New holographic models for QCD and technicolor

Matti Järvinen

University of Crete

Based on: MJ & Elias Kiritsis, arXiv:1112.1261

Strong Interactions Beyond the Standard Model

Bad Honnef – 15 February 2012

Introduction

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

- Often useful (in holography and otherwise): “quenched” or “probe” approximation, $N_f \ll N_c$
- However, many features cannot be captured in such approximations:
  - Phase diagram of QCD at zero temperature, baryon density, and quark mass, varying $x = N_f/N_c$
  - The QCD thermodynamics as a function of x
  - Phase diagram as a function of baryon density

Strong coupling phenomena, hard to analyze $\longrightarrow$ Holographic methods?

Introduction: QCD phases

Phase diagram at zero temperature and chemical potential with massless quarks

- Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed
- Banks-Zaks region, $x = 11/2 - \epsilon$, under perturbative control
- Going for smaller x expect a conformal transition at $x = x_c$

[Miransky; Kaplan, Lee, Son, Stephanov, arXiv:0905.4752]

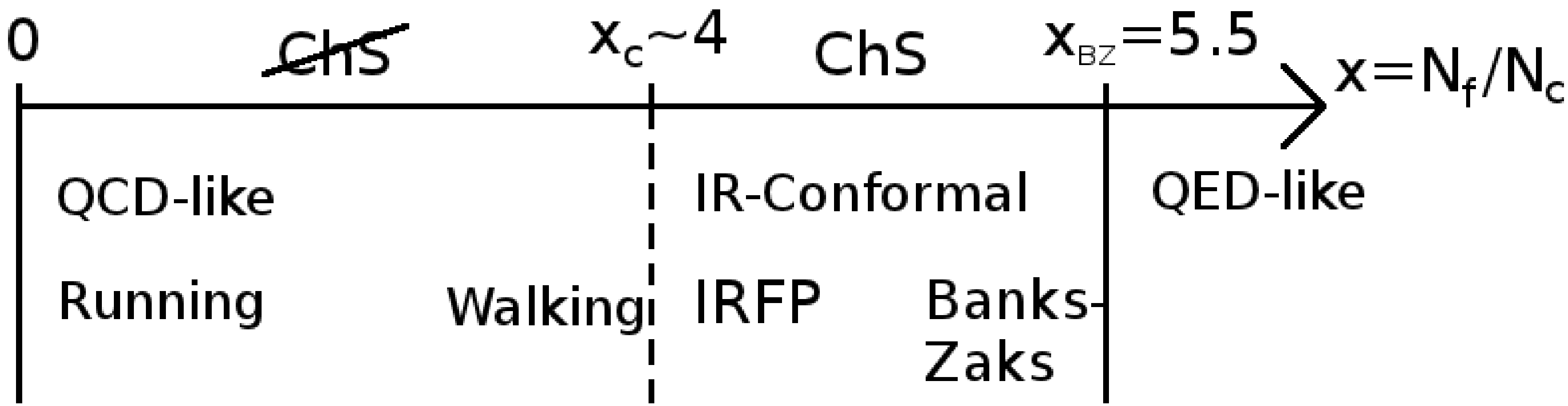


Diagram for $\mathcal{N}=1$ superQCD known, interesting and more complex

Aim of the talk

I present holographic bottom-up models (V-QCD) that describe the QCD phase diagram in the Veneziano limit, with:

[MJ, Kiritsis arXiv:1112.1261]

- ❑ Conformal window for $x_c < x < x_{BZ}$, ChSB for $0 < x < x_c$
- ❑ Critical value $x_c \sim 4$ arising from dynamics
- ❑ Walking backgrounds for x slightly below x_c

We also get:

- ❑ Reasonable behavior for finite (flavor independent) quark mass
- ❑ Miransky/BKT scaling as $x \to x_c$ from below
- ❑ Efimov vacua for $x < x_c$

After choosing the correct action, these features appear almost automatically!

A step back: 5D dilaton gravity

For YM, “improved holographic QCD” (IhQCD): well-tested string-inspired bottom-up model

[Gursoy, Kiritsis, Nitti arXiv:0707.1324, 0707.1349]
[Gubser, Nellore arXiv:0804.0434]

$$\mathcal{S}_{\mathrm{g}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right]$$

with Poincaré invariant metric

$$ds^2 = e^{2A}(dr^2 + \eta_{\mu\nu} x^\mu x^\nu)$$

- Potential $V_g \leftrightarrow$ QCD β-function
  - $A \to \log\mu$ energy scale
  - $e^\phi \to \lambda$ ’t Hooft coupling $g^2 N_c$

$$V_g = \frac{12}{\ell^2}(1 + c_1\lambda + \cdots), \; \lambda \to 0, \quad V_g \sim \lambda^{4/3}\sqrt{\log\lambda}, \; \lambda \to \infty$$

Agrees well with pure YM, both a zero and finite temperature

[Gursoy, Kiritsis, Mazzanti, Nitti; Panero; . . .]

The fusion

① IhQCD: model for glue by using dilaton gravity

② Adding flavor and chiral symmetry breaking via tachyon brane actions (previous talk!)

Consider ①+②, with full backreaction $\Rightarrow$ V-QCD model

The fusion: in terms of actions

$$\mathcal{S}_{\rm V-QCD} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda, T) \sqrt{-\det(g_{ab} + h(\lambda)\partial_a T \partial_b T)}$$

with $V_f(\lambda, T) = V_{f0}(\lambda) \exp(-a(\lambda)T^2)$; $\lambda = e^\phi$

- V-limit $N_c \to \infty$ with $x = N_f/N_c$ fixed: backreacted system
- $T \leftrightarrow \bar{q}q$
- Probe limit $x \to 0 \Rightarrow V_g$ fixed as before
- Must choose V_{f0}, a, and h

The simplest and most reasonable choices do the job!

Matching to QCD: UV

As $\lambda \to 0$ potentials analytic, and we can match:

- $V_g(\lambda)$ with (two-loop) Yang-Mills beta function
- $V_g(\lambda) - xV_{f0}(\lambda)$ with QCD beta function
- $a(\lambda)/h(\lambda)$ with the anomalous dimension of the quark mass/chiral condensate (we will have proper running quark mass!)

The matching allows to mark the BZ point, that we normalize at $x = 11/2$

After this, a single undetermined parameter in the UV: W_0

$$V_g(\lambda) = V_0 + \mathcal{O}(\lambda)\,, \qquad V_{f0}(\lambda) = W_0 + \mathcal{O}(\lambda)$$

$$V_0 - xW_0 = 12/\ell_{\rm UV}^2$$

Matching to QCD: IR

In the IR, the tachyon has to diverge $\Rightarrow$ the tachyon action $\propto e^{-T^2}$ becomes small

- $V_g(\lambda)$ chosen as for Yang-Mills, so that a “good” IR singularity exists, as well as linear confinement
- $V_{f0}(\lambda)$, $a(\lambda)$, and $h(\lambda)$ chosen to produce tachyon divergence: several possibilities
- Phase structure essentially independent of IR choices! (Here “essentially” means that you can also screw this up if you try)

Background analysis

Analysis of the backgrounds (classical vacua) at zero temperature (T is tachyon)

- ❑ Expect two kinds of solutions (Elias' talk), with
  - ① Nontrivial tachyon profile
  - ② Identically vanishing tachyon
- ❑ Fully backreacted system $\Rightarrow$ rich dynamics, analysis complicated . . .

  However, main features can be understood without going to details

Effective potential

For solutions with $T = T_* = \mathrm{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, T_*) \right]$$

IhQCD with an effective potential

$$V_{\mathrm{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, T_*) = xV_g(\lambda) - V_{f0}(\lambda) \exp(-a(\lambda)T_*^2)$$

Minimizing for T_* we obtain $T_* = 0$ and $T_* = \infty$

- $T_* = 0$: $V_{\mathrm{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$
- $T_* \to \infty$: $V_{\mathrm{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

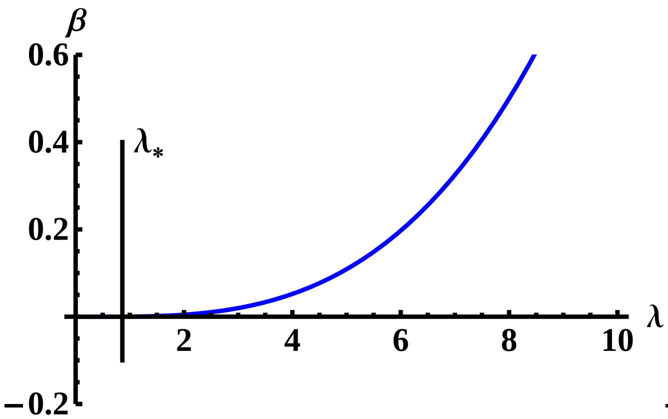
Effective potential

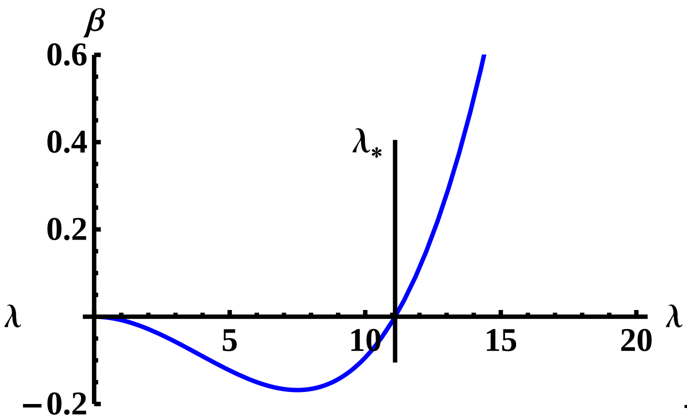
Start from Banks-Zaks region, $T_* = 0$, chiral symmetry conserved ($T \leftrightarrow \bar{q}q$), $V_{\rm eff}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

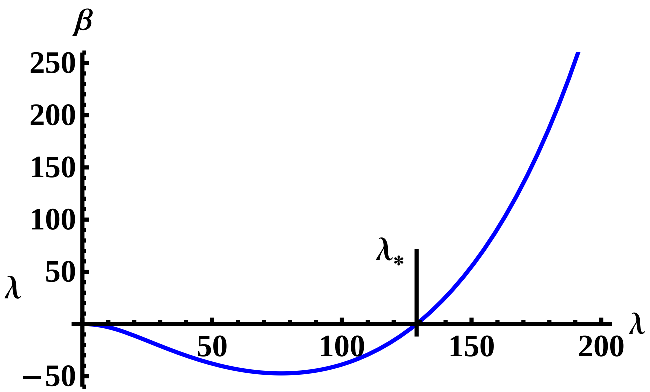
- $V_{\rm eff}$ defines a β-function as in IhQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- Fixed point λ_* runs to ∞ either at finite $x (< x_c)$ or as $x \to 0$

Banks-Zaks | Conformal Window

$x \to 11/2$ | $x > x_c$ | $x < x_c$??



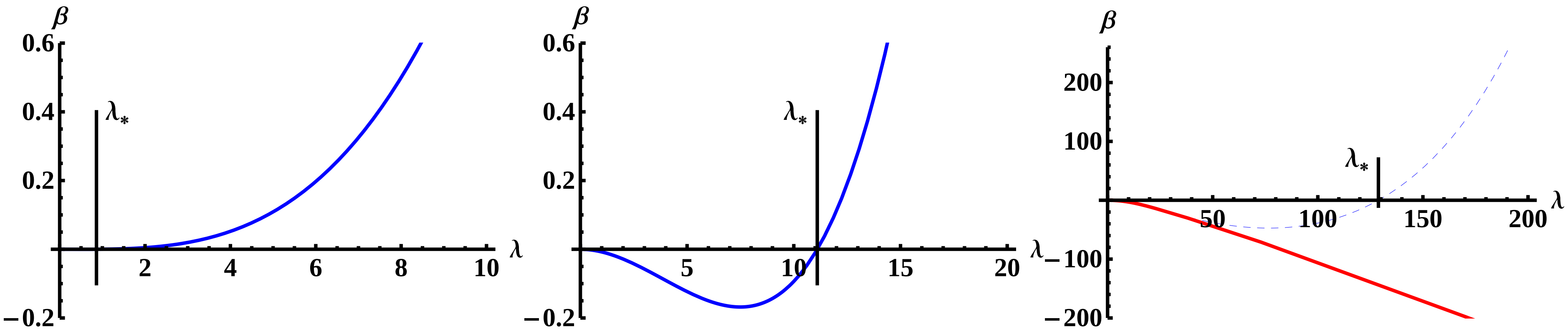




Effective potential

Banks-Zaks | Conformal Window

$x \to 11/2$ | $x > x_c$ | $x < x_c$



$T \equiv 0$ | $T \equiv 0$ | $T \neq 0$

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR T diverges, $V_{\mathrm{eff}} \to V_g \Rightarrow$ dynamics is YM-like

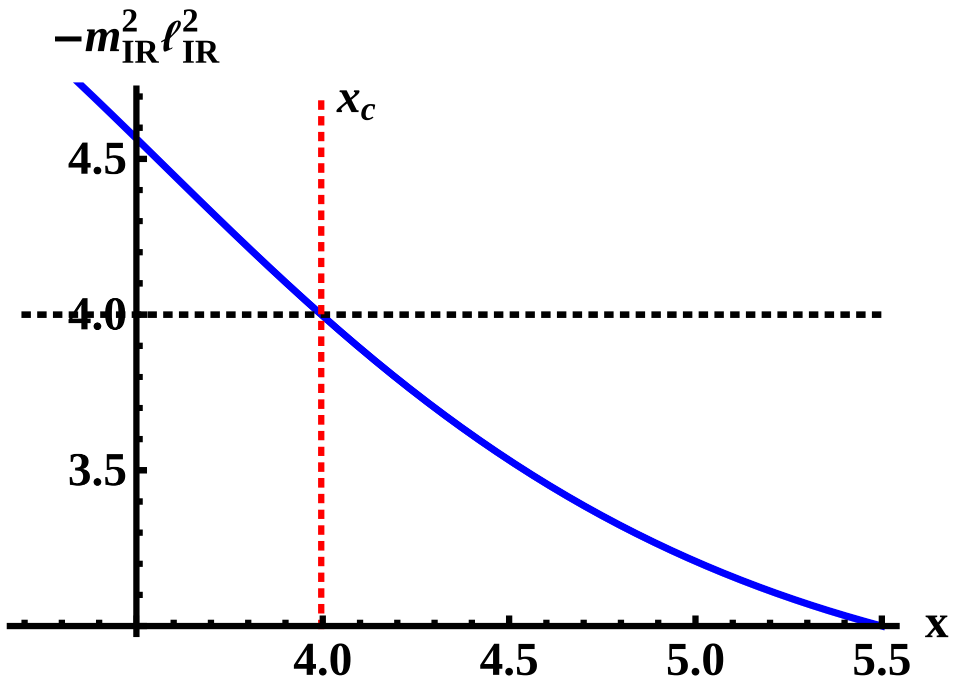
Where is x_c?

How is the edge of the conformal window stabilized?

Tachyon IR mass at $\lambda = \lambda_* \leftrightarrow$ quark mass dimension

$$-m_{\mathrm{IR}}^2 \ell_{\mathrm{IR}}^2 = \Delta_{\mathrm{IR}}(4 - \Delta_{\mathrm{IR}}) = \frac{24a(\lambda_*)}{h(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\mathrm{IR}} - 1$$



Breitenlohner-Freedman

(BF) bound (horizontal line)

$$-m_{\mathrm{IR}}^2 \ell_{\mathrm{IR}}^2 = 4 \;\Leftrightarrow\; \gamma_* = 1$$

defines x_c

Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details . . . the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) < 4 \quad (x > x_c)$:

$$T(r) \sim m_q r^{\Delta_{\rm IR}} + \sigma r^{4-\Delta_{\rm IR}}$$

- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) > 4 \quad (x < x_c)$:

$$T(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR}) \log r + \phi\right]$$

Rough analogy:

Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach

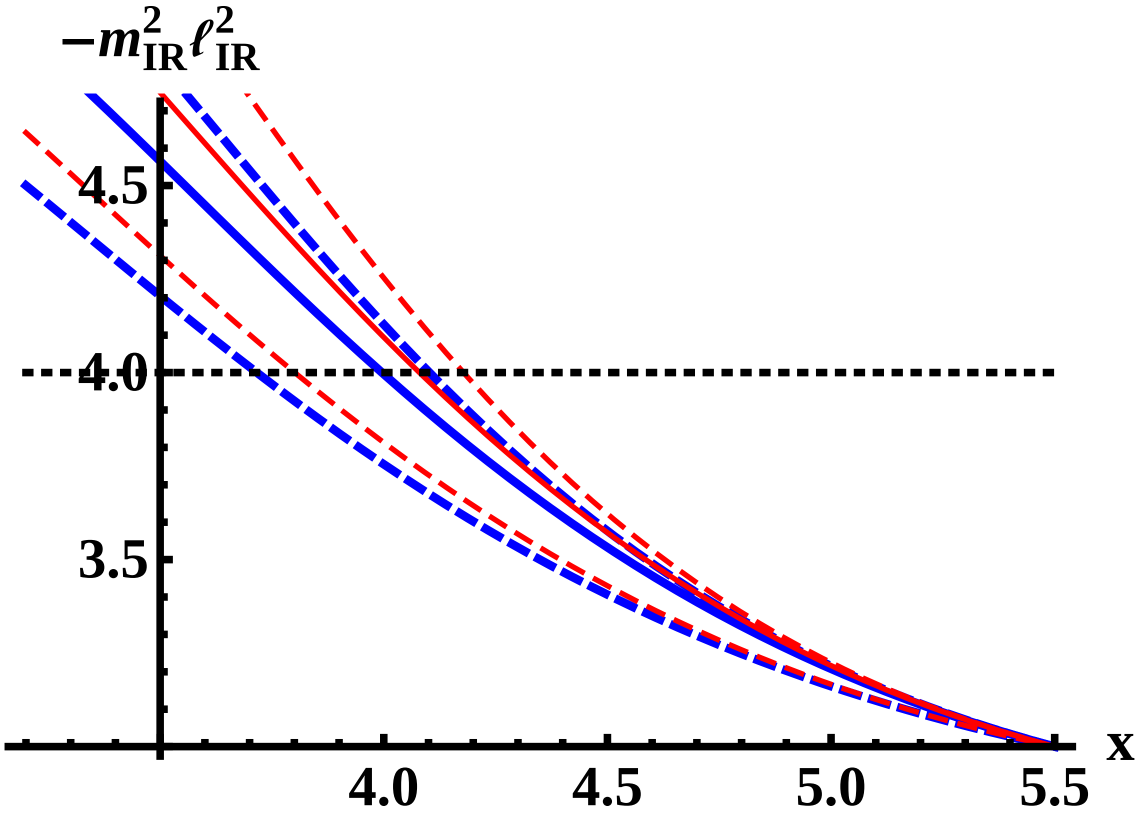
Similar observations have been made in other holographic frameworks

[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

“Prediction” for x_c

Dependence on the UV parameter W_0 and (reasonable) “IR choices” for the potentials

Resulting variation of the edge of conformal window $x_c = 3.7 \ldots 4.2$



γ_* in the conformal window

Comparison to other guesses

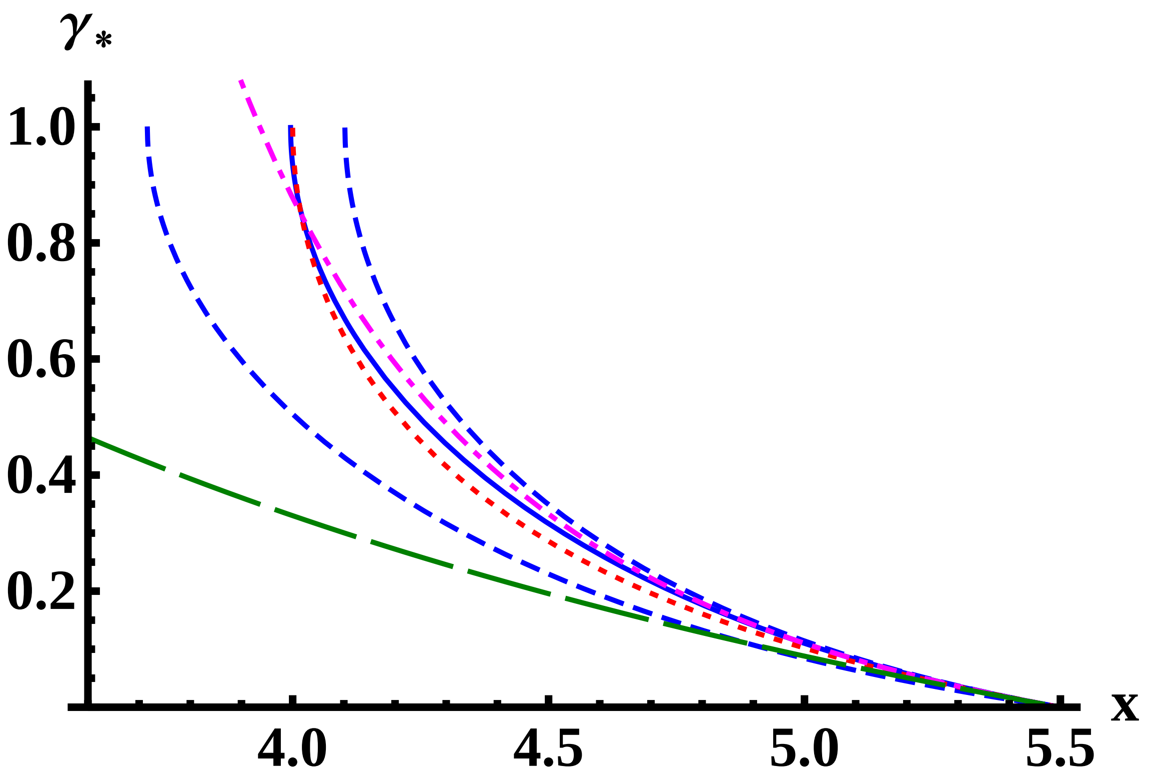
V-QCD (dashed: variation due to W_0)

Dyson-Schwinger

2-loop PQCD

All-orders β

[Pica, Sannino arXiv:1011.3832]



Backgrounds at zero quark mass

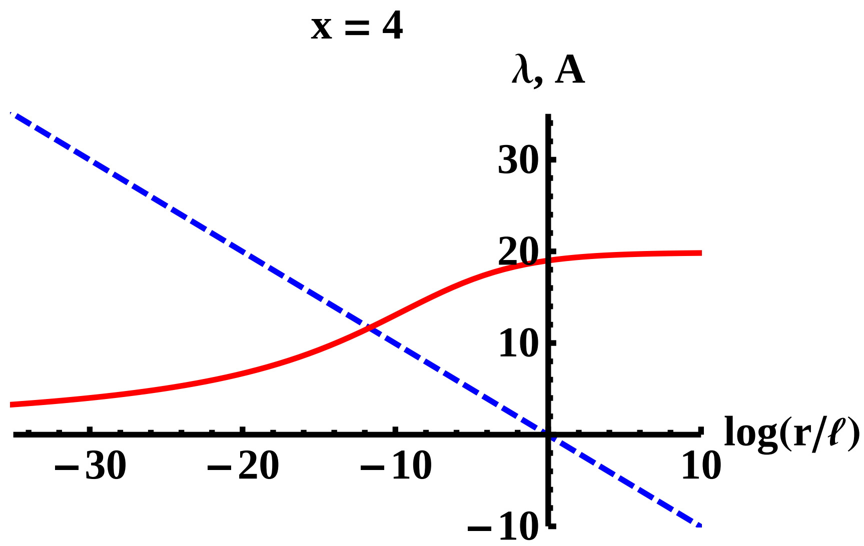
Color code:

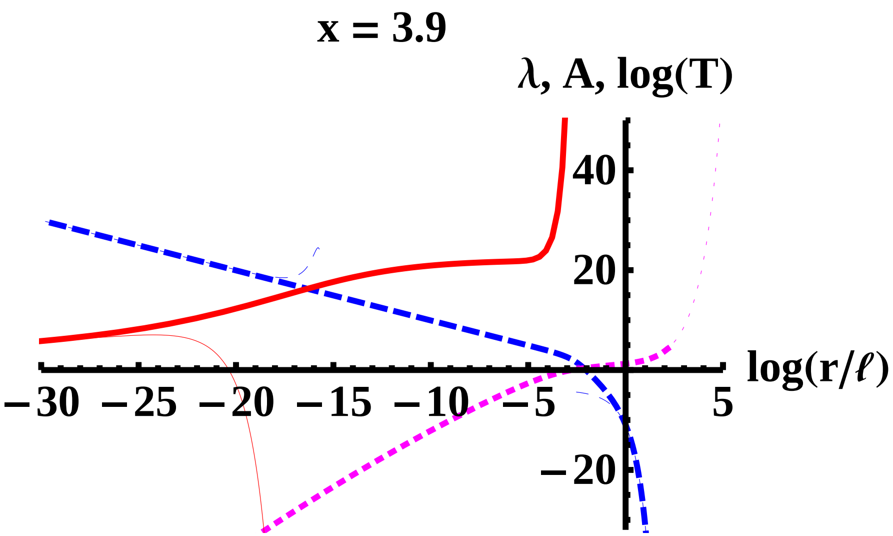
λ, A, T

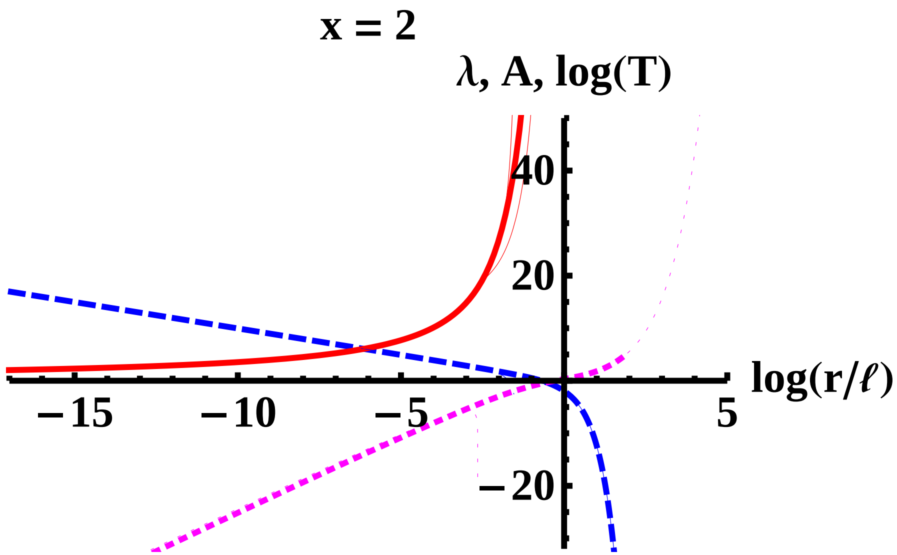
UV: $r = 0$

IR: $r = \infty$

$A \sim \log \mu \sim -\log r$







Metrics similar to earlier simplified approaches

[MJ, Sannino arXiv:0911.2462; Alanen, Kajantie, Tuominen arXiv:1003.5499]

Holographic β-functions

Generalization of the holographic RG flow of IhQCD

$$\beta(\lambda, T) \equiv \frac{d\lambda}{dA}\,; \qquad \gamma(\lambda, T) \equiv \frac{dT}{dA}$$

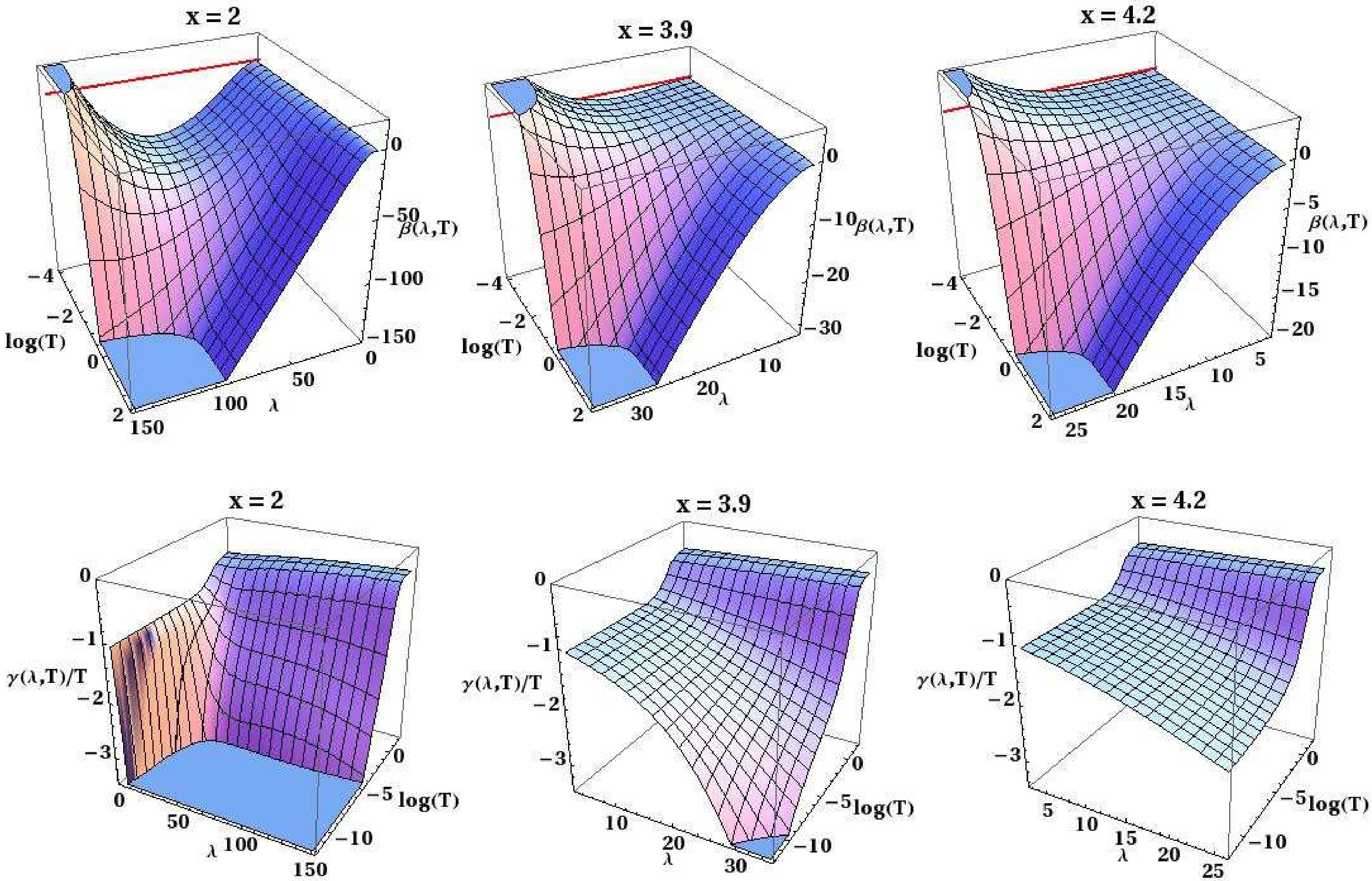
linked to

$$\frac{dg_{\mathrm{QCD}}}{d\log\mu}\,; \qquad \frac{dm}{d\log\mu}$$

The full equations of motion boil down to two first order partial non-linear differential equations for β and γ

Holographic β-functions

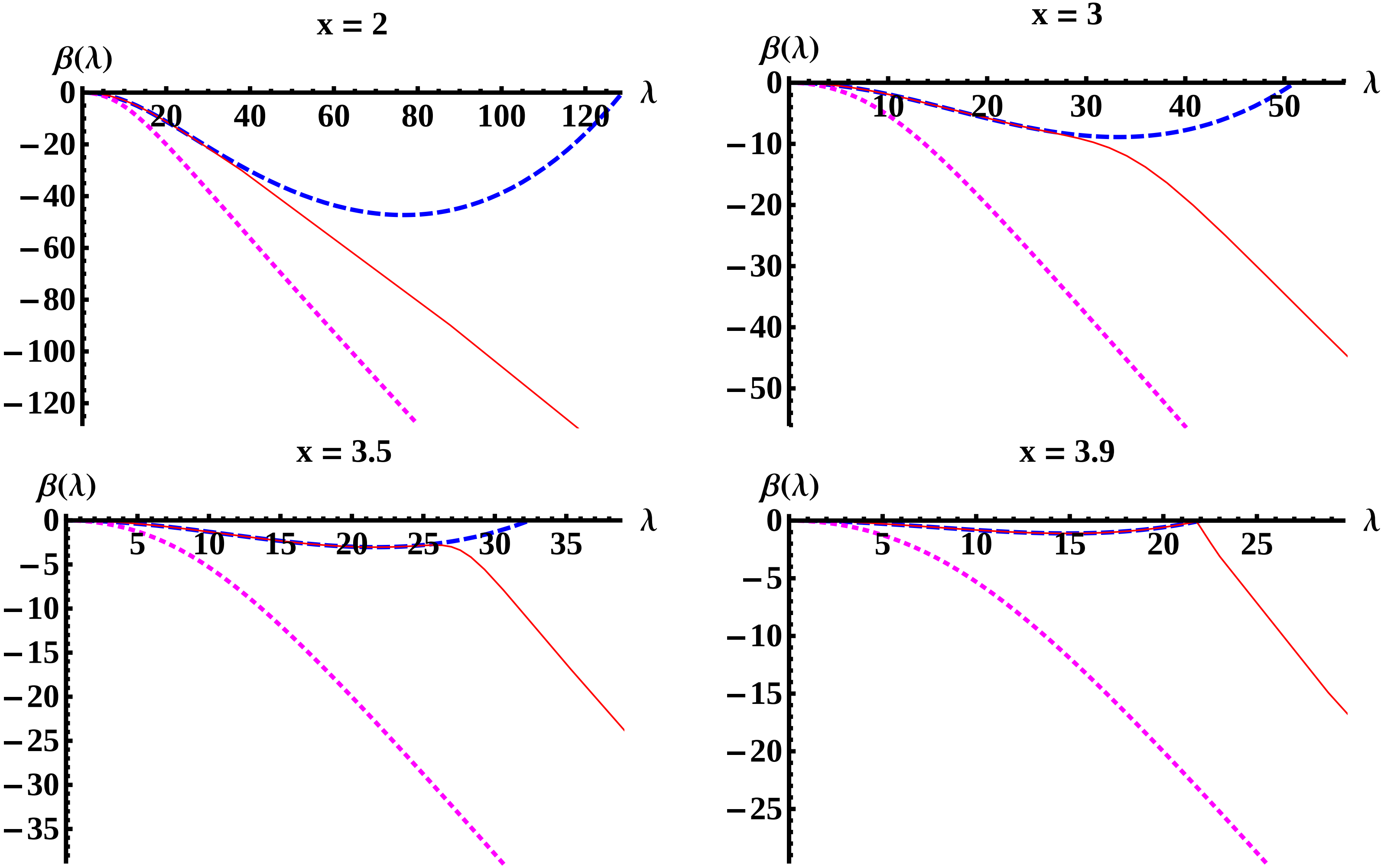
"Good" solutions numerically (unique)



Walking region

Beta functions along the RG flow (evaluated on the background),

zero tachyon, YM $x_c \simeq 3.9959$



Parameters

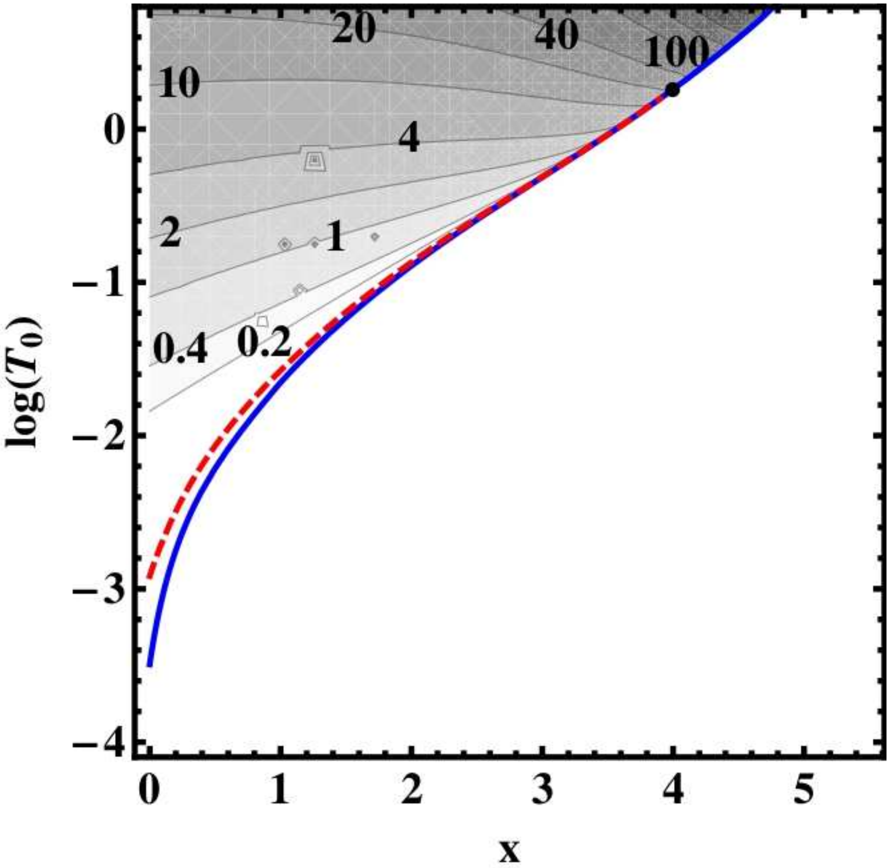
Understanding the generic solutions requires discussing parameters

- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass m: a single (dimensionless) parameter $m/\Lambda_{\rm QCD}$
- In this model, after rescalings, this parameter can be mapped to a parameter (T_0 or r_1) that controls the diverging tachyon in the IR
- x has become continuous in the Veneziano limit

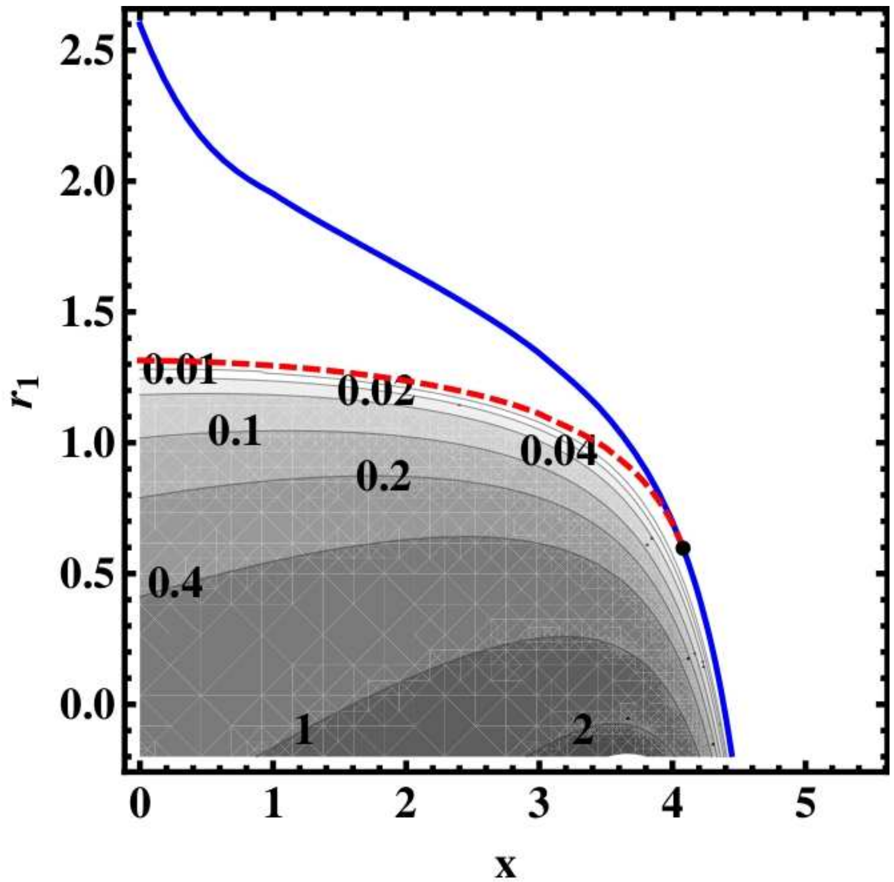
Map of all solutions

All "good" solutions ($T \neq 0$) obtained varying x and T_0 or r_1

Contouring: quark mass (zero mass is the red contour)

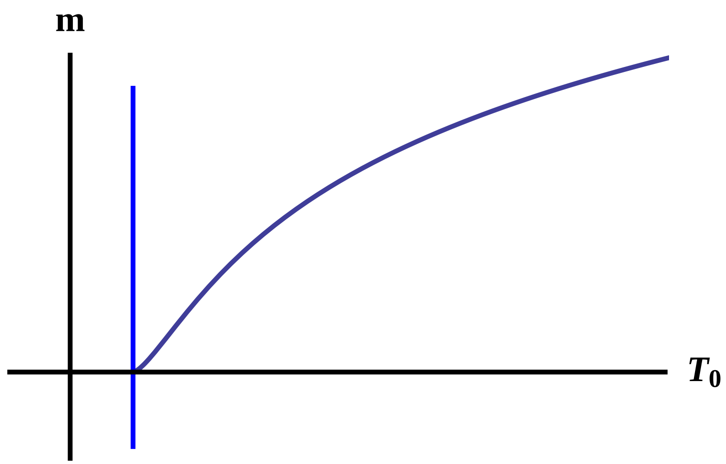


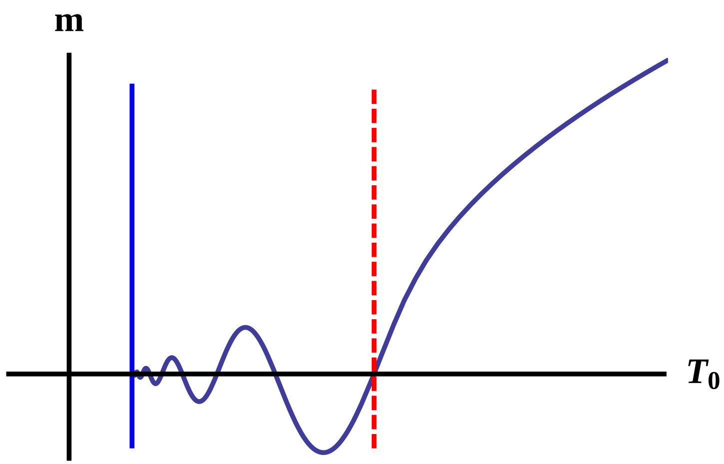
"Potentials I" $\leftrightarrow T_0$



"Potentials II" $\leftrightarrow r_1$

Mass dependence and Efimov vacua





Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $T \equiv 0$
- For $m > 0$, unique "standard" solution with $T \neq 0$

Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $T \equiv 0$ and $m = 0$
- "Standard" stable solution, with $T \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)

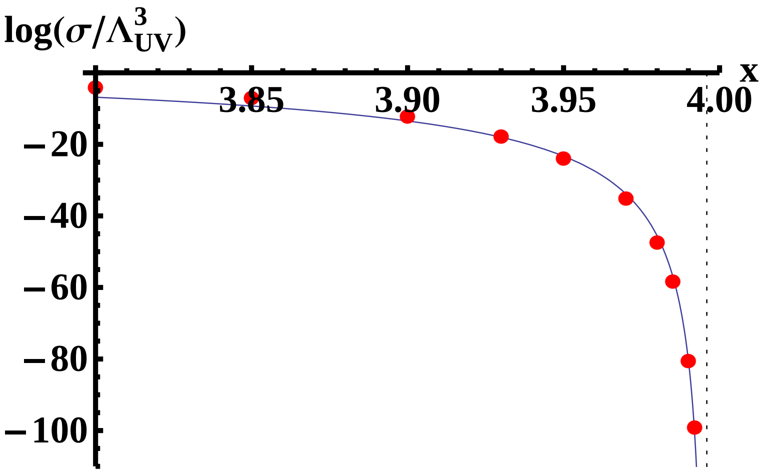
Miransky/BKT scaling

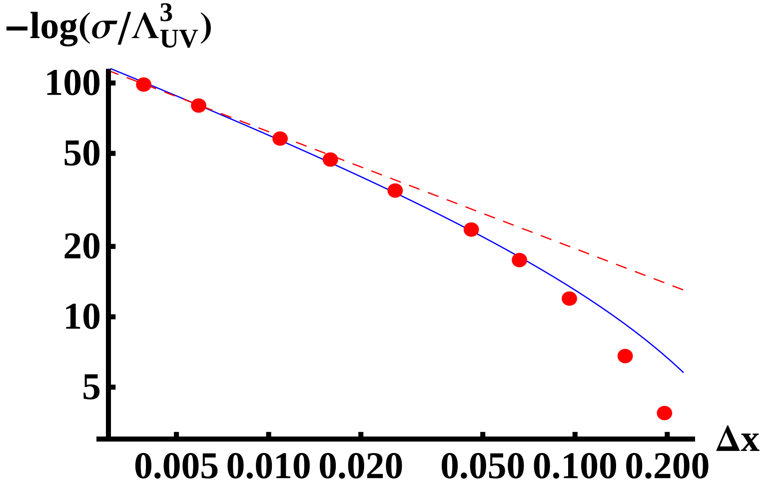
As $x \to x_c$

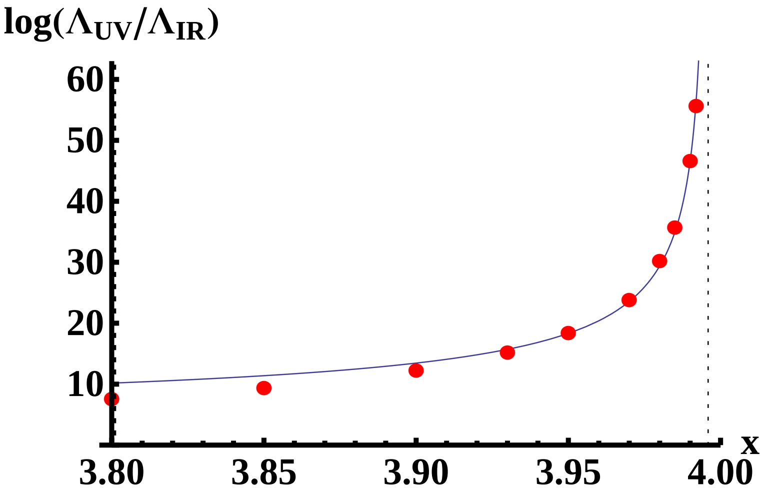
with known κ

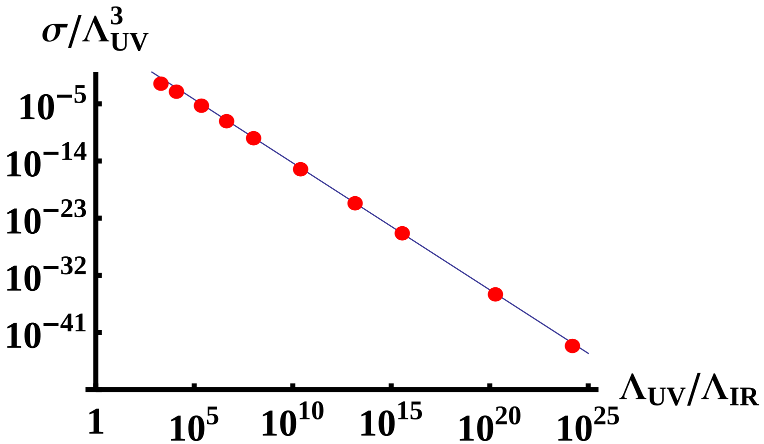
$$\langle \bar{q} q \rangle \sim \sigma \sim \exp(-2\kappa/\sqrt{x_c - x})$$

$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\kappa/\sqrt{x_c - x})$$









Conclusion

- A class of holographic bottom-up models (V-QCD) was obtained by a fusion of IhQCD with tachyonic brane action
- The models capture many interesting features of QCD at finite N_f/N_c
- Work in progress: calculation of mass spectra, oblique corrections, and thermodynamics (with Alho, Arean, Iatrakis, Kajantie, Kiritsis, and Tuominen)
- Lots of work TBD: finite density, hydrodynamics, four-fermion interactions, . . .

Extra slides

Extra slides . . .

sQCD phases

The case of $\mathcal{N}=1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- $x=0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.
- At $0<x<1$, the theory has a runaway ground state.
- At $x=1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x=1+1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1+2/N_c<x<3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f-N_c)$ IR free.
- At $3/2<x<3$, the theory flows to a CFT in the IR. Near $x=3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x=3/2$ the dual magnetic $SU(N_f-N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x>3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\rm IR}(4-\Delta_{\rm IR}) = \frac{24a(l_*)}{h(l_*)(V_g(l_*) - xV_0(l_*))}$$

- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) < 4$:
  $T(r) \sim m_q r^{4-\Delta_{\rm IR}} + \sigma r^{\Delta_{\rm IR}}$
- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) > 4$:
  $T(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR})\log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle
  $\longrightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\longrightarrow$ massless solution

Saturating the BF bound (sketch)

Does the nontrivial (ChSB) massless tachyon solution exist?

Two possibilities:

- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($T \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$

As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, “walking” dynamics

Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda, T) &= V_{f0}(\lambda)e^{-a(\lambda)T^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
h(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
T(r) \sim T_0 \exp\left[\frac{81\,3^{5/6}(115-16x)^{4/3}(11-x)}{812944\,2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

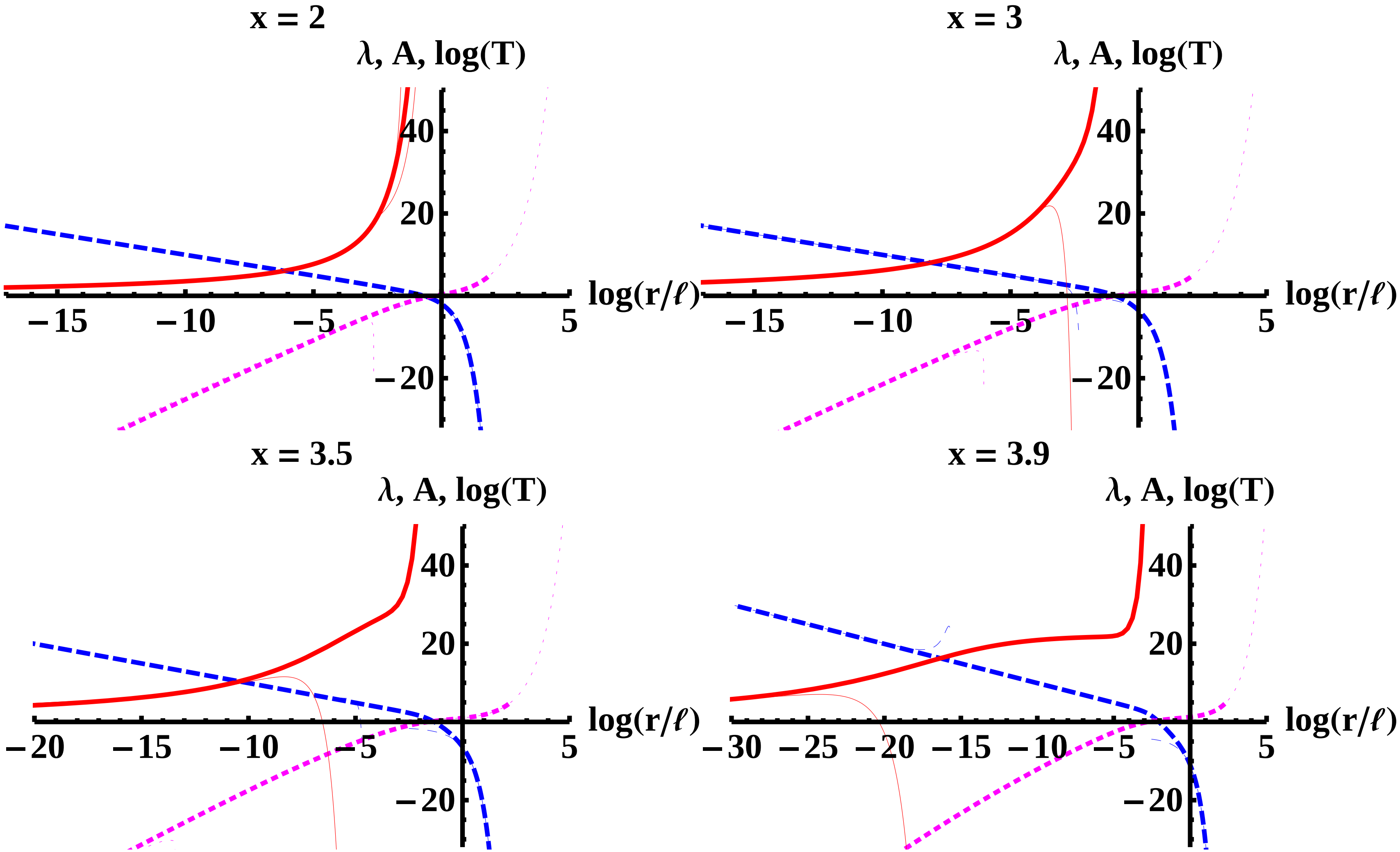
$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda, T) &= V_{f0}(\lambda) e^{-a(\lambda)T^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1 + \frac{115-16x}{216\pi^2}\lambda + \lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
h(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$
T(r) \sim \frac{27\, 2^{3/4} 3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}
$$

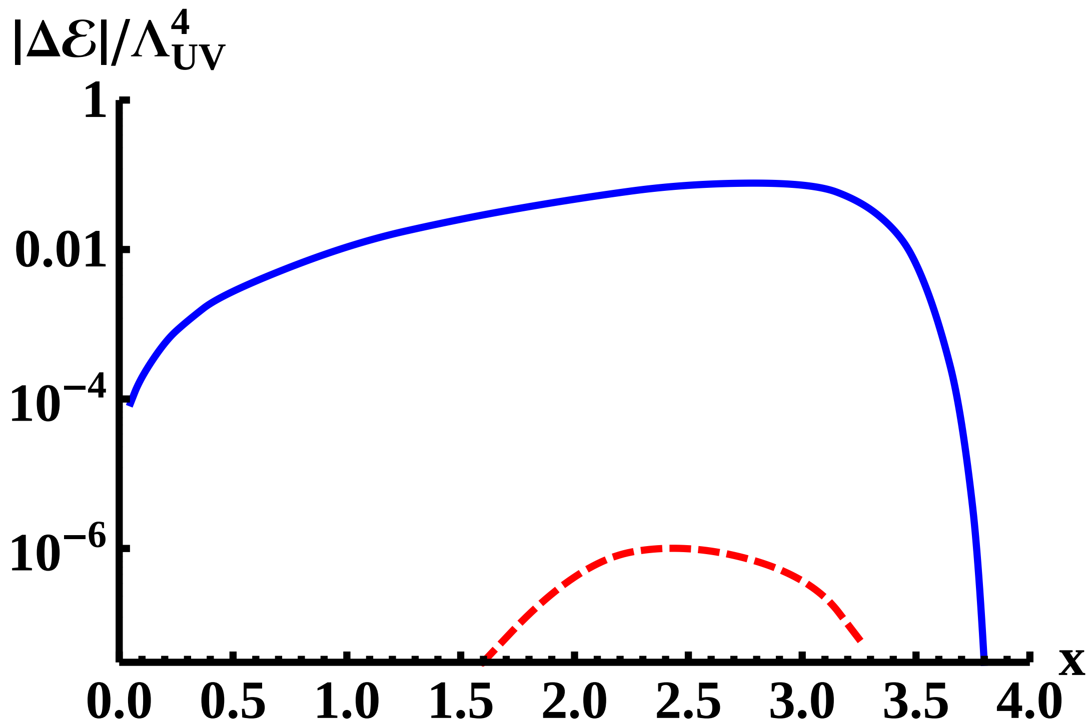
More backgrounds

Massless backgrounds with $x < x_c \simeq 3.9959$ (λ, A, T)



Free energy

The free energy difference between the ChS and ChSB $m_q = 0$ solutions
Chiral symmetry breaking solution favored whenever it exists ($x < x_c$)

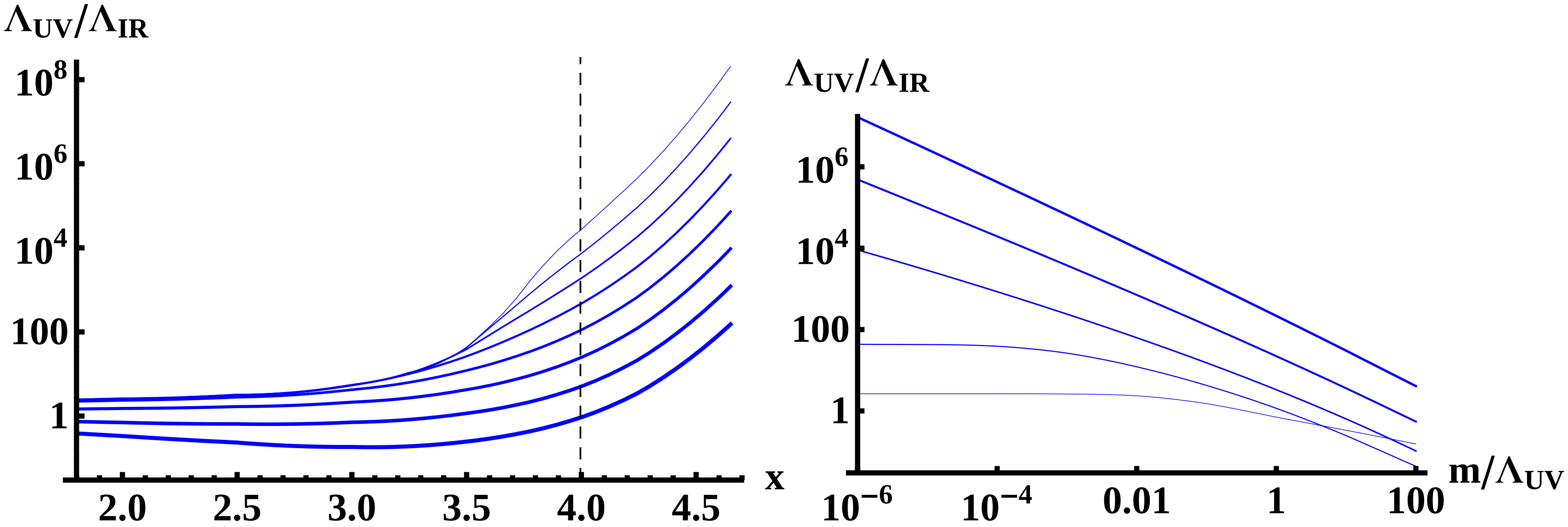


Mass dependence

For $m > 0$ the conformal transition disappears

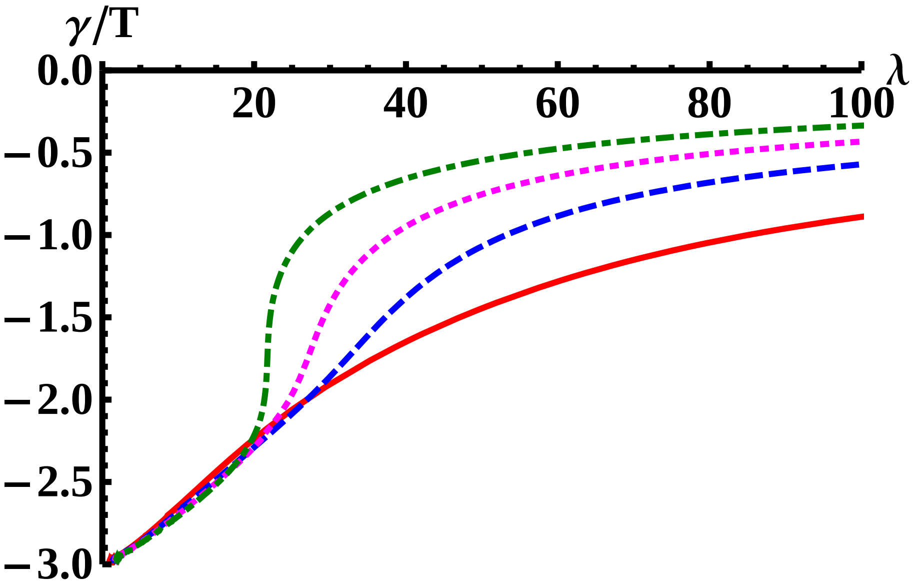
The ratio of typical UV/IR scales $\Lambda_{\rm UV}/\Lambda_{\rm IR}$ varies in a natural way

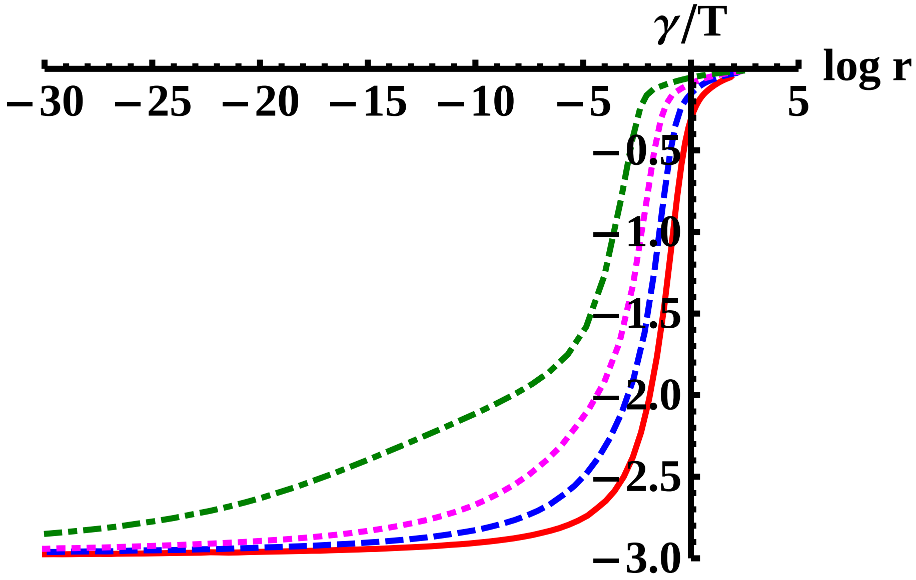
$m/\Lambda_{\rm UV} = 10^{-6}, 10^{-5}, \ldots, 10 \qquad x = 2, 3.5, 3.9, 4.25, 4.5$



γ-functions

Massless backgrounds: gamma functions $\frac{\gamma}{T} = \frac{d \log T}{dA}$





$x = 2, 3, 3.5, 3.9$