Natural Vacuum Alignment from Group Theory

Martin Holthausen

based on
MH, Michael A. Schmidt
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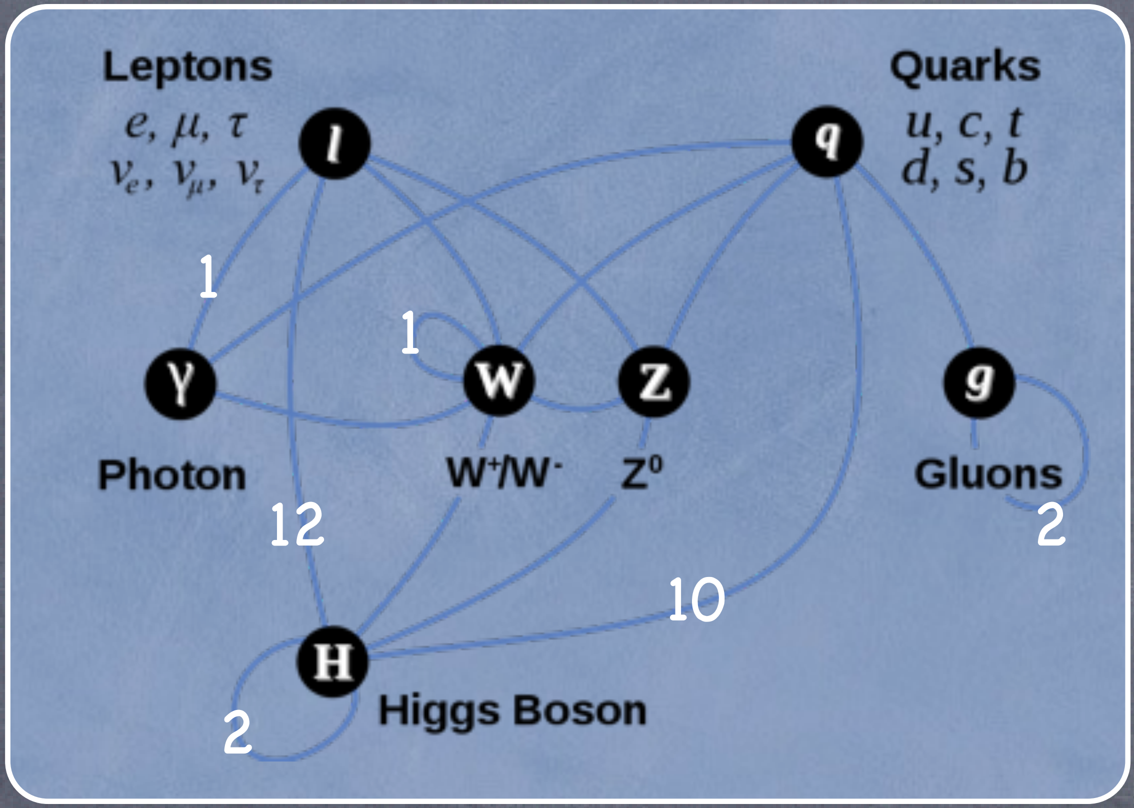
INTERNATIONAL MAX PLANCK RESEARCH SCHOOL
PT FS
FOR PRECISION TESTS OF FUNDAMENTAL SYMMETRIES

Why Flavour Symmetry?

in SM(+Majorana neutrinos) there are a total of 28 Parameters



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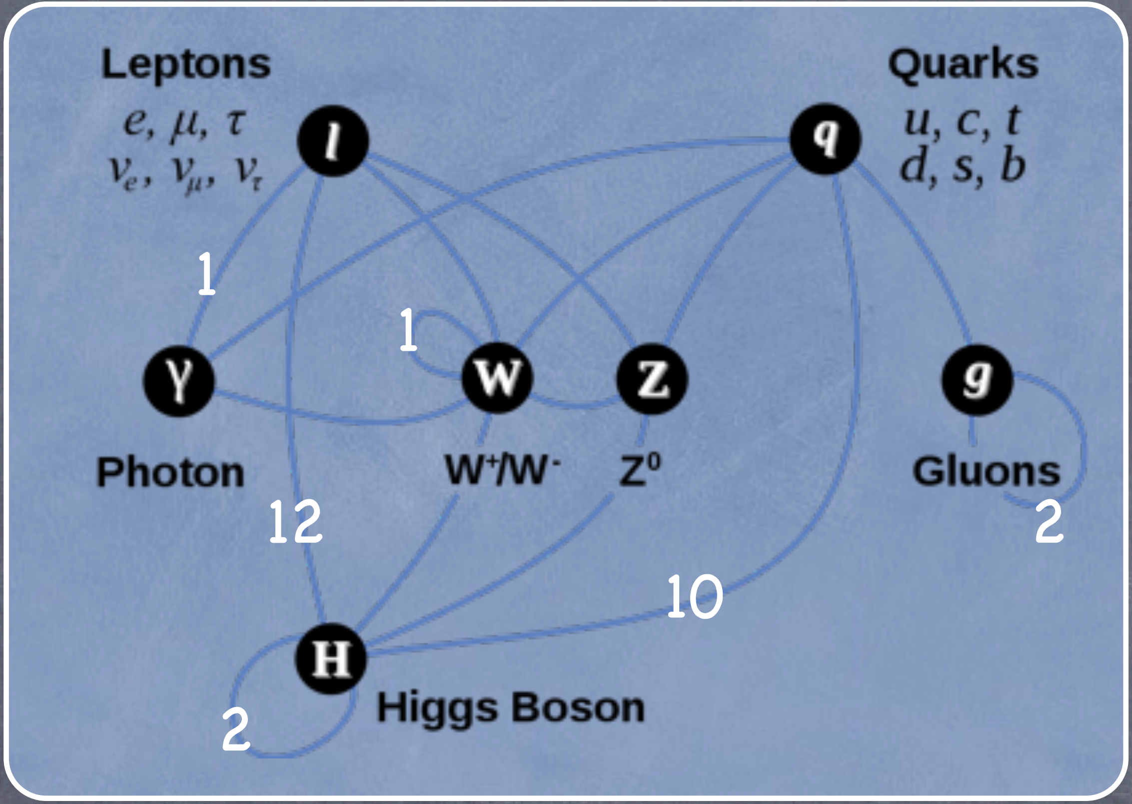

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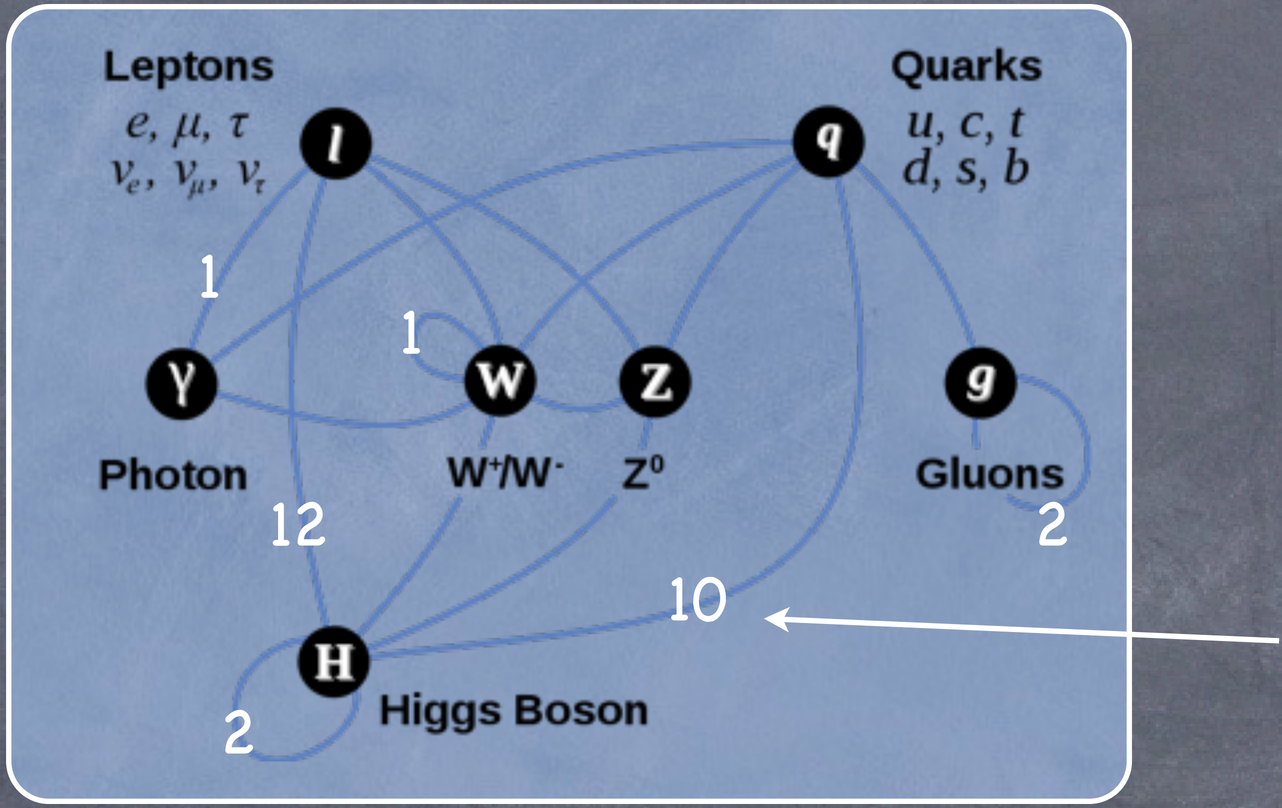

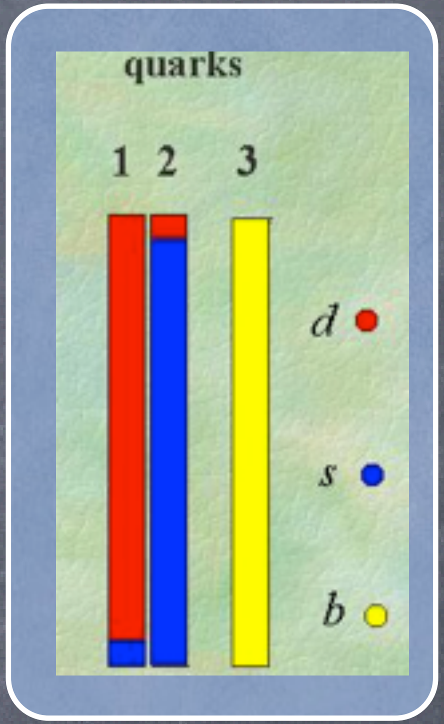

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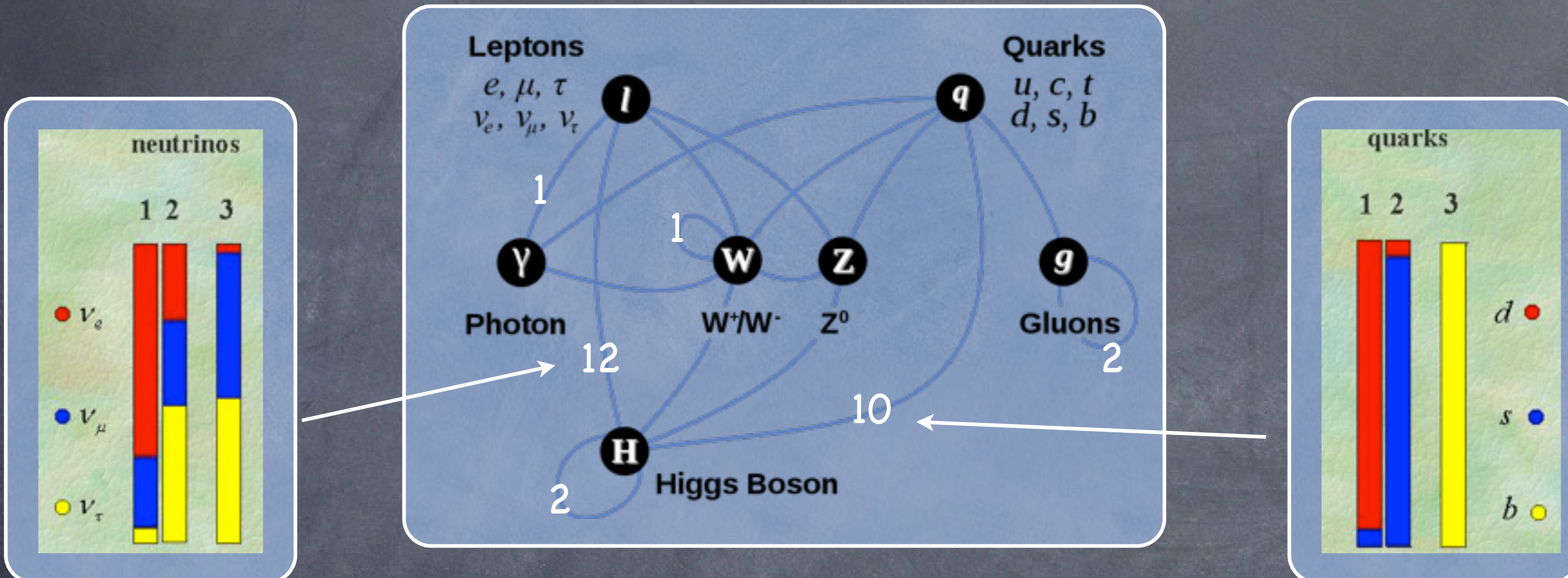


- most of them stem from interactions with the Higgs field, other interactions tightly constrained by symmetry principles
- in quark sector: small mixing angles and hierarchical masses can be explained by Frogatt-Nielsen symmetry
- in lepton sector: two large and one small mixing angle suggestive of non-abelian discrete symmetry

Lepton mixing from discrete groups

G_f

complete flavour group

residual symmetry of ($m_e^\dagger m_e$)

$G_e = Z_3$

$$T m_e m_e^\dagger T^\dagger = m_e m_e^\dagger$$

$$T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(smallest choice, but can also be continuous)

residual symmetry of m_ν

$G_\nu = Z_2 \times Z_2$

$$S m_\nu S^T = m_\nu \qquad U m_\nu U^T = m_\nu$$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(most general choice if mixing angles do not depend on masses & Majorana νs)

misaligned non-communting symmetries lead to

$$U_{PMNS} = U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

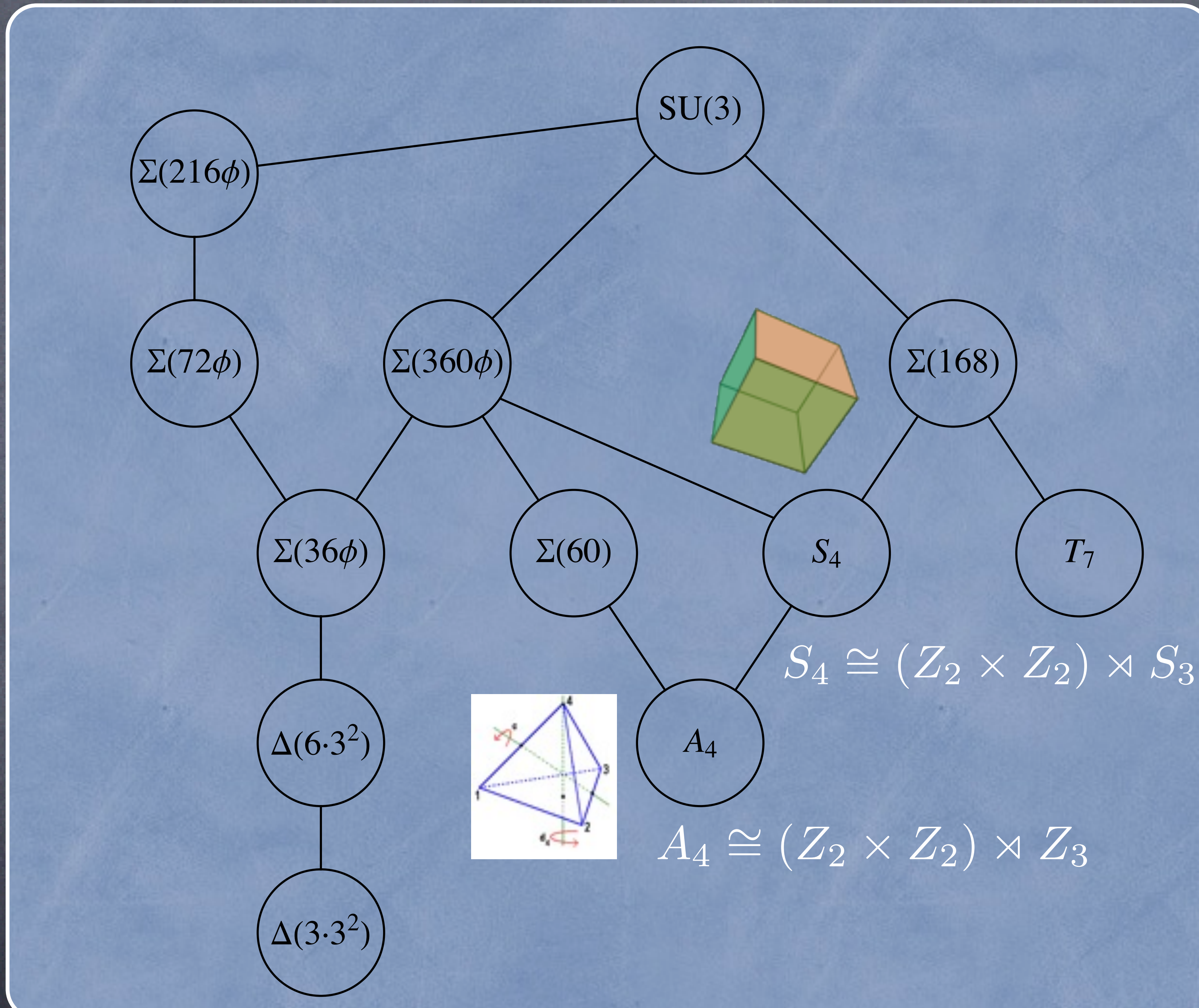
$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$$

[He, Keum, Volkas '06; Lam'07,'08; Altarelli,Feruglio'05]

„tri-bimaximal mixing"(TBM) gives good LO description of lepton mixing

$$\sin^2\theta_{12} = 0.312^{+0.017}_{-0.015}, \quad \sin^2\theta_{23} = 0.52^{+0.06}_{-0.07}, \quad \sin^2\theta_{13} = 0.013^{+0.007}_{-0.006}$$

Candidate Groups



$S_4 \cong (Z_2 \times Z_2) \rtimes S_3$

$A_4 \cong (Z_2 \times Z_2) \rtimes Z_3$

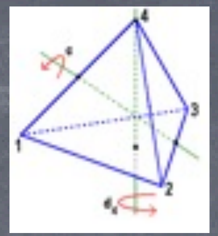
$T_7 \cong Z_7 \rtimes Z_3$

$T' \cong Z_2 . A_4$

$\Delta(27) \cong (Z_3 \times Z_3) \rtimes Z_3$

[Merle,Zwicky 1110.4891]

A_4 Symmetry Group



A_4 is the smallest symmetry group that can lead to TBM mixing:

$$A_4 \cong (Z_2 \times Z_2) \rtimes Z_3 \cong \langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$$

	S	T
$\underline{\mathbf{1}}_\mathbf{1}$	1	1
$\underline{\mathbf{1}}_\mathbf{2}$	1	ω
$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2
$\underline{\mathbf{3}}_\mathbf{1}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

	1	T	T^2	S
$\underline{\mathbf{1}}_\mathbf{1}$	1	1	1	1
$\underline{\mathbf{1}}_\mathbf{2}$	1	ω	ω^2	1
$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2	ω	1
$\underline{\mathbf{3}}$	3	0	0	-1

1-d reps. correspond to reps. of Z_3

$\omega = e^{i2\pi/3}$

$$\underline{\mathbf{3}} \times \underline{\mathbf{3}} = \underline{\mathbf{1}}_\mathbf{1} + \underline{\mathbf{1}}_\mathbf{2} + \underline{\mathbf{1}}_\mathbf{3} + \underline{\mathbf{3}}_S + \underline{\mathbf{3}}_A$$

$$(ab)_{\underline{\mathbf{1}}_\mathbf{1}} = \frac{1}{\sqrt{3}}(a_1 b_1 + a_2 b_2 + a_3 b_3)$$

$$(ab)_{\underline{\mathbf{1}}_\mathbf{2}} = \frac{1}{\sqrt{3}}(a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3) \qquad (ab)_{\underline{\mathbf{1}}_\mathbf{3}} = \frac{1}{\sqrt{3}}(a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3)$$

$$(ab)_{A,\underline{\mathbf{3}}} = \frac{1}{2}\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \qquad (ab)_{S,\underline{\mathbf{3}}} = \frac{1}{2}\begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_3 b_1 + a_1 b_3 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}$$

where $(a_1, a_2, a_3), (b_1, b_2, b_3) \sim \underline{\mathbf{3}}$.

An A_4 Prototype model

- (A_4,Z_4) charge assignments: $l\sim(3,i)$, $e^c\sim(1_1,-i)$, $\mu^c\sim(1_2,-i)$, $\tau^c\sim(1_3,-i)$, $\chi\sim(3,1)$, $\Phi\sim(3,-1)$, $\xi\sim(1,-1)$
- auxiliary Z_4 separates neutral and charged lepton sectors at LO

$(L\chi)e^c$ — $\langle\chi\rangle\sim(1,1,1)$ — A_4 — $\langle\Phi\rangle\sim(1,0,0)$ — $(ll)_3\Phi+(ll)_1\xi$

$Z_3=\langle T\rangle$ $\qquad$ $Z_2=\langle S\rangle$

$$m_e\sim\begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix} \quad \longleftrightarrow \text{TBM} \quad m_\nu\sim\begin{pmatrix} \tilde a & 0 & 0 \\ 0 & \tilde a & \tilde d \\ 0 & \tilde d & \tilde a \end{pmatrix}$$

($Z_2 x Z_2$ symmetry accidental)

Vacuum alignment crucial!

[e.g. Ma,Rajasekaran'01, Babu, Ma, Valle '03, Altarelli,Feruglio, '05,'06]

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- SSB: Minimum Energy State exhibits less symmetry than full Lagrangian

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- Most straightforward case: scalar potential in 4D DOES NOT work:

$$V_\chi = m_0^2 (\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_1 (\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} (\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_2 (\chi\chi)_{\underline{\mathbf{1}}_\mathbf{2}} (\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}}$$

has minima (1,1,1) and (1,0,0). Effect of breaking to Z_2 in another sector can be included by adding:

$$V_{soft,Z_2} = m_A^2 \chi_1^2 + m_B^2 \chi_2^2 + m_C^2 \chi_2 \chi_3$$

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Minimization conditions then give:

$$0 = \left[\frac{\partial V}{\partial \chi_1}\right]_{\chi_i = v'} = \frac{2}{\sqrt{3}}\left(m_0^2 + \sqrt{3}m_A^2\right)\,v' + 4\lambda_1 {v'}^3$$

$$0 = \left[\frac{\partial}{\partial \chi_2}V - \frac{\partial}{\partial \chi_3}V\right]_{\chi_i = v'} = 2\,m_B^2\,v'$$

$$0 = \left[\frac{\partial}{\partial \chi_1}V - \frac{\partial}{\partial \chi_3}V\right]_{\chi_i = v'} = \left(2\,m_A^2 - m_C^2\right)\,v'$$

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- This thus requires $m_A = m_B = m_C = 0$, i.e. all non-trivial contractions between Φ and χ have to vanish in the potential.

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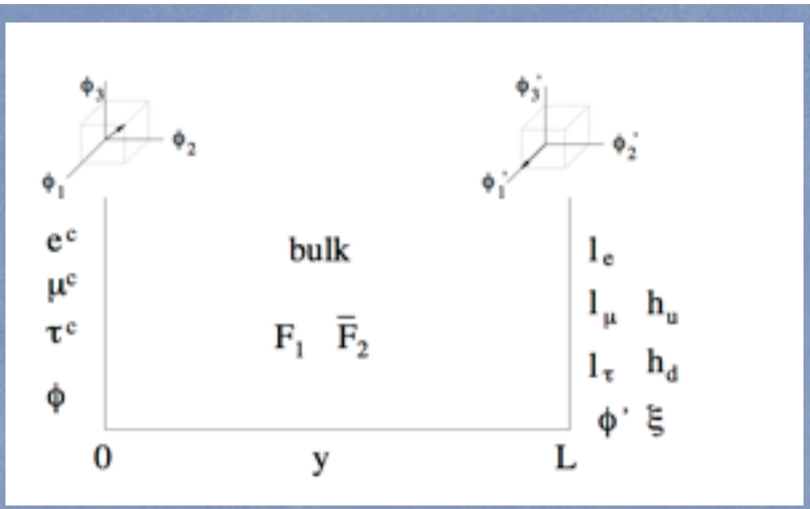
- Breaking to the same subgroup of A_4 can be realized. The non-trivial couplings, i.e. $(\Phi\,\Phi)_3(\chi\chi)_3$ thus force breaking of group to the same subgroup.

- the couplings cannot be forbidden by an internal symmetry that commutes with A_4, as e.g.$(\Phi^\dagger\,\Phi)_3$ is invariant under the commuting symmetry.

Solutions in the Literature

In models with extra dimensions(ED), it is possible to locate the various fields at different locations in the ED, thereby forbidding the cross-couplings.

Altarelli, Feruglio 2005



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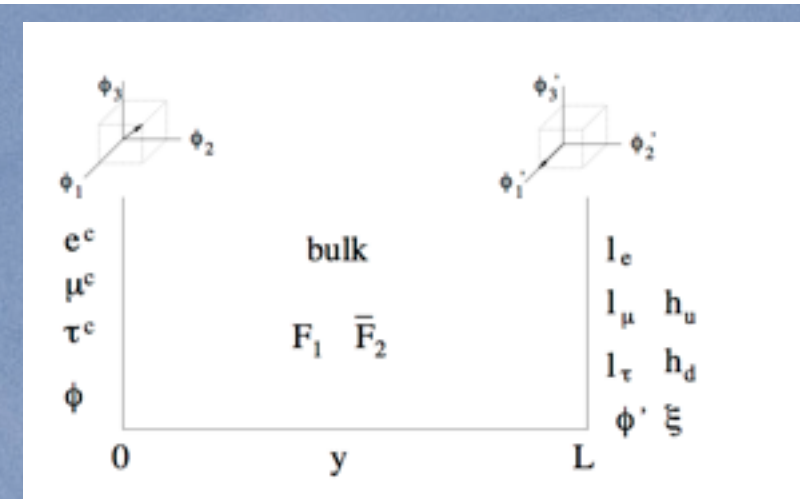

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In SUSY, one has to introduce a continuous R-symmetry and additional fields with R-charge 2(driving fields). These fields enter the superpotential only linearly and allow the vacuum alignment.

Field	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	3	3	1
Z_3	1	ω	ω	ω	1	ω	ω
$U(1)_R$	0	0	0	0	2	2	2

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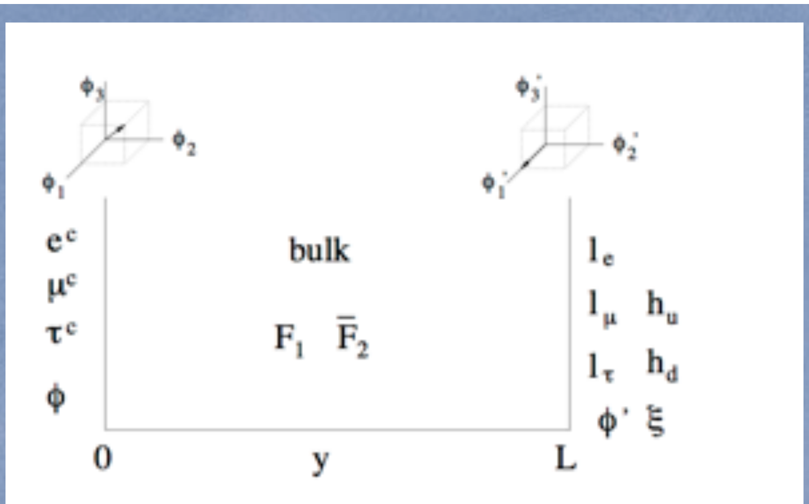

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- leptons transform only under A_4 subgroup
- if one takes $\Phi \sim 16$, vacuum alignment possible as $V = V(\Phi) + V(\chi) + (\Phi\ \Phi)_1 (\chi\chi)_1$
- neutrino masses then generated by coupling to $\langle \Phi^4 \rangle \sim (1,0,0)$

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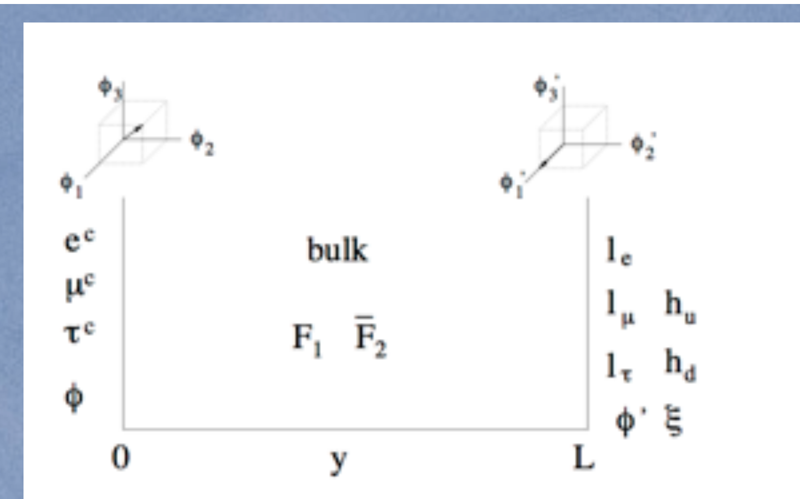

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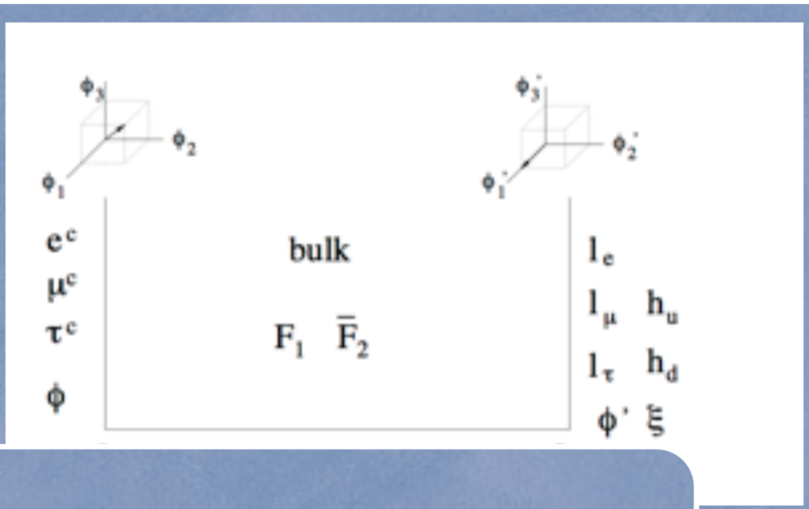

needs high flavour scale, hard to test

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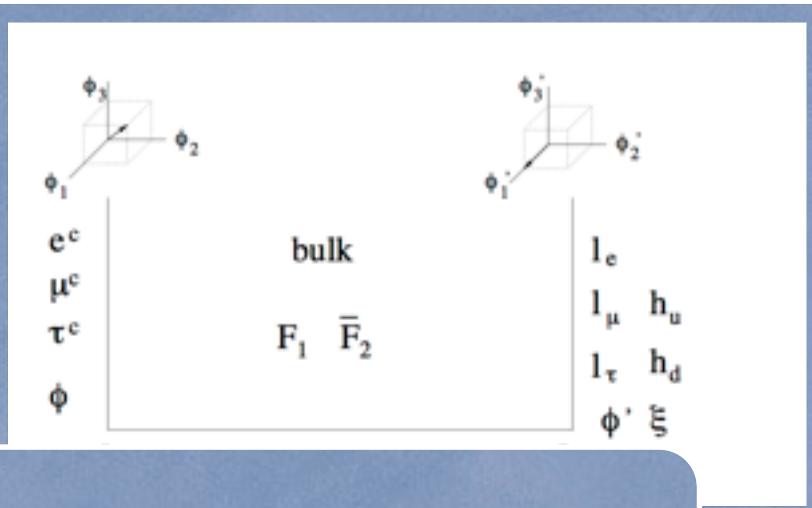

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Babu and Gab[illegible] the properties

- leptons t[illegible]
- if one ta[illegible] $_1(\chi\chi)_1$
- neutrino [illegible]

- Model is fine-tuned/needs special UV completion: different mass entries in neutrino mass matrix stem from operators of very different mass dimensions $(ll)_3\Phi^4+(ll)_1$
- non-minimal(size: 15552), needs large representations

Pro… …ions

In models with…
to locate the v…
the ED, there…

nee… …est

In SUSY, the…
R-symmetry…
charge 2(dri…
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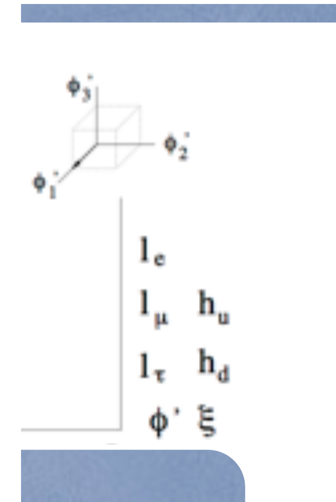

	φ_0	φ_0^S	ξ_0
1	3	3	1
ω	1	ω	ω
0	2	2	2

TABLE II. The character table for $(S_3 \times S_3 \times S_3 \times S_3) \rtimes A_4$.

	1	8	24	32	16	12	54	108	81	72	144	216	96	216	216	108	432	432	648	1296	⋯
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⋯
1′	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⋯
1″	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⋯
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	⋯
$\tilde{1}$	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	⋯
$\tilde{1}'$	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	⋯
$\tilde{1}''$	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1	⋯
$\tilde{3}$	3	3	3	3	3	−3	3	−3	3	−3	−3	3	−3	3	−3	−1	−1	−1	1	1	⋯
4	4	4	4	4	4	2	0	−2	−4	2	2	0	2	0	−2	0	0	0	0	0	⋯
4′	4	4	4	4	4	2	0	−2	−4	2	2	0	2	0	−2	0	0	0	0	0	⋯
4″	4	4	4	4	4	2	0	−2	−4	2	2	0	2	0	−2	0	0	0	0	0	⋯
$\tilde{4}$	4	4	4	4	4	−2	0	2	−4	−2	−2	0	−2	0	2	0	0	0	0	0	⋯
$\tilde{4}'$	4	4	4	4	4	−2	0	2	−4	−2	−2	0	−2	0	2	0	0	0	0	0	⋯
$\tilde{4}''$	4	4	4	4	4	−2	0	2	−4	−2	−2	0	−2	0	2	0	0	0	0	0	⋯
6	6	6	6	6	6	0	−2	0	6	0	0	−2	0	−2	0	2	2	2	0	0	⋯
6′	6	6	6	6	6	0	−2	0	6	0	0	−2	0	−2	0	−2	−2	−2	0	0	⋯
8	8	5	2	−1	−4	6	4	2	0	3	0	1	−3	−2	−1	0	0	0	0	0	⋯
8′	8	5	2	−1	−4	6	4	2	0	3	0	1	−3	−2	−1	0	0	0	0	0	⋯
8″	8	5	2	−1	−4	6	4	2	0	3	0	1	−3	−2	−1	0	0	0	0	0	⋯
$\tilde{8}$	8	5	2	−1	−4	−6	4	−2	0	−3	0	1	3	−2	1	0	0	0	0	0	⋯
$\tilde{8}'$	8	5	2	−1	−4	−6	4	−2	0	−3	0	1	3	−2	1	0	0	0	0	0	⋯
$\tilde{8}''$	8	5	2	−1	−4	−6	4	−2	0	−3	0	1	3	−2	1	0	0	0	0	0	⋯
16	16	−8	4	−2	1	0	0	0	0	0	0	0	0	0	0	4	−2	1	0	0	⋯
16′	16	−8	4	−2	1	0	0	0	0	0	0	0	0	0	0	4	−2	1	0	0	⋯
16″	16	−8	4	−2	1	0	0	0	0	0	0	0	0	0	0	4	−2	1	0	0	⋯
24	24	6	−3	−3	6	12	4	0	0	0	−3	−2	3	1	0	4	1	−2	2	−1	⋯
24′	24	6	−3	−3	6	12	4	0	0	0	−3	−2	3	1	0	−4	−1	2	−2	1	⋯
$\widetilde{24}$	24	6	−3	−3	6	−12	4	0	0	0	3	−2	−3	1	0	4	1	−2	−2	1	⋯
$\widetilde{24}'$	24	6	−3	−3	6	−12	4	0	0	0	3	−2	−3	1	0	−4	−1	2	2	−1	⋯
24″	24	15	6	−3	−12	6	−4	−6	0	3	0	−1	−3	2	3	0	0	0	0	0	⋯
$\widetilde{24}''$	24	15	6	−3	−12	−6	−4	6	0	−3	0	−1	3	2	−3	0	0	0	0	0	⋯
32	32	−4	−4	5	−4	8	0	0	0	−4	2	0	−1	0	0	0	0	0	0	0	⋯
32′	32	−4	−4	5	−4	8	0	0	0	−4	2	0	−1	0	0	0	0	0	0	0	⋯
32″	32	−4	−4	5	−4	8	0	0	0	−4	2	0	−1	0	0	0	0	0	0	0	⋯
$\widetilde{32}$	32	−4	−4	5	−4	−8	0	0	0	4	−2	0	1	0	0	0	0	0	0	0	⋯
$\widetilde{32}'$	32	−4	−4	5	−4	−8	0	0	0	4	−2	0	1	0	0	0	0	0	0	0	⋯
$\widetilde{32}''$	32	−4	−4	5	−4	−8	0	0	0	4	−2	0	1	0	0	0	0	0	0	0	⋯
48	48	−24	12	−6	3	0	0	0	0	0	0	0	0	0	0	−4	2	−1	0	0	⋯
$\widetilde{48}$	48	12	−6	−6	12	0	−8	0	0	0	0	4	0	−2	0	0	0	0	0	0	⋯

	972	144	288	576	432	1296	864	288	432	864	144	288	576	432	1296	864	288	432	864
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1′	1	ω	ω	ω	ω	ω	ω	ω	ω	ω	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2
1″	1	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω^2	ω	ω	ω	ω	ω	ω	ω	ω	ω
3	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\tilde{1}$	1	1	1	1	−1	1	−1	1	−1	−1	1	1	1	−1	1	−1	1	−1	−1
$\tilde{1}'$	1	ω	ω	ω	$-\omega$	ω	$-\omega$	ω	$-\omega$	$-\omega$	ω^2	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	ω^2	$-\omega^2$	$-\omega^2$
$\tilde{1}''$	1	ω^2	ω^2	ω^2	$-\omega^2$	ω^2	$-\omega^2$	ω^2	$-\omega^2$	$-\omega^2$	ω	ω	ω	$-\omega$	ω	$-\omega$	ω	$-\omega$	$-\omega$
$\tilde{3}$	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	1	1	−1	−1	1	1	1	−1	1	1	1	−1	−1	1	1	1	−1

Group extensions and vacuum alignment

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Scan for Small Groups

- using the computer algebra system GAP and its SmallGroups catalogue, we have checked all groups with size smaller than 1000 (11,758,814 groups) and we have found a number of candidates:

- no candidates for T_7 or $\Delta(27)$, maybe because here 3 is complex and there are more couplings that have to be forbidden (also smaller number of possible extensions)

- all candidates in list have non-trivial centre(=element that commute with all other elements), not necessary true for all groups(see e.g. $(S_3)^4 \rtimes A_4$ studied in Babu/Gabriel 2010)

Subgroup H	Order of G	GAP	Structure Description	$Z(G)$
A_4	96	204	$Q_8 \rtimes A_4$	Z_2
	288	860	$T' \rtimes A_4$	Z_2
	384	617, 20123	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes A_4$	Z_2
	576	8273	$(Z_2.S_4) \rtimes A_4$	Z_2
	768	1083945	$(Z_4.Z_4^2) \rtimes A_4$	Z_4
		1085279	$((Z_2 \times Q_{16}) \rtimes Z_2) \rtimes A_4$	Z_2
S_4	192	1494	$Q_8 \rtimes S_4$	Z_2
	384	18133, 20092	$(Z_2 \times Q_8) \rtimes S_4$	Z_2
		20096	$((Z_4 \times Z_2) \rtimes Z_2) \rtimes S_4$	Z_4
	576	8282	$T' \rtimes S_4$	Z_2
		8480	$(Z_3 \times Q_8) \rtimes S_4$	Z_6
	768	1086052, 1086053	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes S_4$	Z_2
	960	11114	$(Z_5 \times Q_8) \rtimes S_4$	Z_{10}
T'	192	1022	$Q_8 \rtimes T'$	Z_2^2
	648	533	$\Delta(27) \rtimes T'$	Z_3
	768	1083573, 1085187	$((Z_2 \times Q_8) \rtimes Z_2) \rtimes T'$	Z_2^2

Groups of the Structure $G \simeq N \rtimes H$, H is subgroup of G

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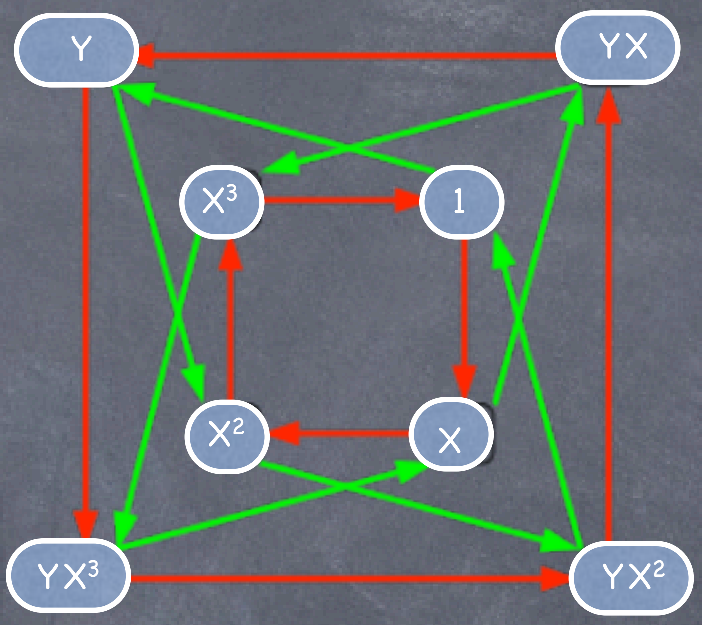
Quotient Group H	Order of G	GAP	Structure Description
A_4	96	201	$Z_2.(Z_2^2 \times A_4)$
	144	127	$Z_2.(A_4 \times S_3)$
	192	1017	$Z_2.(D_8 \times A_4)$
S_4	96	67, 192	$Z_4.S_4$
	144	121, 122	$Z_6.S_4$
	192	187, 963	$Z_8.S_4$
	192	987, 988	$Z_2.((Z_2^2 \times A_4) \rtimes Z_2)$
	192	1483,1484	$Z_2.(Z_2^2 \times S_4)$
	192	1492	$Z_2.((Z_2^4 \rtimes Z_3) \rtimes Z_2)$
T'	192	1007	$Z_2^2.(Z_2^2 \times A_4)$

Groups for which H is not a subgroup of G

Smallest Group



The smallest candidate group that contains A_4 as a subgroup is the semidirect product of the quaternion group Q_8

$$\langle X, Y | X^4 = 1,\ X^2 = Y^2,\ Y^{-1}XY = X^{-1} \rangle$$

with A_4

$$\langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$$

defined by the additional relations (⇔ the homomorphism φ:H→Aut(N) introduced earlier)

$$SXS^{-1} = X, \quad SYS^{-1} = Y^{-1}, \quad TXT^{-1} = YX, \quad TYT^{-1} = X\ .$$

Representations:

		1	T	SYX	SY	Y^2	T^2	TY	S	SX	X	$STYT$
unfaithful A_4 reps for leptons, χ	$\underline{\mathbf{1}}_1$	1	1	1	1	1	1	1	1	1	1	1
	$\underline{\mathbf{1}}_2$	1	ω	1	1	1	ω^2	ω	1	1	1	ω^2
	$\underline{\mathbf{1}}_3$	1	ω^2	1	1	1	ω	ω^2	1	1	1	ω
	$\underline{\mathbf{3}}_1$	3	.	-1	-1	3	.	.	-1	-1	3	.
	$\underline{\mathbf{3}}_2$	3	.	3	-1	3	.	.	-1	-1	-1	.
	$\underline{\mathbf{3}}_3$	3	.	-1	3	3	.	.	-1	-1	-1	.
	$\underline{\mathbf{3}}_4$	3	.	-1	-1	3	.	.	3	-1	-1	.
	$\underline{\mathbf{3}}_5$	3	.	-1	-1	3	.	.	-1	3	-1	.
faithful rep for Φ	$\underline{\mathbf{4}}_1$	4	1	.	.	-4	1	-1	.	.	.	-1
	$\underline{\mathbf{4}}_2$	4	ω^2	.	.	-4	ω	$-\omega^2$	.	.	.	$-\omega$
	$\underline{\mathbf{4}}_3$	4	ω	.	.	-4	ω^2	$-\omega$	.	.	.	$-\omega^2$

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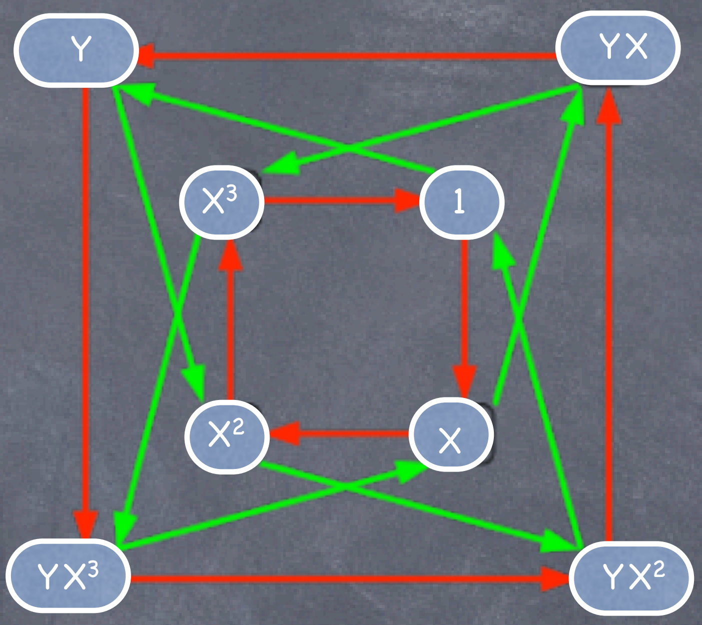

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Representations:

$$\underline{\mathbf{3}}_\mathbf{i} \times \underline{\mathbf{3}}_\mathbf{i} = \underline{\mathbf{1}}_\mathbf{1} + \underline{\mathbf{1}}_\mathbf{2} + \underline{\mathbf{1}}_\mathbf{3} + \underline{\mathbf{3}}_{\mathbf{i}\,S} + \underline{\mathbf{3}}_{\mathbf{i}\,A}$$

$$\underline{\mathbf{3}}_\mathbf{i} \times \underline{\mathbf{3}}_\mathbf{j} = \sum_{\substack{k=1 \\ k \neq i,j}}^{5} \underline{\mathbf{3}}_\mathbf{k} \qquad (i \neq j)$$

$$\underline{\mathbf{3}}_\mathbf{i} \times \underline{\mathbf{4}}_\mathbf{j} = \underline{\mathbf{4}}_\mathbf{1} + \underline{\mathbf{4}}_\mathbf{2} + \underline{\mathbf{4}}_\mathbf{3}$$

$$\underline{\mathbf{4}}_\mathbf{1} \times \underline{\mathbf{4}}_\mathbf{1} = \underline{\mathbf{1}}_{\mathbf{1}\,S} + \underline{\mathbf{3}}_{\mathbf{1}\,A} + \underline{\mathbf{3}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{3}\,S} + \underline{\mathbf{3}}_{\mathbf{4}\,S} + \underline{\mathbf{3}}_{\mathbf{5}\,A}$$

$$\underline{\mathbf{4}}_\mathbf{1} \times \underline{\mathbf{4}}_\mathbf{2} = \underline{\mathbf{1}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{1}\,A} + \underline{\mathbf{3}}_{\mathbf{2}\,S} + \underline{\mathbf{3}}_{\mathbf{3}\,S} + \underline{\mathbf{3}}_{\mathbf{4}\,S} + \underline{\mathbf{3}}_{\mathbf{5}\,A}$$

		S	T	X	Y
A_4 reps	$\underline{\mathbf{1}}_\mathbf{1}$	1	1	1	1
	$\underline{\mathbf{1}}_\mathbf{2}$	1	ω	1	1
	$\underline{\mathbf{1}}_\mathbf{3}$	1	ω^2	1	1
	$\underline{\mathbf{3}}_\mathbf{1}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Φ	$\underline{\mathbf{4}}_\mathbf{1}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$

faithful representation Φ is what we were looking for.
(Φ Φ) only contains non-trivial contraction of the A_4 subgroup.

The model

particle	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
ℓ	1	2	-1/2	$\underline{\mathbf{3}}_{\mathbf{1}}$	i
$e^c + \mu^c + \tau^c$	1	1	1	$\underline{\mathbf{1}}_{\mathbf{1}} + \underline{\mathbf{1}}_{\mathbf{2}} + \underline{\mathbf{1}}_{\mathbf{3}}$	$-$i
H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
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The model

$\langle\chi\rangle = (v', v', v')^T,$

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VEVs:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T,$$

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$$\langle(\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}\rangle = \frac{1}{2}(bc - ad, 0, 0)^T$$

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$$\mathcal{L}_e^{(5)} = y_e(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} e^c \tilde{H}/\Lambda + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} \mu^c \tilde{H}/\Lambda + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}} \tau^c \tilde{H}/\Lambda + \text{h.c.} ,$$

$$\mathbf{m_e} \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}$$

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LO neutral lepton masses:

$$\mathcal{L}_\nu^{(7)} = x_a (\ell H \ell H)_{\underline{\mathbf{1}}_{\mathbf{1}}} (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}/\Lambda^3 + x_d (\ell H \ell H)_{\underline{\mathbf{3}}_{\mathbf{1}}} \cdot (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}/\Lambda^3 + \text{h.c.}\ .$$

$$\mathbf{m_e} \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}$$

TBM

$$\mathbf{m_\nu} \sim \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a} \end{pmatrix}$$

($Z_2 \times Z_2$ symmetry accidental)

The model

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H	1	2	1/2	$\underline{\mathbf{1}}_{\mathbf{1}}$	1
χ	1	1	0	$\underline{\mathbf{3}}_{\mathbf{1}}$	1
ϕ_1	1	1	0	$\underline{\mathbf{4}}_{\mathbf{1}}$	1
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$$\mathcal{L}_e^{(5)} = y_e(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} e^c \tilde{H}/\Lambda + y_\mu(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}} \mu^c \tilde{H}/\Lambda + y_\tau(\ell\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}} \tau^c \tilde{H}/\Lambda + \text{h.c.}\ ,$$

LO neutral lepton masses:

$$\mathcal{L}_\nu^{(7)} = x_a(\ell H \ell H)_{\underline{\mathbf{1}}_{\mathbf{1}}} (\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}/\Lambda^3 + x_d(\ell H \ell H)_{\underline{\mathbf{3}}_{\mathbf{1}}} \cdot (\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}/\Lambda^3 + \text{h.c.}\ .$$

- additional 4_1 necessary to get correct symmetry breaking (otherwise only breaking to A_4)
- same # of d.o.f. as in case of complex triplet and singlet, no additional driving fields necessary
- low flavour symmetry breaking scale possible, testable

Scalar Potential & Vacuum Alignment

The most general scalar potential invariant under the flavour symmetry is given by

$$V(\chi,\phi_1,\phi_2) = V_\chi(\chi) + V_\phi(\phi_1,\phi_2) + V_{\rm mix}(\chi,\phi_1,\phi_2)$$

with

$$\begin{aligned} V_\phi(\phi_1,\phi_2) =& \mu_1^2(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}} + \alpha_1(\phi_1\phi_1)^2_{\underline{\mathbf{1}}_\mathbf{1}} + \sum_{i=2,3}\alpha_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}} \\ &+\mu_2^2(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}} + \beta_1(\phi_2\phi_2)^2_{\underline{\mathbf{1}}_\mathbf{1}} + \sum_{i=2,3}\beta_i(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}} \\ &+\gamma_1(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}} + \sum_{i=2,3,4}\gamma_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_\mathbf{i}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_\mathbf{i}} \\ V_\chi(\chi) =& \mu_3^2(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \rho_1(\chi\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_1(\chi\chi)^2_{\underline{\mathbf{1}}_\mathbf{1}} + \lambda_2(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{2}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}} \\ V_{\rm mix}(\chi,\phi_1,\phi_2) =& \zeta_{13}(\phi_1\phi_1)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} + \zeta_{23}(\phi_2\phi_2)_{\underline{\mathbf{1}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{1}} \end{aligned}$$

- Potential has an accidental symmetry $[(Q_8 \rtimes A_4)\times A_4]\times Z_4$
  - invariant under independent transformations of Φ and χ
- note that couplings such as $\chi\cdot(\phi_1\phi_2)_{\underline{\mathbf{3}}_\mathbf{1}}$ are forbidden by the auxiliary Z_4 symmetry that separates the charged and neutral lepton sectors

Scalar Potential & Vacuum Alignment

Scalar Potential & Vacuum Alignment

- Characterization of Minima

  - If there is a minimum, in which the symmetry generator $Q \in G$ is left unbroken, i.e. $Q \langle \Phi_i \rangle = \langle \Phi_i \rangle$,there are degenerate minima $\langle \Phi'_i \rangle = g \langle \Phi'_i \rangle$ that leave gQg^{-1} unbroken, with $g \in G$.

  - The physically distinct minima are therefore characterized by the conjugacy class $G\ Q = \{ gQg^{-1} : g \in G \}$ Only conjugacy classes with an eigenvalue +1 can lead to a non-trivial little group.

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- For 4_1 there are 5 such classes

There are three physically distinct minima of ϕ_1, that preserve a Z_2 subgroup:

- $\langle \phi_1 \rangle = \frac{1}{\sqrt{2}}(a, a, b, -b)^T$ results in the little group $\langle S \rangle$,
- $\langle \phi_1 \rangle = (0, a, b, 0)^T$ in $\langle SY \rangle$ and
- $\langle \phi_1 \rangle = \frac{1}{\sqrt{2}}(-a, b, -a, b)^T$ in $\langle SYX \rangle$.

In addition, there is one preserving a Z_3 subgroup:

- $\langle \phi_1 \rangle = \frac{1}{\sqrt{2}}(a, a, a, b)^T$ preserves $\langle T \rangle$ (as well as $\langle T^2 \rangle = \langle T \rangle$).

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- $\langle\phi_1\rangle = \frac{1}{\sqrt{2}}(-a,b,-a,b)^T$ in $\langle SYX\rangle$.

$$\Longrightarrow \quad \langle(\phi_1\phi_2)_{\underline{\mathbf{3}}_{\mathbf{1}}}\rangle = \frac{1}{2}(bc-ad,0,0)^T$$

$$\langle(\phi_1\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}\rangle = \frac{1}{2}(ac+bd)$$

(a,b) replaced by (c,d) in Φ_2

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Scalar Potential & Vacuum Alignment

Minimum Conditions

$$a\left(\alpha_+\left(a^2+b^2\right)+\alpha_-\left(a^2-b^2\right)+\gamma_+\left(c^2+d^2\right)+\gamma_-\left(c^2-d^2\right)+U_1\right)+\Gamma bcd=0$$
$$b\left(\alpha_+\left(a^2+b^2\right)-\alpha_-\left(a^2-b^2\right)+\gamma_+\left(c^2+d^2\right)-\gamma_-\left(c^2-d^2\right)+U_1\right)+\Gamma acd=0$$
$$c\left(\beta_+\left(c^2+d^2\right)+\beta_-\left(c^2-d^2\right)+\gamma_+\left(a^2+b^2\right)+\gamma_-\left(a^2-b^2\right)+U_2\right)+\Gamma abd=0$$
$$d\left(\beta_+\left(c^2+d^2\right)-\beta_-\left(c^2-d^2\right)+\gamma_+\left(a^2+b^2\right)-\gamma_-\left(a^2-b^2\right)+U_2\right)+\Gamma abc=0$$
$$v'\left(4\sqrt{3}\lambda_1 v'^2+3\rho_1 v'+2\mu_3^2+\zeta_{13}(a^2+b^2)+\zeta_{23}(c^2+d^2)\right)=0$$

with $\xi_+=\frac{\xi_1}{2},\xi_-=\frac{\xi_2+\xi_3}{2\sqrt{3}}$ for $\xi=\alpha,\beta$ for $\langle S\rangle$, similar relations for $\langle SY\rangle$, $\langle SYX\rangle$ $\quad U_i=\frac{1}{2}\left(\mu_i^2+\sqrt{3}\zeta_{i3}\,v'^2\right)$

$\gamma_+=\frac{\sqrt{3}\gamma_1+\gamma_4}{4\sqrt{3}},\quad \gamma_-=\frac{\gamma_2+\gamma_3}{4\sqrt{3}}$ and $\Gamma=\frac{\gamma_4}{\sqrt{3}}$

- eleven minimization conditions reduce to these 5 equations for 5 VEVs there is therefore generally a solution
- note that e.g. a=b=0 or c=d=0 is also a solution, here the singlet and triplet contraction vanishes
- we have performed a numerical study to show that there is finite region of parameter space where $\langle S\rangle$ is the global minimum

Higher Order Corrections

- NLO Corrections to vacuum potential

$$V^{(5)} = \sum_{L,M=1}^{2} \sum_{i,j=2}^{4} \frac{\delta_{ij}^{(LM)}}{\Lambda} \chi \cdot \left\{ (\phi_L \phi_L)_{\underline{\mathbf{3}}_\mathbf{i}} \cdot (\phi_M \phi_M)_{\underline{\mathbf{3}}_\mathbf{j}} \right\}_{\underline{\mathbf{3}}_\mathbf{1}} +$$

$$+ \frac{\chi^3}{\Lambda} \left(\delta_1^{(3)} \chi^2 + \delta_2^{(3)} (\phi_1 \phi_1)_{\underline{\mathbf{1}}_\mathbf{1}} + \delta_3^{(3)} (\phi_2 \phi_2)_{\underline{\mathbf{1}}_\mathbf{1}} \right)$$

$\delta_{ij}^{(LM)} = 0$ for $i \geq j$

- leads to shifts in VEVs

$$\langle \chi \rangle = (v' + \delta v'_1, v' + \delta v'_2, v' + \delta v'_2)^T,$$

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} (a + \delta a_1, a + \delta a_2, b + \delta a_3, -b + \delta a_4)^T,$$

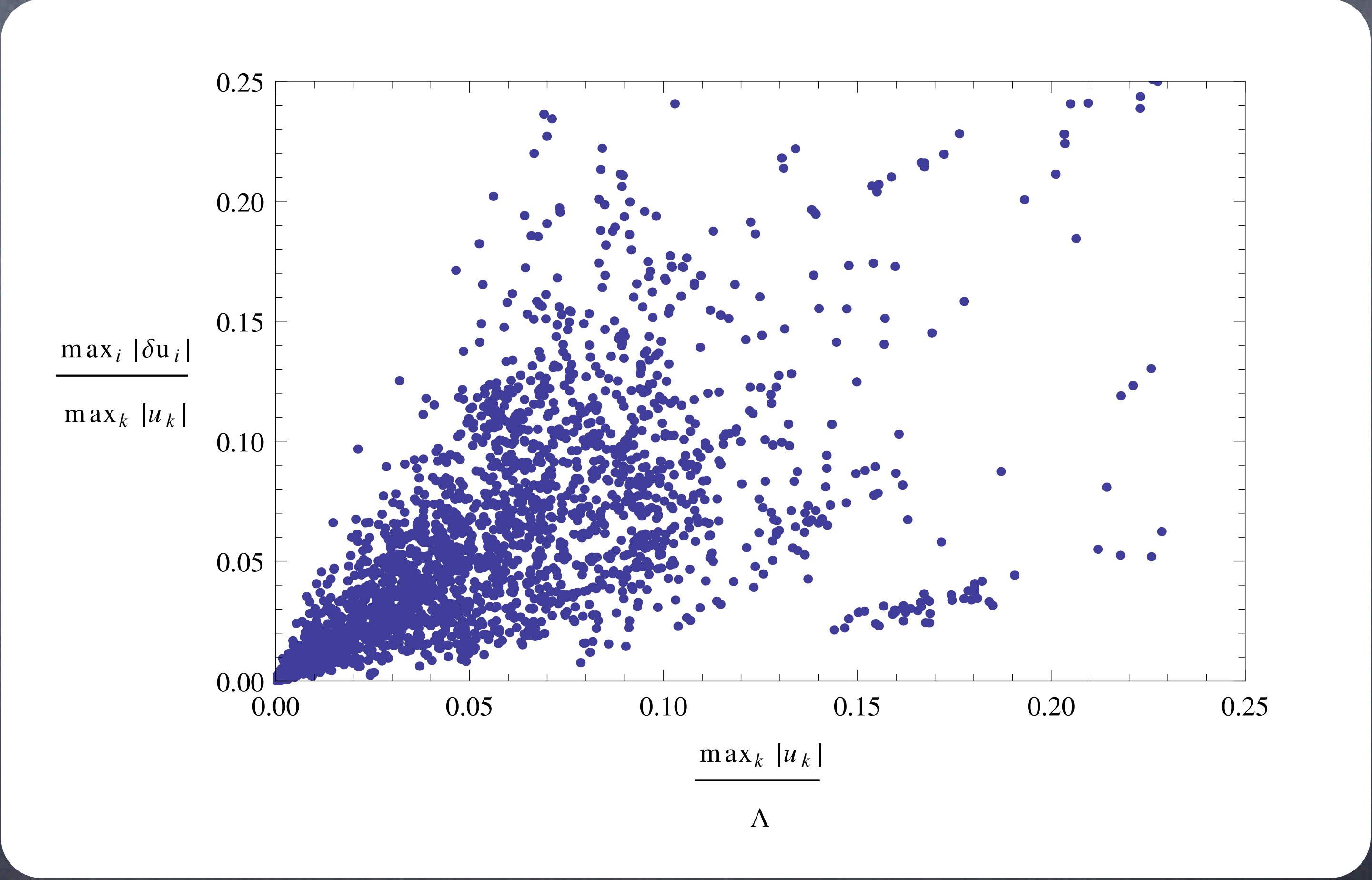
$$\langle \phi_2 \rangle = \frac{1}{\sqrt{2}} (c + \delta b_1, c + \delta b_2, d + \delta b_3, -d + \delta b_4)^T$$

- generic size of shifts

$$\frac{\delta u}{u} \sim \frac{u}{\Lambda}$$

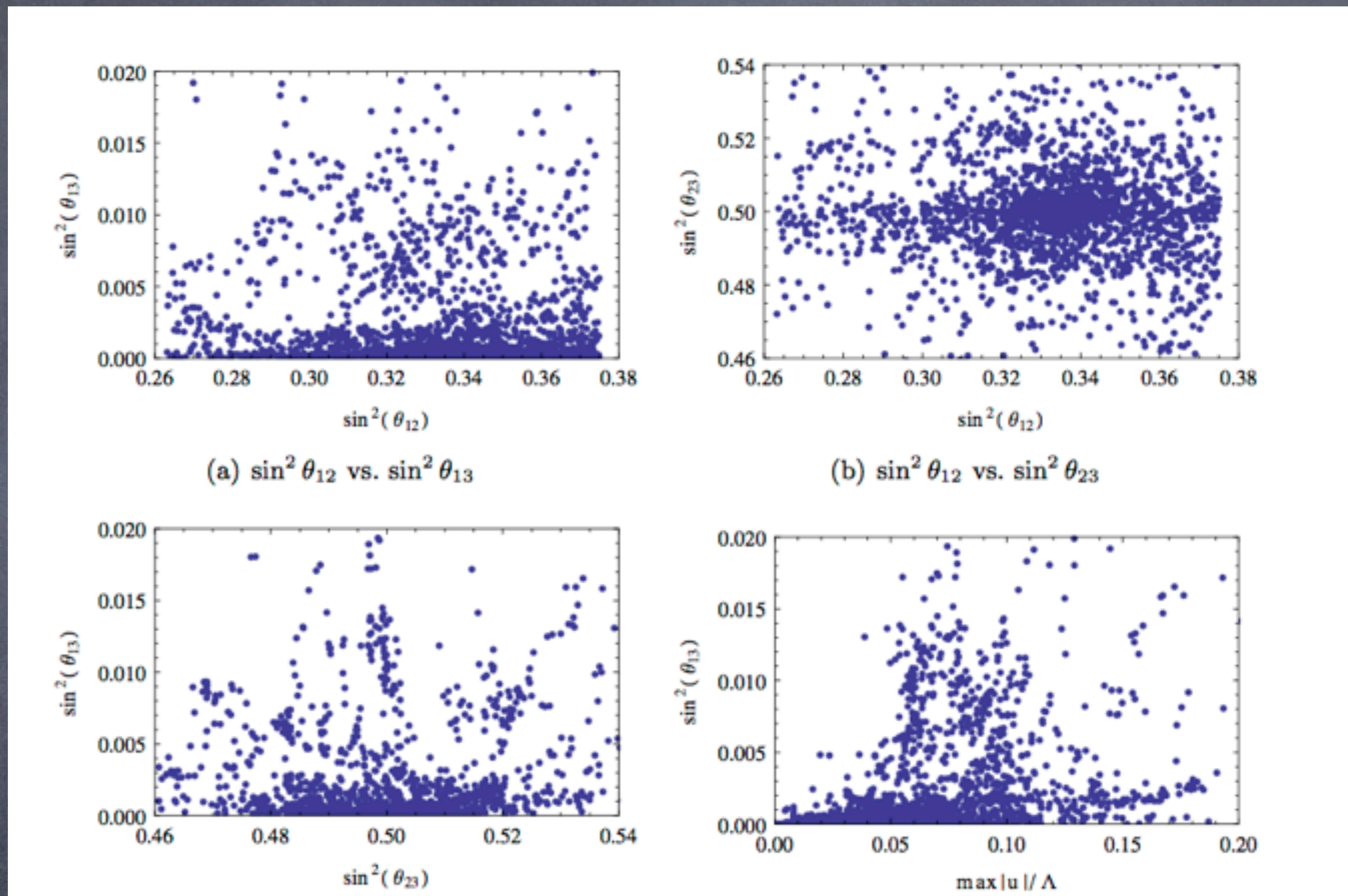
$$\langle \chi_2 \rangle - \langle \chi_3 \rangle = \mathcal{O}(1/\Lambda^2)$$

VEV alignment not destroyed!



generic size of shifts for scalar potential parameters of order one

Higher Order Corrections



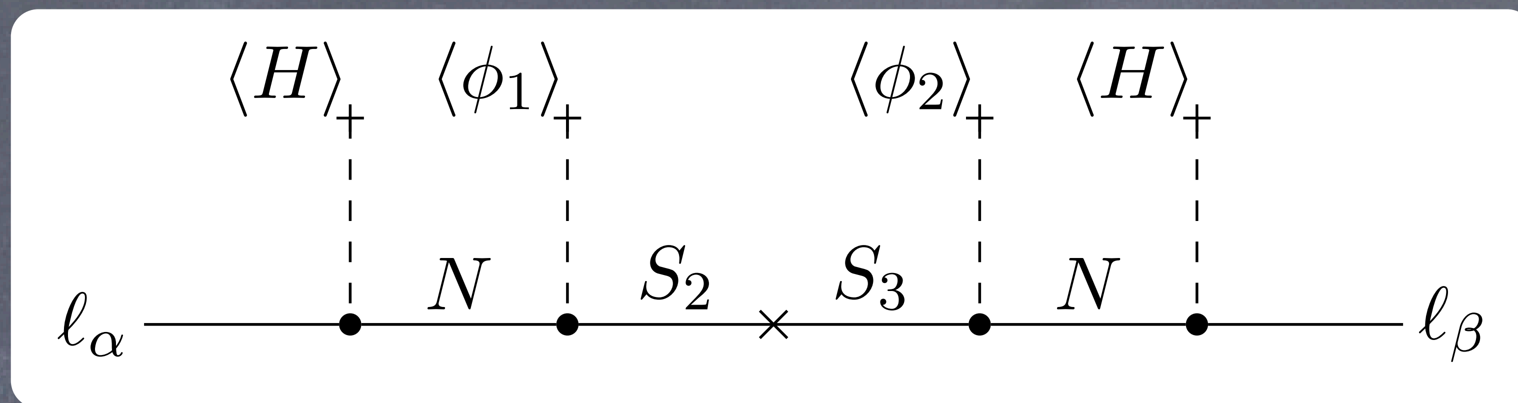
(a) $\sin^2\theta_{12}$ vs. $\sin^2\theta_{13}$
(b) $\sin^2\theta_{12}$ vs. $\sin^2\theta_{23}$
(c) $\sin^2\theta_{13}$ vs. $\sin^2\theta_{23}$
(d) $\sin^2\theta_{13}$ vs. $\frac{\max_i u_i}{\Lambda}$

- $\sin^2\Theta_{13}\approx .1$ as suggested by T2K can be accomodated at NLO
- or by introducing additional non-trivial singlet field $\xi \sim (1_2, i)$[does not destroy VEV alignment]

[Lin'10, Shimizu,Tanimoto, Watanabe'11,Luhn,King'11]

UV Completion

For Seesaw UV completion, introduce fermionic singlets N~ $(3_1,-i)$, S_2~ $(4_2,i)$, S_3~ $(4_3,-i)$:



$$\mathcal{L} = x_{\ell N} \ell H N + x_{N2} N S_2 \phi_1 + x_{N3} N S_3 \phi_2 + m S_2 S_3 + x_{23} S_2 S_3 \chi + \text{h.c.} \,,$$

generates singlet masses (N):

$$m_N = \frac{x_{N2} x_{N3}}{m} \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix} \quad \text{with} \quad A = -2(ac+bd) \quad \text{and} \quad B = \mathrm{i}\sqrt{3}(bc-ad) \;.$$

the light neutrino mass matrix $m_\nu = x_{\ell N}^2 v^2 m_N^{-1}$ is of TBM form

$$U_\nu^T m_\nu U_\nu = \text{diag}(\frac{1}{B+A}, \frac{1}{A}, \frac{1}{B-A})$$

Accidental degeneracy of m_1 and m_3 is lifted by introduction of additional S_2 or S_3.

Mathematica Package Discrete

We have developed a Mathematica Package that can be used to facilitate model building using discrete groups. It has the features:

- has access to groups catalogue of GAP, which contains all groups one would ever want to use

Initialization

```
In[8]:= Needs["Discrete`ModelBuildingTools`"];
```

```
In[11]:= Group = MBloadGAPGroup["AlternatingGroup(4)"];
starting GAP generating AlternatingGroup(4)...
...finished

StructureDescription:A4

Size of Group:12

Number of irreps: 4
```

Dimensions of irreps:

1	2	3	4
1	1	1	3

Character Table:

1	1	1	1
1	1	$e^{-\frac{2 i \pi}{3}}$	$e^{\frac{2 i \pi}{3}}$
1	1	$e^{\frac{2 i \pi}{3}}$	$e^{-\frac{2 i \pi}{3}}$
3	-1	0	0

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```
In[193]:= χ = MBgetRepVector[Group, 4, χc]
          L = MBgetRepVector[Group, 4, Lc]

Out[193]= {{}, {}, {}, {{χc1, χc2, χc3}}}

Out[194]= {{}, {}, {}, {{Lc1, Lc2, Lc3}}}
```

```
In[195]:= MBmultiply[Group, χ, L]
Out[195]= {{{(Lc1 χc1 + Lc2 χc2 + Lc3 χc3)/√3}},
  {{1/6 (2 √3 Lc1 χc1 - (3 i + √3) Lc2 χc2 - (-3 i + √3) Lc3 χc3)}},
  {{1/6 (2 √3 Lc1 χc1 - (-3 i + √3) Lc2 χc2 - (3 i + √3) Lc3 χc3)}},
  {{Lc3 χc2, Lc1 χc3, Lc2 χc1}, {Lc2 χc3, Lc3 χc1, Lc1 χc2}}}
```

```
In[197]:= MBmultiply[Group, {χ, χ, χ, L, L}][[1]]
Out[197]= {{(Lc1^2 + Lc2^2 + Lc3^2) χc1 χc2 χc3},
  {1/3 (Lc2 Lc3 χc1 + Lc1 Lc3 χc2 + Lc1 Lc2 χc3) (χc1^2 + χc2^2 + χc3^2)},
  {(Lc1 Lc3 χc2 χc3^2 + Lc2 χc1 (Lc3 χc2^2 + Lc1 χc1 χc3))/√3},
  {(Lc2 Lc3 χc1 χc3^2 + Lc1 χc2 (Lc3 χc1^2 + Lc2 χc2 χc3))/√3},
  {1/(6 √3) (Lc1 Lc3 χc2 (-(-3 i + √3) χc1^2 + 2 √3 χc2^2 - (3 i + √3) χc3^2) +
     Lc2 (Lc1 χc3 (-(3 i + √3) χc1^2 - (-3 i + √3) χc2^2 + 2 √3 χc3^2) +
       Lc3 χc1 (2 √3 χc1^2 - (3 i + √3) χc2^2 - (-3 i + √3) χc3^2)))},
```

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- reduce set covariants to a smaller set of independent covariants
- calculate flavon potentials

```
In[200]:= MBgetFlavonPotential[Group, χ, 4, λ]
          2
          3
          4
```

Out[200]=
$$\lambda 3n1\, \chi c1\, \chi c2\, \chi c3 + \frac{\lambda 2n1\, (\chi c1^2 + \chi c2^2 + \chi c3^2)}{\sqrt{3}} + \lambda 4n1\, (\chi c1^4 + \chi c2^4 + \chi c3^4) + \frac{1}{3}\lambda 4n2\, (\chi c2^2\, \chi c3^2 + \chi c1^2\, (\chi c2^2 + \chi c3^2))$$

[see also SUtree, Merle Zwicky]

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- available at http://projects.hepforge.org/discrete/

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