Some Progress on the Construction of Technicolor and Extended Technicolor Models

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Motivations for Dynamical Electroweak Symmetry Breaking

There are several motivations for considering dynamical electroweak symmetry breaking (EWSB). Standard Model (SM) Higgs mechanism for EWSB works but leaves some questions:

To get EWSB, one sets $\mu^2 < 0$ in the scalar potential of the SM Lagrangian, $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$, yielding $\langle \phi \rangle = \binom{0}{v/\sqrt{2}}$. But why should μ^2 be negative rather than positive?

μ^2 and hence $m_H^2 = -2\mu^2 = 2\lambda v^2$ with $v = (2/g)m_W = 246$ GeV are unstable to large radiative corrections from much higher energy scales - gauge hierarchy problem, fine-tuning needed to keep the scalar light.

The SM Yukawa mechanism for generating fermion masses, with $m_f \simeq y_f v/\sqrt{2}$, accomodates these masses, but one must use a large range of Yukawa coupling values, from $O(1)$ for top quark to 10^{-5} for electron mass (with further inputs necessary to explain light neutrino masses). What is the origin of this large range of values?

Moreover, in two major previous cases where fundamental scalar fields were used in phenomenologically modelling spontaneous symmetry breaking, the underlying physics involved bilinear fermion condensates:

Superconductivity: the Ginzburg-Landau free energy functional was a successful phenomenological description, using complex scalar field ϕ with $V = c_2|\phi|^2 + c_4|\phi|^4$, with $c_2 \propto (T - T_c)$, so for $T < T_c$, $c_2 < 0$ and $\langle \phi \rangle \neq 0$. But the underlying origin of superconductivity is the dynamical formation of a condensate of Cooper pairs $\langle ee \rangle$ in BCS theory.

Gell-Mann and Lévy constructed a reasonable phenomenological model, the σ model, for spontaneous chiral symmetry breaking (SχSB) in hadronic physics, with $V = (\mu^2/2)\vec{\phi}^2 + (\lambda/4)\vec{\phi}^4$, where $\vec{\phi} = (\sigma, \vec{\pi})$. In this model, one produces SχSB by the choice $\mu^2 < 0$, leading to $\langle \sigma \rangle = f_\pi \neq 0$. But the underlying origin of SχSB in QCD is the dynamical formation of a $\langle \bar{q}q \rangle$ condensate.

These examples suggest the possibility that the underlying physics responsible for EWSB may also be a dynamically induced fermion condensate.

Indeed, there is one known source of dynamical EWSB via a fermion condensate: the $\langle \bar{q}q \rangle$ condensate in QCD breaks electroweak symmetry.

Consider, for simplicity, QCD with $N_f = 2$ massless quarks, u, d. This theory has a global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry. The quark condensate $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \langle \bar{q}_R q_L \rangle$ transforms as an $I_w = 1/2$, $|Y| = 1$ operator and breaks this symmetry to the diagonal, vectorial isospin SU(2)$_V$. The resultant Nambu-Goldstone bosons (NGB's) - $\pi^{\pm}$ and π^0 - are absorbed to become the longitudinal components of the $W^{\pm}$ and Z, giving them masses:

$$m_W^2 = \frac{g^2 f_\pi^2}{4}\,, \qquad m_Z^2 = \frac{(g^2 + g'^2) f_\pi^2}{4}$$

With $f_\pi \sim 93$ MeV, this yields $m_W \simeq 30$ MeV, $m_Z \simeq 33$ MeV. These masses satisfy the tree-level relation $\rho = 1$, where $\rho = m_W^2/[m_Z^2 \cos^2\theta_W]$. (A gedanken world in which this is the only source of EWSB is discussed in Quigg and RS, Phys. Rev. D79, 096002 (2009)).

While the scale here is too small by $\sim 10^3$ to explain the observed W and Z masses, it suggests how to construct a model with dynamical EWSB.

Basics of Technicolor

Technicolor (TC) is an asymptotically free vectorial gauge theory with gauge group that can be taken as $\mathrm{SU}(N_{TC})$ and a set of fermions $\{F\}$ with zero Lagrangian masses, transforming according to some representation(s) of G. The TC interaction becomes strong at a scale Λ_{TC} of order the electroweak scale, confining and producing a chiral symmetry breaking technifermion condensate (Weinberg, Susskind, 1979); recent review: Sannino, Acta Phys. Polon., arXiv:0911.0931).

Assign technifermions so L (R) components form SU(2)$_L$ doublets (singlets). Minimal choice: "one-doublet" (1DTC) model with fund. rep. for technifermions uses

$$\begin{pmatrix} F_u^\tau \\ F_d^\tau \end{pmatrix}_L \qquad F_{uR}^\tau, \quad F_{dR}^\tau$$

with TC indices τ and $Y = 0$ $(Y = \pm 1)$ for SU(2)$_L$ doublet (singlets).

The SU(N_{TC}) TC theory is asymptotically free, so as energy scale decreases, α_{TC} increases, eventually producing condensates; for generic N_{TC}, these are $\langle \bar{F}_u F_u \rangle$, $\langle \bar{F}_d F_d \rangle$ transforming as $I_w = 1/2$, $|Y| = 1$, breaking EW symmetry at Λ_{TC}.

Just as in the QCD example above, the W and Z pick up masses, but now involving the TC scale:

$$m_W^2 \simeq \frac{g^2 \, F_{TC}^2 \, N_D}{4} \, , \qquad m_Z^2 \simeq \frac{(g^2 + g'^2) \, F_{TC}^2 \, N_D}{4}$$

again satisfying the tree-level relation $\rho = 1$ because of the I_w and Y of $\langle \bar{F} F \rangle$. Here $F_{TC} \sim \Lambda_{TC}$ is the TC analogue to $f_\pi \sim \Lambda_{QCD}$ and $N_D =$ number of SU(2)$_L$ technidoublets. For this minimal example, $N_D = 1$, so $F_{TC} = 250$ GeV. One may also add SM-singlet technifermions to this model, as discussed further below.

Another class of TC models that was studied in the past (but is now disfavored) used one SM family of technifermions (1FTC)

$$\begin{pmatrix} U^{a\tau} \\ D^{a\tau} \end{pmatrix}_L \quad U_R^{a\tau}, \quad D_R^{a\tau}$$

$$\begin{pmatrix} N^{\tau} \\ E^{\tau} \end{pmatrix}_L \quad N_R^{\tau}, \quad E_R^{\tau}$$

(a, τ color, TC indices) with usual Y assignments. Similar condensate formation, with approx. equal condensates $\langle \bar{F} F \rangle$ for $F = U^a, \ D^a, \ N, \ E$, generating dynamical technifermion masses $\Sigma_{TC} \sim \Lambda_{TC}$, analogous to constituent quark mass $\sim \Lambda_{QCD}$ in QCD. Resultant m_W^2 and m_Z^2 given by formula above with $N_D = N_c + 1 = 4$, so $F_{TC} \simeq 125$ GeV for 1FTC.

Technicolor has several appealing properties:

- Given the asymptotic freedom of the TC theory, the condensate formation and hence EWSB are automatic, as in QCD, and do not require a specific parameter choice like $\mu^2 < 0$ in the SM.

- Because TC has no fundamental scalar field, there is no hierarchy problem.

- Because $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_L \rangle$, technicolor explains why the chiral part of G_{SM} is broken and the residual exact gauge symmetry, $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{em}$, is vectorial (also explained in SM).

However, TC by itself is not a complete theory; to give masses to quarks and leptons (which are technisinglets), one must communicate the EWSB in the TC sector to these SM fermions. For this purpose, one embeds TC in a larger, extended technicolor (ETC) gauge theory with ETC gauge bosons transforming SM fermions into technifermions (Dimopoulos and Susskind; Eichten and Lane, 1979-80).

An ETC theory thus gauges the SM fermion generation index and combines it with TC gauge indices in the full ETC symmetry group.

To satisfy constraints on flavor-changing neutral current (FCNC) processes, ETC gauge bosons must have large masses. These masses are envisioned as arising from sequential breaking of the ETC chiral gauge symmetry.

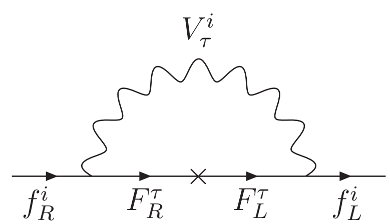
Diagrams for generating SM fermion masses involve virtual exchanges of ETC gauge bosons, so resultant masses depend on inverse powers of $m_{ETC,i}$. To account for the hierarchy in the three generations of SM fermion masses, the ETC theory should break sequentially at three corresponding scales, $\Lambda_1 > \Lambda_2 > \Lambda_3$, e.g., $\Lambda_1 \simeq 10^3$ TeV, $\Lambda_2 \simeq 50 - 100$ TeV, $\Lambda_3 \simeq$ few TeV.

The ETC theory is constructed to be asymptotically free, so as energy decreases from a high scale, ETC coupling α_{ETC} grows, eventually becomes large enough to form condensates that sequentially break the ETC symmetry to a residual exact subgroup, which is the TC gauge group; so $G_{ETC} \supset G_{TC}$.

An ETC theory is much more ambitious than the SM or MSSM because a successful ETC model would predict the entries in the SM fermion mass matrices and the resultant values of the quark and lepton masses and mixings. It would explain longstanding mysteries like the mass ratios m_e/m_μ, m_u/m_d, m_d/m_s, etc. Not surprisingly, no fully realistic ETC model has yet been constructed, and TC/ETC models face many stringent constraints.

Mass Generation Mechanism for Fermions

The ETC gauge bosons enable SM fermions, which are TC singlets, to transform into technifermions and back. This provides a mechanism for generating SM fermion masses. The figure shows a one-loop graph contributing to diagonal entries in mass matrix for SM fermion f^i. Basic ETC vertex is $f^i \to f^j + V^i_j$, with $V^i_j =$ ETC gauge boson, $1 \le i, j \le 5$; here we distinguish the first three ETC indices, which refer to SM fermion generations, and additional ETC indices that are TC indices, by denoting the latter as τ (with any color indices suppressed):



Rough estimate:

$$M^{(f)}_{ii} \simeq \frac{2\alpha_{ETC} C_2(R)}{\pi} \int dk^2 \frac{k^2 \Sigma_{TC}(k)}{[k^2 + \Sigma_{TC}(k)^2][k^2 + M_i^2]}$$

where $M_i \simeq (g_{ETC}/2)\Lambda_i \simeq \Lambda_i$ is the mass of the ETC gauge bosons that gain mass at scale Λ_i, $C_2(R) =$ quadratic Casimir invariant. For Euclidean $k \gg \Lambda_{TC}$, $\Sigma_{TC}(k) \simeq \Sigma_{TC}(0)[\Sigma_{TC}(0)/k]^{2-\gamma}$. In walking TC (WTC), γ may be $\sim O(1)$, so

$\Sigma_{TC}(k) \simeq \Sigma_{TC}(0)^2/k$; contrast with QCD, where $\Sigma(k) \simeq \Sigma(0)^3/k^2$ for $k \gg \Lambda_{QCD}$. In general, the TC/ETC calculation of $M_{ii}^{(f)}$ gives

$$M_{ii}^{(f)} \simeq \frac{\kappa\, C_2(R)\, \eta\, \Lambda_{TC}^3}{\Lambda_i^2}$$

where $\kappa \simeq O(10)$ is a numerical factor from the integral and η is a RG factor, discussed further below, that enhances the mass. This is only a rough estimate, since ETC coupling is strong, so higher-order diagrams are also important.

The sequential breaking of the ETC symmetry at the highest scale, Λ_1, the intermediate scale, Λ_2, and the lowest scale, Λ_3, thus produces the generational hierarchy in the SM fermion masses. Since these ETC scales enter as inverse powers in the resultant SM fermion masses and since Λ_1 is the largest ETC scale, it follows that first-generation fermion masses are the smallest, and since Λ_3 is the smallest ETC scale, third-generation fermion masses are the largest.

There are mixings among the interaction eigenstates of the ETC gauge bosons to form mass eigenstates. Insertions of these on ETC gauge boson lines lead to CKM and lepton mixing (Appelquist and RS, Phys. Lett. B 548, 204 (2002); Appelquist and RS, Phys. Rev. Lett. 90, 201801 (2003); Appelquist, Piai, RS, Phys. Rev. D 69, 015002 (2004); Christensen and RS, Phys. Rev. D 74, 015004 (2006)).

Since SM fermion masses arise dynamically, the running mass $m_{f_i}(p)$ of a SM fermion of generation i is constant up to the ETC scale Λ_i and has the power-law decay (Christensen and RS, Phys. Rev. Lett. 94, 241801 (2005)):

$$m_{f_i}(p) \sim m_{f_i(0)} \frac{\Lambda_i^2}{p^2}$$

for Euclidean momenta $p \gg \Lambda_i$ (neglect subdominant logarithmic factors).

Thus, e.g., the third-generation quark masses $m_t(p)$ and $m_b(p)$ decay like Λ_3^2/p^2 for $p \gg \Lambda_3$, while the first-generation quark masses $m_u(p)$ and $m_d(p)$ are hard up to the much higher scale Λ_1, eventually decaying like Λ_1^2/p^2 for $p \gg \Lambda_1$.

UV to IR Evolution and Walking (Quasi-Conformal) TC

TC models that behaved simply as scaled-up versions of QCD were excluded by their inability to produce sufficiently large fermion masses (especially for the third generation) without having ETC scales so low as to cause excessively large FCNC.

Modern TC theories are constructed to have a coupling g_{TC} that gets large, but runs slowly ("walks") over an extended interval of energy (WTC) (Holdom, Yamawaki et al., Appelquist, Wijewardhana...).

This walking (quasi-conformal) behavior arises naturally from an approximate IR zero of the perturbative beta function:

$$\beta(\alpha_{TC}) = \frac{d\alpha_{TC}}{dt} = -\frac{\alpha_{TC}^2}{2\pi}\left(b_1 + \frac{b_2\,\alpha_{TC}}{4\pi} + O(\alpha_{TC}^2)\right)$$

where $t = \ln\mu$, with $b_1 > 0$ - asymp. freedom. For sufficiently many technifermions, $b_2 < 0$, so β has a second zero (approximate IR fixed point of RG) at $\alpha_{TC} = -4\pi b_1/b_2 \equiv \alpha_{IR}$.

If $N_f < N_{f,cr}$ (depending on technifermion rep. of G_{TC}, R), as the theory evolves from the UV to IR, α_{TC} gets large, but runs slowly because β approaches this zero at α_{IR}. For TC, we want to choose N_f so that α_{IR} is slightly greater than the minimal value α_{cr} for technifermion condensation. Then the TC theory has quasi-conformal behavior, with a large $\alpha_{TC}(\mu)$ over an extended interval of energies μ.

As $\alpha_{TC}(\mu)$ eventually exceeds α_{cr} at $\mu \sim \Lambda_{TC}$, the technifermion condensate $\langle \bar{F} F \rangle$ forms, the technifermions gain dynamical masses, and in the low-energy theory at smaller μ, they are integrated out, so the TC beta function changes, and α_{TC} evolves away from α_{IR} which is thus an approximate IR fixed point (IRFP).

Because WTC has approx. dilatational invariance, which is dynamically broken by the $\langle \bar{F} F \rangle$ condensate, it has been suggested that this could lead to a light approx. Nambu-Goldstone boson (NGB), the techidilaton (Yamawaki..Goldberger, Grinstein, and Skiba; Sannino...; Appelquist and Bai; see also Bardeen et al.; Holdom and Terning). This might be as light as 125 GeV and could have couplings to SM fields similar to those of the SM Higgs.

Currently, there are initial indications in ATLAS and CMS data of a possible state at about 125 GeV, seen in several channels. If confirmed, this might be the SM Higgs, but it might instead be a technidilaton. Further experimental and theoretical studies are necessary to decide this question.

For $N_f > N_{f,cr}$, the theory would evolve from the UV to the IR in a chirally symmetric manner, without ever producing $\langle \bar{F} F \rangle$, so the (initially massless) technifermions remain massless, and the IRFP is exact. This IR-conformal phase ("conformal window") is of basic field-theoretic interest, although for TC model-building, we should choose the technifermion content so that we are in the phase with SχSB, as is necessary for EWSB.

Walking TC has several desirable features.

- SM fermion masses are enhanced by the factor

$$\eta_i = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_i} \frac{d\mu}{\mu} \gamma(\alpha_{TC}(\mu)) \right]$$

  If γ is approximately constant over this range of μ, then $\eta_i = (\Lambda_i/\Lambda_{TC})^\gamma$, which can be substantially larger than 1.

- Hence, one can increase ETC scales Λ_i for a fixed m_{f_i}, reducing FCNC effects.

To analyze this, study the Dyson-Schwinger (DS) equation for the fermion propagator; for $\alpha > \alpha_{cr}$, this yields a nonzero sol. for a dynamically generated fermion mass. Simple ladder approx. to DS eq. gives $\alpha_{cr} C_2(R) \sim O(1)$, where R is fermion rep. Detailed studies for higher-dim. R: Sannino, Dietrich, Ryttov...

As number of technifermions, N_f, increases, α_{IR} decreases, and $N_f \nearrow N_{f,cr}$ as $\alpha_{IR} \searrow \alpha_{cr}$. This yielded the estimate $N_{f,cr} \simeq 4N_{TC}$.

Corrections to the conventional DS equation analysis to take account of confinement and instantons tend to cancel each other, as regards $N_{f,cr}$ (Brodsky and RS, Phys. Lett. 666, 95 (2008)). For discussion on condensates, see Roberts' talk at this conf.

Lattice gauge simulations provide a fully nonperturbative determination of $N_{f,cr}$ and measurement of the anomalous dimension γ that describes the running of m and the bilinear operator, $\bar{F}F$ as a function of $\ln \mu$. In recent years, intensive work using lattice methods to determine these quantities for SU(3), SU(2), and various fermion representations, including fundamental, adjoint, and 2-index symmetric tensor reps.

Higher-loop corrections to UV $\to$ IR evolution of gauge theories

Because of the strong-coupling nature of the physics at an approximate IRFP of interest to TC theories, there are generically significant higher-order corrections to results obtained from the two-loop β function.

This motivates the calculation of the location of the IR zero in β and the value of $\gamma = \gamma(\alpha)$ for an SU(N) gauge thy. evaluated at $\alpha = \alpha_{IR}$ to higher-loop order. We have done this to 3-loop and 4-loop order (Ryttov and RS, PRD 83, 056011 (2011), arXiv:1011.4542; see also Pica and Sannino, PRD 83,035013 (2011), arXiv:1011.5917). This is of general field-theoretic interest, beyond the specific application to technicolor.

We have recently extended this analysis to an $\mathcal{N} = 1$ supersymmetric SU(N) theory in Ryttov and RS, arXiv:1202.1297. First discuss nonsupersymmetric theory.

Although the coefficients in the beta function at 3-loop and higher-loop order are scheme-dependent, the results give a measure of the accuracy of the 2-loop calculation of the IR zero, and similarly with the value of γ evaluated at this IR zero. We use the $\overline{MS}$ scheme, for which the coefficients of β and γ have been calculated up to 4-loop order by Vermaseren, Larin, and van Ritbergen.

Analytic and numerical results are presented in our paper; here we only list numerical results. We find that for given SU(N) ($N \equiv N_{TC}$) and fermion content for which $\exists$ IR zero of β, the 3- and 4-loop values of α_{IR} are smaller than the 2-loop value.

Results for N_f technifermions in the fundamental rep. of SU(N) for $N = 2,\ 3$:

N	N_f	$\alpha_{IR,2\ell}$	$\alpha_{IR,3\ell}$	$\alpha_{IR,4\ell}$
2	7	2.83	1.05	1.21
2	8	1.26	0.688	0.760
2	9	0.595	0.418	0.444
2	10	0.231	0.196	0.200
3	10	2.21	0.764	0.815
3	11	1.23	0.578	0.626
3	12	0.754	0.435	0.470
3	13	0.468	0.317	0.337
3	14	0.278	0.215	0.224
3	15	0.143	0.123	0.126
3	16	0.0416	0.0397	0.0398

Similarly, we find that for given N, R, and N_f, the value of γ calculated to 3-loop and 4-loop order and evaluated at the value of α_{IR} calculated to the same order is somewhat smaller than the 2-loop value:

For N_f technifermions in $R =$ fundamental rep. of SU(N) for $N = 2,\ 3$:

N	N_f	$\gamma_{2\ell}(\alpha_{IR,2\ell})$	$\gamma_{3\ell}(\alpha_{IR,3\ell})$	$\gamma_{4\ell}(\alpha_{IR,4\ell})$
2	7	(2.67)	0.457	0.0325
2	8	0.752	0.272	0.204
2	9	0.275	0.161	0.157
2	10	0.0910	0.0738	0.0748
3	10	(4.19)	0.647	0.156
3	11	1.61	0.439	0.250
3	12	0.773	0.312	0.253
3	13	0.404	0.220	0.210
3	14	0.212	0.146	0.147
3	15	0.0997	0.0826	0.0836
3	16	0.0272	0.0258	0.0259

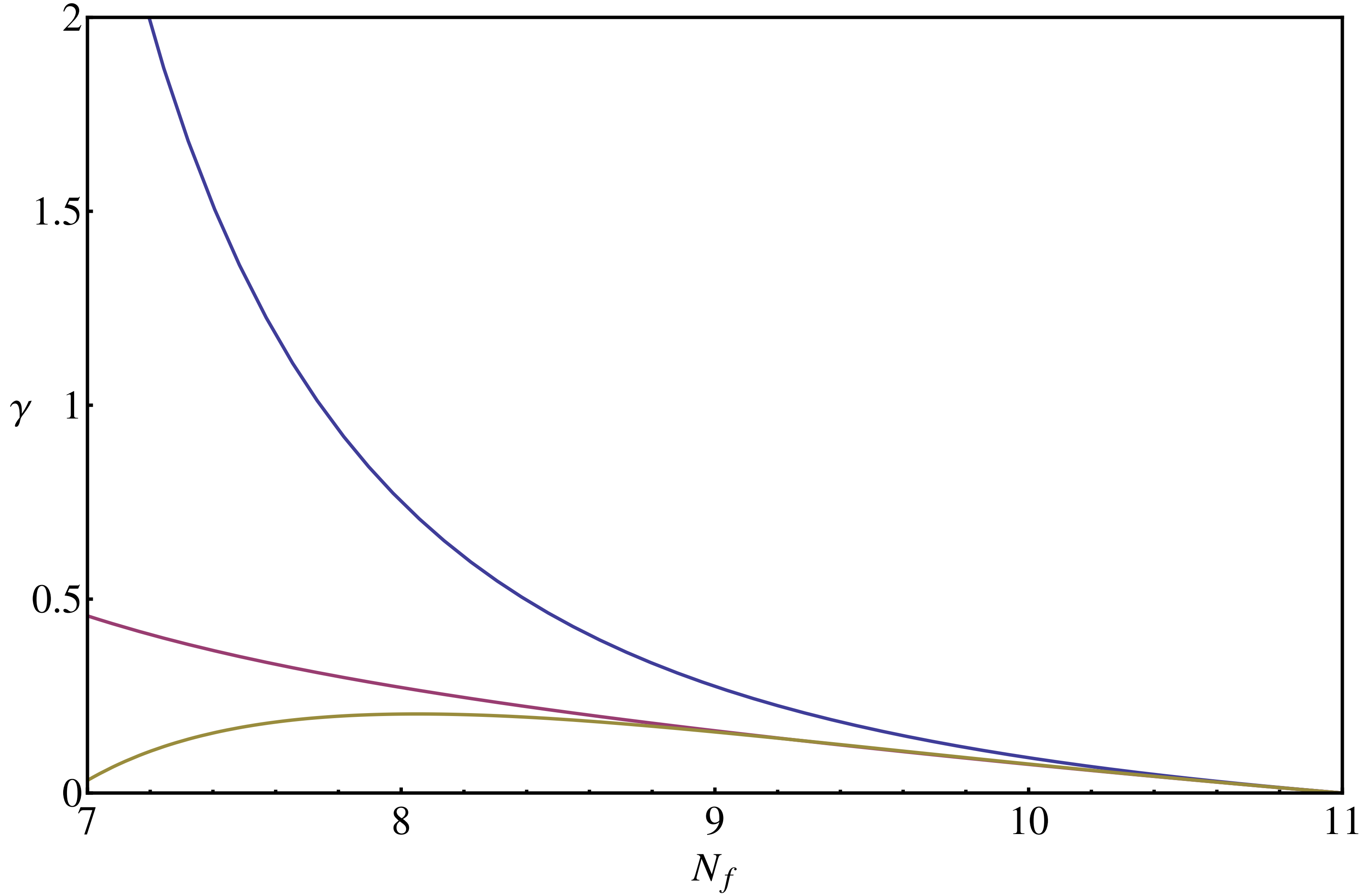


Figure 1: Anomalous dimension γ for SU(2) for N_f fermions in the fundamental representation; (i) blue: 2-loop; (ii) red: 3-loop; (iii) brown: 4-loop calculation ($N_{f,max} = 11$).

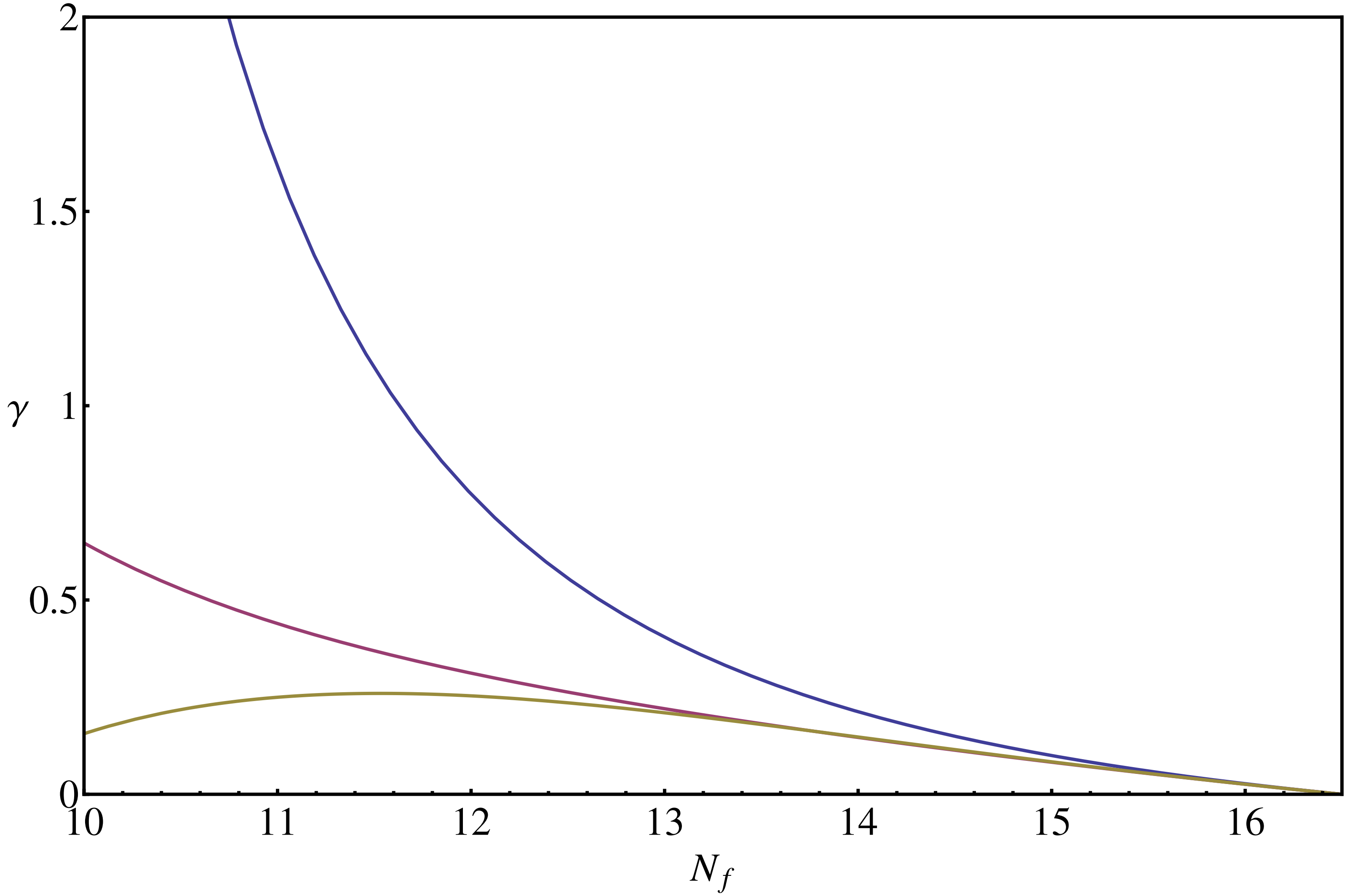


Figure 2: Anomalous dimension $\boldsymbol{\gamma}$ for SU(3) for $\boldsymbol{N_f}$ fermions in the fundamental representation; (i) blue: 2-loop; (ii) red: 3-loop; (iii) brown: 4-loop calculation ($\boldsymbol{N_{f,max} = 16.5}$).

The value of our higher-loop calculations to 3-loop and 4-loop order is evident from these figures. A necessary condition for a perturbative calculation to be reliable is that higher-order contributions do not modify the result too much. One sees from the tables and figures that as N_f decreases and hence α_{IR} increases, there is a substantial decrease in α_{IR} and γ when one goes from 2-loop to 3-loop order, but for a reasonable range of N_f, the 3-loop and 4-loop results are close to each other.

Thus, our higher-loop calculations of α_{IR} and γ allow us to probe the theory reliably down to smaller values of N_f, stronger couplings. Of course, because of the increase in α_{IR} as N_f decreases, perturbative calcs. of α_{IR} and γ eventually get less reliable. Values of γ in parentheses are unphysically large.

In the phase with confinement and SχSB, α_{IR} is only an approximate IRFP and γ is only an effective quantity describing the theory at scales μ where α is near to α_{IR}. In the conformal phase, an IRFP is exact (although our perturbative calculation of it is only approximate), and γ describes the scaling of the bilinear $\bar{F}F$ at this IRFP.

Some examples of comparison with lattice measurements:

For SU(3) with $N_f = 12$, from the table above,

$$\gamma_{IR,2\ell} = 0.77, \qquad \gamma_{IR,3\ell} = 0.31, \qquad \gamma_{IR,4\ell} = 0.25$$

Lattice results:

$\gamma = 0.414 \pm 0.016$ (Appelquist, Fleming, Lin, Neil, Schaich, PRD 84, 054501 (2011), arXiv:1106.2148, analyzing data of Kuti et al., PLB 703, 348 (2011), arXiv:1104.3124, inferring consistency with conformality)

$\gamma \sim 0.35$ (DeGrand, arXiv:1109.1237, also analyzing Kuti et al. data).

So here the 2-loop value is slightly larger than, and the 3-loop and 4-loop values closer to, these lattice measurements.

Thus, this improved agreement with lattice results using our higher-order calculations also shows the value of these computations.

We have also carried out these higher-loop calculations for fermions in larger representations. For fermions in the adjoint representation, $N_f \le 2$ to maintain asymptotic freedom. For $N_f = 2$ we find

N	$\alpha_{IR,2\ell,adj}$	$\alpha_{IR,3\ell,adj}$	$\alpha_{IR,4\ell,adj}$
2	0.628	0.459	0.493
3	0.419	0.306	0.323

N	$\gamma_{2\ell,adj}(\alpha_{IR,2\ell,adj})$	$\gamma_{3\ell,adj}(\alpha_{IR,3\ell,adj})$	$\gamma_{4\ell,adj}(\alpha_{IR,4\ell,adj})$
2	0.820	0.543	0.571
3	0.820	0.543	0.561

For SU(2) with $N_f = 2$ fermions in the adjoint rep., lattice results include (caution: various groups quote uncertainties differently):

$\gamma = 0.49 \pm 0.13$ (Catterall, Del Debbio et al., arXiv:1010.5909, PoS(Lat2010) 057)

$\gamma = 0.31 \pm 0.06$ (DeGrand, Shamir, Svetitsky, PRD 83, 074507 (2011)

$\gamma = 0.17 \pm 0.05$ (Appelquist et al., PRD 84, 054501 (2011), arXiv:1106.2148)

$-0.6 < \gamma < 0.6$ (Catterall, Del Debbio, et al., arXiv:1108.3794)

It is of interest to carry out a similar analysis in an asymptotically free $\mathcal{N} = 1$ supersymmetric gauge theory with vectorial chiral superfield content $\Phi, \ \tilde{\Phi}$ in the $R, \ \bar{R}$ reps. for various R, since here $N_{f,cr}$ is known (Seiberg for $F = R$; Ryttov and Sannino for higher R).

We have done this for an SU(N) gauge theory in Ryttov and RS, arXiv:1202.1297. In the susy case, there is a bound $\gamma \le 1$ for a theory in the IR conformal phase. Insofar as perturbative calculations are reliable, they indicate that γ increases (from 0) as N_f decreases from $N_{f,max}$. So we get a perturbative estimate for $N_{f,cr}$ by setting the perturbatively calculated $\gamma = 1$ and solving for N_f.

For example, for $R = F$, fundamental rep., $N_{f,max} = 3N$, $N_{f,cr} = (1/2)N_{f,max} = (3/2)N$. Perturbative estimates are approx. 1.3 to 1.4 times larger than exact result. Similar results for higher-dim. reps.

So via this comparison, we find that perturbative results slightly overestimate the value of $N_{f,cr}$ compared with the exact results, i.e., slightly underestimate the size of the IR-conformal phase.

Some Constraints on TC/ETC Models

Early studies of ETC considered the TC theory as an effective low-energy theory and added various plausible four-fermion operators linking SM fermions and technifermions.

Part of our work has focused on constructing reasonably UV-complete ETC models that predict the forms and coefficients of the four-fermion operators in the effective low-energy technicolor theory.

Typically, ETC is arranged to be an asymptotically free chiral gauge theory, and includes a set of SM-singlet, ETC-nonsinglet fermions chosen so that as the scale decreases from the deep UV, the ETC gauge coupling becomes large enough to produce condensates of these SM-singlet fermions, which break the ETC gauge symmetry.

Since this involves strongly coupled gauge interactions, it is not precisely calculable, but the pattern of condensate formation can be plausibly determined by the most attractive channel (MAC) criterion. Some studies include Appelquist and Terning, PRD 50, 2116 (1994); Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003); Appelquist, Piai, RS, PRD 69, 015002 (2004); Christensen and RS, PRD 74, 015004 (2006); Ryttov and RS PRD 81, 115013 (2010); Ryttov and RS, PRD 84, 056009 (2011).

To account for the three generations of SM fermion masses, there is a sequential breaking of the ETC gauge symmetry, at the three scales Λ_i, $i = 1,\ 2,\ 3$. Although the full ETC theory is chiral, we focus here on ETC models with vectorial couplings to quarks and charged leptons, denoted VSM ETC models.

At the highest scale, Λ_1, G_{ETC} breaks to H_{ETC}, and the gauge bosons in the coset G_{ETC}/H_{ETC} gain masses $\sim g_{ETC}\Lambda_1 \sim \Lambda_1$, and so forth for the breakings at the two lower scales Λ_2 and Λ_3.

Studies of reasonably UV-complete models showed how not just diagonal, but also off-diagonal, elements of SM fermion mass matrices could be produced, via nondiagonal propagator corrections to ETC gauge bosons, $V^i_\tau \to V^j_\tau$, where $i,\ j$ are generation indices and τ is a TC index (Appelquist, Piai, RS, PRD 69, 015002 (2004)).

A feature that was found in these studies of reasonably UV-complete ETC models was the presence of approximate residual generational symmetries that naturally suppress these ETC gauge boson propagator corrections and hence also off-diagonal elements of SM fermion mass matrices.

Further, a possible mechanism to account for the very small neutrino masses was presented. This made use of suppressed Dirac and Majorana neutrino masses leading to a low-scale seesaw (Appelquist and RS, PLB 548, 204 (2002); PRL 90, 201801 (2003)).

TC/ETC theories are constrained by FCNC processes. These can be suppressed by making the ETC breaking scales Λ_i sufficiently large, but this is restricted by the requirement that one not cause excessive suppression of SM fermion masses.

One insight from studies of reasonably UV-complete ETC models was that the approximate residual generational symmetries suppress the FCNC effects.

For example, consider $K^0 - \bar{K}^0$ mixing and resultant $K_L - K_S$ mass difference $\Delta m_{K_L K_S}$. SM contribution consistent with experimental value $\Delta m_{K_L K_S}/m_K \simeq 0.7 \times 10^{-14}$.

Simple effective Lagrangian used in early studies without a UV-complete ETC theory: $\mathcal{L}_{eff} = c[s\gamma_\mu d]^2$ with coefficient $c \sim 1/\Lambda_{ETC}^2$, usually with just a single generic ETC scale.

Now in terms of ETC eigenstates, an $s\bar{d}$ in a $\bar{K}^0$ produces a V_1^2 ETC gauge boson, but this cannot directly yield a $d\bar{s}$ in the final-state K^0; the latter is produced by a V_2^1. So this requires either the ETC gauge boson mixing $V_1^2 \to V_2^1$ or the related mixing of ETC quark eigenstates to produce mass eigenstates.

The ETC gauge boson propagator insertion ${}_2^1\Pi_1^2$ required for this breaks the generational symmetries associated with the $i = 1$ and $i = 2$ generations, and hence

$$|{}_2^1\Pi_1^2| \lesssim \Lambda_2^2$$

Therefore, the contribution to $\bar{K}^0 \longrightarrow K^0$ transition from $V_1^2 \longrightarrow V_2^1$:

$$|c| \lesssim \frac{1}{\Lambda_1^2}\,{}_2^1\Pi_1^2\,\frac{1}{\Lambda_1^2} \sim \frac{\Lambda_2^2}{\Lambda_1^2}\,\frac{1}{\Lambda_1^2} \ll \frac{1}{\Lambda_1^2}$$

With above values for Λ_1 and Λ_2, the suppression factor is $(\Lambda_2/\Lambda_1)^2 \simeq 10^{-2}$. So rather than the naive result $\Delta m_{K_L K_S}/m_K \sim \Lambda_{QCD}^2/\Lambda_1^2$, this yields the considerably smaller result

$$\frac{\Delta m_{K_L K_S}}{m_K} \sim \frac{\Lambda_2^2\,\Lambda_{QCD}^2}{\Lambda_1^4} \sim 10^{-15}$$

which agrees with experimental limits on new-physics contributions.

Similar analysis applies to ETC contributions to a number of other FCNC processes. Some studies of FCNC constraints that take account of these approximate generational symmetries include Appelquist, Piai, RS, PLB 593, 175 (2004); PLB 595, 442 (2004); Appelquist, Christensen, Piai, RS, PRD 70, 093010 (2004). TC theories may also provide viable dark matter candidates, as discussed by Nussinov, Sannino...

It remains challenging to construct a TC/ETC model (e.g. VSM type) that does everything that is demanded of it, including sufficient suppression of FCNC effects and accounting for realistic quark, charged lepton, and neutrino masses and quark and lepton mixing. Example of a recent study, invoking additional interactions: Ryttov and RS, Phys. Rev. D 82, 055012 (2010).

Constraints from precision electroweak data: $\Delta\rho = \alpha_{em}(m_Z)T$ and S, where

$$\frac{\alpha_{em} S}{\sin^2(2\theta_W)} = \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

S is sensitive to heavy fermion loop contributions to Z propagator.

From experimental data, SM fits obtain allowed regions in S and T, depending on an assumed value of m_H mass; in general, $S \lesssim 0.2$ (90 % CL).

Naive perturbative estimate (which is not applicable, since TC is nonperturbative at scale m_Z):

$$(\Delta S)_{TC,pert.} \simeq \frac{dim(R_{TC})\, N_D}{6\pi}$$

where $dim(R_{TC})$ is the dimension of the TC fermion rep., e.g., $dim(R_{TC}) = N_{TC}$ for fundamental. If TC were QCD-like, nonperturbative effects would yield $(\Delta S)_{TC} \simeq 2(\Delta S)_{TC,pert.}$ (Peskin-Takeuchi, 1990), which, together with too-small SM fermion masses, showed that TC could not be a scaled-up QCD-like theory.

A viable TC model must have a reduction in $(\Delta S)_{TC}$ wrt. its QCD-like value. This motivates building TC models with the minimal content of SU(2)$_L$-nonsinglet technifermions.

Studies of Dyson-Schwinger equations have shown that $(\Delta S)_{TC}$ (per EW doublet) is somewhat reduced in walking TC as compared with its QCD-like value: Harada, Kurachi and Yamawaki, Prog. Theor. Phys. 115, 765 (2006); Kurachi and RS, Phys. Rev. D 74, 056003 (2006); Kurachi, RS, Yamawaki, Phys. Rev. D 76, 035003 (2007). Further analytic work in Sannino, Phys. Rev. D82, 081701 (2010).

Lattice simulations (Appelquist et al., (LSD Collab.), with SU(3) with $N_f = 6$, fund. rep., have also found a reduction in $(\Delta S)_{TC}$ (per EW doublet) wrt. its QCD-like value (PRL 106, 231601 (2011)).

In general, the constraint from the S parameter remains a crucial and stringent one for TC/ETC theories.

Collider signals for TC/ETC theories and constraints from early LHC data

Although TC/ETC theories have been constrained indirectly from flavor physics, precision electroweak quantities, and searches at the Tevatron, key tests are now forthcoming with the data from the LHC.

Some collider signals for TC depend on the type of model. A general signature that applies to all technicolor models results from the property that the technihadrons include a techni-ρ, denoted ρ_{TC}. In QCD the ρ couples strongly to $\pi\pi$ and decays to $\pi\pi$ with a large width, so also in technicolor.

In TC, the technipions are absorbed to become the longitudinal components of the $W^{\pm}$ and Z. Hence at sufficiently high energy the scattering of longitudinally polarized W and Z's will be enhanced by resonant s-channel contributions:

$$W_L^+ W_L^- \to \rho_{TC}^0 \to W_L^+ W_L^-$$

$$W_L^+ Z_L \to \rho_{TC}^+ \to W_L^+ Z_L$$

The ρ_{TC} mass, $m_{\rho_{TC}}$ mass can be roughly estimated from

$$\frac{m_{\rho_{TC}}}{m_\rho} \simeq \frac{\Lambda_{TC}}{\Lambda_{QCD}} \simeq \frac{f_{TC}}{f_\pi}\left(\frac{N_c}{N_{TC}}\right)^{1/2}$$

where $f_{TC} \simeq 250$ GeV for a one-doublet TC theory. With $f_\pi = 93$ MeV and $m_\rho = 775$ MeV, this yields

$$m_{\rho_{TC}} \simeq (2.0 \text{ TeV})\left(\frac{N_c}{N_{TC}}\right)^{1/2}$$

Studies of meson masses in WTC (Kurachi and RS, JHEP 12, 034 (2006)) obtained an approx. 30 % increase in $m_{\rho_{TC}}/m_\rho$ relative to this QCD-like estimate, suggesting that $m_{\rho_{TC}} \simeq (2.6 \text{ TeV})\sqrt{N_c/N_{TC}}$ in a WTC theory.

By analogy with $\rho \to \pi\pi$ in QCD, the ρ_{TC} would decay as $\rho^0_{TC} \to W^+W^-$ and $\rho^+_{TC} \to W^+Z$. For the width of such a technihadron, a rough estimate is

$$\frac{\Gamma_{\rho_{TC}}}{\Gamma_\rho} \sim \frac{\Lambda_{TC}}{\Lambda_{QCD}}$$

so, with $\Gamma_\rho \simeq 150$ MeV, one has $\Gamma_{\rho_{TC}} \sim 250$ GeV. Similar for other technihadrons.

LHC can search for this resonant behavior, but this will require substantially more integrated luminosity than the present $\int \mathcal{L}dt = 5$ fb^{-1} per experiment. Recent analysis of TC signatures for the LHC: J. Andersen et al., arXiv:1104.1255; estimates suggest that clear observation of this resonant behavior may require $\int \mathcal{L}dt \sim 50 - 100$ fb^{-1} at $\sqrt{s} = 14$ TeV.

For an example of how current LHC data are useful in constraining technicolor models, consider the one-family TC model. This is already in some tension with precision electroweak constraints, since it yields $(\Delta S)_{TC,pert.} = N_{TC} N_D/(6\pi)$. Now $N_D = N_c + 1 = 4$ in this one-family model, so even if one takes the minimum value, $N_{TC} = 2$, this is $(\Delta S)_{TC,pert.} = 4/(3\pi) = 0.4$ (reduced somewhat in WTC).

The one-family TC model makes two predictions for techni-hadrons that are tested with current LHC data.

The first is a large number of pseudo-NGB's (PNGB's). If one neglects ETC effects and uses the fact that the SM gauge interactions are weak at the EW scale, then a generic one-family TC model has an $\mathrm{SU}(8)_L \times \mathrm{SU}(8)_R$ global chiral symmetry (where $8 = N_w(N_c + 1)$). The technifermion condensates break this to SU(8)$_V$, yielding 63 (P)NGB's, of which 3 NGB's are eaten. The PNGB's gain masses from color and ETC interactions that break the above global chiral symmetry, but some of them include color-nonsinglet states and could have masses of order several 100 GeV. There is no evidence for these at the LHC.

In particular, the one-family TC model predicts color-octet $\bar{Q}_a(T_\alpha)^a_b Q^b$ pseudoscalar and vector states, where Q are techniquarks and $T_\alpha, \alpha = 1, ..., 8$ are SU(3)$_c$ generators. The mass of the color-octet $\rho_{TC}^{(8)}$ can be estimated as above, with $f_{TC} \simeq 125$ GeV, yielding $m_{\rho_{TC}^{(8)}} \simeq 1.3 \sqrt{N_c/N_{TC}}$ TeV. Walking might raise this mass slightly, as noted above.

CMS and ATLAS have set lower bounds on color-octet resonances of 2.5 TeV. Although the mass estimates for TC theories have significant uncertainties owing to the strongly coupled nature of the TC physics, this causes tension with the one-family TC model.

Note that minimal technicolor models are consistent with these LHC data. They use only color-singlet technifermions, so there are no color-nonsinglet technihadrons, and they have only one SU(2)$_L$ doublet of left-handed technifermions (with corresponding SU(2)$_L$-singlet right-handed technifermions), so the resultant three SM-nonsinglet NGB's are all eaten by the $W^\pm$ and Z and there are no residual SM-nonsinglet (P)NGB's. Because they use a minimal SM-nonsinglet technifermion content, they also have the potential to yield an acceptably small value of S.

Some Further Model-Building Results

LHC results thus motivate further study of TC (1DTC) models with minimal technifermion content, consisting of one (color-singlet) SU(2)$_L$ doublet with corresponding right-handed components. In general, these models have additional SM-singlet technifermions.

A number of studies of this type of theory using higher-dim. reps. for technifermions by Sannino and coworkers (Dietrich, Tuominen, Ryttov, incl. Dietrich, Sannino, and Tuominen, PRD 72, 055001 (2005), Sannino review in arXiv:0911.0931). A recent study using technifermions in fundamental rep. is Ryttov, RS, PRD 84, 056009 (2011).

Another question concerns the extent to which one can embed TC, ETC in a theory having higher gauge unification, using dynamical symmetry breaking. This would be desirable in order to explain features not explained by the standard model:

- unification of quarks and leptons
- charge quantization

We have shown how, in principle, this is possible, using an extended strong-EW gauge group $\mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ (Appelquist and RS, PRL 90, 201801 (2003)).

Prospects for possible higher unification of both TC and SM symmetries have also been studied; recent discussions include Christensen and RS, PR D72, 035013 (2005); Gudnason, Ryttov, Sannino, PR D76, 015005 (2007); Chen and RS, PR D78, 035002 (2008); Chen, Ryttov, and RS, PR D82, 116006 (2010)).

Conclusions

Dynamical electroweak symmetry breaking via technicolor is an interesting and well-motivated possibility. Its embedding in ETC is very ambitious and encounters a number of challenges. So far, model-building work has shown

- how a new gauge interaction that becomes strongly coupled on the TeV scale naturally produces EWSB, W and Z masses
- how an associated large but slowly running gauge coupling can result from an approximate IR fixed point, enhancing fermion mass generation and leading to reduction in TC modifications of precision electroweak quantities; higher-loop calculations have been valuable in understanding the UV to IR evolution of such a theory
- how fermion generations could arise, by sequential breaking of ETC symmetry; how the EWSB could be communicated to the fermions and hence how quark and lepton masses could arise
- Dynamical EWSB has distinctive experimental signatures that can be probed at the LHC, including resonant scattering of longitudinally polarized W and Z, and also a possible light technidilaton.