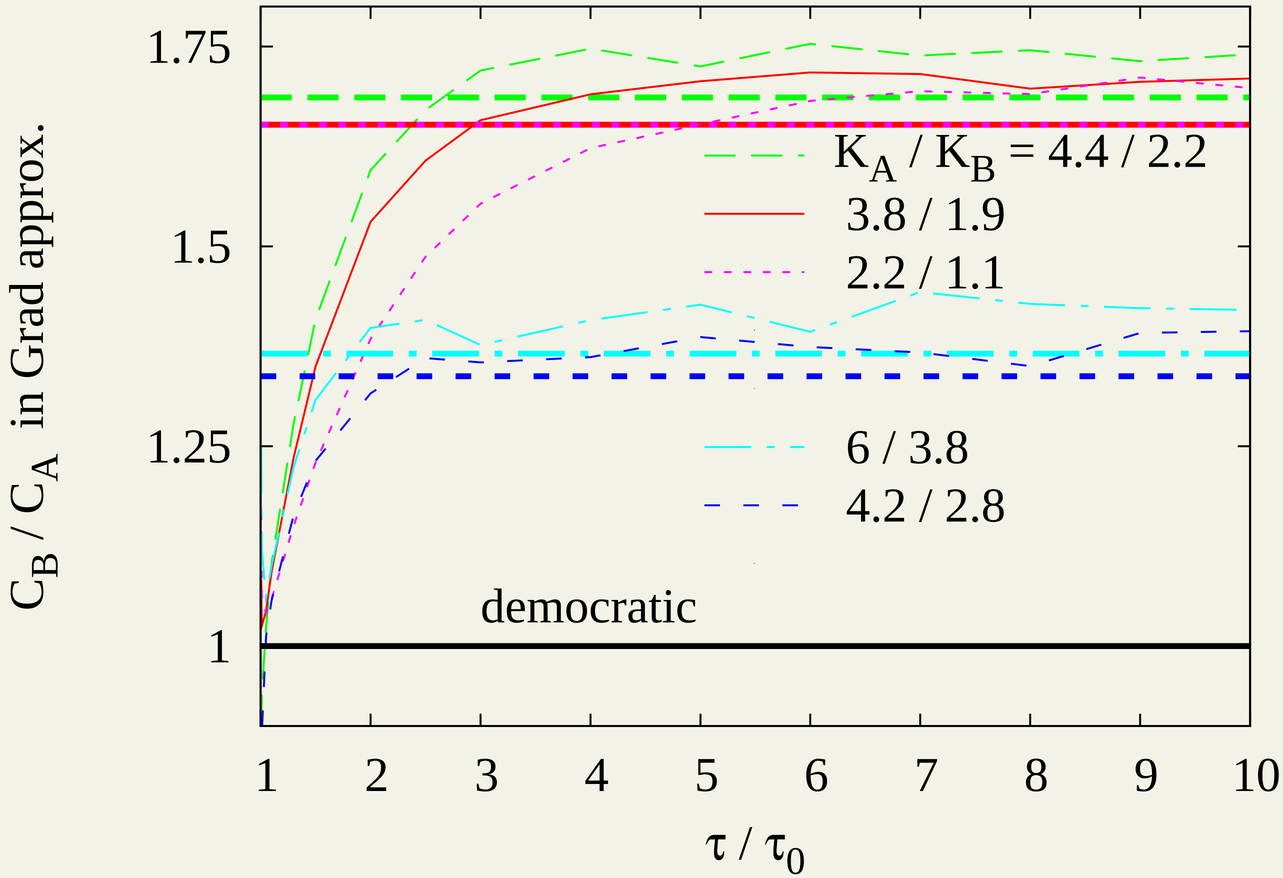
$$
\begin{aligned}
Q[\chi] \quad = \quad & \frac{T^2}{2} \int\limits_1 f_{1,eq} \chi_1 P_1 \cdot P_1 \\
& + \frac{1}{2} \int\limits_1 \int\limits_2 \int\limits_3 \int\limits_4 f_{1,eq} f_{2,eq} \, \chi_1 \left(\chi_3 P_3 \cdot P_1 + \chi_4 P_4 \cdot P_1 \right. \\
& \qquad \qquad \qquad \left. - \chi_1 P_1 \cdot P_1 - \chi_2 P_2 \cdot P_1 \right) W_{12 \to 34}
\end{aligned} \tag{1}
$$

where

$$P_i \cdot P_j = \frac{1}{T^4} \left[(\vec{p}_i \cdot \vec{p}_j)^2 - \frac{1}{3} p_i^2 p_j^2 \right]$$

4 numerical integrals to compute (isotropic case)

$$\Rightarrow \qquad \left(\frac{C_B}{C_A}\right)^{Grad}_{lin.response} = \frac{5K_A + 2(K_{A(B)} + K_{B(A)})}{5K_B + 2(K_{A(B)} + K_{B(A)})}$$



$$\delta f_i = C_i\,(p_T/T)^2(\mathrm{sh}^2 y - 1/2) f_i^{eq}$$

$$\pi_{L,i}\,/\,p_i = 8C_i$$

linear response with $\delta f \propto p^2$ "gets" late-time $\pi^{\mu\nu}$ sharing within 10%

one-component massive gas also reproduced, caught typo in de Groot et al kinetic theory book

$$\eta^{Grad} = \frac{15z^2 K_2^2(z) h^2(z)}{16[(15z^2+2)K_2(2z) + (3z^3+49z)K_3(2z)]} \cdot \frac{T}{\sigma}$$

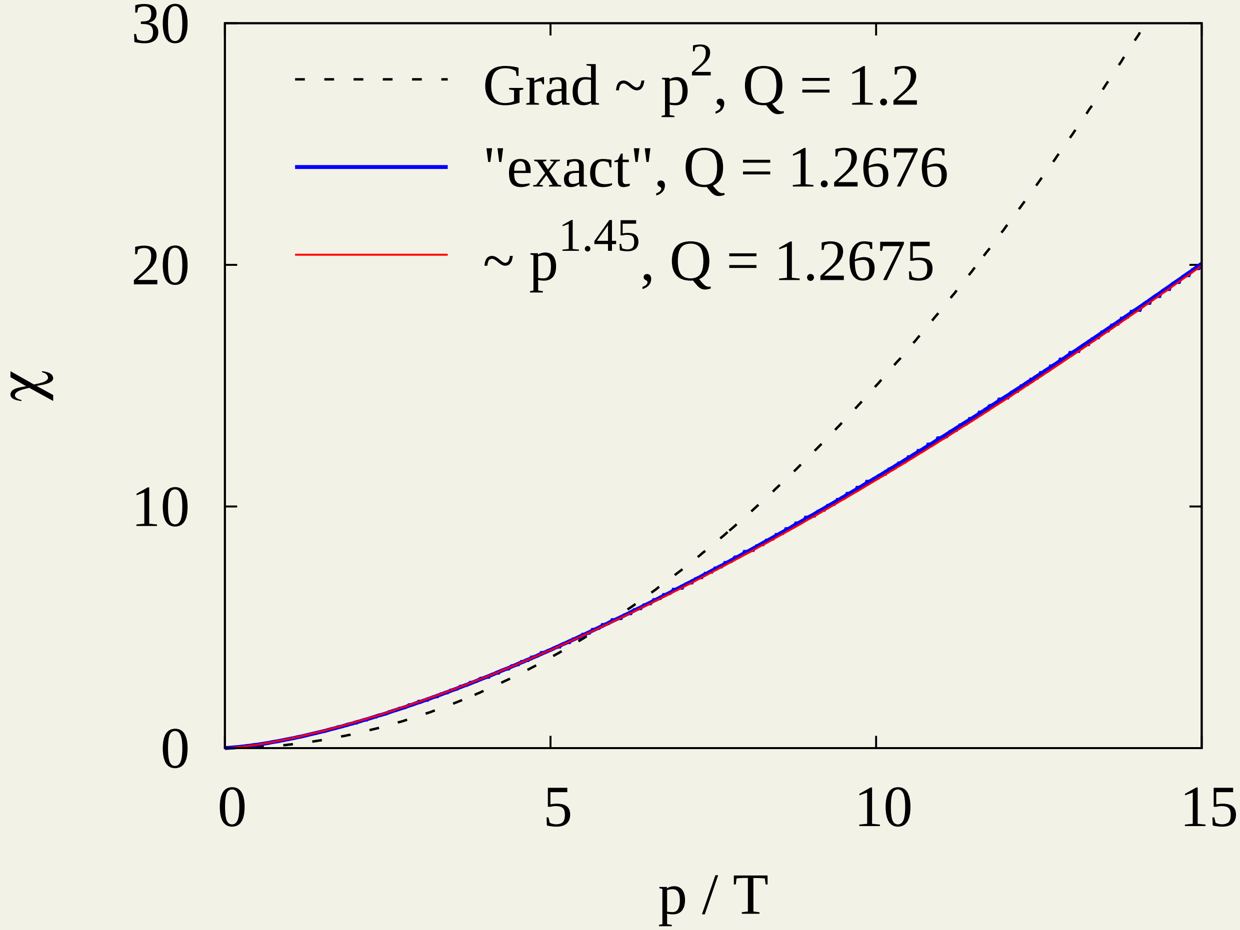
where

$$h(z) \equiv \frac{zK_3(z)}{K_2(z)} , \qquad\qquad z \equiv m/T$$

Momentum dependence - Grad inconsistent with linear response!

e.g., one-component massless gas with isotropic $\sigma = const$**:**

DM ('11)

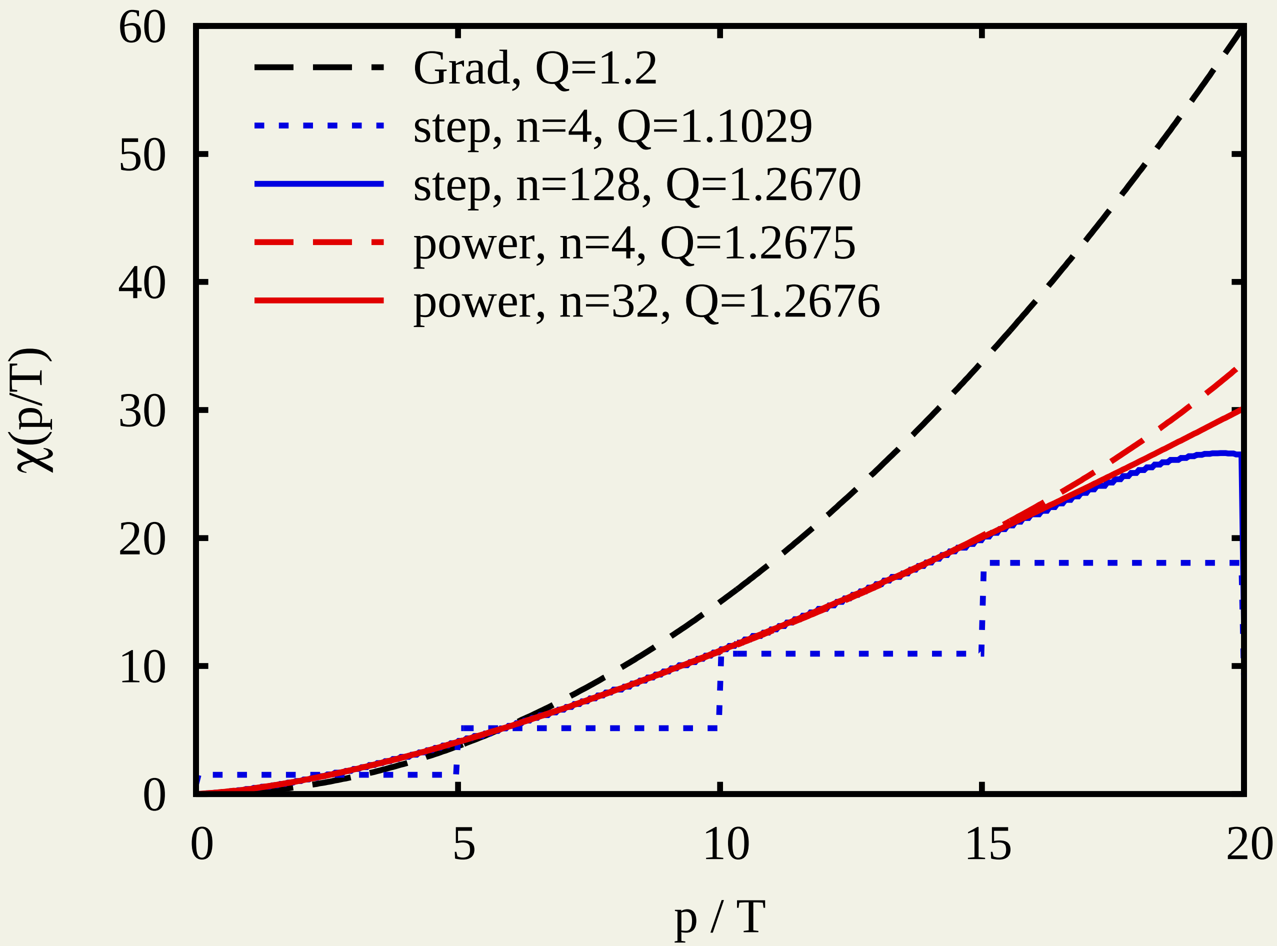


$$\delta f = \chi(p/T)\frac{T}{n\sigma}\frac{\pi_{\mu\nu}\hat{p}^\mu\hat{p}^\nu}{\eta T} f_{eq}$$

$$\eta \simeq \frac{\mathbf{1.2676}T}{\sigma}$$

Duslin et al **get similar** $\sim p^{1.5}$ **but with forward-peaked** $2 \to 2$ **and** $1 \leftrightarrow 2$

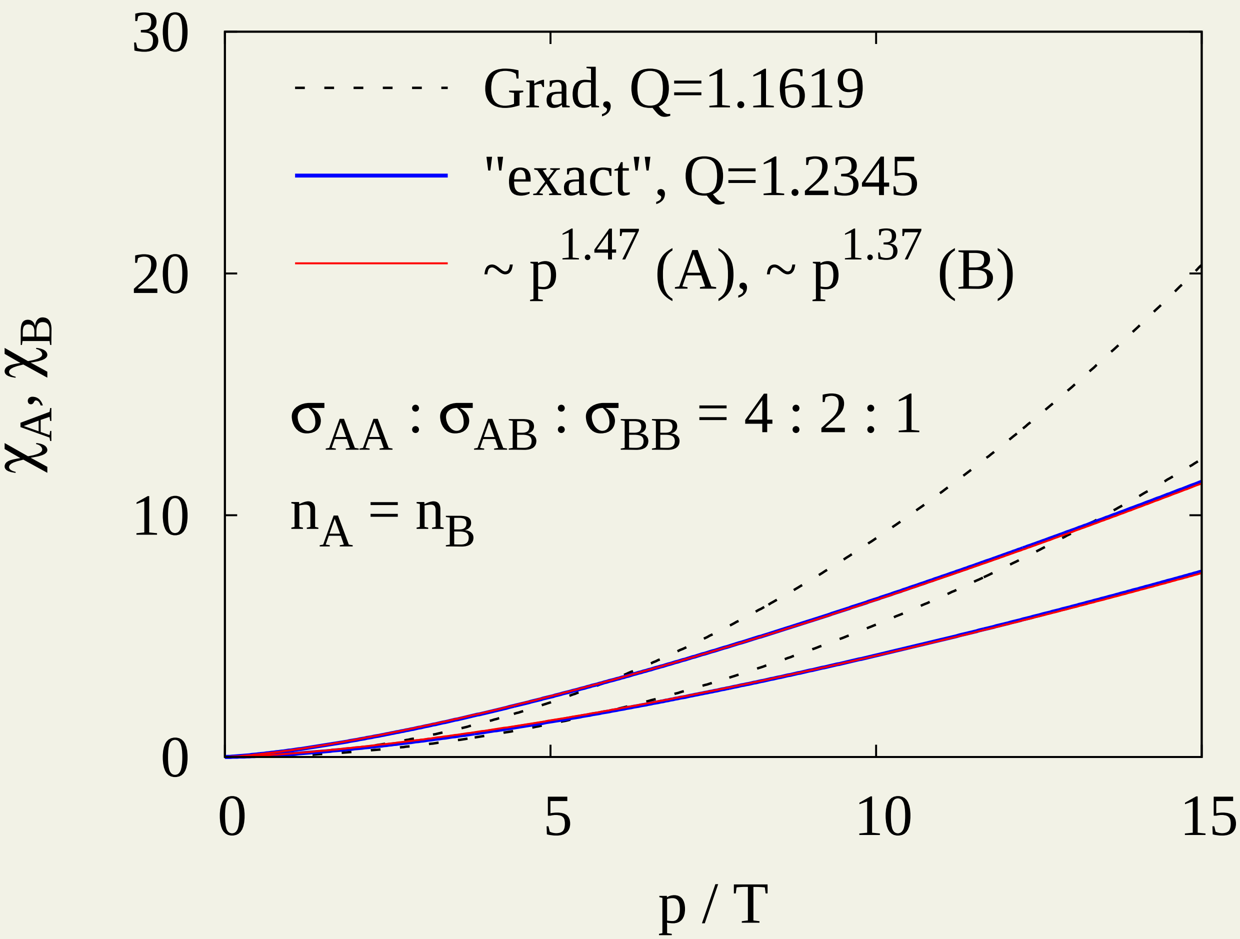
confirmed convergence



$$\eta = Q_{max} \frac{T}{\sigma}$$

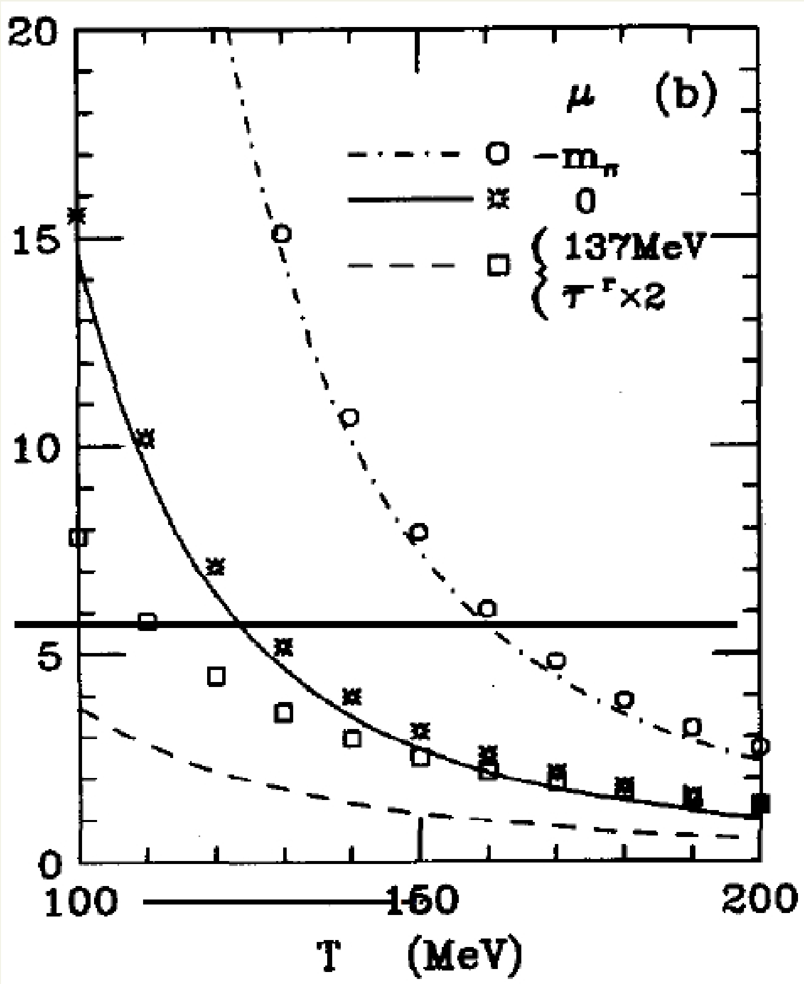
two-component, massless - not quadratic either

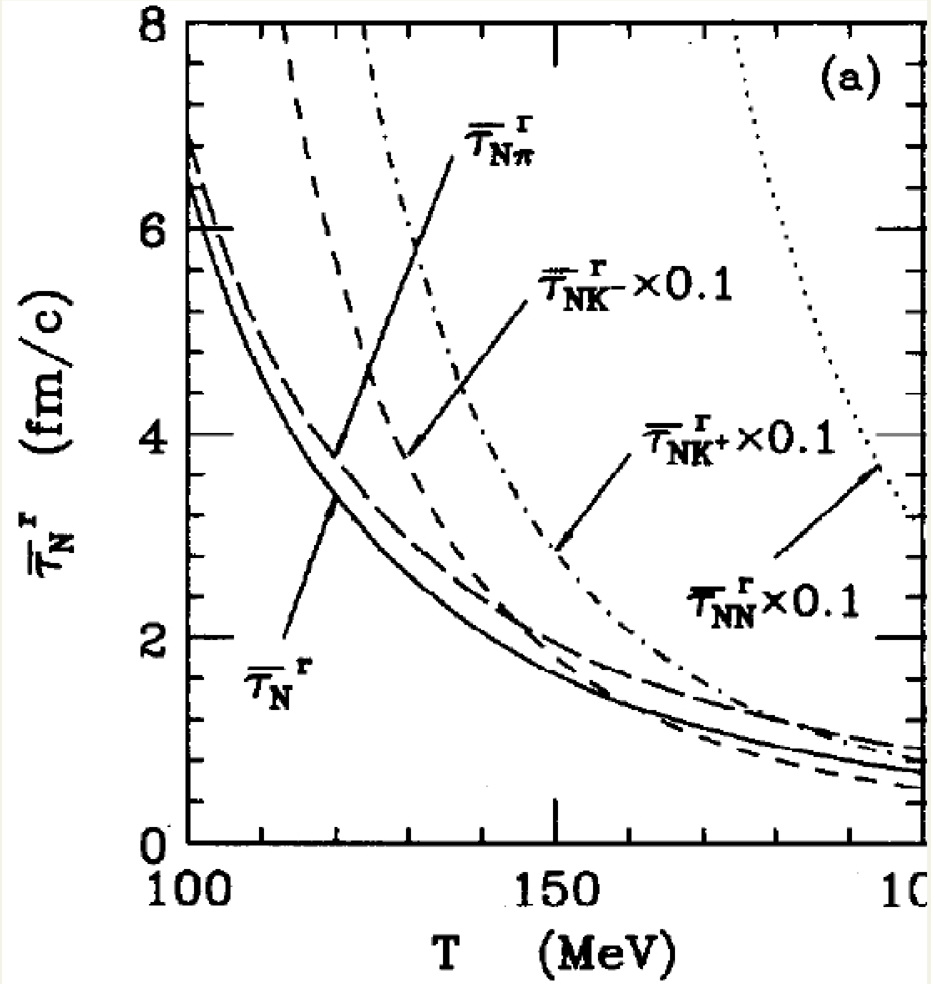
DM ('11)



Apply to hadron gas. Approximation: $\pi - N$ system.

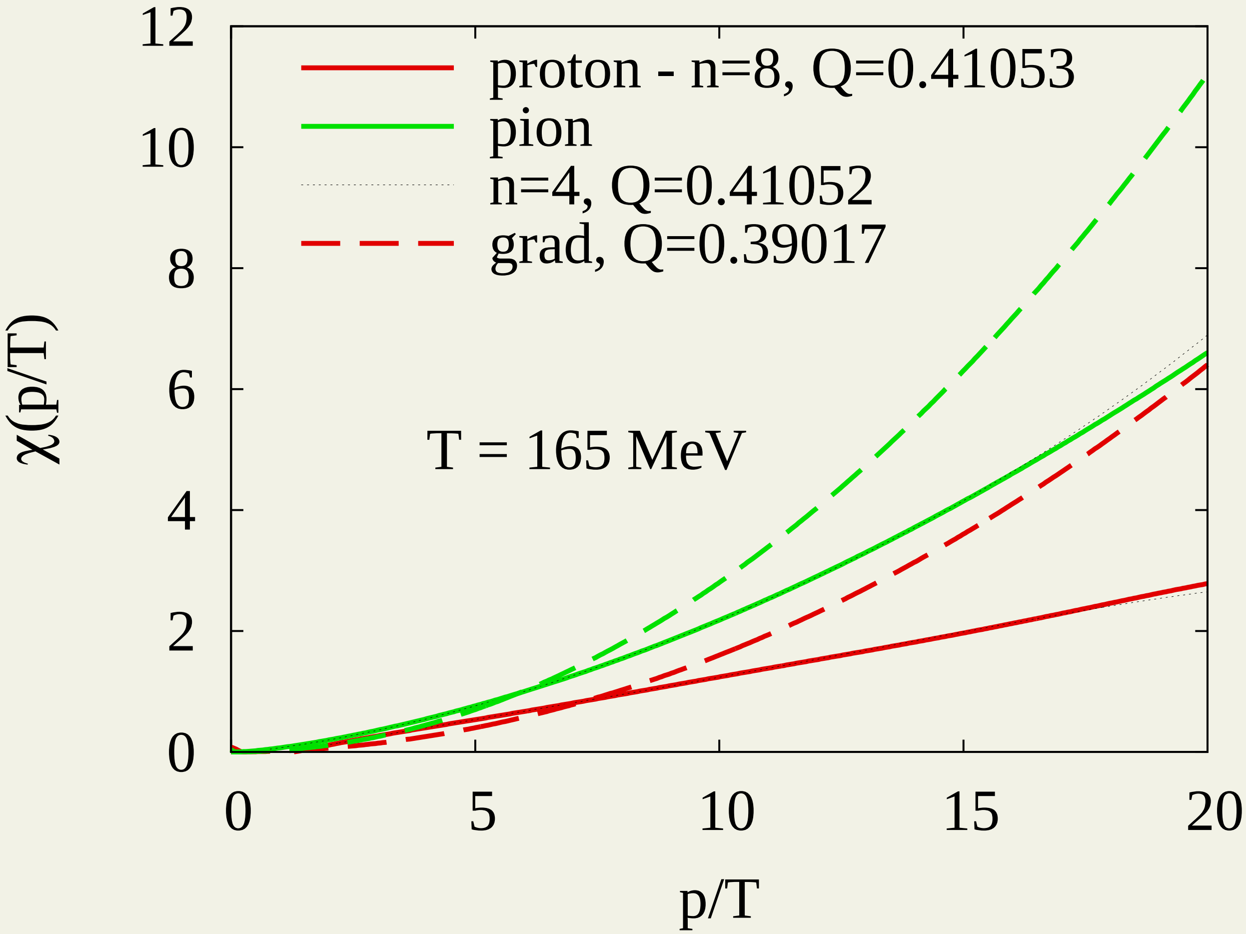
Prakash et al, Phys.Rep. 227 ('93): $\bar{\tau}_{\pi\pi} \approx 1/\bar{\nu}_{\pi\pi}$ $\bar{\tau}_{\pi N} \approx 1/\bar{\nu}_{\pi N} \sim \tau_{\pi\pi}/2$





quite well captured with isotropic $\sigma_{eff}^{\pi\pi} = 30mb$, $\sigma_{eff}^{\pi N} = 50mb$, $\sigma_{eff}^{NN} = 20mb$

pion-proton system



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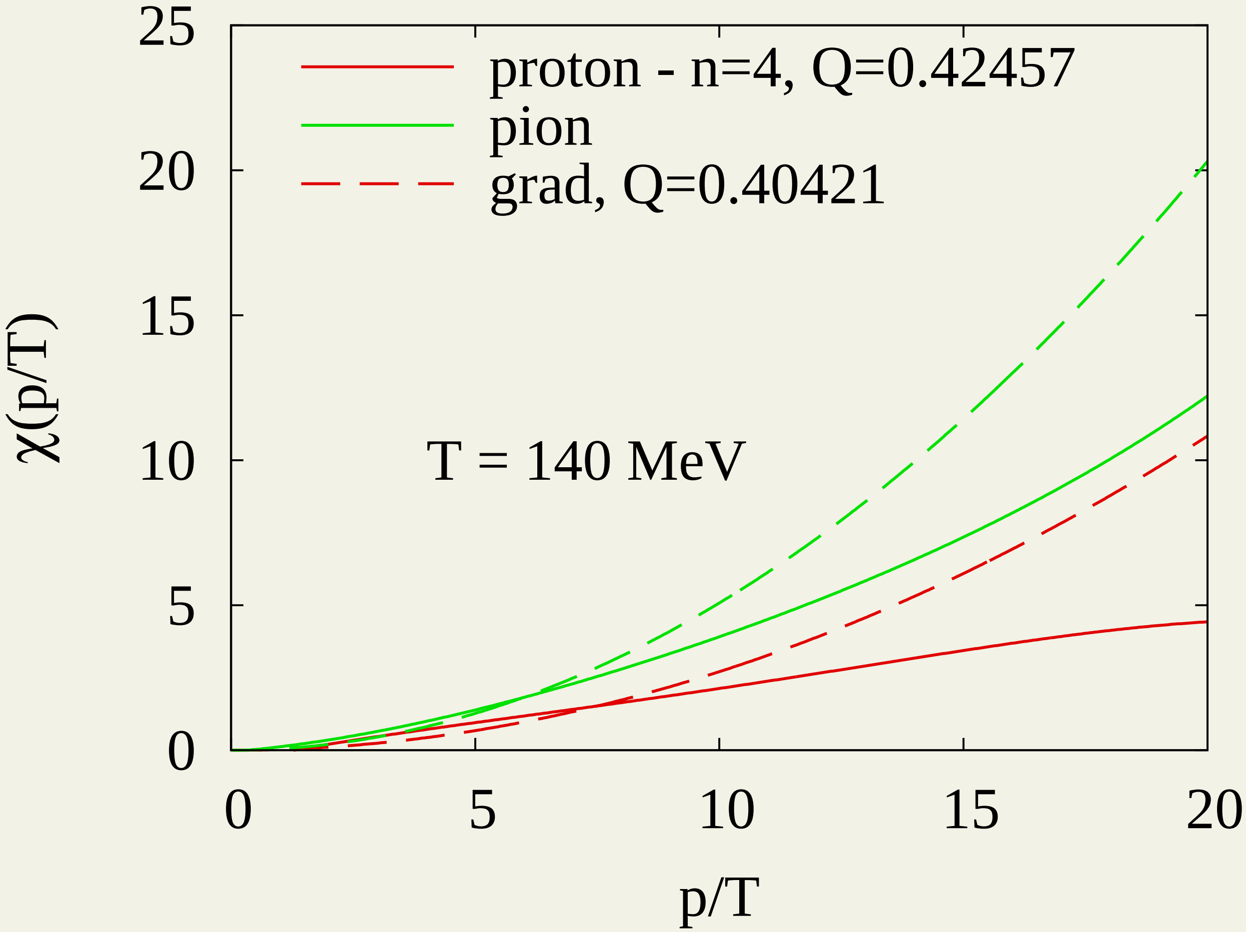

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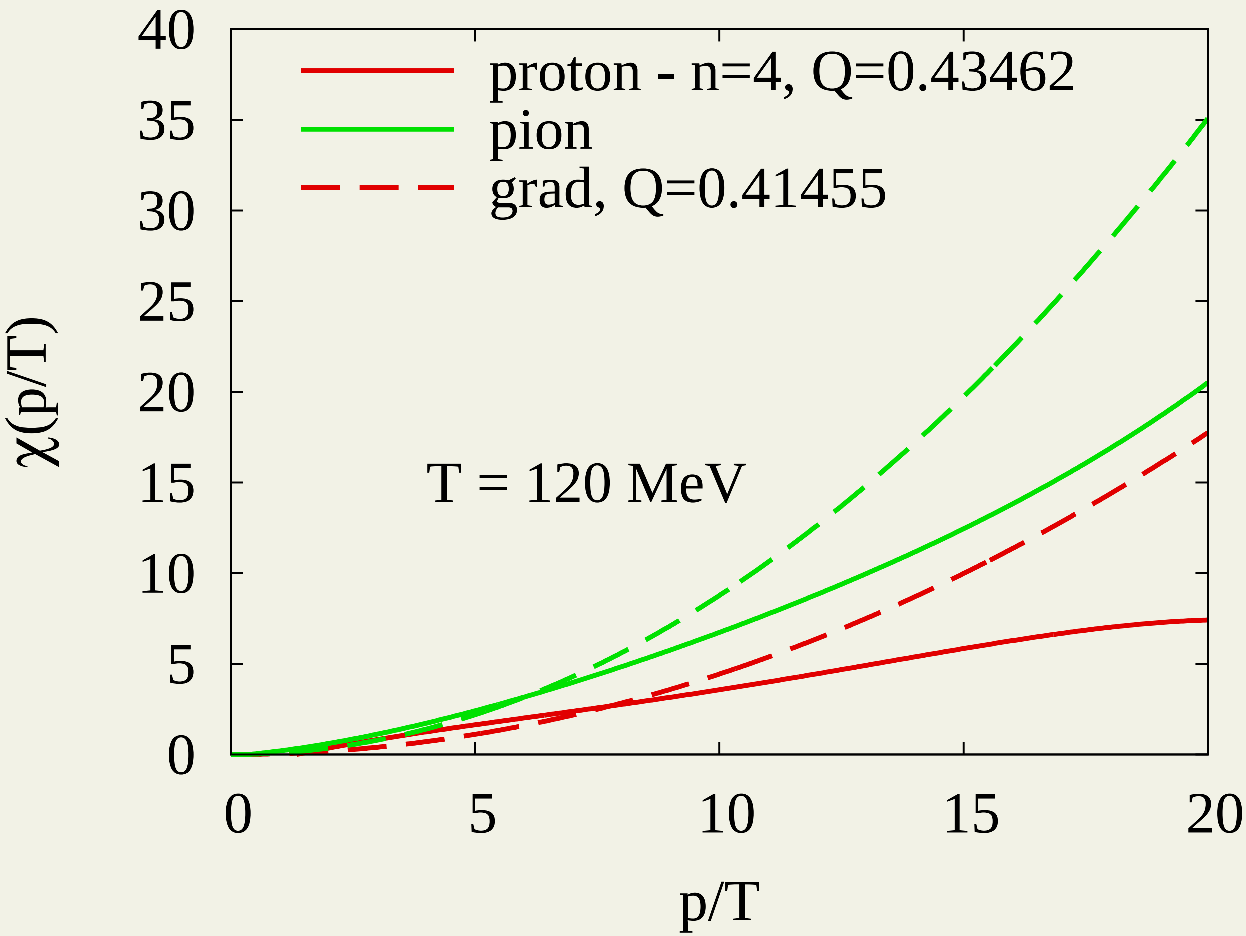


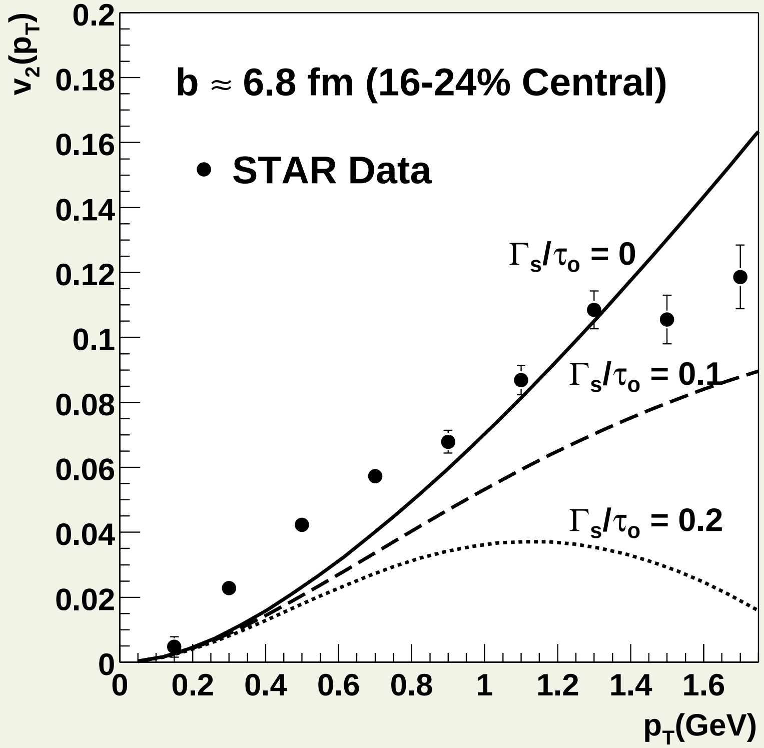
smaller correction for protons

slower than quadratic, proton almost linear

illustrate effect on $v_2(p_T)$ using Navier-Stokes shear stress estimate

$$\pi^{\mu\nu}_{NS} = \eta[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}(\partial u)]$$

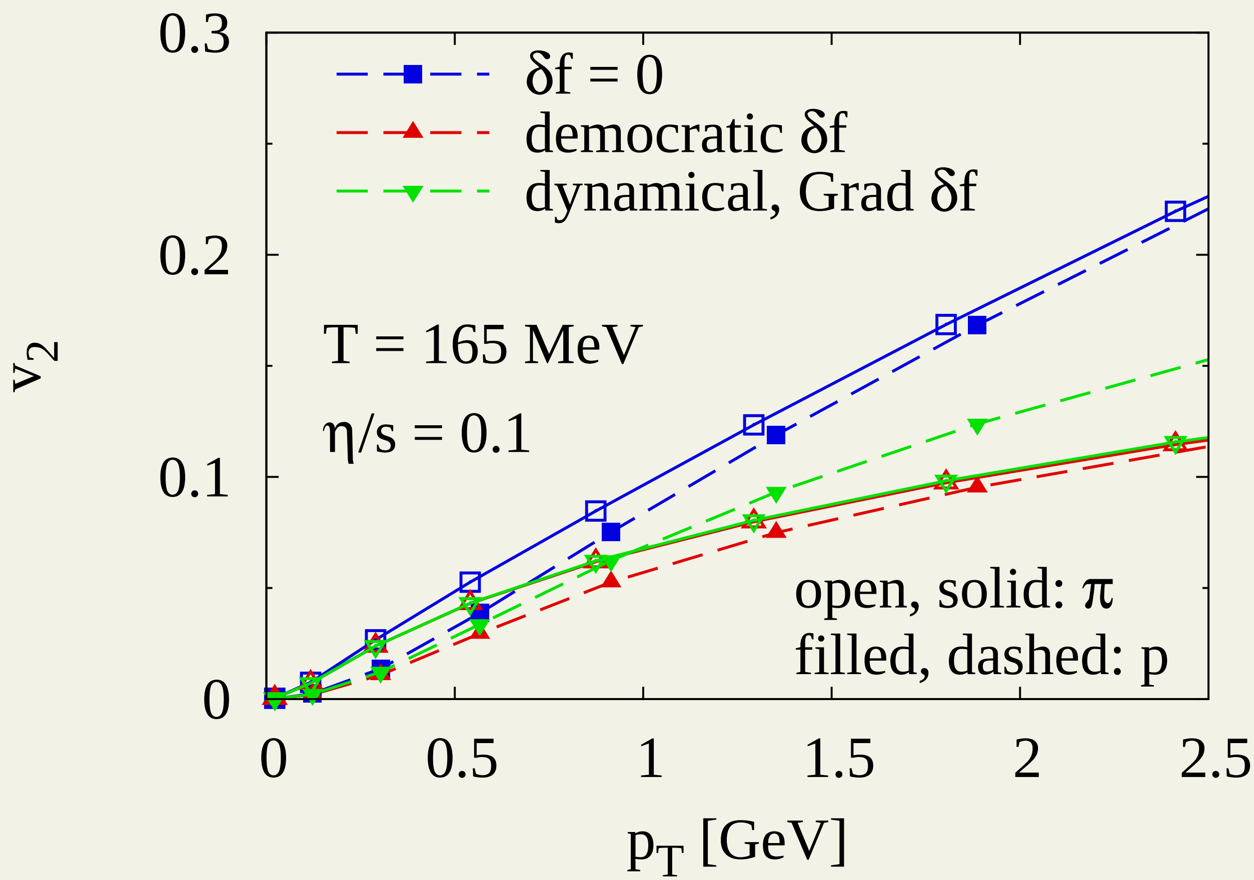
'a la' Teaney, PRC68 ('03)



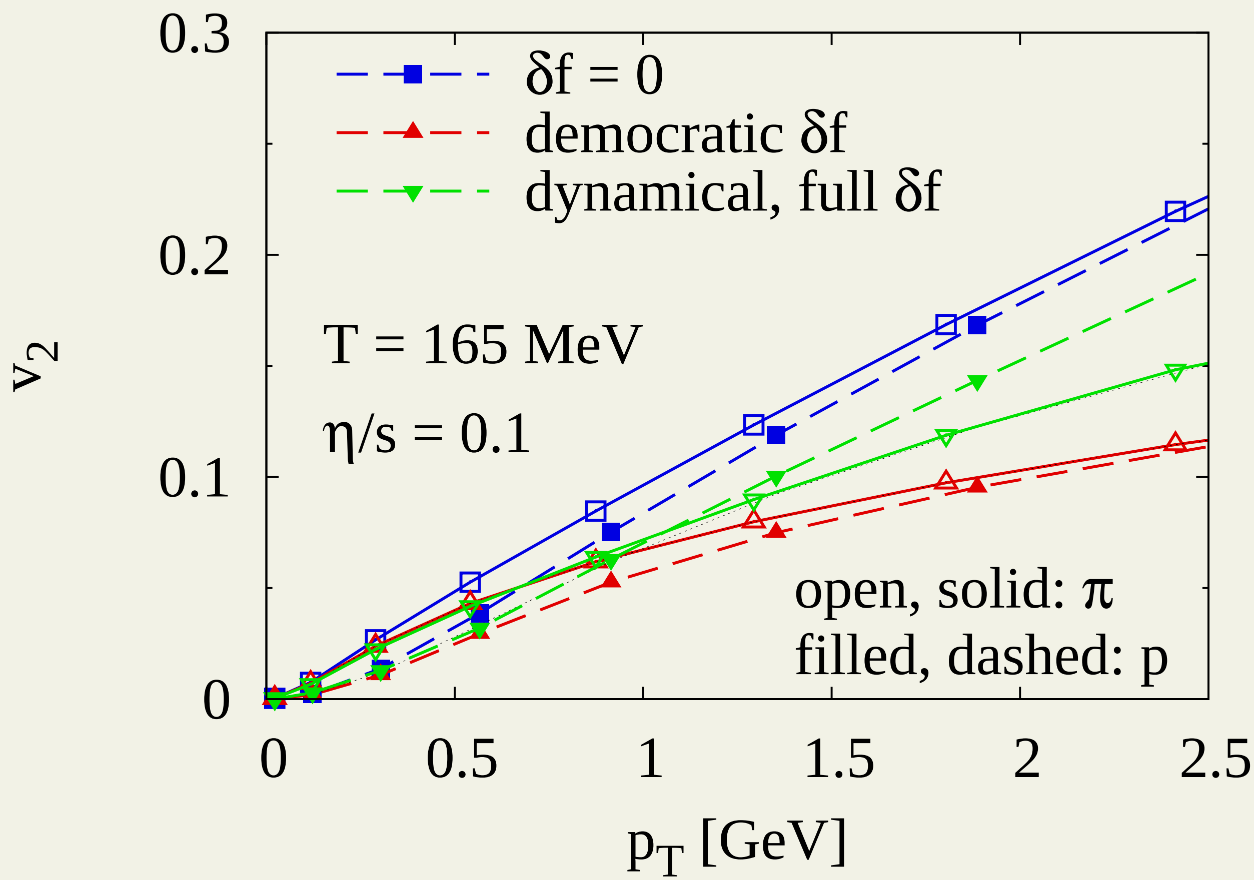
but with real hydro AZHYDRO-0.2p2 (instead of Blast wave)

(EOS s95p-v1, Glauber profile, $\tau_0 = 0.5$ fm)

Au+Au, b=7 fm - without resonance decays Dynamical GRAD $\delta f_i \propto C_i p^2$



Au+Au, b=7 fm - without resonance decays FULL dynamical $\delta f_i \propto \chi_i(p)$



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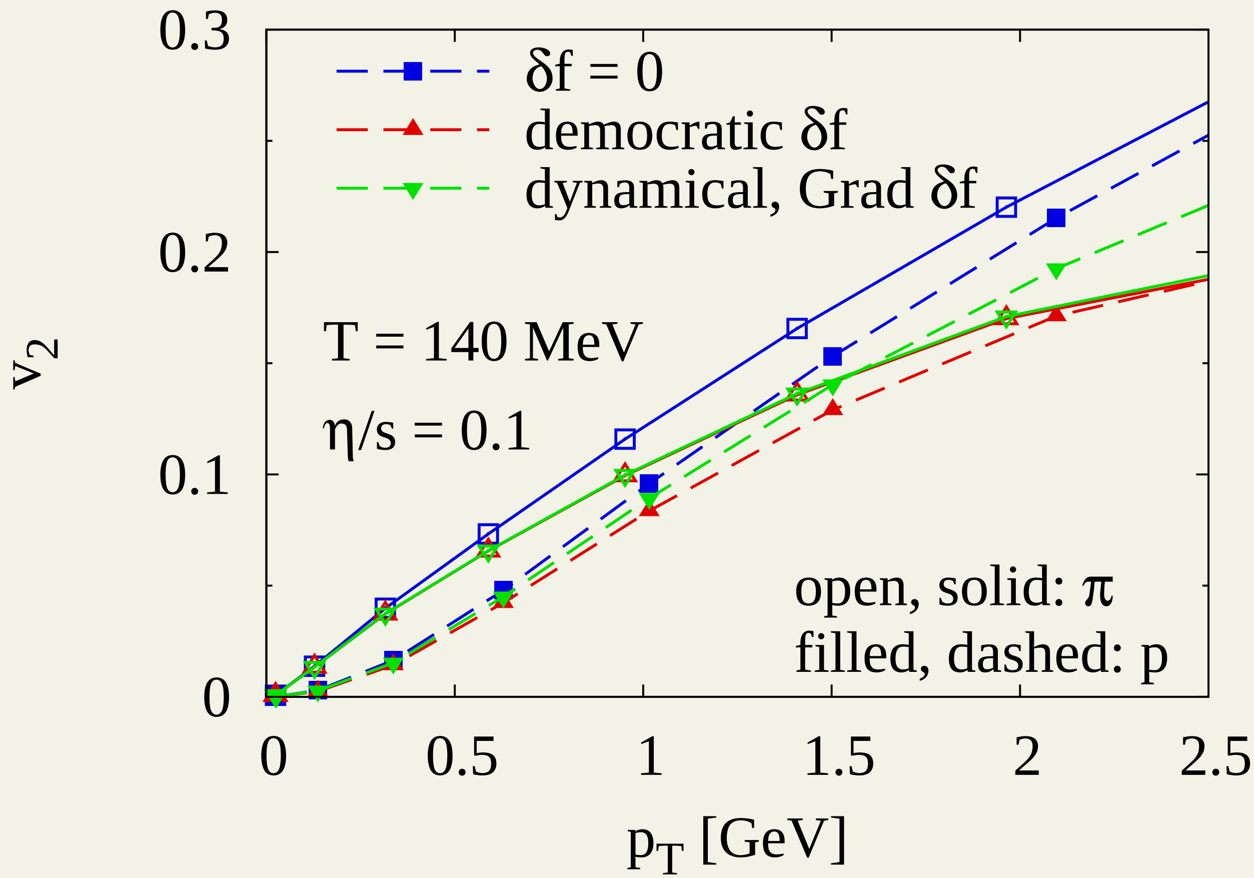

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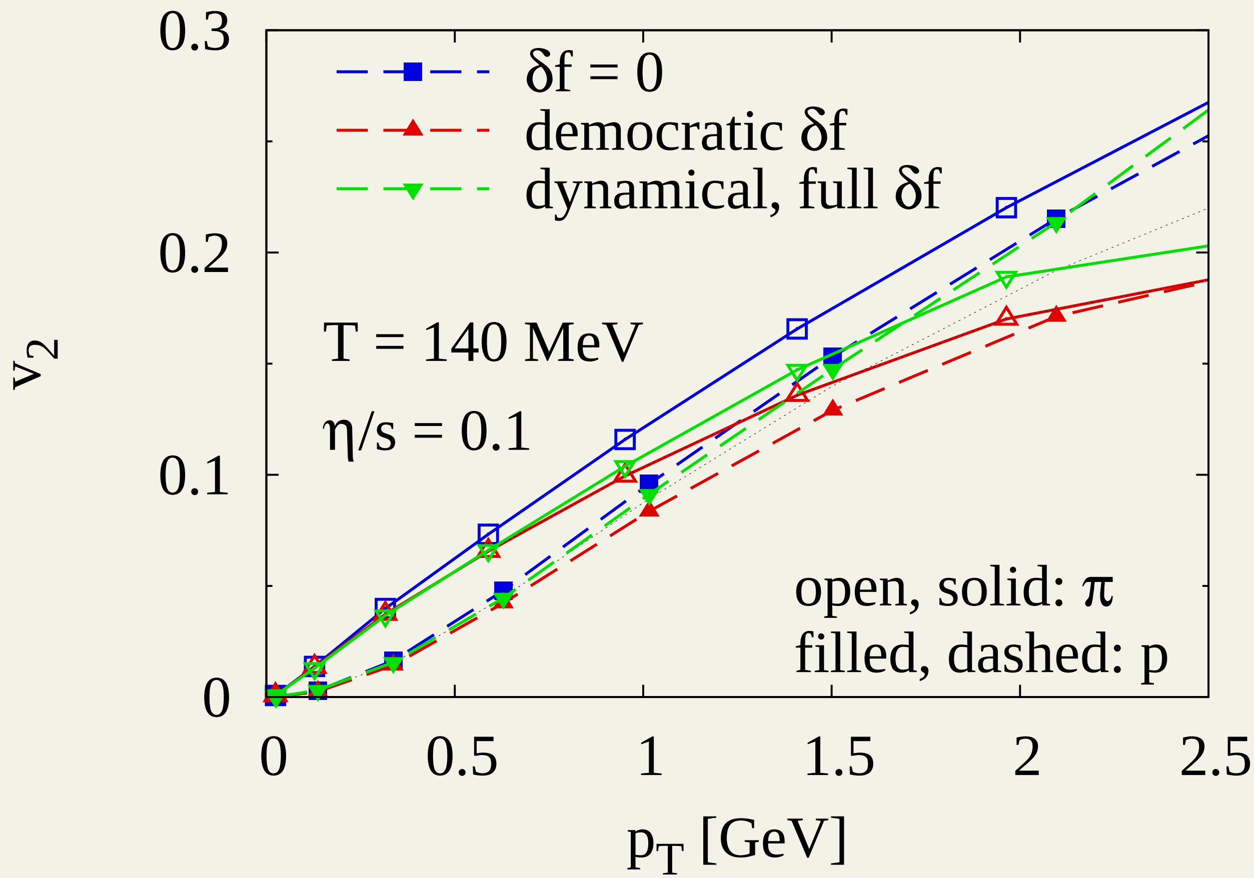

significant reduction of $\pi - p$ splitting at $T_{switch} = 140$ and 165 MeV

smaller viscous suppressions at higher momenta compared to Grad ansatz

Summary

- **We have a new, tested radiative $3 \leftrightarrow 2$ on-shell transport solver MPC/Grid. Results with pQCD matrix elements soon.**

- **Identical particle results from viscous hydrodynamics are commonly obtained with unphysical assumptions ("democratic Grad" ansatz), ignoring microscopic dynamics in hadron gas.**

  From a dynamical freezeout approach, viscous corrections for protons are smaller than for pions, and in general viscous effects are weaker at moderate momenta, than with the democratic Grad prescription.

- **more work to be done on both fronts**

  ... see update at Quark Matter 2012

$$\boldsymbol{E\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}}\textbf{!}$$