

# Large-scale structure challenges dilaton gravity in a 5D brane scenario with AdS bulk

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# Outline

introduction

at the brane: effective Einstein-like equation

AdS<sub>5</sub> bulk: large-scale structure at the brane?

conclusions

## beyond General Relativity

- ▶ ongoing search for a unified description of
  - gravity
  - gauge interactions of the Standard Model
  - ↔ *string theories* as the most promising proposal
- ▶ *low-energy effective action* in string theories
  - *dilaton* ( $\phi$ ): scalar field accompanying gravity
  - at the leading order (when restricted to gravity and the dilaton)
  - ↔ Einstein gravity coupled to the dilaton
- ▶ additional spatial dimensions
  - required by the string theories' formulation
  - have to be compactified or warped
  - ↔ *dilaton gravity in a 5D brane scenario*

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## scalar-tensor theories of gravity & conformal frames

► *dilaton gravity*: a scalar-tensor theory of gravity

↪ can be formulated in various *conformally-related frames*

○ gravitational Lagrangians differ e.g. in the coefficient of the Ricci scalar

↪ (generically) scalar field dependent coefficients

○ Einstein frame:  $\mathcal{L} = \frac{1}{2\kappa} \mathcal{R} + \dots$  (coefficient: a constant)

○ Jordan frame: e.g.  $\mathcal{L} = \frac{1}{16\pi} \phi \mathcal{R} + \dots$   
(coefficient: a polynomial function of the scalar field)

○ string frame: e.g.  $\mathcal{L} = e^{-\phi} \frac{\alpha_1}{2} \mathcal{R} + \dots$   
(coefficient: an exponential function of the dilaton)

○ related ( $g_{\mu\nu}$  &  $\tilde{g}_{\mu\nu}$ ) by a conformal (Weyl) transformation:  $g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu}$

## non-minimal matter-dilaton coupling

- ▶ if a matter term  $\mathcal{L}_m$  is included into the Lagrangian in one frame
  - ↻ conformal transformation to another frame will change its coefficient
  - ↔ if constant in one frame, it will become dilaton dependent in others
- ▶ which **conformal frame** is the *natural physical* frame?
  - ↻ no clear consensus
  - ↔ in which frame the matter-dilaton coupling should be minimal?
- ▶ thus: a general **non-minimal coupling**  $f(\phi) \mathcal{L}_m$   
of the **dilaton** to the **matter content** of the universe (localized on the brane)

## the aim of the game

- ▶ framework:
  - dilaton gravity in a 5D brane scenario
  - non-minimal matter-dilaton coupling  $f(\phi) \mathcal{L}_m$
- ▶ take **assumptions** crucial to many models in the modern literature:
  - *bulk*: exact anti de Sitter type spacetime (AdS<sub>5</sub>)
  - *brane*: matter content of the universe described by a **perfect fluid**
- ▶ and **answer** the **question**:

*can the **large-scale structure of the universe**  
exist on the brane in an AdS-type bulk?*



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## dilaton gravity at the brane with general matter-dilaton coupling

- ▶ dilaton gravity in a 5D brane scenario: (Einstein frame)

$$\mathcal{L} = \frac{\alpha_1}{2} \left[ \mathcal{R} - \frac{2}{3} \nabla^\sigma \partial_\sigma^{(5)} \phi - \frac{1}{3} (\partial^{(5)} \phi)^2 \right] - V(\phi) + [f(\phi) \mathcal{L}_m + \lambda(\phi)] \delta_B$$

○  $\mathcal{R} - \frac{2}{3} \nabla^\sigma \partial_\sigma^{(5)} \phi - \frac{1}{3} (\partial^{(5)} \phi)^2$ : 5D dilaton gravity

○  $\mathcal{L}_m$ : (brane localized) matter content of the universe

$\lambda(\phi)$ : 'cosmological constant'-type term on the brane

↔ position of the co-dimension 1 brane: Dirac delta type distribution  $\delta_B$

○  $f(\phi) \mathcal{L}_m$ : (non-minimal) coupling of the dilaton  $\phi$  to brane localized matter  $\mathcal{L}_m$

- ▶ induced (projected) brane metric:  $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$  (covariant approach)

○  $n^\mu$ : vector field orthonormal to the brane at its position

○  $g_{\mu\nu}$ :  $\mathcal{R}_{\mu\nu}{}^{\rho\sigma}$  &  $\nabla_\mu$  vs  $h_{\mu\nu}$ :  $R_{\mu\nu}{}^{\rho\sigma}$  &  $D_\mu$

- ▶ assume a  $\mathbb{Z}_2$  symmetry for the bulk (with its fixed point at the brane position)

○ usually imposed 'automatically'

○ crucial for the existence of the effective brane equations

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⊙ crucial for the existence of the effective brane equations

## at the brane: effective Einstein-like equation

- consequently, the *effective Einstein-like equation* at the brane reads

$$R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \bar{G}(\phi) \tau_{\mu\nu} - h_{\mu\nu} \bar{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} - E_{\mu\nu} \\ + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial\phi)^2$$

$$\circ \bar{G}(\phi) = \frac{-1}{48\pi\alpha_1^2} f(\phi)\lambda(\phi) \quad (\text{effective brane Newton's constant})$$

$$\circ \tau_{\mu\nu} = h_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta h^{\mu\nu}}, \tau_\phi = \frac{f'(\phi)}{f(\phi)} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta \phi} \quad (\text{brane localized sources})$$

$$\circ \bar{\Lambda}(\phi) = \frac{1}{2\alpha_1} V - \frac{f^2}{4\alpha_1^2} \left[ \frac{3}{4} \tau_\phi^2 + \frac{3\lambda'}{2f} \tau_\phi - \frac{\lambda^2}{3f^2} + \frac{3\lambda'^2}{4f^2} \right] \quad (\text{eff. brane cosmol. const.})$$

$$\circ \pi_{\mu\nu} = -\tau_{\mu\rho} \tau_\nu^\rho + \frac{1}{3} \tau \tau_{\mu\nu} + \frac{1}{2} h_{\mu\nu} \tau_\rho^\sigma \tau_\sigma^\rho - \frac{1}{6} h_{\mu\nu} \tau^2 \\ (\text{terms quadratic in the brane energy-momentum tensor})$$

$$\circ \text{bulk's influence on the brane gravity: } E_{\mu\nu} = n^\alpha h_\mu^\beta n^\gamma h_\nu^\delta C_{\alpha\beta\gamma\delta} \\ (\text{bulk Weyl tensor projected on the brane (generically non-vanishing)})$$

- consistency condition** (on the brane sources):  $D_\lambda (f(\phi) \tau_\mu^\lambda) = f(\phi) \tau_\phi (\partial_\mu \phi)$   
(brane: 'generalized' covariant conservation of the energy-momentum tensor)

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## on the brane: spatial derivative of the energy density

- ▶ OR: inhomogeneous perfect fluid on the brane in AdS<sub>5</sub> bulk?

- ▶ assumptions:

- bulk: exact anti de Sitter type spacetime: AdS<sub>5</sub> →  $E_{\mu\nu} = 0$

(no bulk influence on the brane gravity)

- brane (matter content of the universe): perfect fluid →  $\tau_{\mu\nu} = \rho_m t_\mu t_\nu + p_m \gamma_{\mu\nu}$

( $\gamma_{\mu\nu}$ : 3d spatial metric,  $\rho_m$ : (dark) matter & radiation)

- ▶ calculus ingredients:

- 4D Bianchi identity:  $D^\nu (R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R) = 0$

- effective gravitational (Einstein-like) equation at the brane:

$$R_{\mu\nu} - \frac{1}{2} h_{\mu\nu} R = 8\pi \bar{G}(\phi) \tau_{\mu\nu} - h_{\mu\nu} \bar{\Lambda}(\phi) + \frac{f^2(\phi)}{4\alpha_1^2} \pi_{\mu\nu} - E_{\mu\nu} \\ + \frac{2}{9} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{5}{36} h_{\mu\nu} (\partial\phi)^2$$

- consistency condition (on the brane sources):  $D_\lambda (f(\phi) \tau_\mu^\lambda) = f(\phi) \tau_\phi (\partial_\mu \phi)$

## on the brane: late universe

- ▶ consequently, the spatial derivative of the matter energy density reads

$$\rho_{m,i} = - \left( \frac{f'}{f} \rho_m - \frac{\lambda'}{f} \right) \phi_{,i} + \frac{\alpha_1^2}{3f^2(\rho_m + \rho_m)} \left[ D^\nu \partial_i \phi - \dot{\phi}^{-1} \phi_{,i} D^\nu \partial_t \phi \right] (\partial_\nu \phi)$$

- ↪ imposes a strict condition on the matter content of the universe  
(constraint on spatial inhomogeneities in brane matter)

- ▶ (at least) late universe: terms  $\mathcal{O}((\partial\phi)(D\partial\phi))$  can be neglected, as

$$\odot \dot{\phi}_0 \lesssim 2.4 H_0 \simeq 1.8 (10^{10} \text{ yr})^{-1}$$

(derived: model-independent bound set by current observational data)

$$\odot |\ddot{\phi}_0| \ll \dot{\phi}_0^2 \text{ can be assumed / expected}$$

(otherwise: currently observed  $\phi_0 \approx \text{const}$  would be another coincidence problem)

$$\odot \text{ typical models: } |\phi_{,i}| \lesssim c_1 |\dot{\phi}|$$

( $c_1 > 0$  and of order 1)

(any initial inhomogeneities of the dilaton washed out by inflation)

- ▶ hereafter:  $\lambda \neq \lambda(\phi)$  ('cosmological constant'-type term in the energy-momentum tensor on the brane)  
(only a contribution to the effective brane cosmological constant  $\bar{\Lambda}(\phi)$ )

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## late universe: spatial derivative of the energy density

- ▶ hence for the **late universe** we obtain

$$\rho_{m0,i} \simeq -\frac{f'}{f} \rho_{m0} \phi_{0,i}$$

spatial **inhomogeneities** in **matter** energy density are highly **constrained**

for the common assumptions of **AdS<sub>5</sub>** bulk and **perfect fluid** on the brane

- ▶ inhomogeneous perfect fluid ( $\rho_{m,i} \neq 0$ ) on the brane? *only if:*  
**matter** content of the universe coupled **non-minimally** ( $f' \neq 0$ ) to the *dilaton*
- ▶ if dilaton spatially homogeneous: no matter inhomogeneities  
 ↪ already:  $\dot{\phi}_0 \lesssim 2.4 H_0 \simeq 1.8 (10^{10} \text{ yr})^{-1}$

*let's quantify the constraint on  $\rho_{m0,i}$ :*

- ▶ current observational limits:  $|\dot{\bar{G}}_0/\bar{G}_0| < (10^{11} \text{ yr})^{-1}$  ( $\bar{G} = \bar{G}(\phi)$ )  
 (pulsar timing, solar system, stellar, cosmological constraints)

$$\Leftrightarrow \left| \frac{f'}{f} \phi_{0,i} \right| \lesssim 3.3 c_1 (10^5 \text{ Mpc})^{-1} \quad (\text{for } |\phi_{0,i}| \lesssim c_1 |\phi|) \quad \text{resulting in}$$

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## confrontation with large-scale structure (LSS) data

compare: the *model's prediction* (constraint) on  $\rho_{m0,i}$  with the *observational data*

- ▶ galaxy distribution: probed by **galaxy redshift surveys**
  - (e.g. Sloan Digital Sky Survey (SDSS))
  - (addressed: content and statistical properties of the LSS)
- ▶ approximations:
  - (*aim*: just an estimation - allowing for the comparison)
  - ⌚ for the spatial derivative:  $\rho_{m,i} \simeq \frac{\rho_m(x_1) - \rho_m(x_2)}{|x_1 - x_2|}$
  - ⌚ LSS surveys probe the overall baryonic matter distribution
  - ⌚ spatial distributions of baryonic and dark matter similar
    - (typical of dark matter models)

↪ and the outcome is . . .

$$\rho_{m,i} (\text{max. model prediction}) \ll \rho_{m,i} (\text{LSS data})$$

(within the entire range of measured scales)

i.e. **brane scenario of dilaton gravity with AdS<sub>5</sub> bulk**

(and matter content of the universe described by a **perfect fluid**)

means **NO** today's **large scale structure**

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## conclusions

- dilaton gravity studied in a 5D brane scenario
  - brane localized matter coupled to dilaton non-minimally:  $f(\phi)\mathcal{L}_m$
  - ↪ derived: effective gravitational equations at the brane
- *can large-scale structure of the universe exist on the brane?*

*(inhomogeneous matter content of the universe)*

  - ↪ investigated for AdS<sub>5</sub> bulk & perfect fluid on the brane *(matter content of the universe)*
- ↔ spatial derivative of matter energy density *constrained*
- non-minimal dilaton-matter coupling essential
- result quantified with current limits
 

from (non-)variation of the Newton's constant
- and confronted with observational data from galaxy surveys
- ↔ up to scales of the order of 10<sup>4</sup> Mpc *(larger scales: measurements consistent with 0)*

**NO** large-scale structure as observed today

dilaton gravity brane scenario ruled out? for exact AdS<sub>5</sub> bulk only!