Bad Honnef 2012

Hadronic contribution to the muon $g-2$ from a Dyson-Schwinger perspective

Tobias Göcke

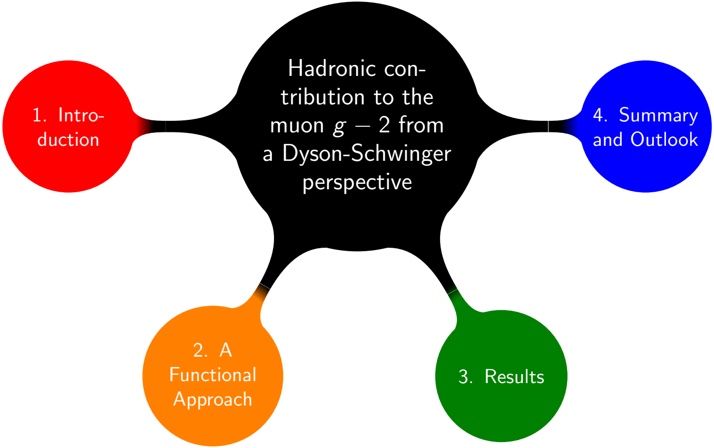
Together with C. S. Fischer (JLU) and R. Williams (Complutense, Madrid)

[Fischer, TG, Williams, arXiv:1009.5297]
[TG, Fischer, Williams, arXiv:1012.3886]
[TG, Fischer, Williams, arXiv:1107.2588]

HELMHOLTZ GEMEINSCHAFT

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

UNIVERSITAS COMPLUTENSIS MATRITENSIS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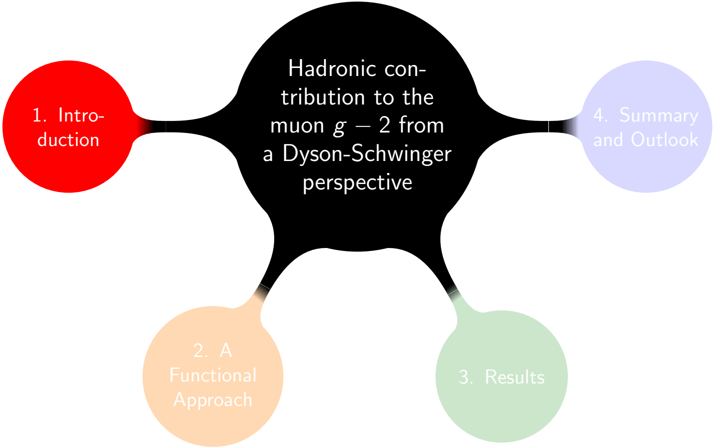
1. Introduction
2. A Functional Approach
3. Results
4. Summary and Outlook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1. Introduction

a_μ is. . .

- Precisely determined by experiment and accurately predicted by Standard Model
- Precision test for the Standard Model
- More sensitive to contributions from high scales than a_e since : $m_\mu \gg m_e$
- Sensitive to QCD and potential 'new physics' contributions
- Deviation between experiment and theory?

So one has to . . .

- get the SM predictions under control
- use non-perturbative methods for QCD-contributions.

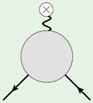
1. Introduction

Magnetic moment $\vec{\mu}$, g-factor

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

- Dirac equation: $g = 2$
- Schwinger: anomalous part $a_\mu = \frac{g-2}{2} \approx \alpha/2\pi \approx 0.00116$

Relativistic QFT

-  $= (-ie)\bar{u}(p\prime) \left[F_1(q^2)\gamma_\alpha + iF_2(q^2)\frac{\sigma_{\alpha\beta}q^\beta}{2m_\mu} \right] u(p)$
- $a_\mu := F_2(0) = \frac{g_\mu - 2}{2}$

1. Introduction

Precision tests of the standard model

contribution	$a_\mu[10^{-11}]$
Experiment	116 592 089(63)
SM	116 591 828(49)
hadronic LO	6 949(43)
hadronic LbL	105(26)
exp. - th.	261(80)

[B. L. Roberts, arXiv:1001.2898 (2010)]
[Hagiwara, Liao, Martin, Nomura, Teubner, arXiv:1105.3149 (2011)]

New Physics?

- 3.3σ effect
- Lattice? First results but needs time

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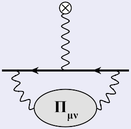
1. Introduction

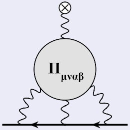
Two Hadronic Contributions

hadronic vacuum polarization



- one-scale problem
- known from dispersion relation
- leading contribution

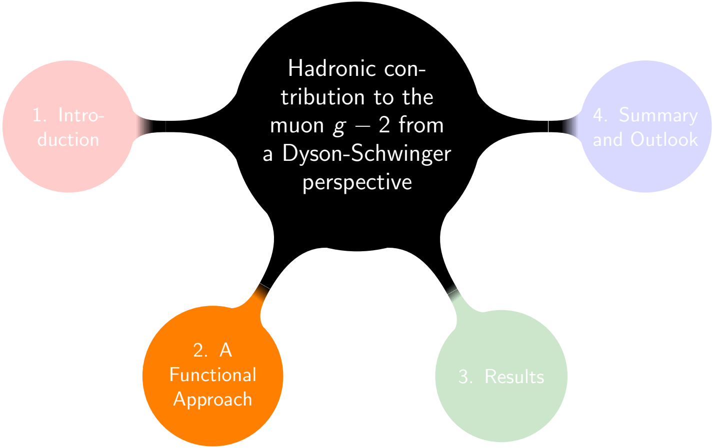
hadr. light by light scattering



- two-scale problem
- only accessible through theory
- systematic uncertainty?

How to deal with these objects?

- $\longrightarrow$ calculate both from the same approach
- use the 'little brother' as test case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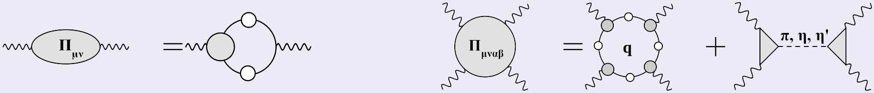
2. A Functional Approach - Overview

Diagrammatic representation

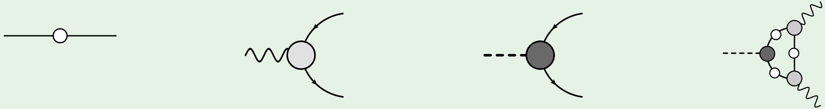


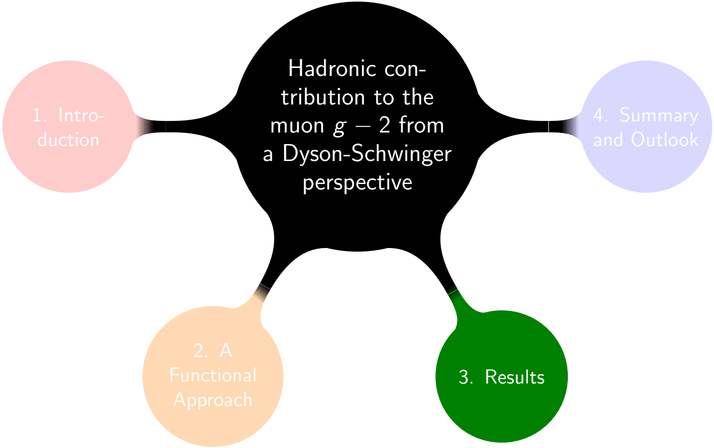
- Model of quark-gluon interaction:

  [Maris and Roberts, arXiv:nucl-th/9708029]
  [Maris and Tandy, arXiv:nucl-th/9905056]
  [Maris and Tandy, arXiv:nucl-th/9910033]

- one description for high and low energy → important for two-scale problem

building blocks

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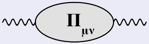
3. Results - Hadronic Vacuum Polarization

- 

  $= \Pi_{\mu\nu}(q) = \left(\delta_{\mu\nu} q^2 - q_\mu q_\nu\right) \Pi(q^2)$
- Adler function: $D(q) = -d\Pi / d \ln q^2$
- two parameter sets, u and d quark masses fixed to the
  I:pion mass II: rho mass

- use u, d, s, c, b quarks (isospin-limit: $m_u = m_d$)
- all parameters fixed by meson phenomenology
  - model interaction $\rightarrow \langle \bar{\psi}\psi \rangle$ and f_π
  - $m_{u/d} \rightarrow m_\pi$ or m_ρ
  - $m_s \rightarrow m_K$ or m_ϕ
  - $m_{c/b} \rightarrow c\bar{c}$ and $b\bar{b}$ vector states

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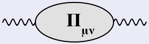
- 

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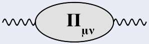
- 

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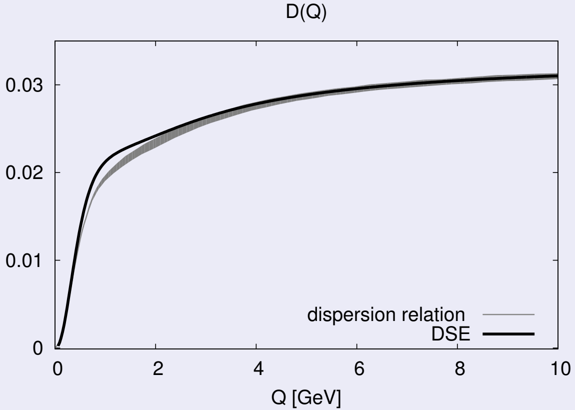
3. Results - Hadronic Vacuum Polarisation

Adler function

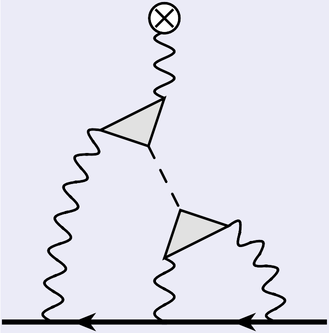


$a_\mu^{\mathrm{HVP,DSE}} = (6760 - 7440) \times 10^{-11}$ | $a_\mu^{\mathrm{HVP,disp.rel.}} = 6903.0(52.6) \times 10^{-11}$

[TG, Fischer, Williams, arXiv:1107.2588 (2011)] [Jegerlehner and Nyffeler ,arXiv:0902.3360 (2009)]

3. Results - LbL - π, η, η' Pole

pseudoscalar (PS) exchance



- π only
  $$a_\mu^{\pi-\mathrm{pole}} = 58(7) \times 10^{-11}$$
- π, η and η'
  $$a_\mu^{\mathrm{PS-pole}} = 80.7(12) \times 10^{-11}$$

[Fischer, TG, Williams, arXiv:1009.5297]
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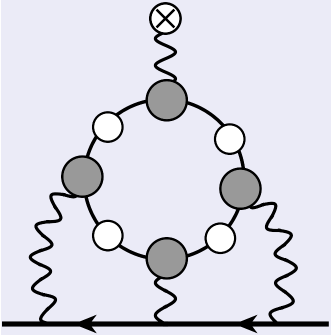
good agreement with existing approaches

3. Results - LbL - Quark Loop

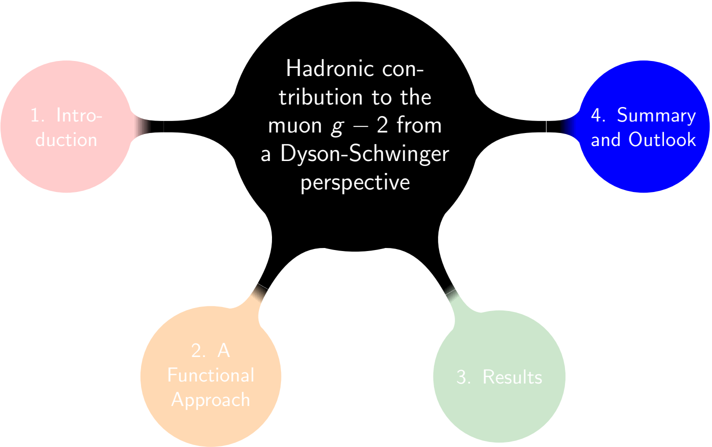
quark loop



- bare vertex γ_μ
  $$a_\mu^{LBL,bare} = 61(2) \times 10^{-11}$$
- γ_μ with 1st Ball-Chiu dressing
  $$a_\mu^{LBL,1BC} = 107(2) \times 10^{-11}$$
- Only 'full' self-consistent vertex from BSE will be conclusive $\rightarrow$ numerically quite challenging

[Fischer, TG, Williams, arXiv:1009.5297]
[TG, Fischer, Williams, arXiv:1012.3886]

- enhancement compared to existing approaches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5. Summary and Outlook

summary

DSE calculation of $g-2$

- HVP
  - $a_\mu^{\mathrm{HVP,DSE}} = (6760 - 7440) \times 10^{-11}$
  - $a_\mu^{\mathrm{HVP,disp.rel.}} = 6949(43) \times 10^{-11}$
  - Adler function can be described reasonably
- LbL
  - $a_\mu^{\mathrm{PS-pole}} = 81(12) \times 10^{-11}$
  - enhancement of quark-loop overlooked?
  - but no final answer yet

[Hagiwara, Liao, Martin, Nomura, Teubner, arXiv:1105.3149 (2011)]

5. Summary and Outlook

outlook

- complete quark-loop calculation
- overcome resonance expansion (π^0, η, η') pole dominance

- Thank You for the attention!

supported by

- DFG under grant No. Fi 970/8-1
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