# Universal Aspects of Gauge Theories with Many Flavors

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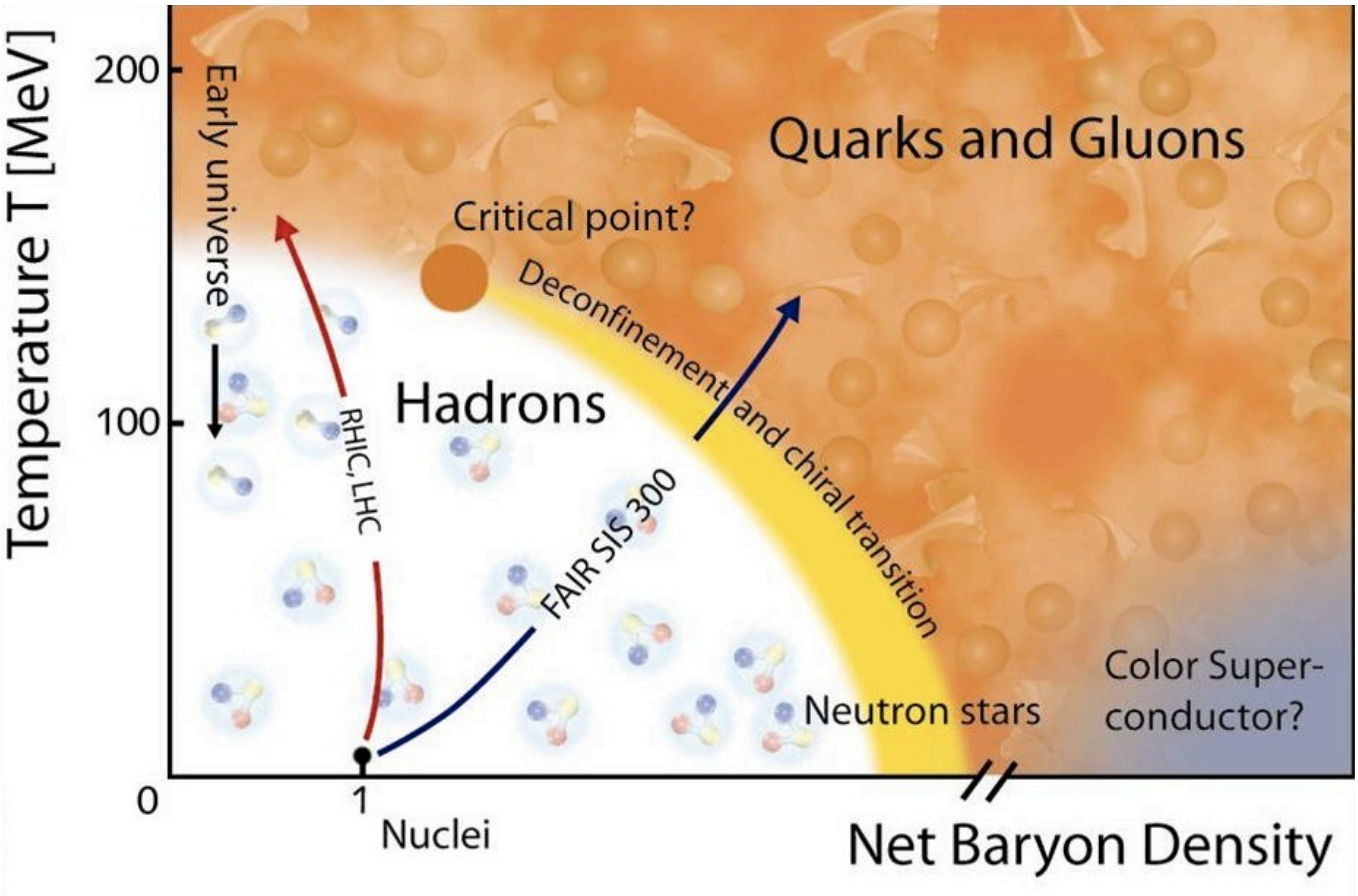
JB, J. Phys. G.: Nucl. Part. Phys. 39 (2012) 033001, arxiv:1108.4449;
JB, C. S. Fischer, H. Gies, Phys. Rev. D84:034045,2011;
JB, H. Gies, JHEP 1006:060, 2010; Phys. Lett. B645:52, 2007; JHEP 0606:024,2006;

FAIR, www.gsi.de



# Insights from Critical Behavior

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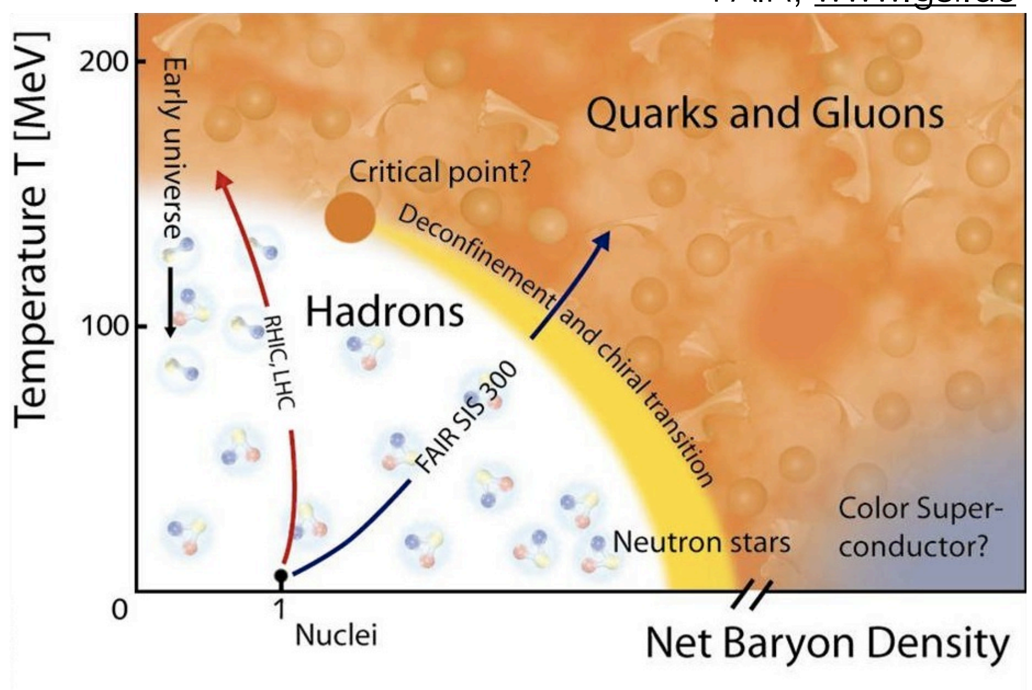
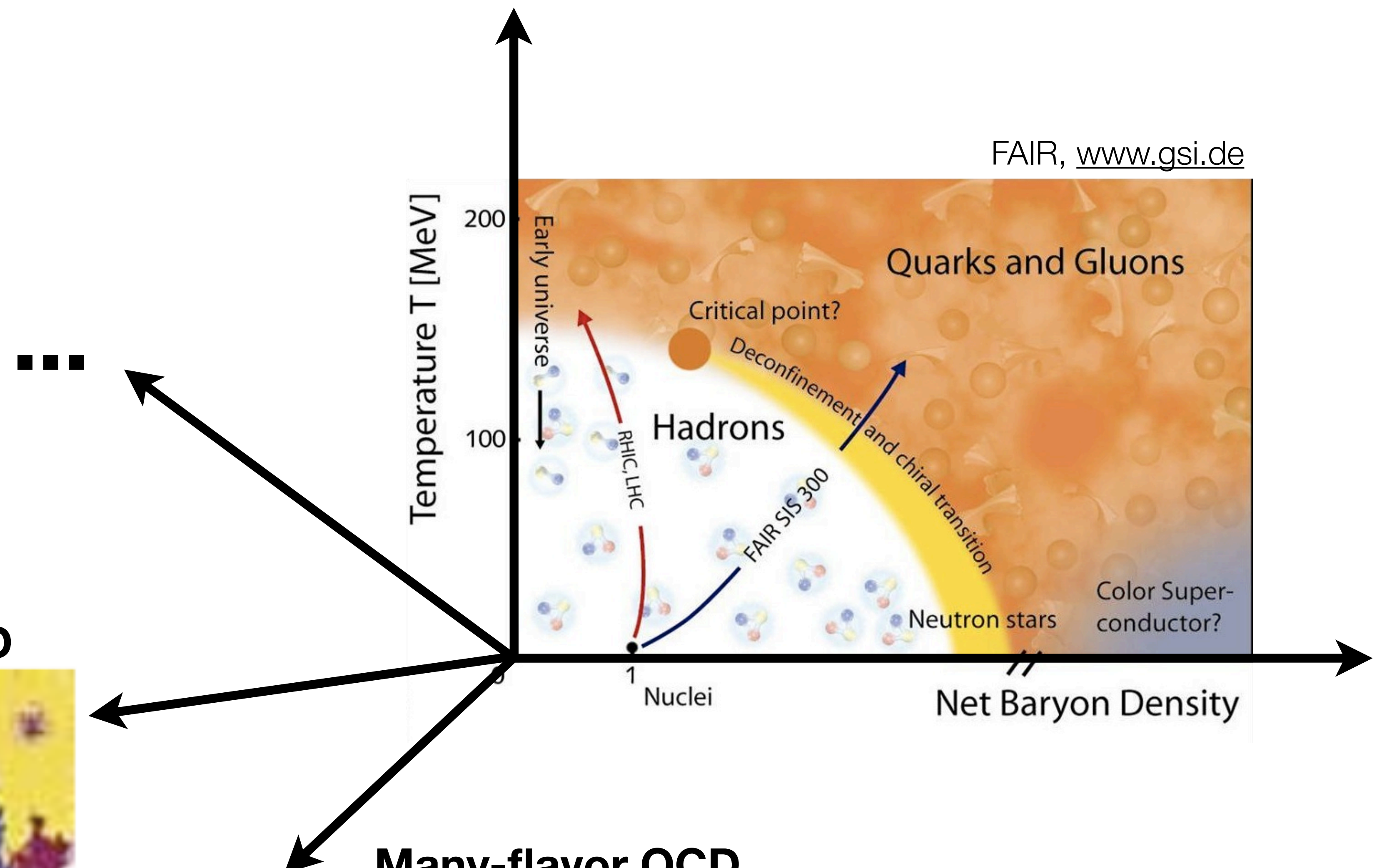



universal behavior close to, e. g, the chiral phase transition:

$$|\langle \bar{q} q \rangle|^{1/3} \sim |T - T_\chi|^\beta$$

(dynamics is governed by the critical exponents which are determined by the symmetries and the dimensionality)

# Phase transitions (may) occur in various “directions”




**Many-color QCD**

decoupling of gluons
and matter degrees of freedom

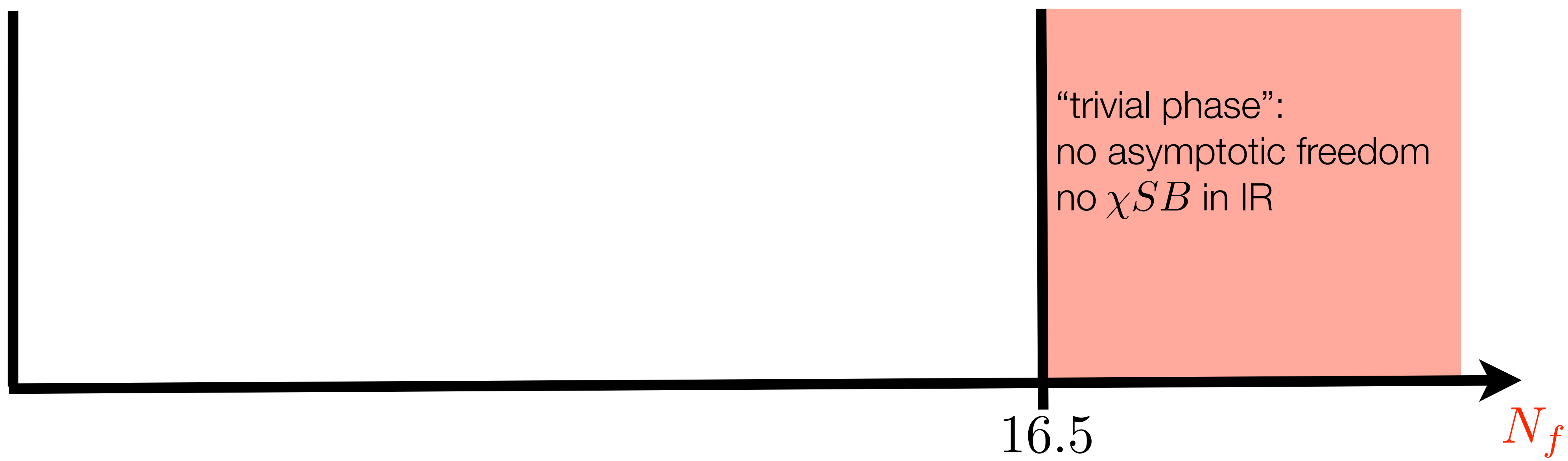
**Many-flavor QCD**

weak-coupling limit

**relevant for:**

- beyond standard model applications
(see e. g. Weinberg ’74, Appelquist ‘86, Sannino ’09)
- condensed-matter physics, e. g. graphene, QED3, ...
- QCD: for example, reveal mechanism for chiral symmetry breaking

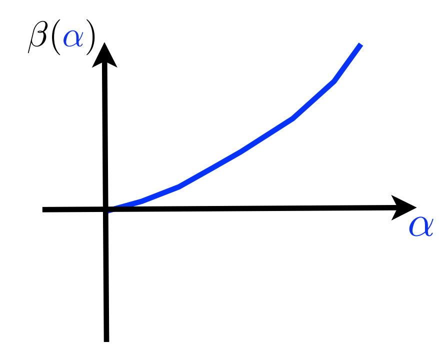
# Many-flavor QCD




- one-loop $\beta$-function

$$\partial_t \frac{g^2}{4\pi} = \partial_t \alpha \equiv \beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11N_c - 2N_f)}^{b_1}\,\alpha^2$$

- $b_1 < 0 \Longrightarrow N_f > \frac{11}{2} N_c \overset{N_c=3}{=} 16.5$ (QCD is NOT asymptotically free)



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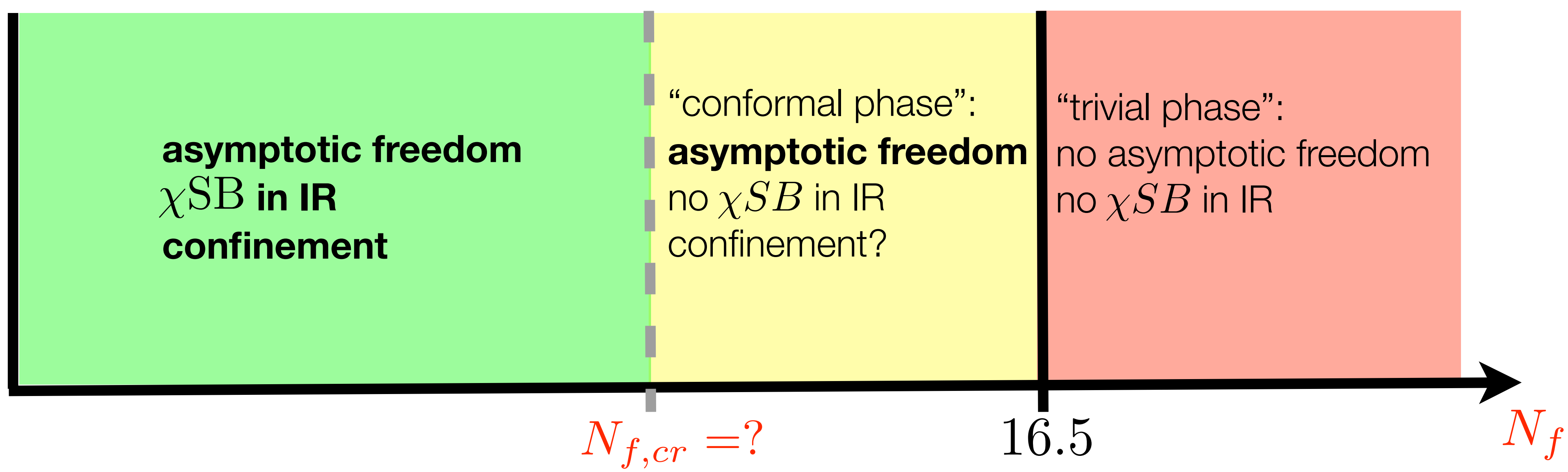


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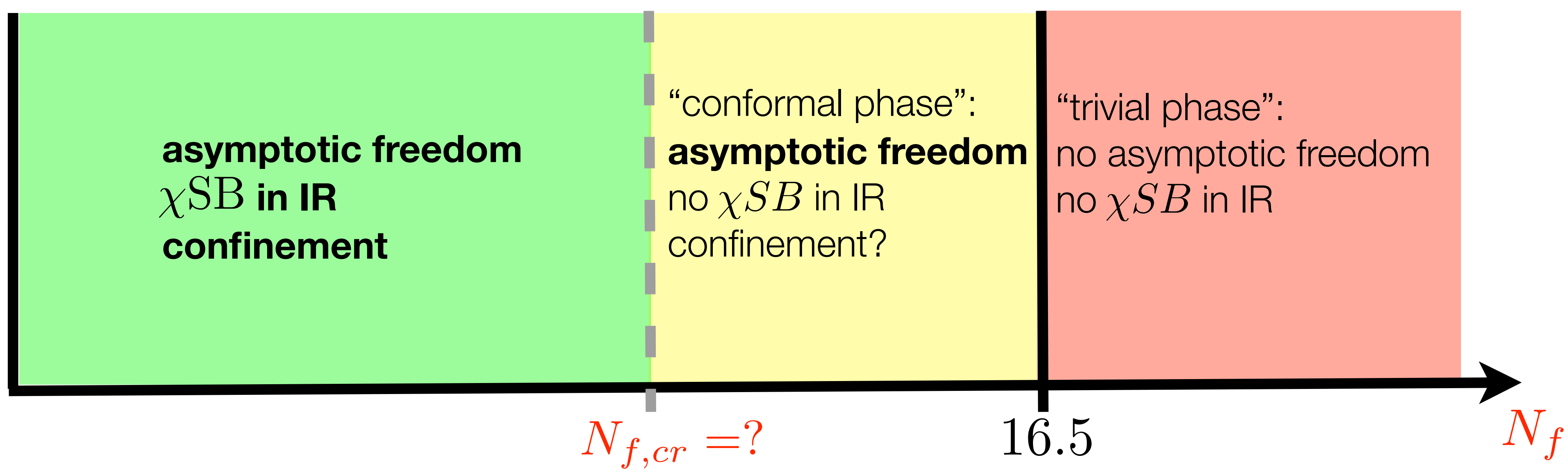
- $b_1 > 0$: QCD is asymptotically free

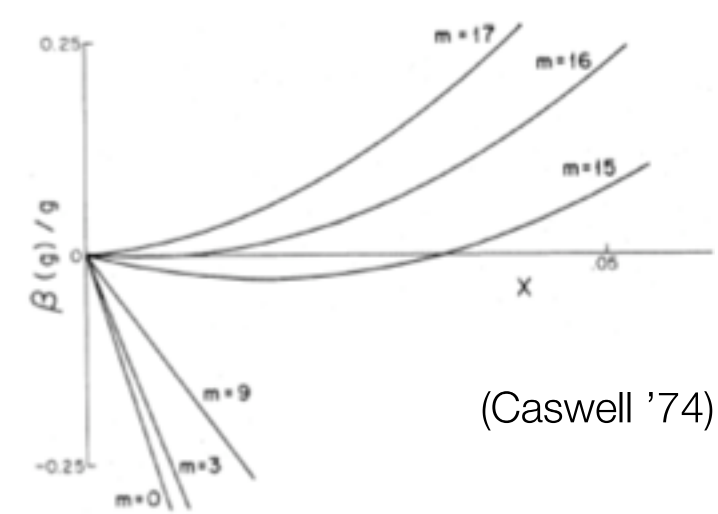
# Many-flavor QCD




- two-loop $\beta$-function

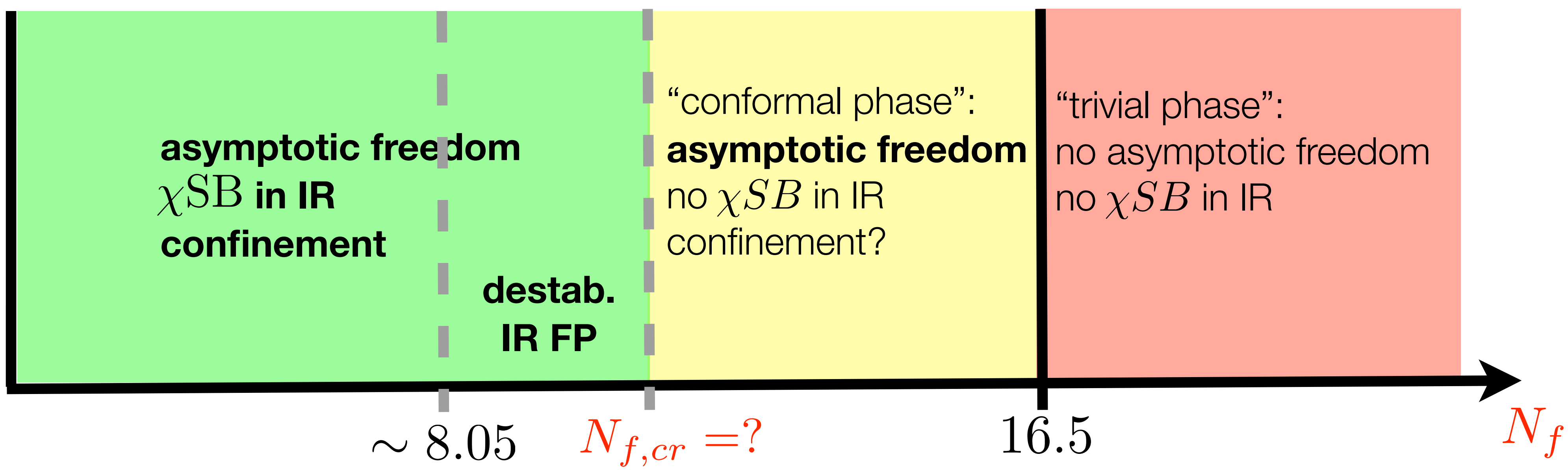
$$\partial_t \alpha \equiv \beta(\alpha) = -\overbrace{\frac{1}{6\pi}(11N_c - 2N_f)}^{b_1}\,\alpha^2 - \overbrace{\frac{1}{8\pi^2}\left(\frac{34N_c^3 + 3N_f - 13N_c^2 N_f}{3N_c}\right)}^{b_2}\,\alpha^3$$

- non-trivial infrared fixed point $\alpha_*$ for $8.05 \lesssim N_f < 16.5$
(Caswell '74; Banks & Zaks '82)




(Caswell '74)

# Many-flavor QCD

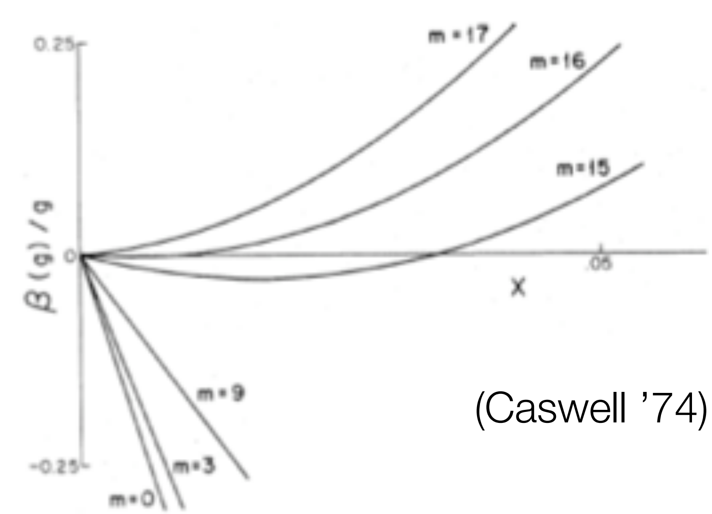



• Caswell-Banks-Zaks fixed gets destabilized due to **chiral symmetry breaking:**

$$g^2 > g^2_{\rm cr} \text{ : fermions acquire mass, i. e. } N_f^{\rm eff.} \to 0$$

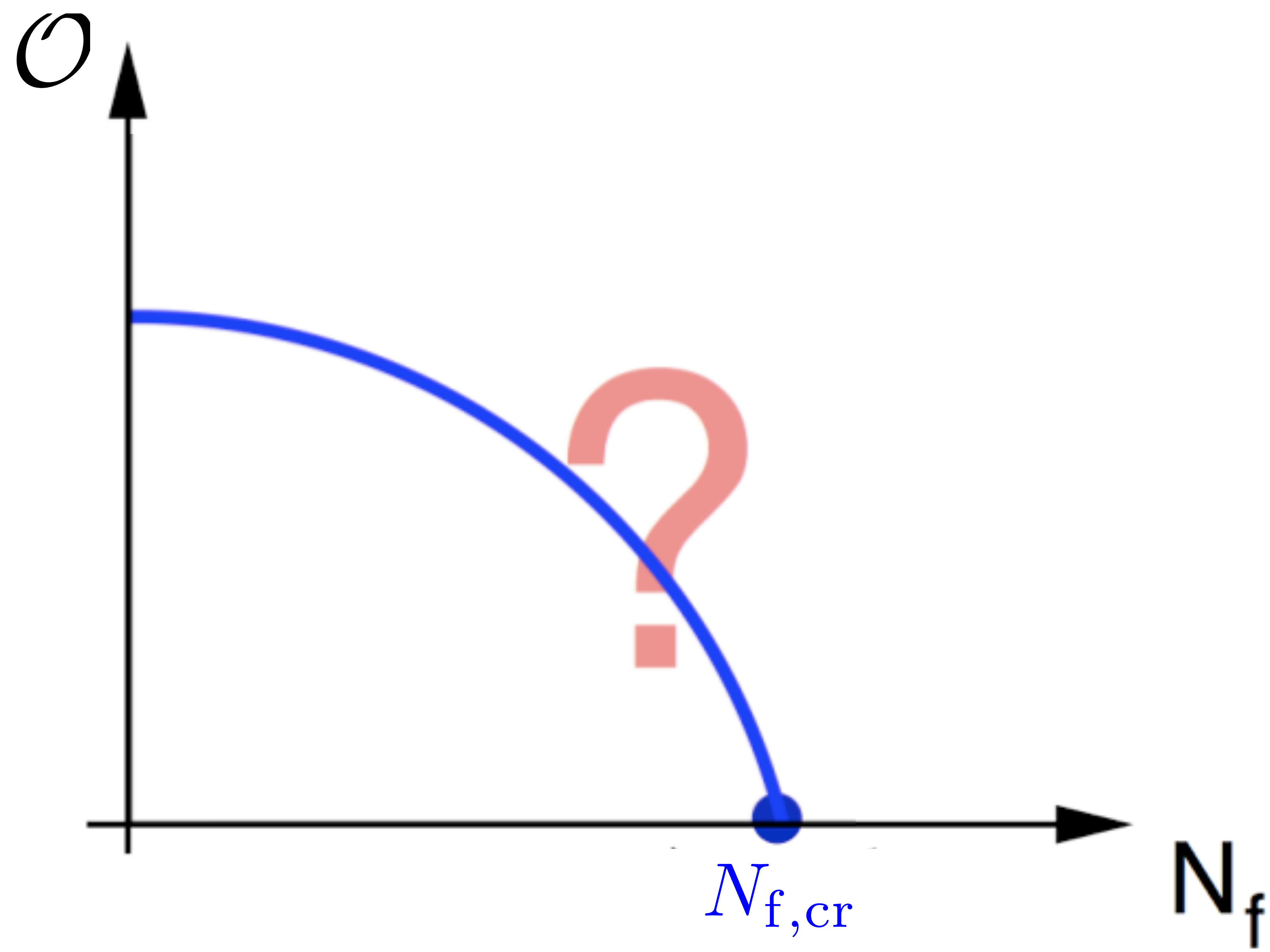
• lower end of the **conformal window** is determined by the onset of **chiral symmetry breaking**

• cf. **quantum phase transition (QPT)** in 3d QED
(R. D. Pisarski '84)




(Caswell '74)

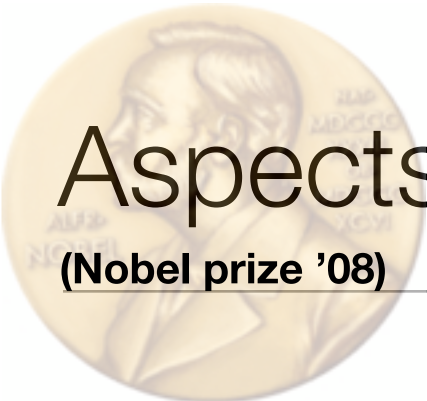
# How do chiral observables scale close to the QPT?




scaling of observables $\mathcal{O} = f_\pi\,, \langle\bar\psi\psi\rangle\,, T_\chi\,, \ldots$ in gauge theories with many fermions, such as $\mathrm{QED}_3$, QCD, ...?

# Outline

- Introduction
- Scaling behavior of QCD with many flavors
  - Miransky scaling
  - Beyond Miransky scaling
- Scaling in the $(N_{\rm c}, N_{\rm f})$-plane
- Conclusions and outlook



# Aspects of the NJL model: Brief Reminder

**(Nobel prize '08)**

- classical action of the NJL model:

$$S = \int_x \{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\lambda}_\sigma [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] \}$$

- spontaneous symmetry breaking if quark condensate is non-vanishing: $\langle \bar{\psi}\psi \rangle \neq 0$

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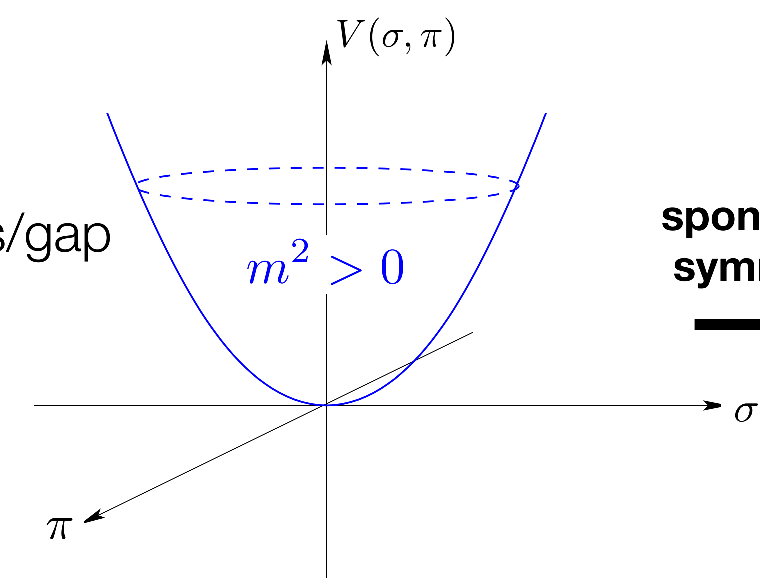
- bosonization of the NJL model yields $(\sigma = -2\bar{\lambda}_\sigma \bar{\psi}\psi\,, \pi = -2\bar{\lambda}_\sigma \bar{\psi}\gamma_5\psi)$

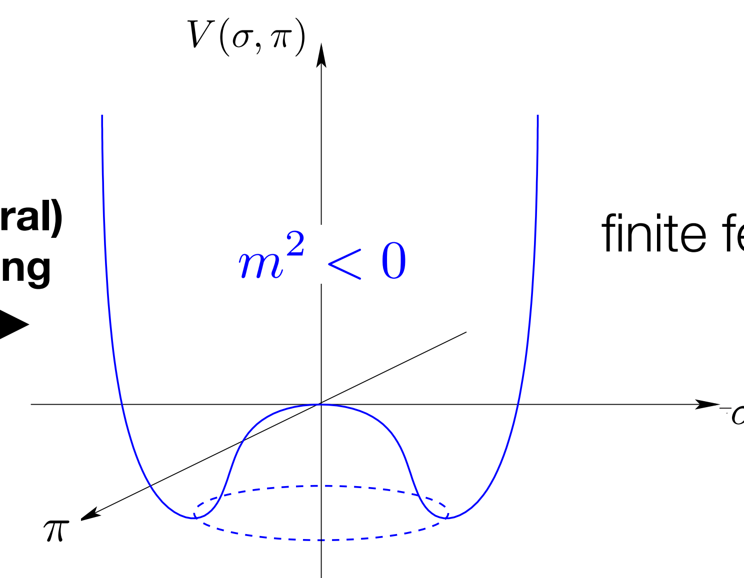
$$S = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \bar{\psi}(\sigma + \mathrm{i}\gamma_5 \pi)\psi - \frac{1}{\bar{\lambda}_\sigma}(\sigma^2 + \pi^2) \right\}$$

⟹ $\bar{\lambda}_\sigma$ is inverse proportional to the scalar mass parameter, $m^2 \propto \frac{1}{\bar{\lambda}_\sigma}$

no fermion mass/gap
$\langle \bar{\psi}\psi \rangle = 0$



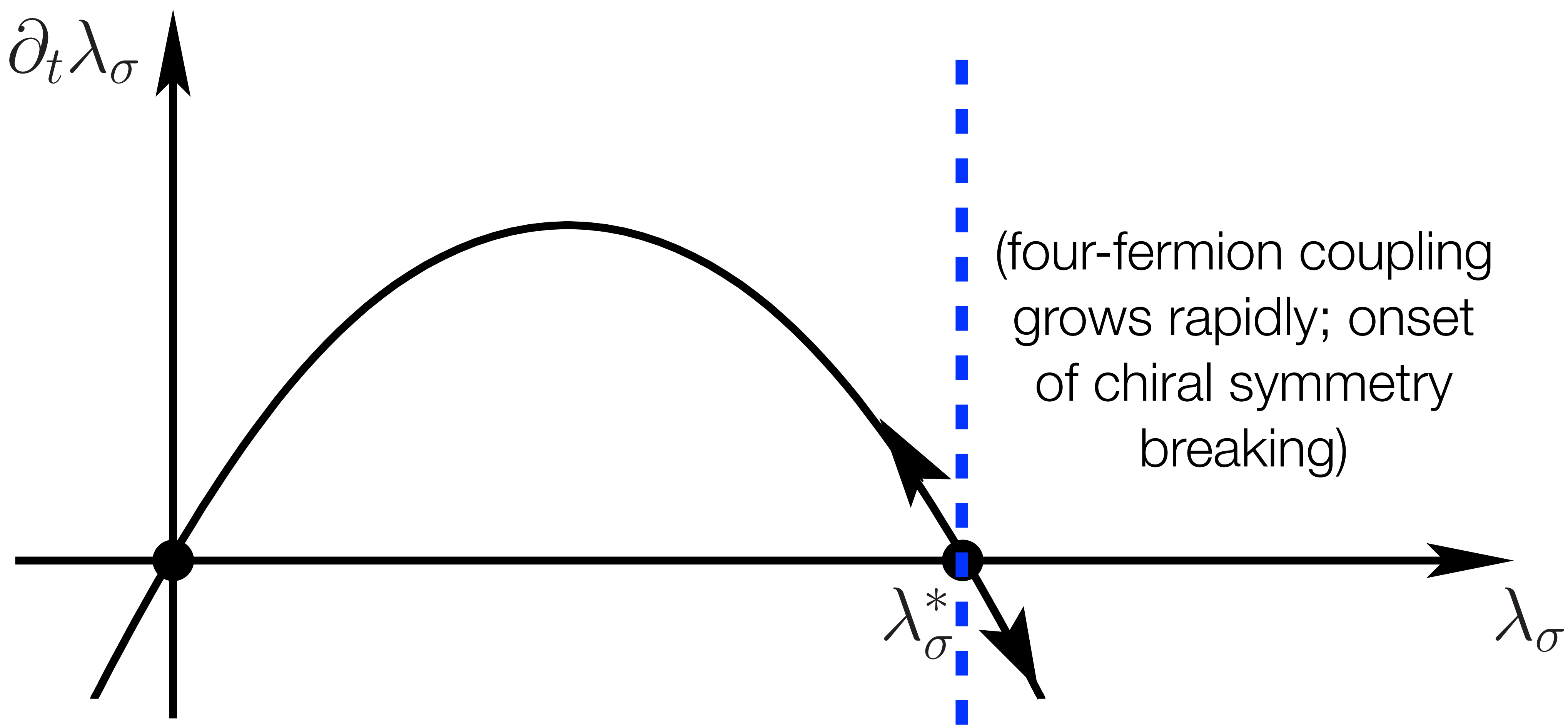

**spontaneous (chiral) symmetry breaking** ⟶



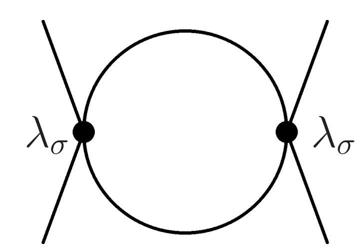

finite fermion mass/gap
$\langle \bar{\psi}\psi \rangle \neq 0$

phase transition can be induced by
changing an "external" parameter, e. g. the temperature, **number of flavors**, ...

# NJL model: RG flow of Four-Fermion Interactions

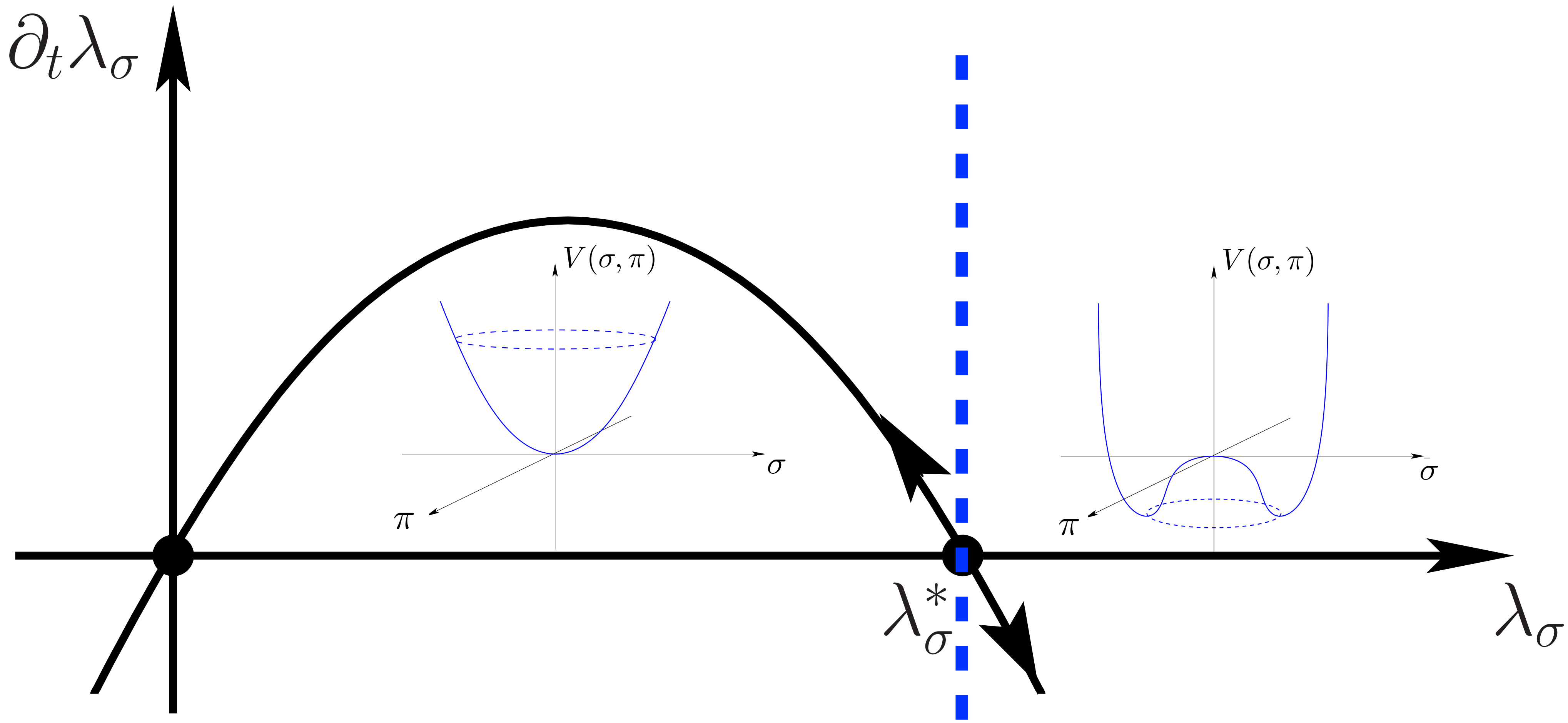



$$\partial_t\lambda_\sigma \equiv k\partial_k\lambda_\sigma \simeq 2\lambda_\sigma -$$



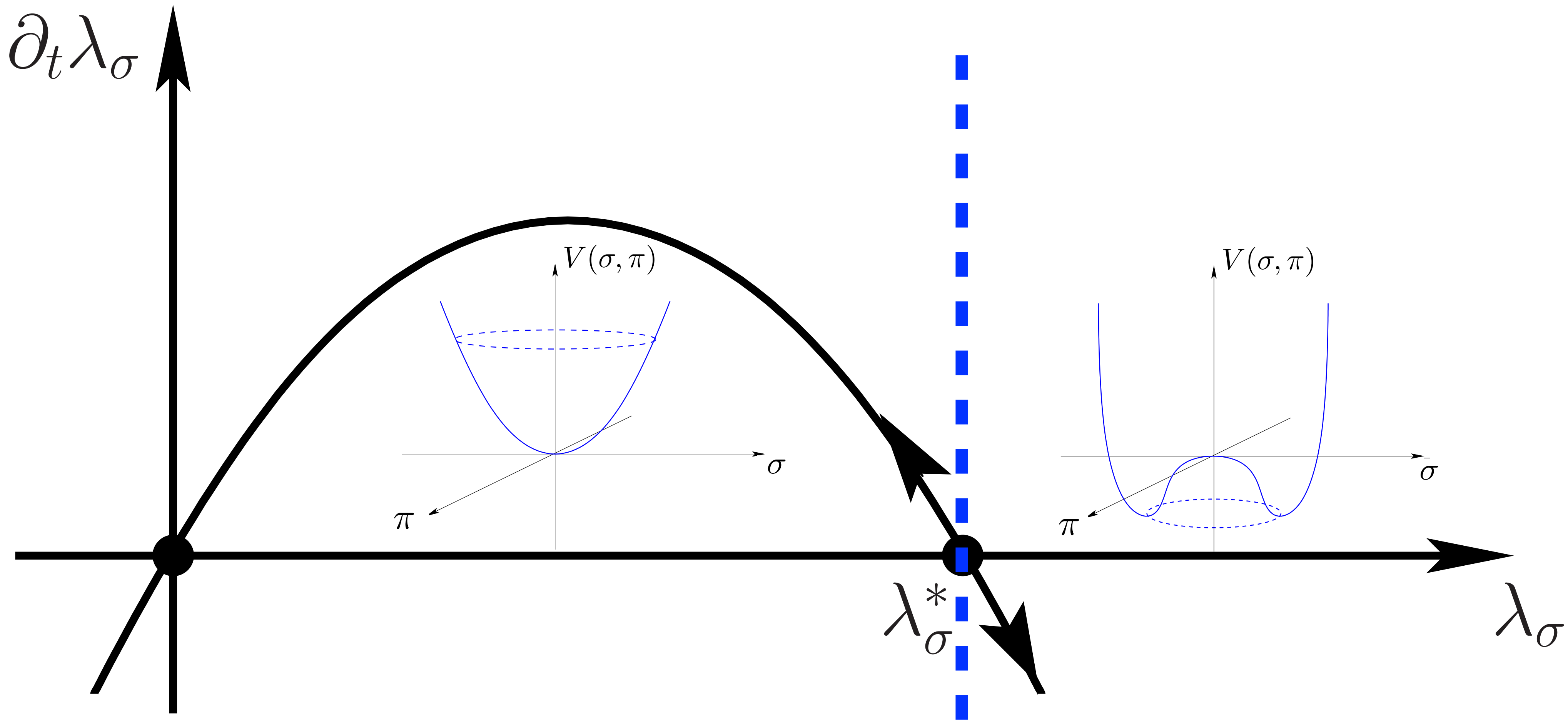

k is the Renormalization Group (RG) scale;
k~p ("momentum scale")

# NJL model: RG flow of Four-Fermion Interactions




initial condition of the differential equation determines whether chiral symmetry is spontaneously broken or not

# NJL model: RG flow of Four-Fermion Interactions




scale for (chiral) low-energy observables is set by the scale $k_{\rm SB}$ at which the four-fermion coupling **diverges**, $1/\lambda_\sigma(k_{\rm SB}) = 0$ :

$$m_{\rm f} \sim k_{\rm SB}\,, \quad f_\pi \sim k_{\rm SB}\,, \quad |\langle\bar\psi\psi\rangle|^{\frac{1}{3}} \sim k_{\rm SB}\,, \quad T_\chi \sim k_{\rm SB}\,, \ldots$$

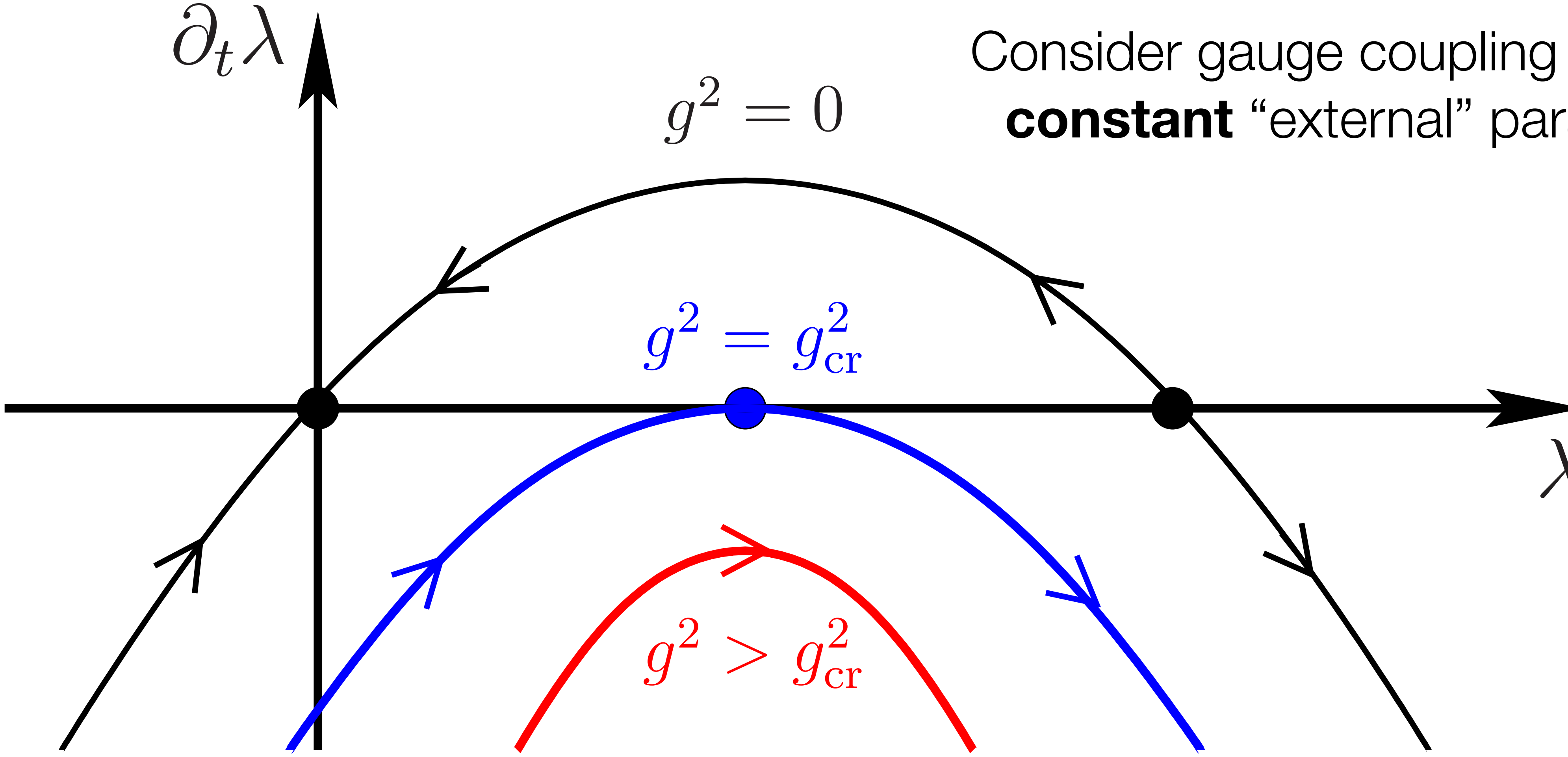
# Role of gauge degrees of freedom?

$$S = \int_x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \left( \mathrm{i}\partial\!\!\!/ - g A\!\!\!/ \right) \psi + \lambda (\bar{\psi} \mathcal{O} \psi)^2 + \dots \right\}$$

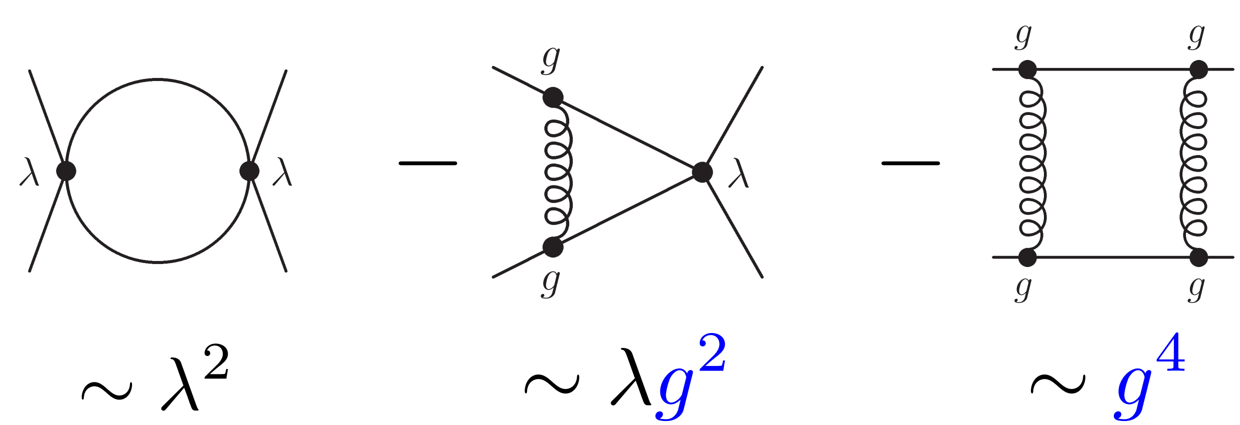
# RG Flow of Four-Fermion Interactions in QCD

(H. Gies & J. Jaeckel ’05; JB & H. Gies ’05, ’06)

Consider gauge coupling $g^2$ as a **constant** “external” parameter




$$\partial_t \lambda \equiv k\partial_k \lambda \simeq 2\lambda -$$



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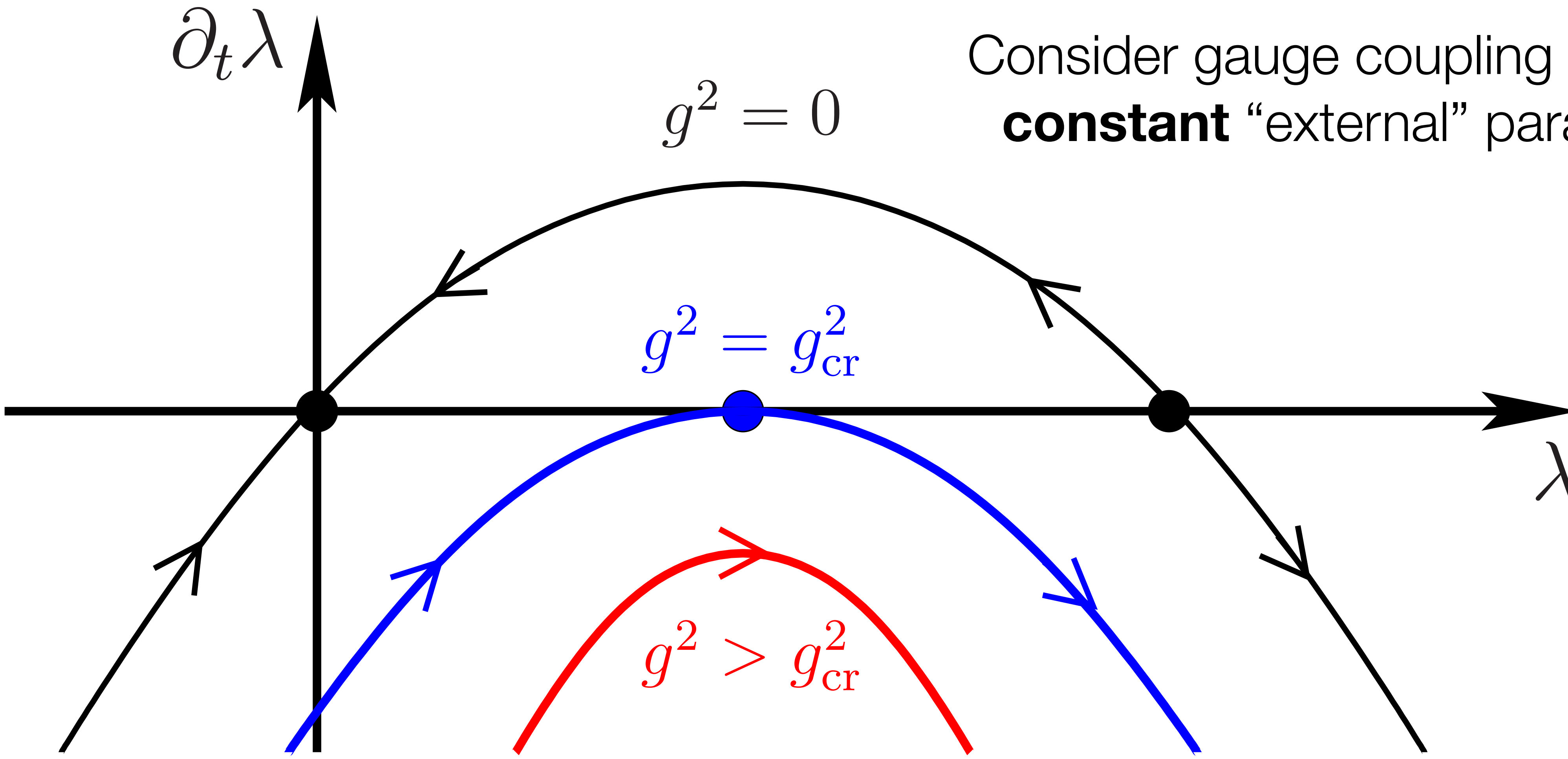
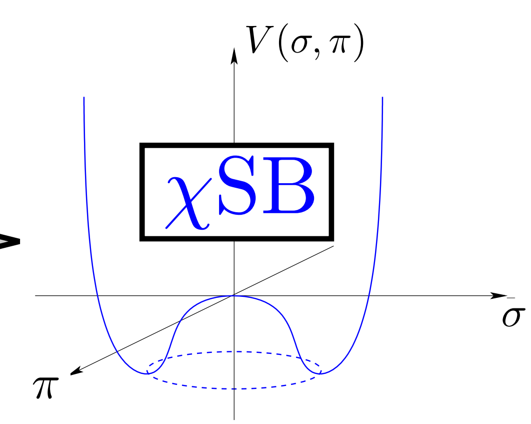



Consider gauge coupling $g^2$ as a **constant** "external" parameter

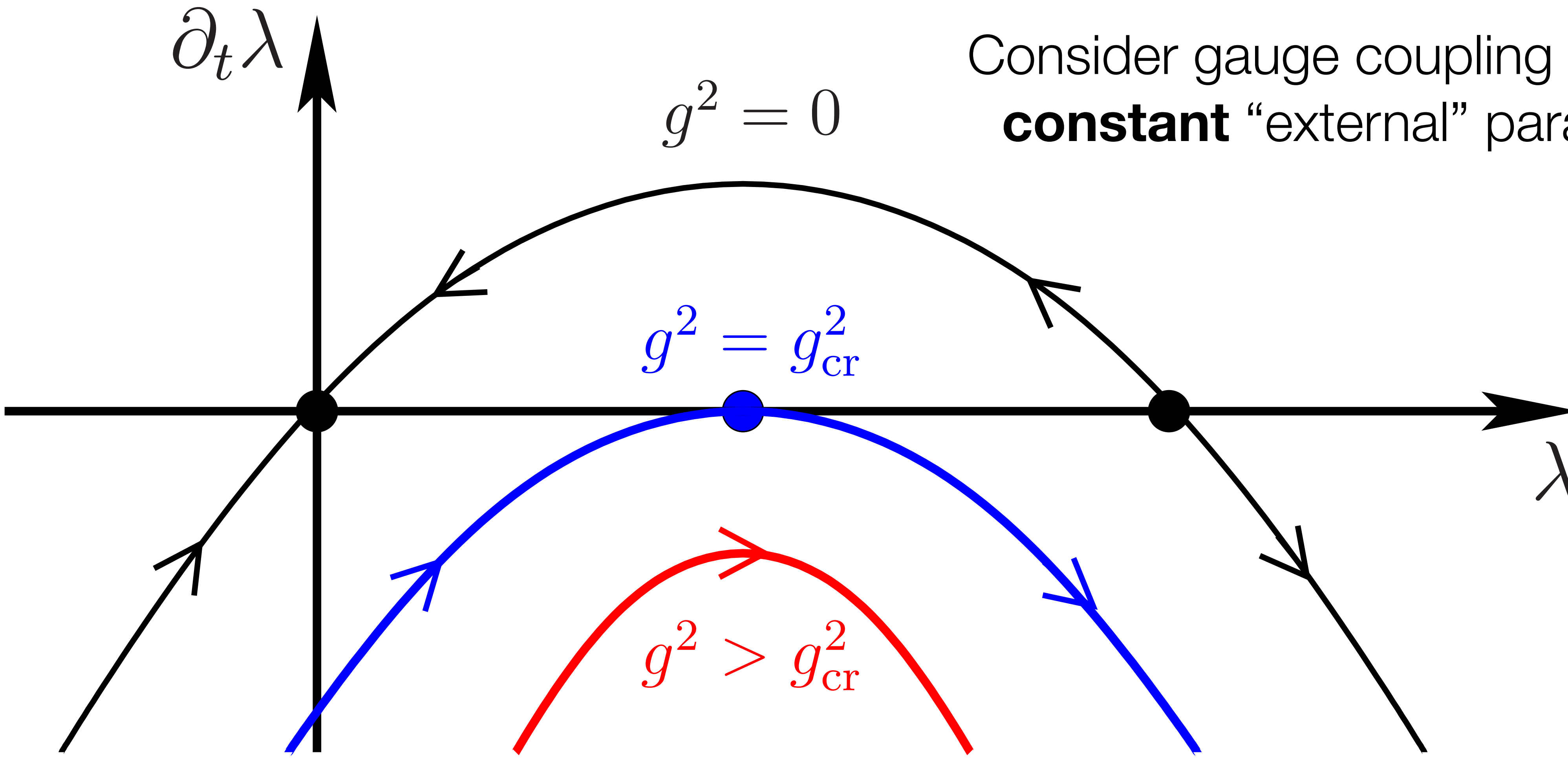
**critical** gauge coupling $g^2_{\rm cr}$ :

if $g^2 > g^2_{\rm cr}$ ⟹ **no** fixed points ($\lambda \to \infty$) ⟹



# RG Flow of Four-Fermion Interactions in QCD

(Kosterlitz '74, Miransky '85, Kaplan et al. '09;  JB, C. S. Fischer H. Gies, '10)




Consider gauge coupling $g^2$ as a **constant** “external” parameter

**symmetry breaking scale**

$$k_{\rm SB} \propto \Lambda \theta(g^2 - g^2_{\rm cr}) \exp\left( - \frac{\rm const.}{\sqrt{g^2 - g^2_{\rm cr}}} \right)$$

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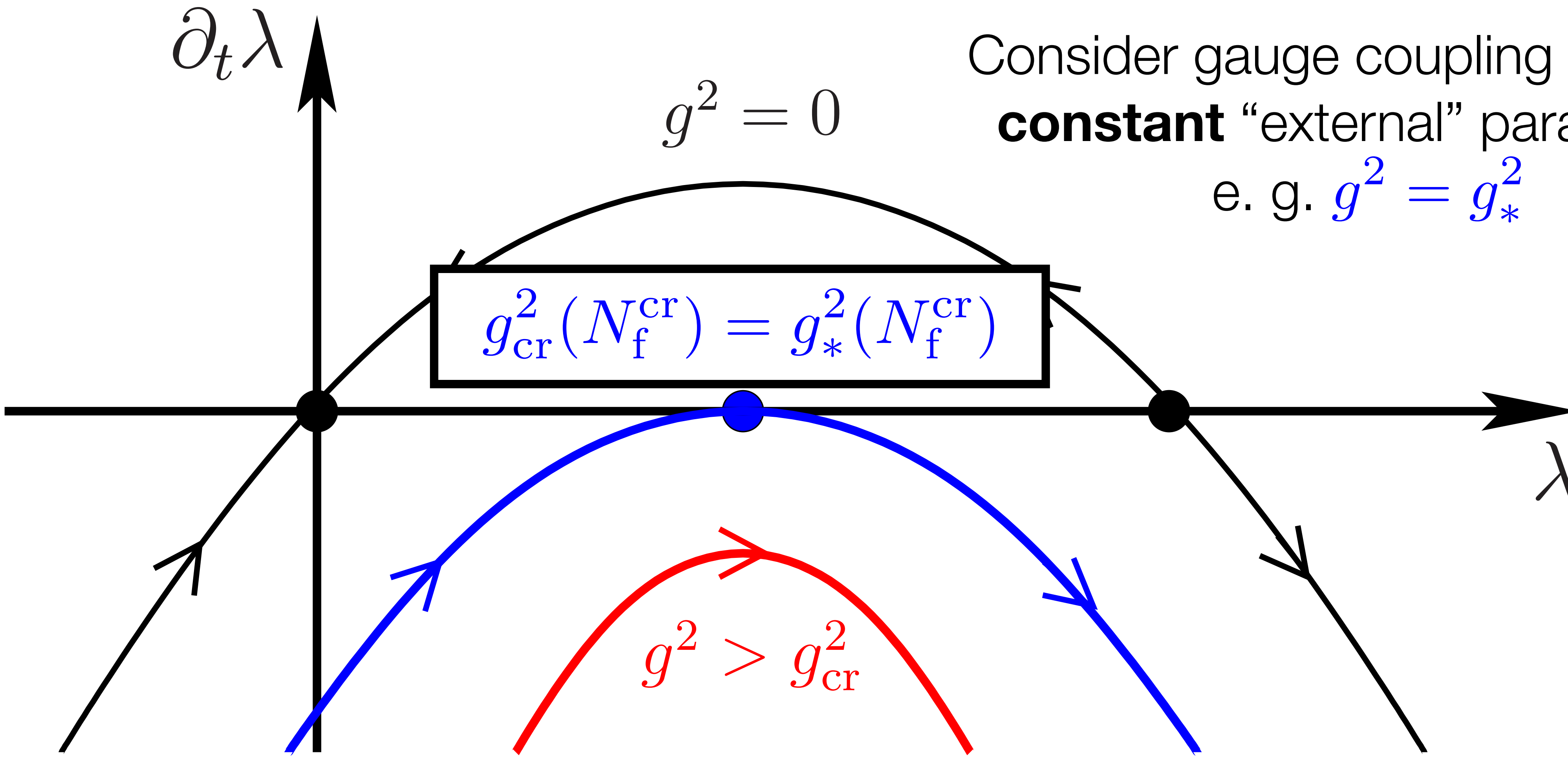


Consider gauge coupling $g^2$ as a **constant** “external” parameter, e. g. $g^2 = g^2_*$

**symmetry breaking scale**

$$k_{\mathrm{SB}} \propto \Lambda \theta(N_{\mathrm{f}}^{\mathrm{cr}} - N_{\mathrm{f}}) \exp\left(-\frac{\mathrm{const.}}{\sqrt{|N_{\mathrm{f}}^{\mathrm{cr}} - N_{\mathrm{f}}|}}\right)$$

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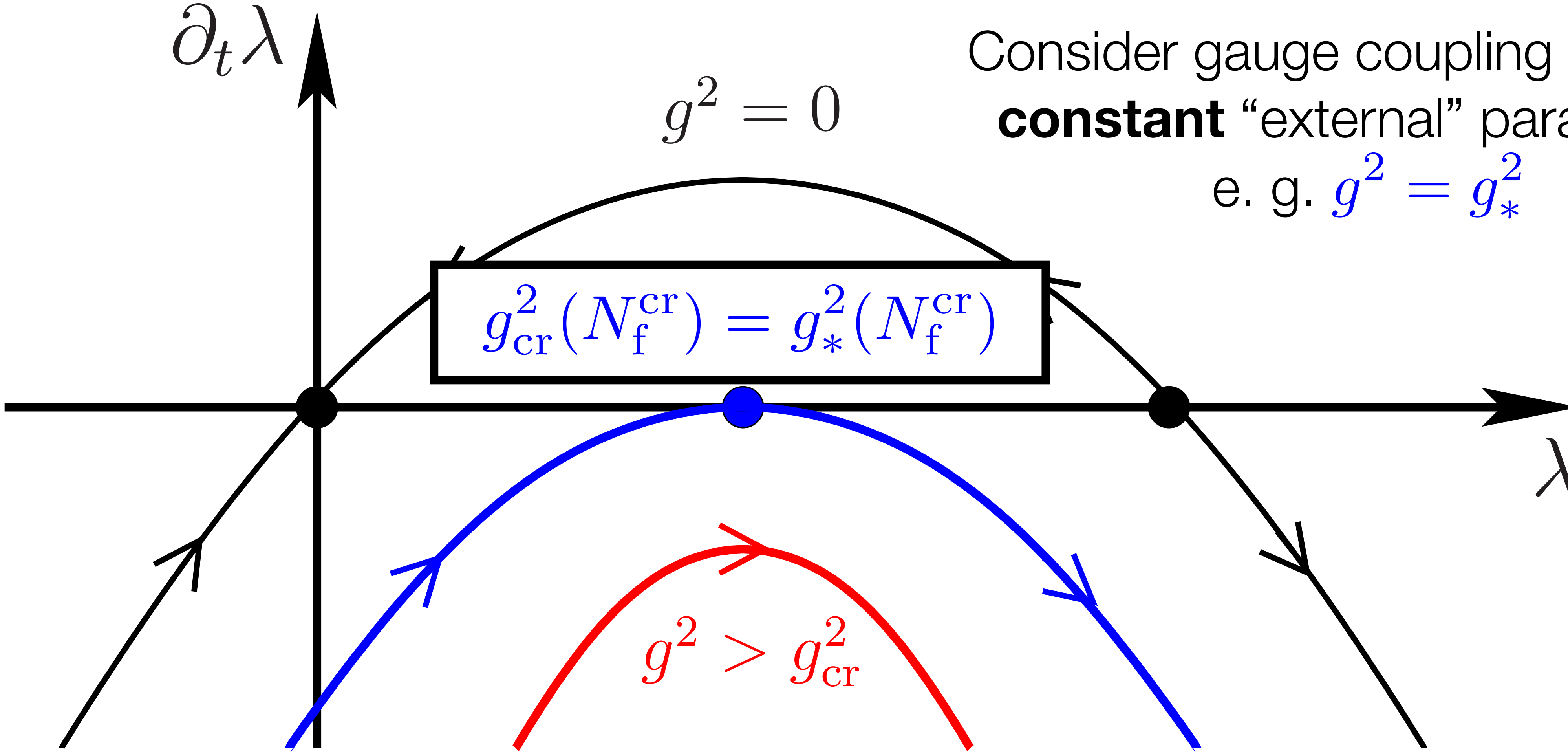


Consider gauge coupling $g^2$ as a **constant** “external” parameter, e. g. $g^2 = g^2_*$

**scaling of (chiral) observables close the quantum phase transition:**

$$\mathcal{O} \sim k_{\rm SB} \propto \Lambda \theta(N_{\rm f}^{\rm cr} - N_{\rm f}) \exp\left( - \frac{\rm const.}{\sqrt{|N_{\rm f}^{\rm cr} - N_{\rm f}|}} \right)$$

In general, the gauge coupling is **not** a **constant** “external” parameter ...

How does this change the well-known scaling behavior?

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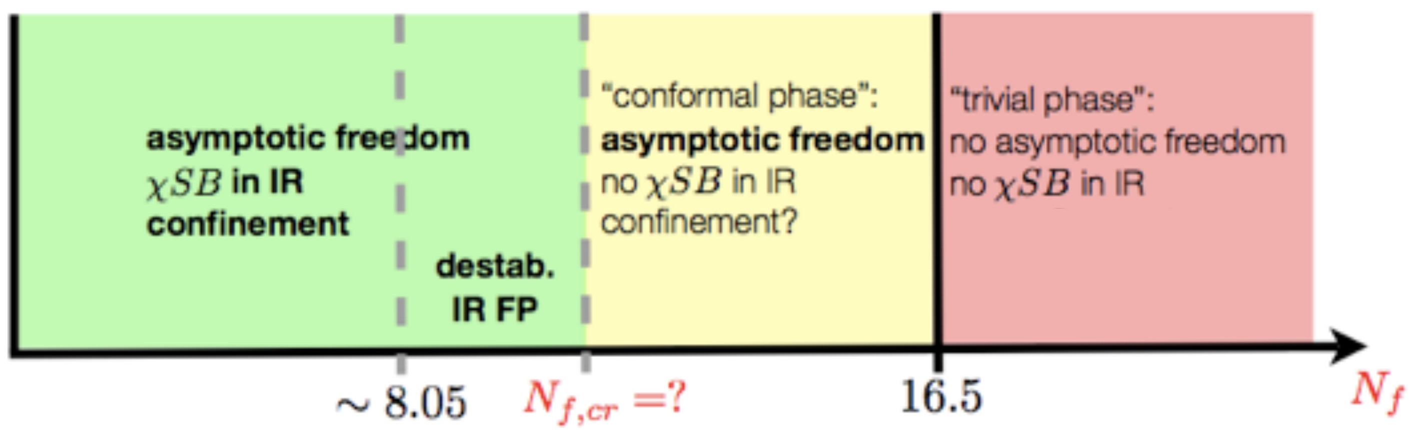
# Towards criticality & the role of the running coupling

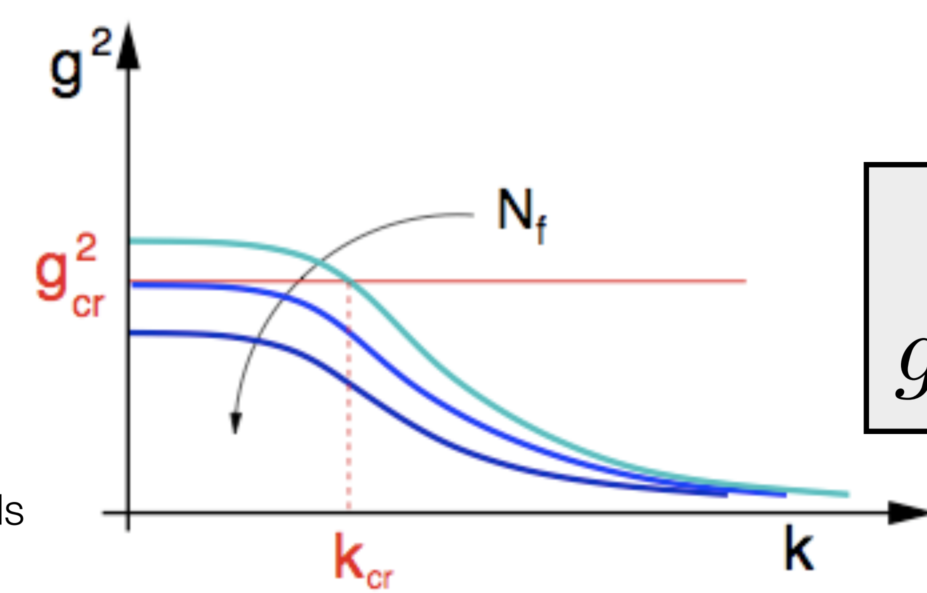
- lower end of the **conformal window** is determined by the onset of **chiral symmetry breaking**




- **chiral symmetry breaking** requires the strong coupling to exceed a critical value

**Note that**
$$k_{\rm cr} \geq k_{\rm SB}$$

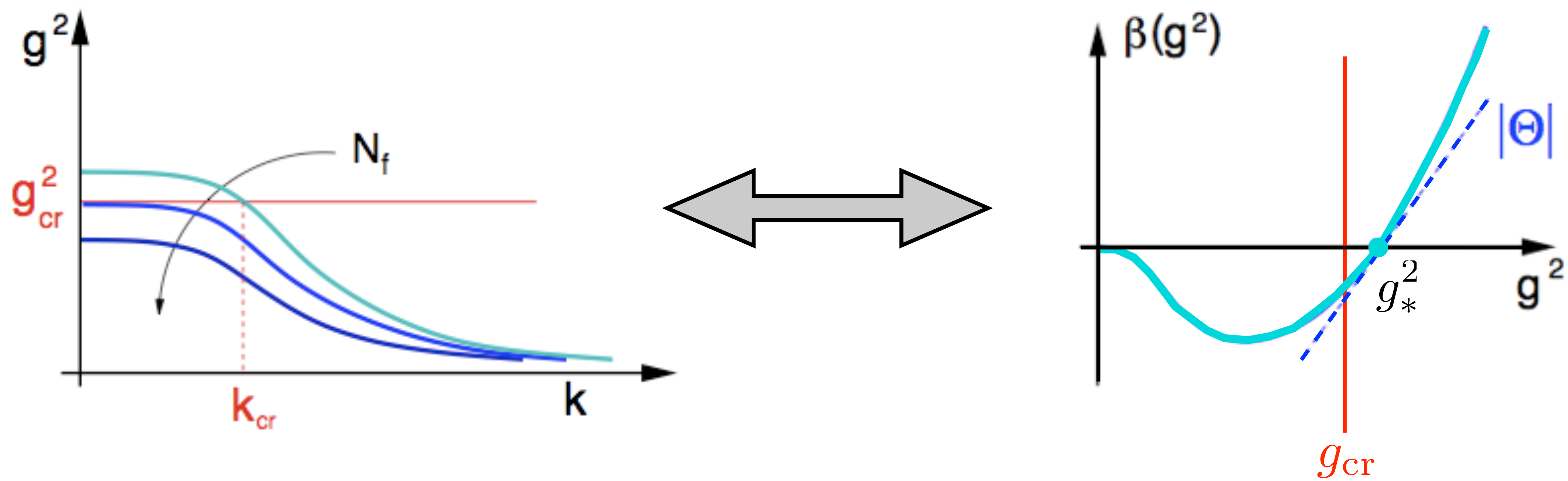
(symmetry breaking scale $k_{\rm SB}$ depends on "critical scale" $k_{\rm cr}$ )




**Initial condition:**
$$g^2(k=\Lambda, N_{\rm f}) = \text{const.}$$

# Towards criticality & the role of the running coupling

(JB, H. Gies '05, '06, '09)




- RG flow in the vicinity of the fixed point $g_*$ is governed by the **universal** critical exponent $\Theta$:

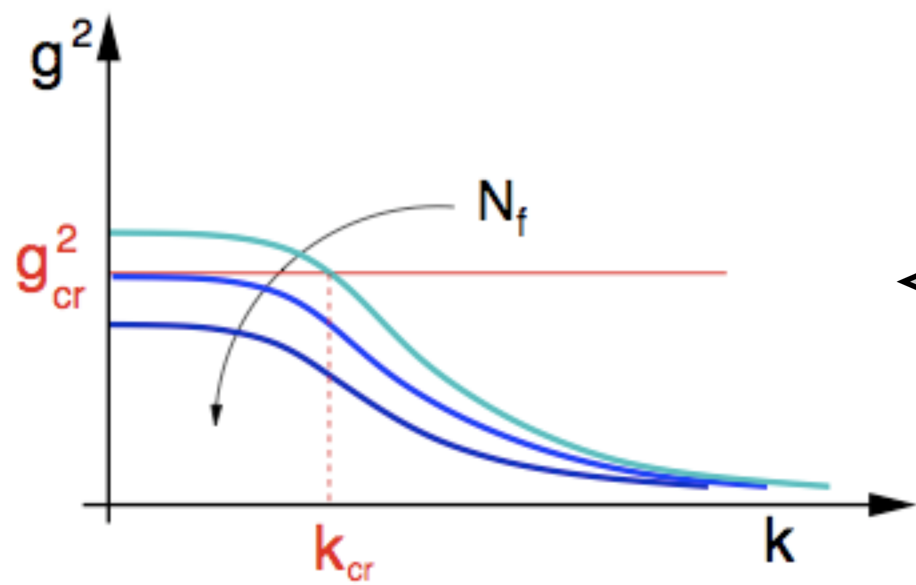
$$k\partial_k g^2 = \beta(g^2) = -\Theta(g^2 - g_*^2) + \ldots$$

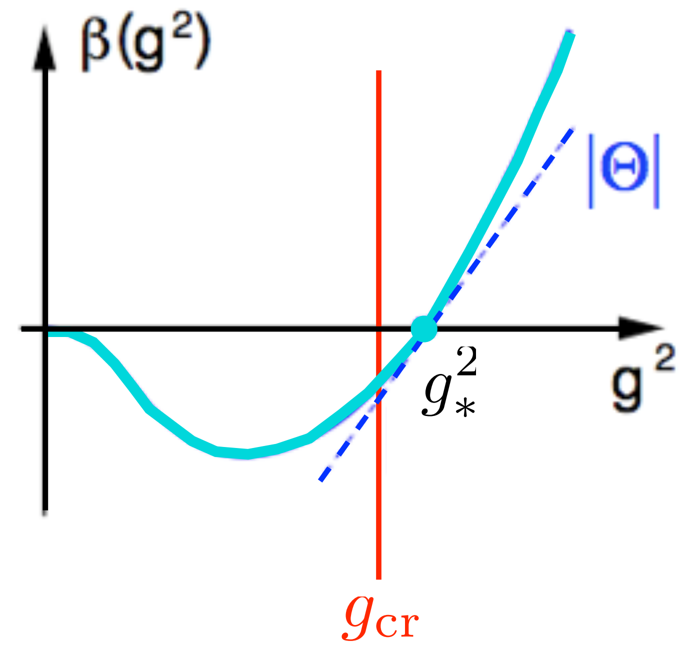
- solution in the fixed-point regime:

$$g^2(k) = g_*^2 - \left(\frac{k}{\Lambda}\right)^{|\Theta|}$$

# Towards criticality & the role of the running coupling

(JB, H. Gies '05, '06, '09)



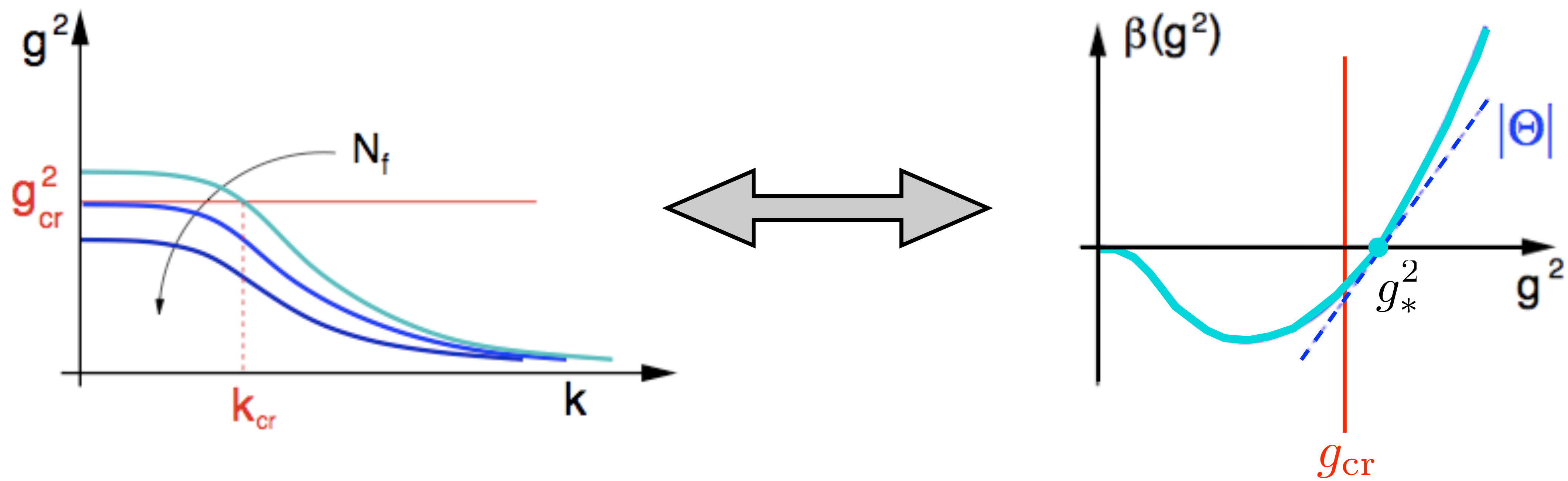




- $g^2(k) \stackrel{!}{=} g^2_{\rm cr}$: onset of $\chi{\rm SB}$ at $k_{\rm cr} \simeq \Lambda(g^2_* - g^2_{\rm cr})^{\frac{1}{|\Theta|}}$

- proportionality: $g^2_* \sim N_f$

# Towards criticality & the role of the running coupling

(JB, H. Gies '05, '06, '09)




- $g^2(k) \stackrel{!}{=} g^2_{\rm cr}$: onset of $\chi$SB at $k_{\rm cr} \simeq \Lambda (g_*^2 - g_{\rm cr}^2)^{\frac{1}{|\Theta|}}$

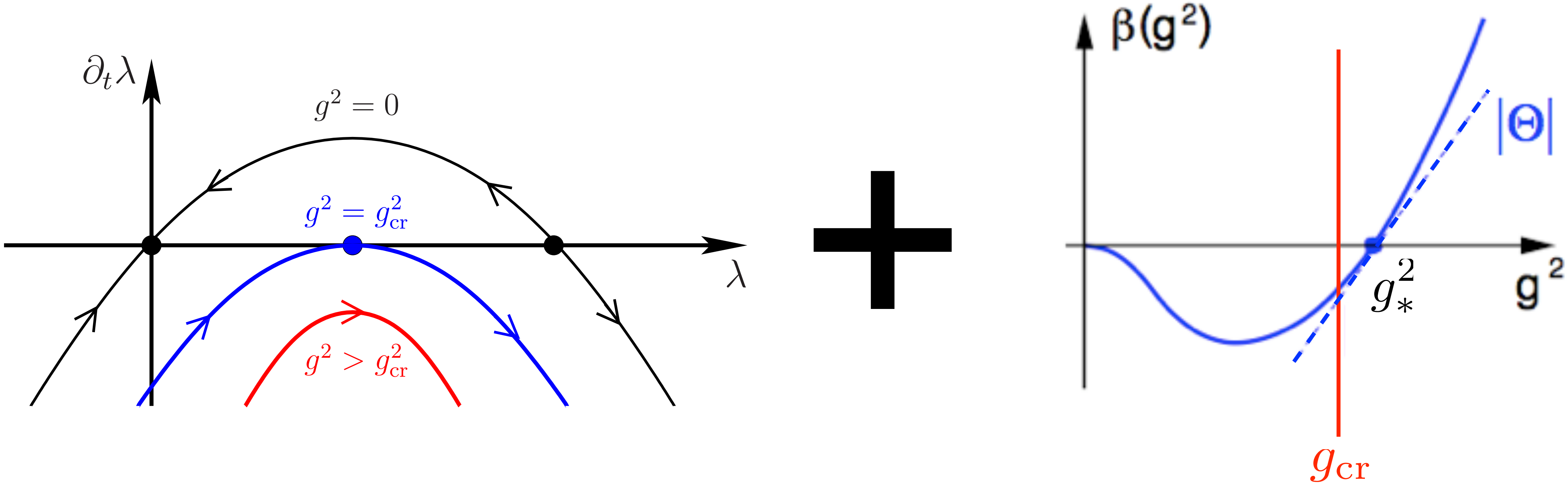
- proportionality: $g^2_* \sim N_f$

- "critical scale" scales as

$$k_{\rm cr} \simeq \Lambda |N_{\rm f} - N_{\rm f}^{\rm cr}|^{\frac{1}{|\Theta|}} \quad \text{with} \quad \Theta = \Theta(N_{\rm f}^{\rm cr})$$

# Beyond Miransky Scaling

(JB, C. S. Fischer, H. Gies, ’10)



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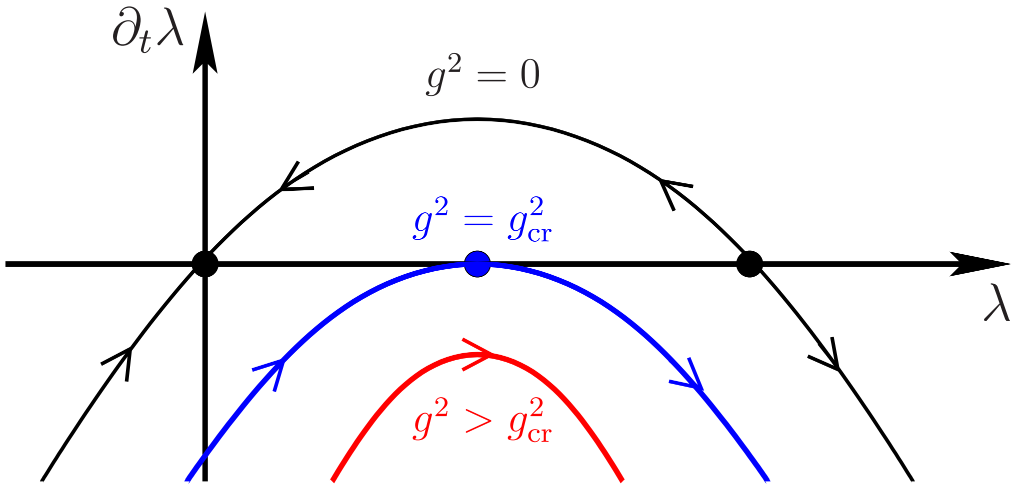


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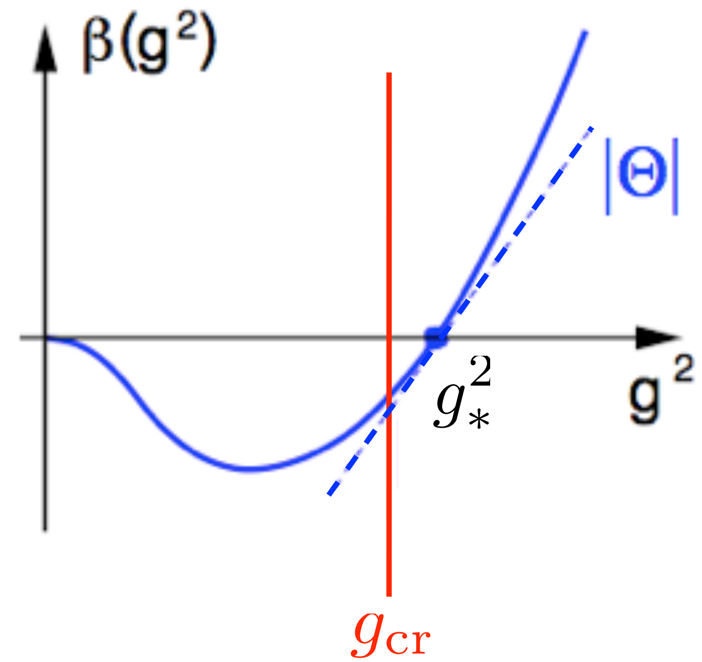



- symmetry breaking scale (in accordance with an improved DSE study by Jarvinen & Sannino '10)

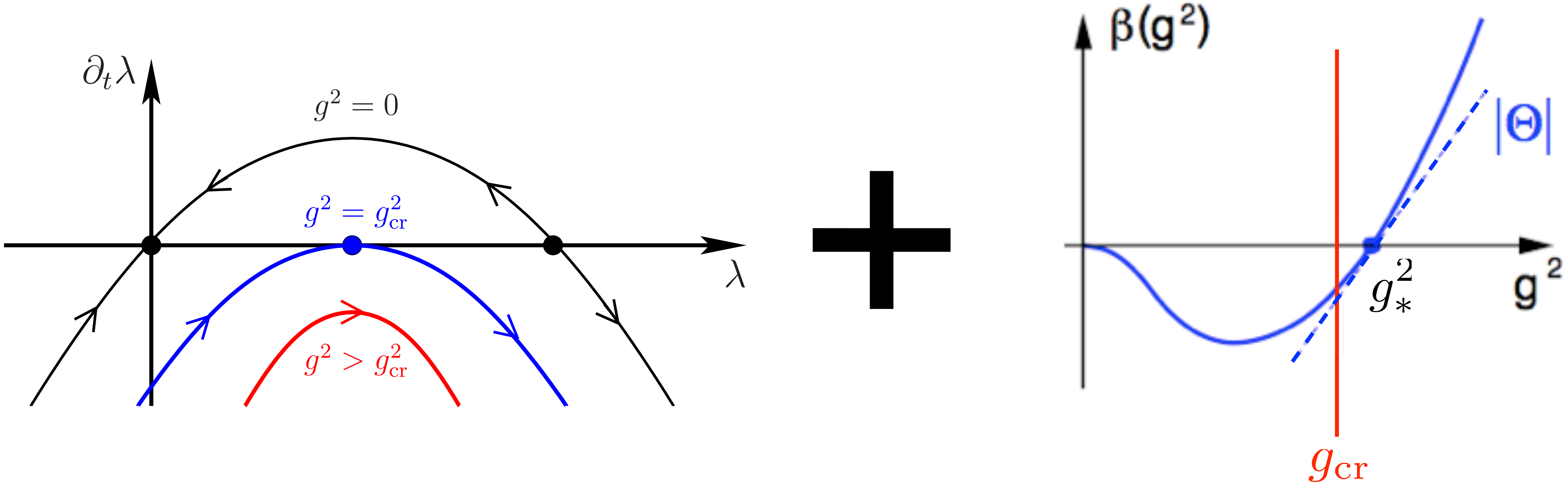
$$k_{\rm SB} \propto \Lambda |N_{\rm f}^{\rm cr} - N_{\rm f}|^{\frac{1}{|\Theta|}} \exp\left( - \frac{\rm const.}{\sqrt{|N_{\rm f}^{\rm cr} - N_{\rm f}|}} \right) \theta(N_{\rm f}^{\rm cr} - N_{\rm f})$$

- chiral observables (reminder):

$$m_{\rm f} \sim k_{\rm SB}\,, \quad f_\pi \sim k_{\rm SB}\,, \quad |\langle \bar{\psi}\psi \rangle|^{\frac{1}{3}} \sim k_{\rm SB}\,, \quad T_\chi \sim k_{\rm SB}\,, \ldots$$

# Beyond Miransky Scaling

(JB, C. S. Fischer, H. Gies, '10)




- symmetry breaking scale

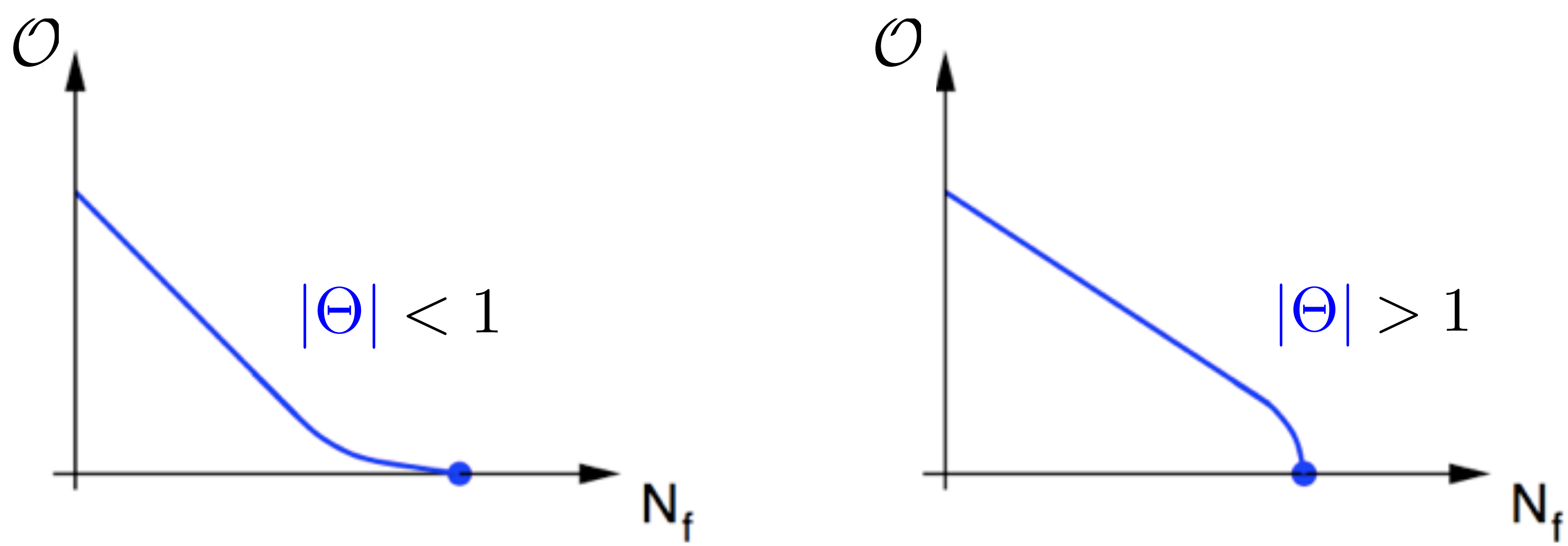
$$k_{\rm SB} \propto \Lambda |N_{\rm f}^{\rm cr} - N_{\rm f}|^{\frac{1}{|\Theta|}} \exp\left(-\frac{\rm const.}{\sqrt{|N_{\rm f}^{\rm cr} - N_{\rm f}|}}\right) \theta(N_{\rm f}^{\rm cr} - N_{\rm f})$$

- Power-law scaling: $|\Theta| \ll 1$ ("slowly walking ...")
- Miransky scaling: $|\Theta| \gg 1$

# Shape of the phase boundary: Many flavors

(JB, C. S. Fischer, H. Gies, ’10)

$$\mathcal{O} \sim k_{\mathrm{SB}} \propto \Lambda |N_{\mathrm{f}}^{\mathrm{cr}} - N_{\mathrm{f}}|^{\frac{1}{|\Theta|}} \exp\left( - \frac{\mathrm{const.}}{\sqrt{|N_{\mathrm{f}}^{\mathrm{cr}} - N_{\mathrm{f}}|}} \right) \theta(N_{\mathrm{f}}^{\mathrm{cr}} - N_{\mathrm{f}})$$




- relation between two **universal** quantities
- relation between **IR gauge dynamics** and **scaling behavior of chiral observables**
- parameter-free prediction

What is $N_f^{\rm cr}$?

What is $\Theta(N_f^{\rm cr})$?

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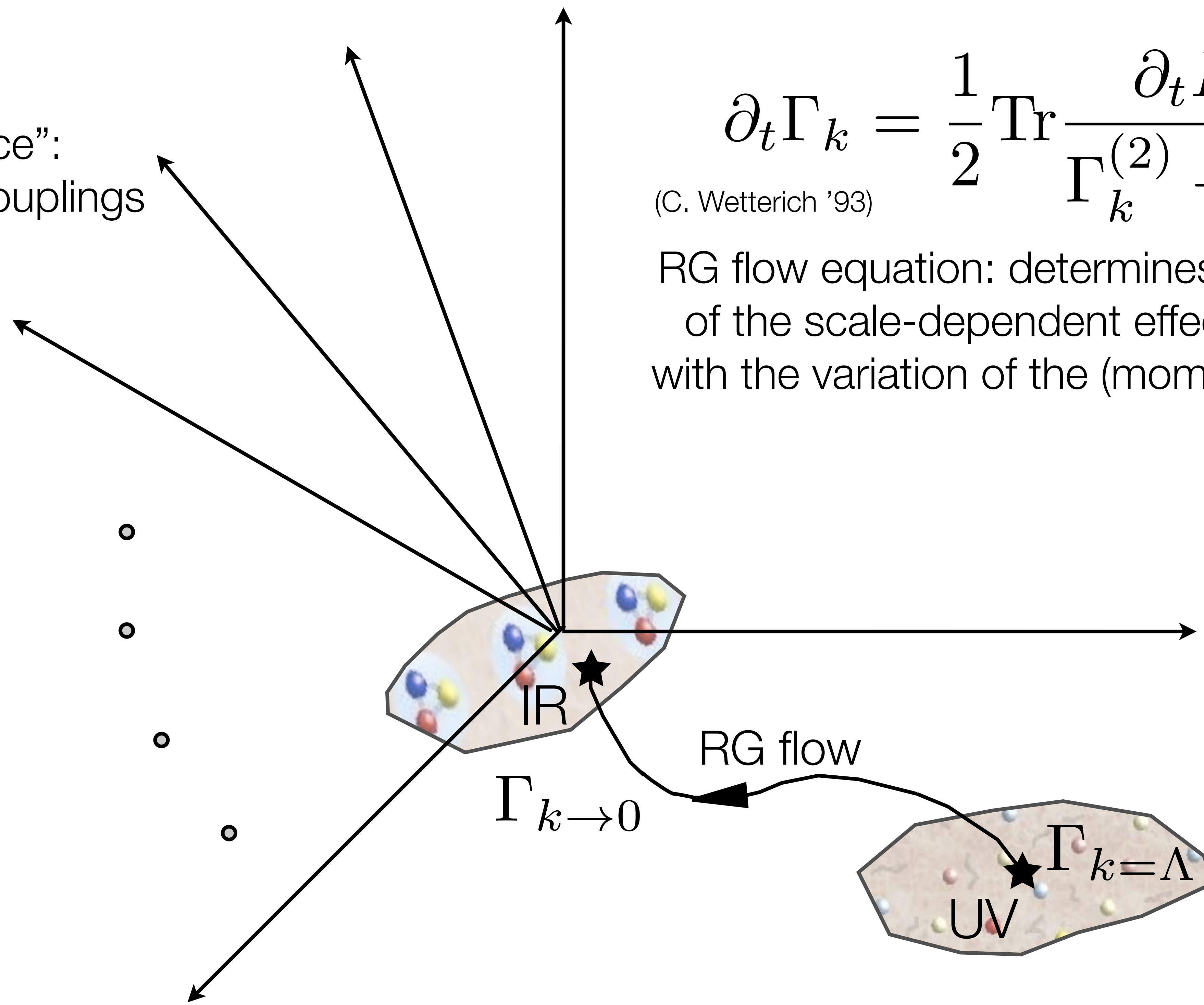
# Functional Renormalization Group

"Theory space":
spanned by all couplings

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

(C. Wetterich '93)

RG flow equation: determines the change of the scale-dependent effective action with the variation of the (momentum) scale



# RG flow for the chiral QCD sector

- effective action:

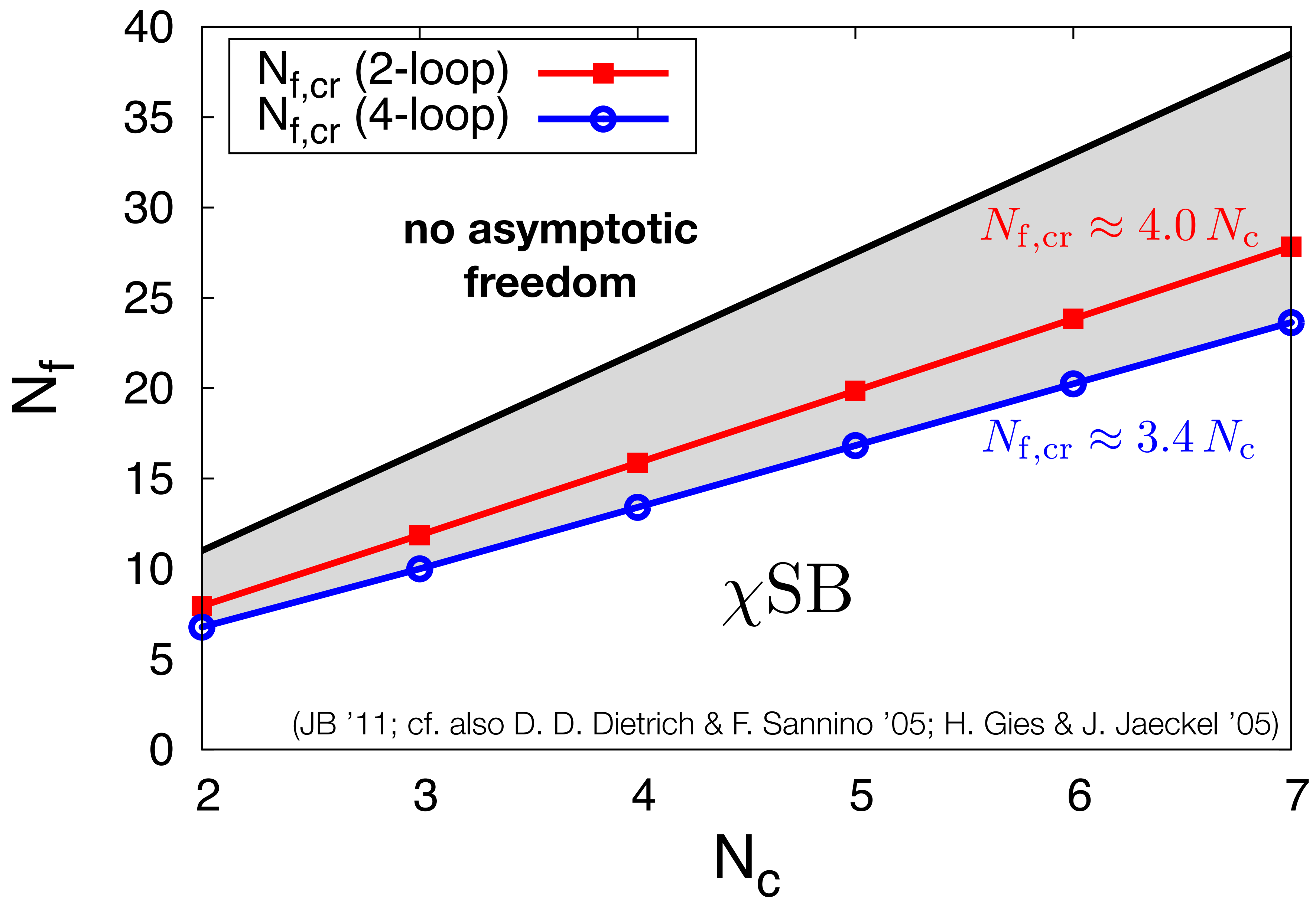
$$\Gamma_k = \int_x \Big\{ \frac{\bar{g}^2}{g^2} F^a_{\mu\nu} F^a_{\mu\nu} + w_2 (F^a_{\mu\nu} F^a_{\mu\nu})^2 + w_3 (F^a_{\mu\nu} F^a_{\mu\nu})^3 + \ldots \Big\}$$
$$+ \int_x \Big\{ \bar{\psi} (\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/) \psi + \frac{1}{2} \Big[ \frac{\lambda_-}{k^2} (\mathrm{V}-\mathrm{A}) + \frac{\lambda_+}{k^2} (\mathrm{V}+\mathrm{A})$$
$$+ \frac{\lambda_\sigma}{k^2} (\mathrm{S}-\mathrm{P}) + \frac{\lambda_{\mathrm{VA}}}{k^2} [2(\mathrm{V}-\mathrm{A})^{\mathrm{adj}} + (1/N_c)(\mathrm{V}-\mathrm{A})] \Big] \Big\}$$

- no Fierz-ambiguity

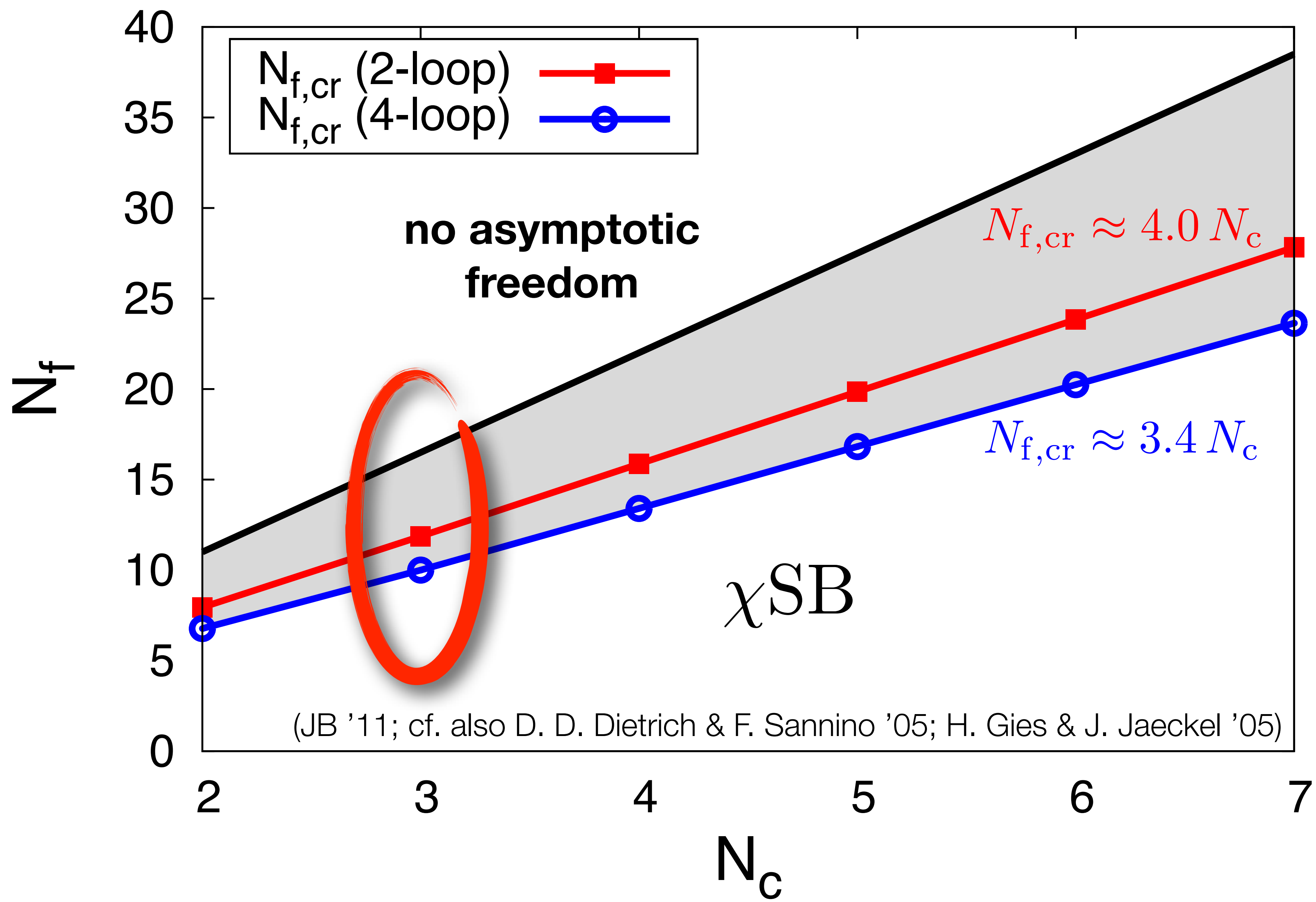
- four-fermion interactions ($\lim_{\Lambda \to \infty} \lambda_i = 0$): "QCD universality class"

- truncation checks: momentum dependencies, regulator dependencies, higher order interactions (H. Gies, J. Jaeckel, C. Wetterich '04, H. Gies, C. Wetterich '02, JB '08)

# Many-flavor QCD

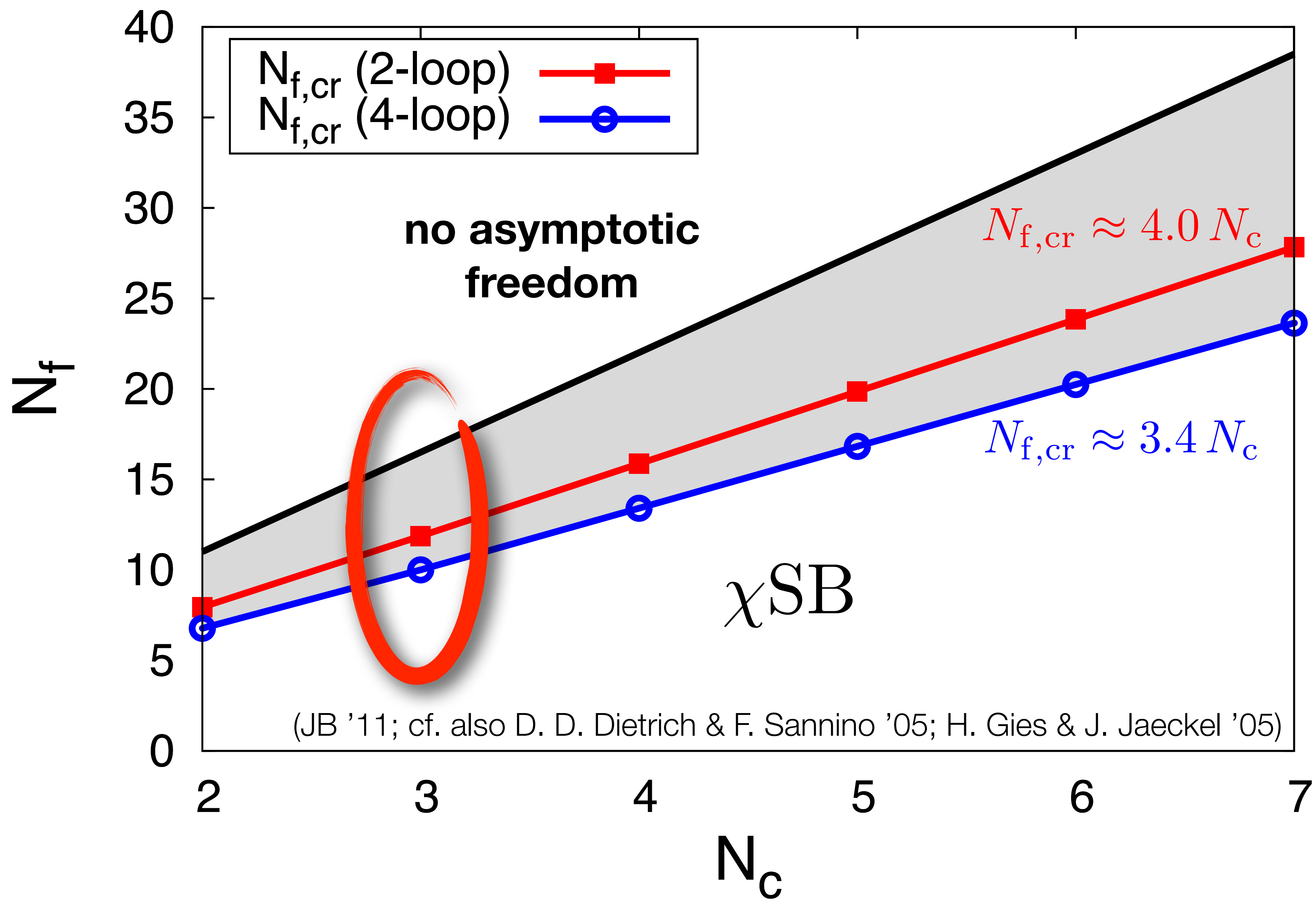


# Many-flavor QCD




- critical number (RG error estimate): $N_{\rm f,cr} \simeq 10\,..\,12$ (H. Gies & J. Jaeckel '05; JB & H. Gies '05, '06)
- "conformal phase" for $N_{\rm f,cr} < N_f < 16.5$: asymptotic freedom but no $\chi SB$

# Many-flavor QCD

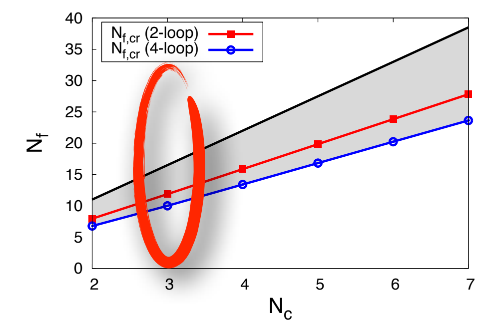


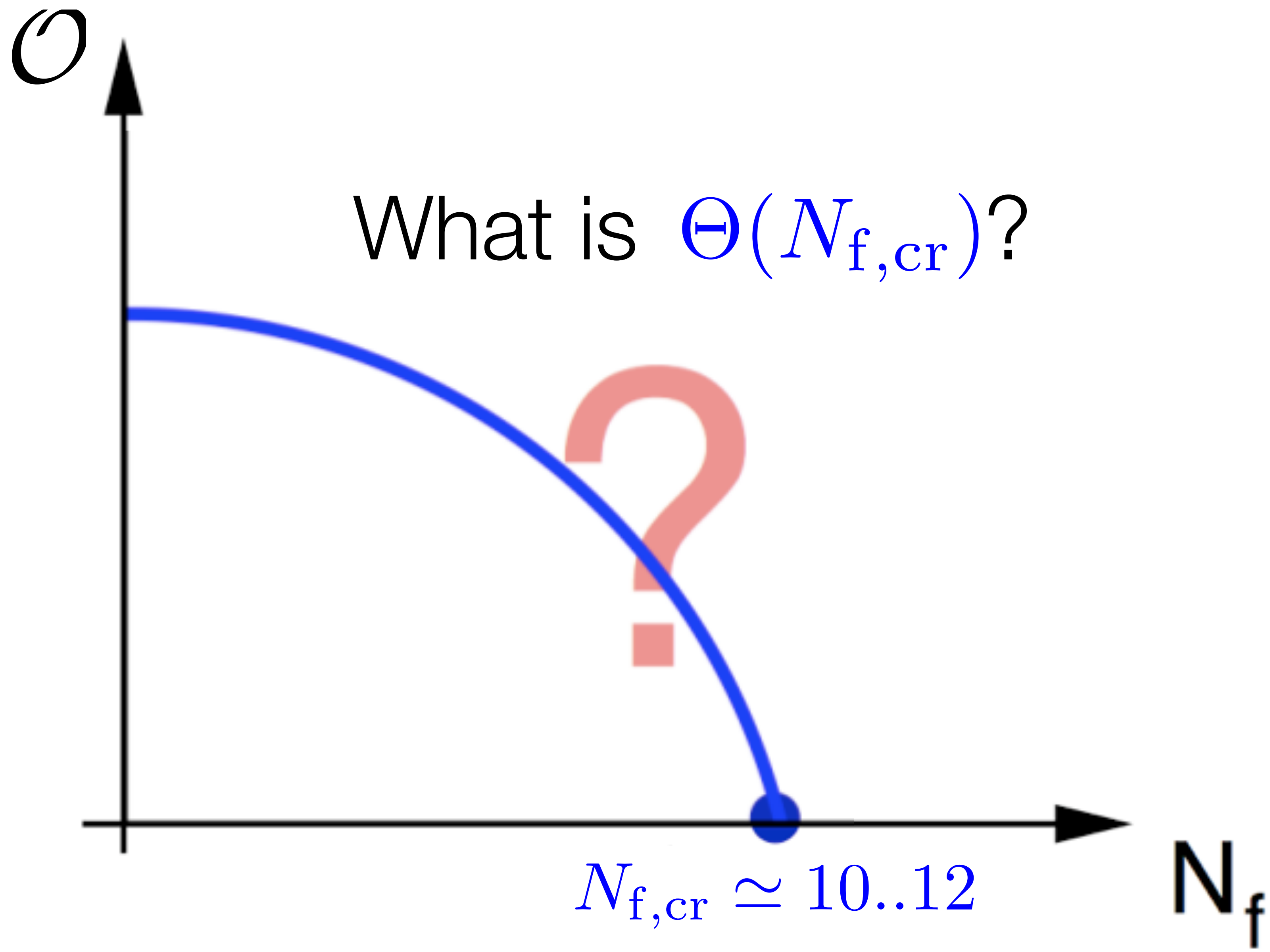

(Miransky & Yamawaki '96; Appelquist et al. '96; Sannino & Schechter '99; Sannino & Tuominen '05; Dietrich & Sannino '07; Ryttov & Sannino '07,'08,'09; Sannino '08; Fukano & Sannino '10)

- consistent with rainbow-ladder approximation & SUSY inspired all-order $\beta$-function
- state-of-the-art lattice studies: $9 < N_{f,cr} \lesssim 13$

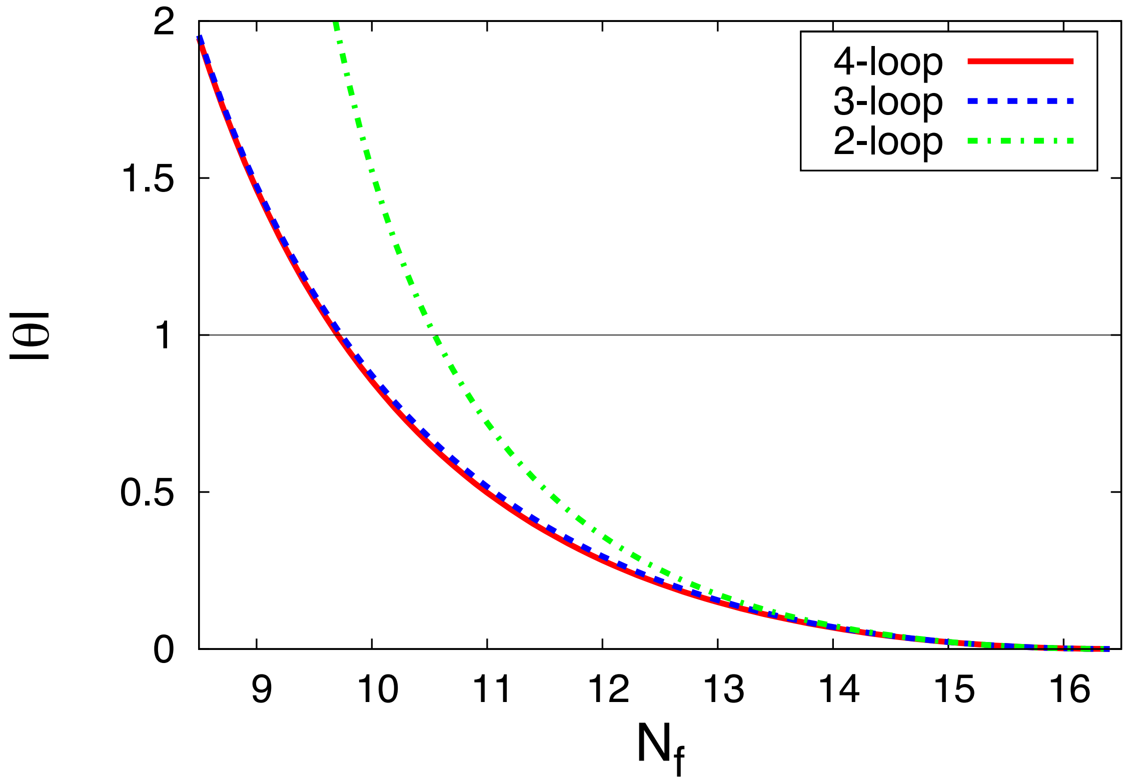
(Appelquist, Fleming, Neil '08, '09; Deuzeman, Lombardo, Pallante '08; Fodor et al. '08, '09 Fodor, Holland, Kuti, Nogradi, Schroeder '09; Jin, Mawhinney '09)

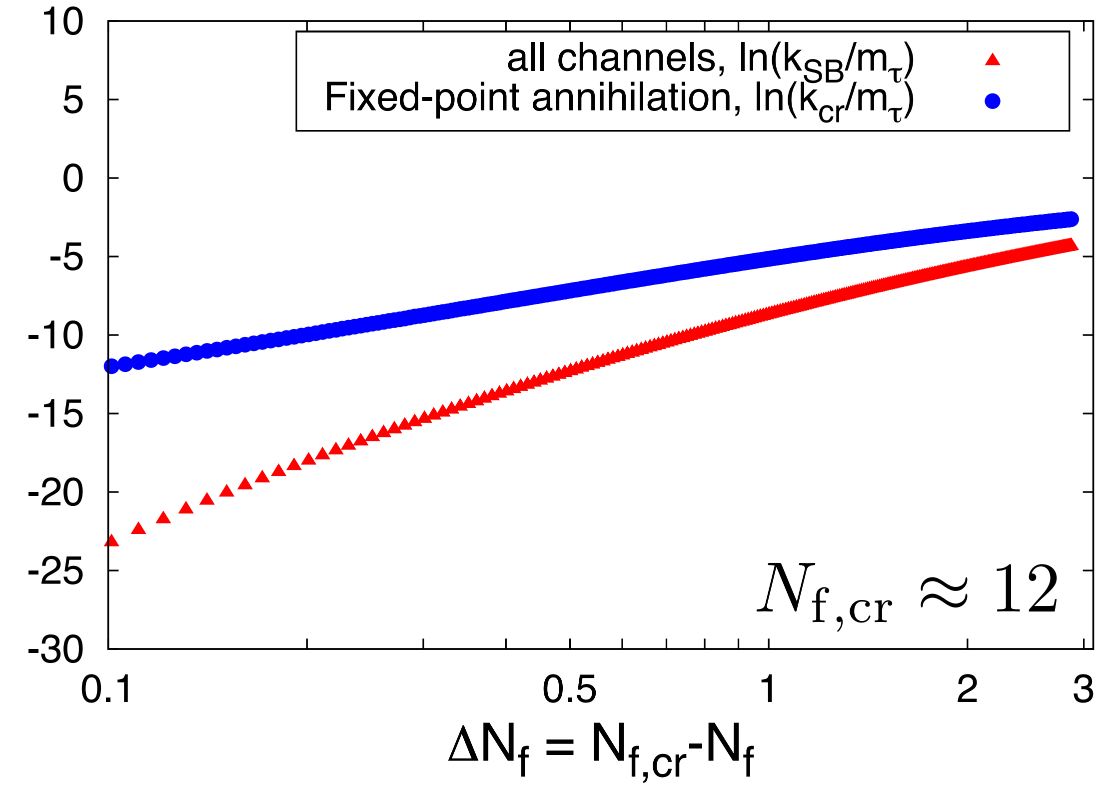
# Scaling Behavior Close to the QPT?



# Many-flavor QCD and Scaling: The 3-color case



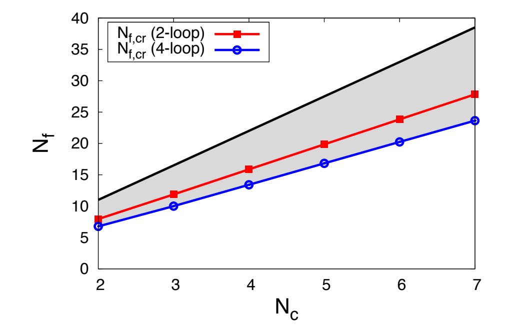




$$k_{\rm SB} \propto \Lambda |N_{\rm f}^{\rm cr} - N_{\rm f}|^{\frac{1}{\Theta}} \exp\left(-\frac{\rm const.}{\sqrt{|N_{\rm f}^{\rm cr} - N_{\rm f}|}}\right) \theta(N_{\rm f}^{\rm cr} - N_{\rm f})$$

$k_{\rm cr}$ power-law behavior

$k_{\rm cr} \geq k_{\rm SB}$ superposition of power-law and exponential/“Miransky” behavior

# Many-flavor QCD and Scaling




| $N_c$ | 2 | 3 | 4 | 5 | 6 | 7 (JB '11) |
|---|---|---|---|---|---|---|
| $N_{f,cr}^{one}$ | 7.6 | 11.7 | 15.7 | 19.7 | 23.6 | 27.6 |
| $N_{f,cr}^{all}$ | 7.9 | 11.9 | 15.9 | 19.9 | 23.8 | 27.8 |
| $N_{f,cr}^{4-loop}$ | 6.8 | 10.0 | 13.4 | 16.8 | 20.2 | 23.6 |
| $\Delta N_f^{2-loop}$ | 0.36 | 0.27 | 0.31 | 0.36 | 0.42 | 0.48 |

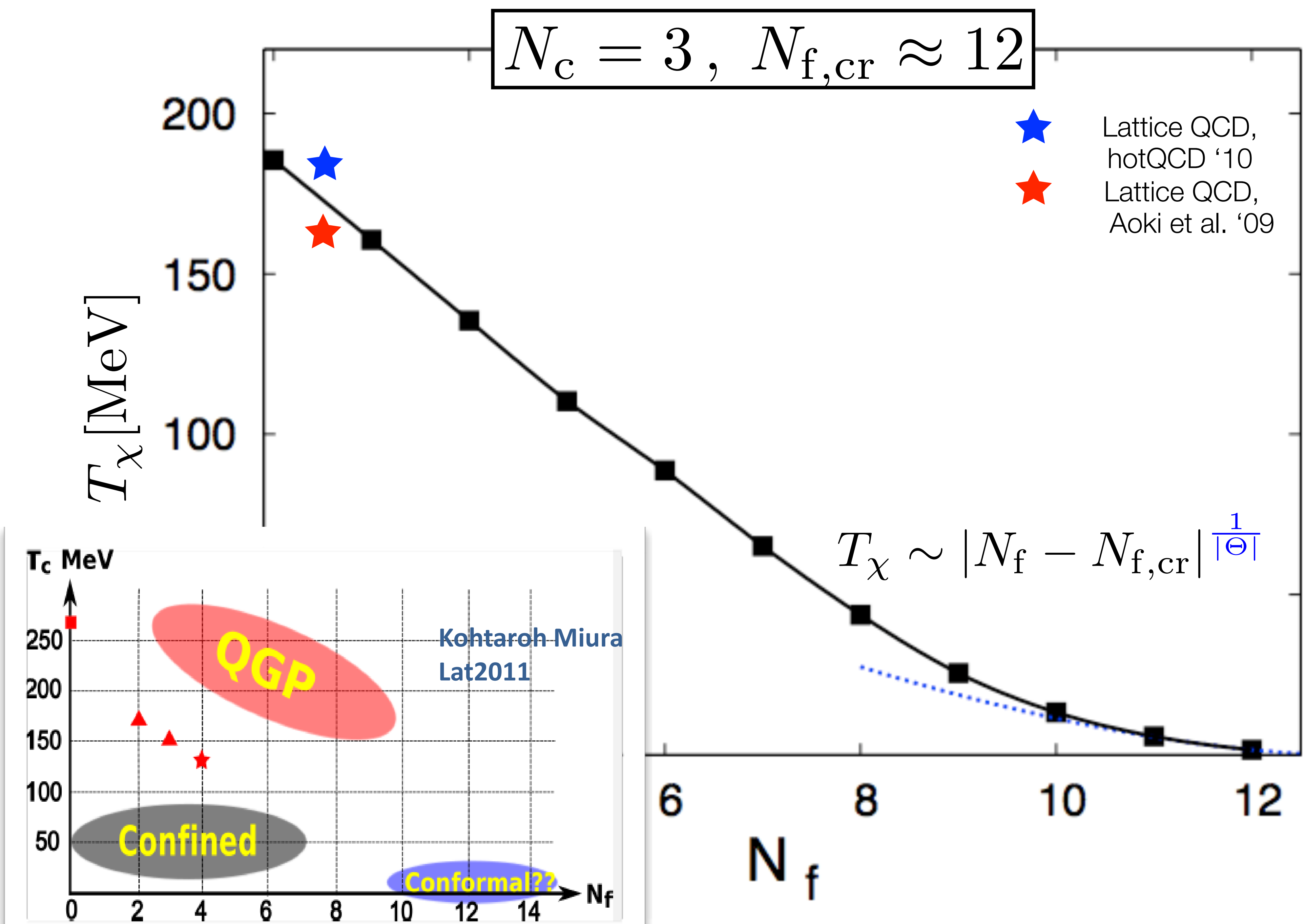
**size of the regime in which Miransky scaling dominates**

Power-law scaling might be more relevant for lattice simulations which probe the theory at integer $N_f$

(JB, C. S. Fischer, H. Gies, '10)

# Many-flavor QCD at finite temperature

(JB, H. Gies '05,'06, '09)




**(under investigation using lattice simulations, cf. Deuzeman, Lombardo, Miura, Pallante)**

# Conclusions

- **universal corrections** to Miransky (BKT) scaling
- critical number of quark flavors: $N_{\mathrm{f,cr}} \simeq (3.4 \ldots 4.0) \times N_{\mathrm{c}}$
- scaling of physical observables near $N_{\mathrm{f,cr}}$ is determined by the underlying IR fixed point scenario (**testable prediction!**)

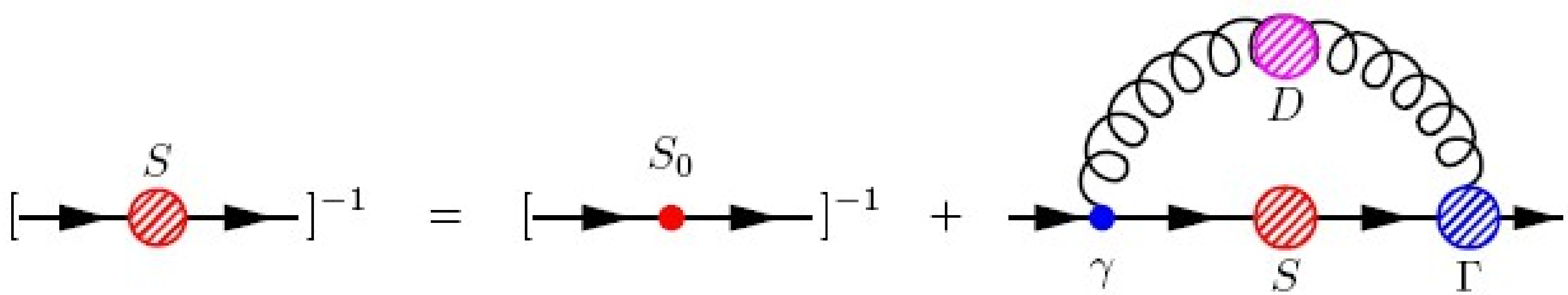
# Outlook

- corrections to scaling due to (current) quark mass (and finite volume) (cf. Dietrich '10)
- testing other theories: e. g. QED3 (together with C. S. Fischer, H. Gies, L. Jansen, D. Roscher), adjoint matter, ...
- confining dynamics vs. chiral dynamics in aQCD (together with T. K. Herbst, to appear soon)

# Dyson-Schwinger Equations and Miransky Scaling

(Miransky ’85; Miransky, Yamawaki ’97)

- Dyson-Schwinger equation for the fermion propagator [ physik.uni-graz.at/itp/sicqft ]




- approximations: $\gamma = \Gamma\,;\quad g^2 = \text{const.}\,;\quad g^2 - g^2_{\text{cr}} \sim |N_{\text{f}}^{\text{cr}} - N_{\text{f}}| + \ldots$

- symmetry breaking scale:

$$k_{\text{SB}} \propto \Lambda\theta(N_{\text{f}}^{\text{cr}} - N_{\text{f}}) \exp\left(-\frac{\pi}{2\epsilon\sqrt{|\alpha_1||N_{\text{f}}^{\text{cr}} - N_{\text{f}}|}}\right)$$