Danish Institute for Advanced Study
University of Southern Denmark

S parameter at non-zero temperature and chemical potential

Ulrik Ishøj Søndergaard

CP³ - Origins

Particle Physics & Origin of Mass

February 2012

Søndergaard, Pica, Sannino: Phys. Rev. D 84, 075022 (2011)

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Electroweak Precision Observables

α

Electron anomalous magnetic moment

G_F

Muon lifetime

m_Z

Resonance

Classic review:
M. E. Peskin and T. Takeuchi, Phys. Rev. D 46, 381 (1992)

At Lagrangian level

$$\mathcal{L} = \mathcal{L}(g, g', v)$$

$m_H^2 \quad \lambda$

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Broken phase:

$$\mathcal{L} = \mathcal{L}(e_0, s_0, v)$$

Experimental setup: External fermions are light

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$\Longrightarrow$ $q^\mu q^\nu$-term in propagators suppressed

$\Longrightarrow$ ‘direct’ BSM corrections suppressed

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Tree-Level expression   Example: Charged Current

$$\mathcal{M}_{WW} = \frac{e_0^2}{2s_0^2} l_+ \frac{1}{q^2 - m_{w,0}^2} l_-$$

Tree-Level expression Example: Charged Current

$$\mathcal{M}_{WW} = \frac{e_0^2}{2s_0^2} J_+ \frac{1}{q^2 - m_{w,0}^2} J_-$$

All Orders Vacuum Polarization

$$\mathcal{M}_{WW} = \frac{e_*^2}{2s_*^2} J_+ \frac{Z_{W,*}}{q^2 - m_{w,*}^2} J_-$$

$\star$'ed quantities depend on q^2

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+ Most SM corrections can be absorbed into these variables (including direct corrections)

Choosing a scheme

$$s_W \equiv ?$$

M. Awramik et al., Phys. Rev. D69, 053006 (2004), hep-ph/0311148 ← SM m_W calc. + others

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'On shell' $\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$

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Better: $\sin 2\theta_w = \sqrt{\frac{4\pi\alpha_{sm*}(m_Z^2)}{\sqrt{2}G_F m_Z^2}}$

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We can now compute any EW observable as an expression of s_W, α, G_F and m_Z

Up to corrections from new physics!

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Ex:
$$\frac{\frac{m_W^2}{m_Z^2} - \cos^2\theta_w}{} = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}\underbrace{16\pi\left(\Pi'_{33}(0) - \Pi'_{3Q}(0)\right)}_{\equiv S} + c^2\underbrace{\frac{4\pi}{s^2c^2m_z^2}\left(\Pi_{11}(0) - \Pi_{33}(0)\right)}_{\equiv T} + \frac{c^2 - s^2}{4s^2}\underbrace{16\pi\left(\Pi'_{11}(0) - \Pi'_{33}(0)\right)}_{\equiv U}\right]$$

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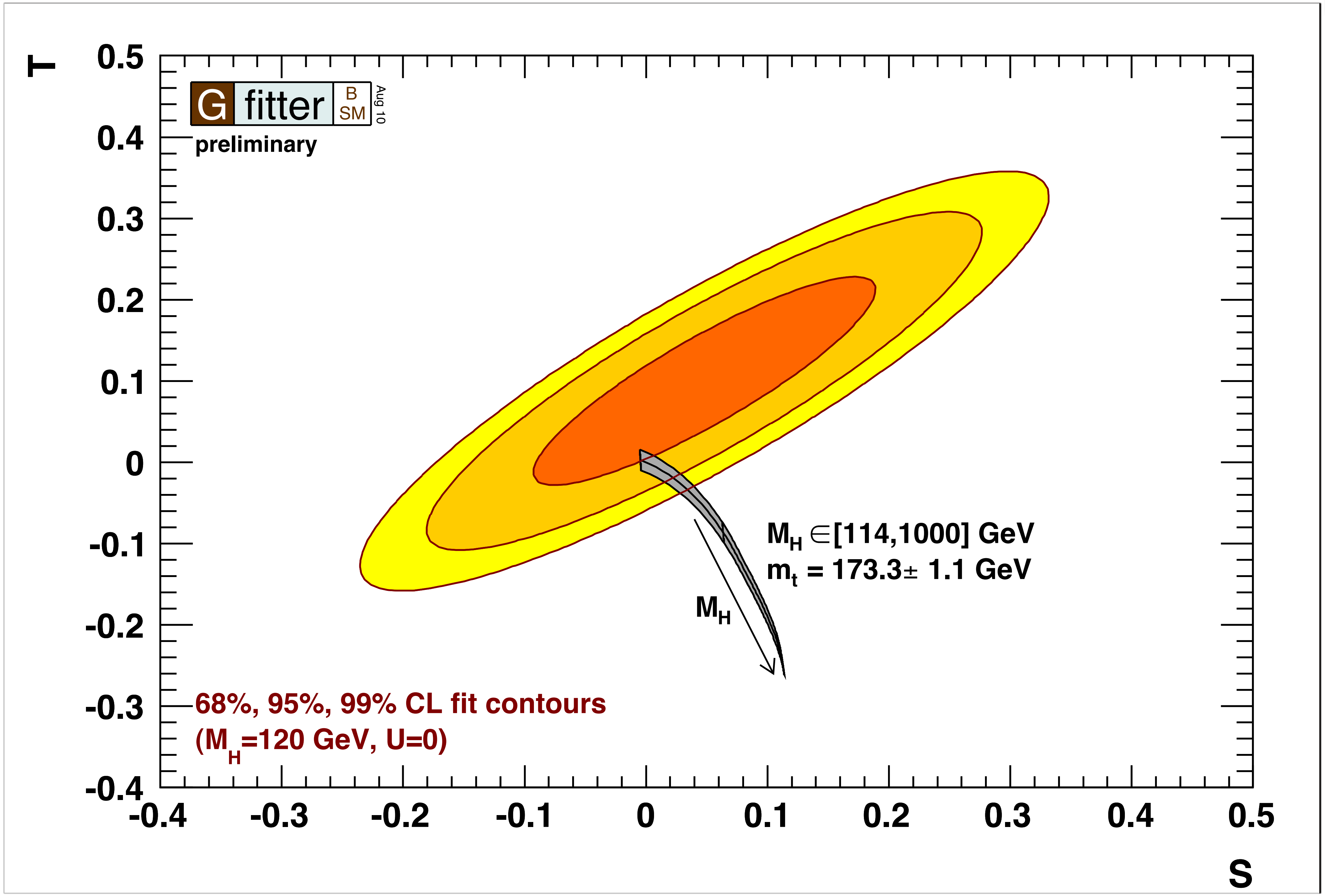

T
S
Gfitter BSM
Aug 10
preliminary
$M_H \in [114,1000]$ GeV
m_t = 173.3± 1.1 GeV
M_H
68%, 95%, 99% CL fit contours
(M_H=120 GeV, U=0)

GFITTER

Definitions

Peskin-Takeuchi

$$S = 16\pi\left(\Pi'_{33}(0) - \Pi'_{3Q}(0)\right)$$

$$= -16\pi\Pi'_{3Y}(0)$$

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$q^2 \to 0$ ↑

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$T \to 0$

$\mu \to 0$

We will not do this

$$S = -16\pi \frac{\Pi_{3Y}(q^2, x, T, \mu) - \Pi_{3Y}(0, x, T, \mu)}{q^2}$$

$q_0 = x|\mathbf{q}|, \quad q^2 = q_0^2 - \mathbf{q}^2$ $\qquad x = \coth\eta \geq 1$

S at zero T and μ

Degenerate technifermions: No Y dependence

$$S(q^2/m_{TF}^2) = \frac{\sharp}{6\pi}\left[1 + \frac{1}{10}\frac{q^2}{m_{TF}^2} + \frac{1}{70}\frac{q^4}{m_{TF}^4} + \mathcal{O}\left(\frac{q^6}{m_{TF}^6}\right)\right]$$

$$\sharp = d[r]N_{TF}/2$$

fund: $= N_{TC}N_{TF}/2$

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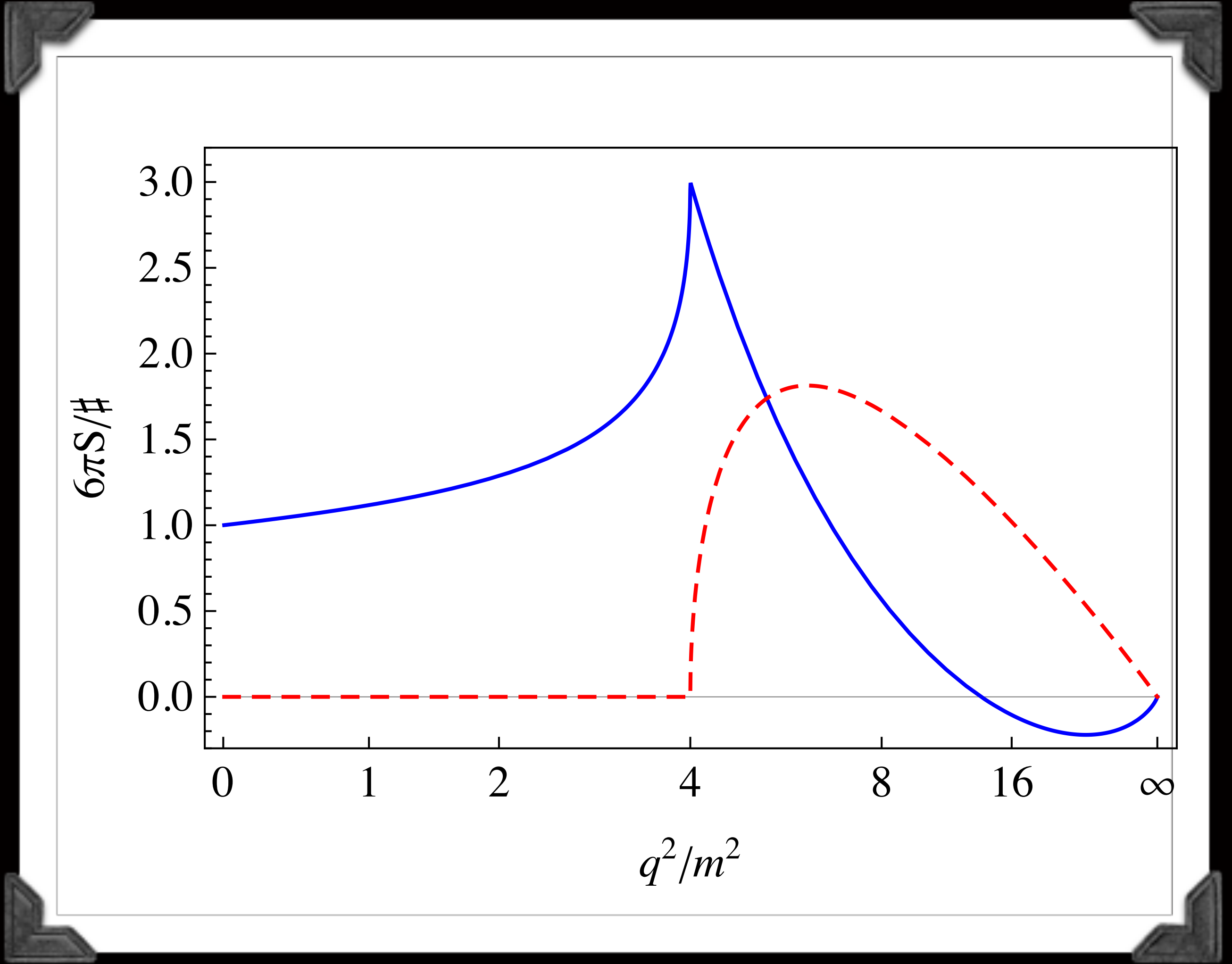

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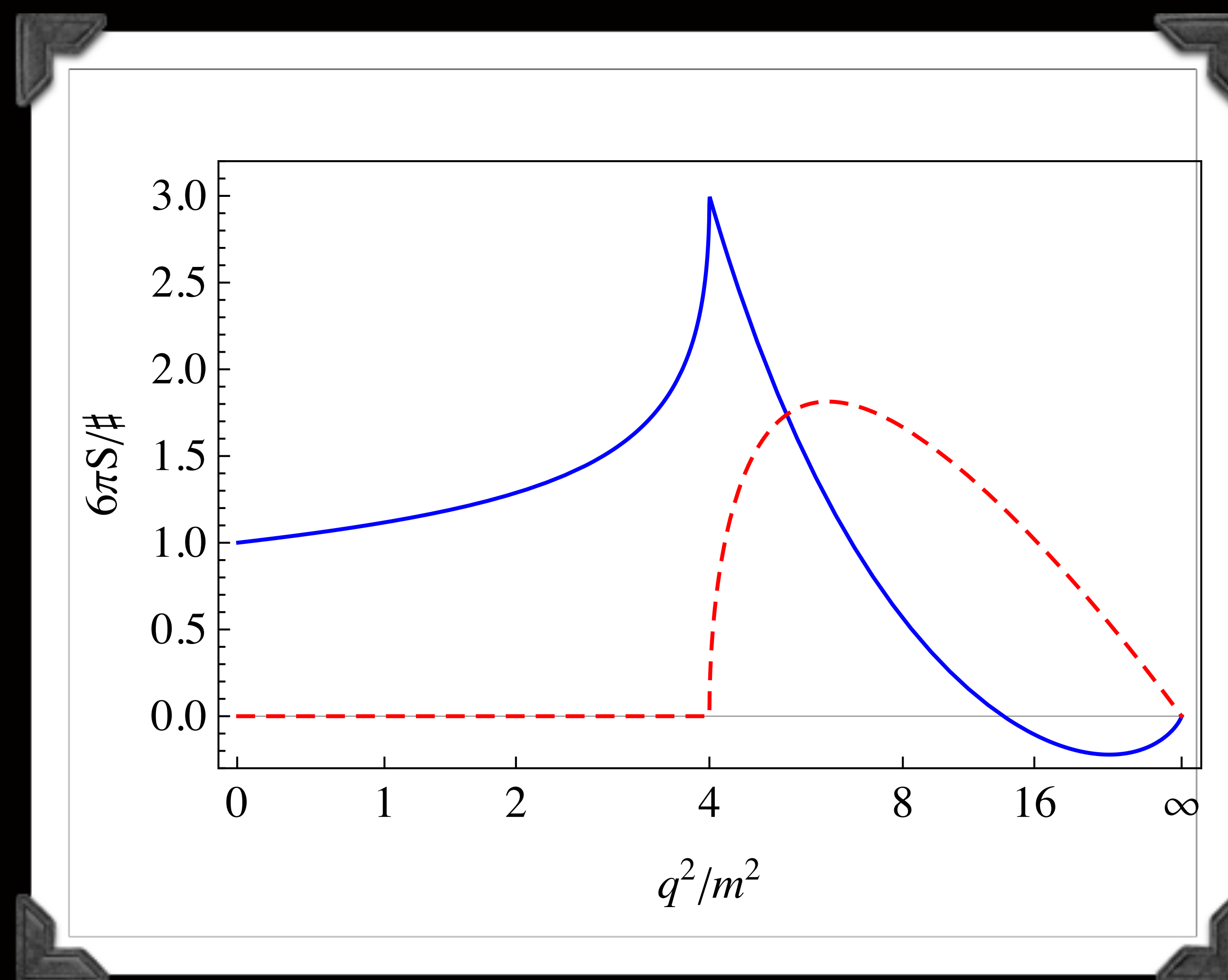
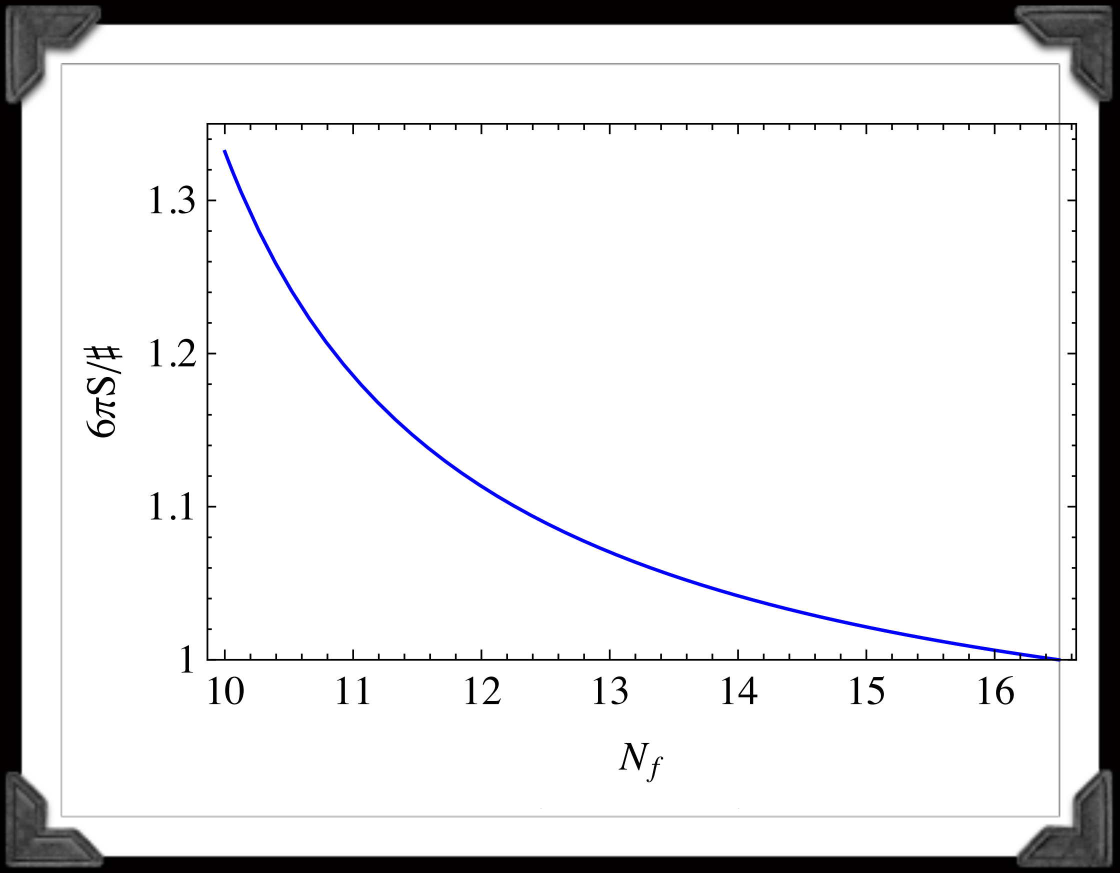
sxc.hu:1128191

$$S(0) = \frac{\sharp}{6\pi}$$



2-loop perturbation



$$> \frac{\sharp}{6\pi}$$

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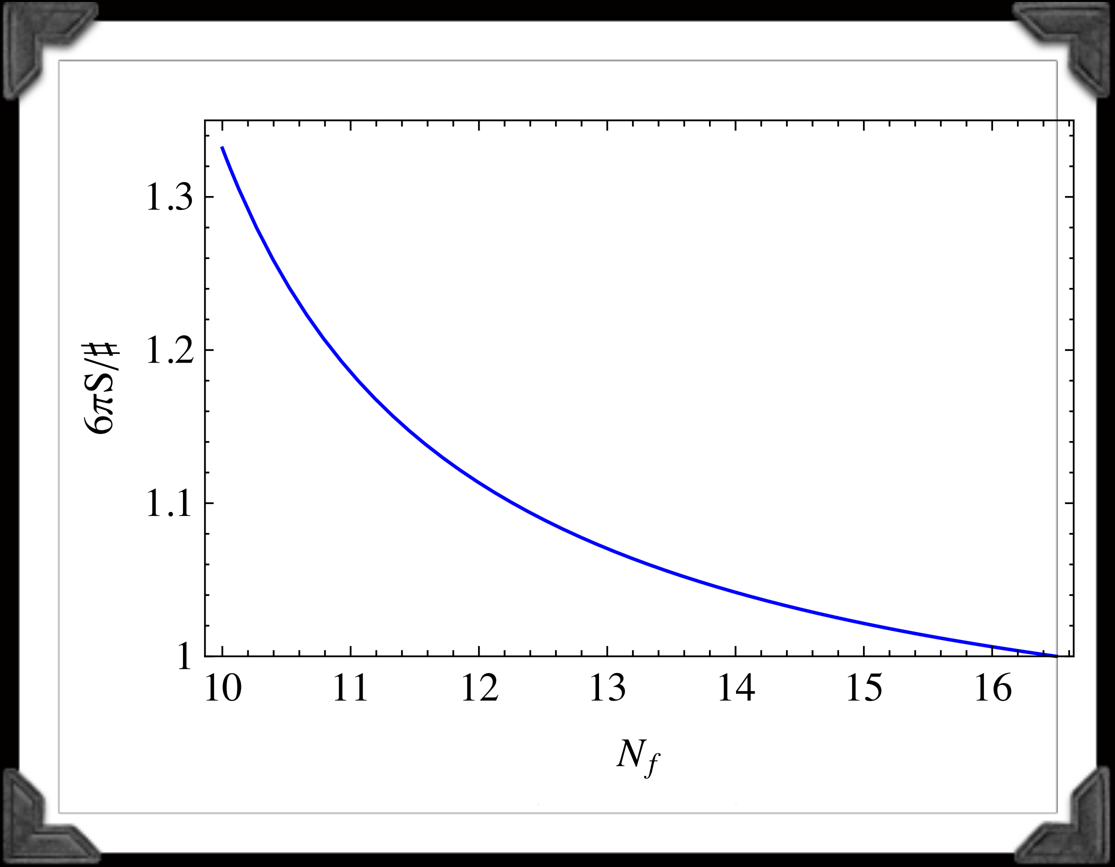

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WSR + vector dominance + Large N rescaling

$$S(0) \simeq 1.57 \frac{\sharp}{6\pi} > \frac{\sharp}{6\pi}$$

Peskin, Takeuchi, Phys. Rev. D 46, 381 (1992)

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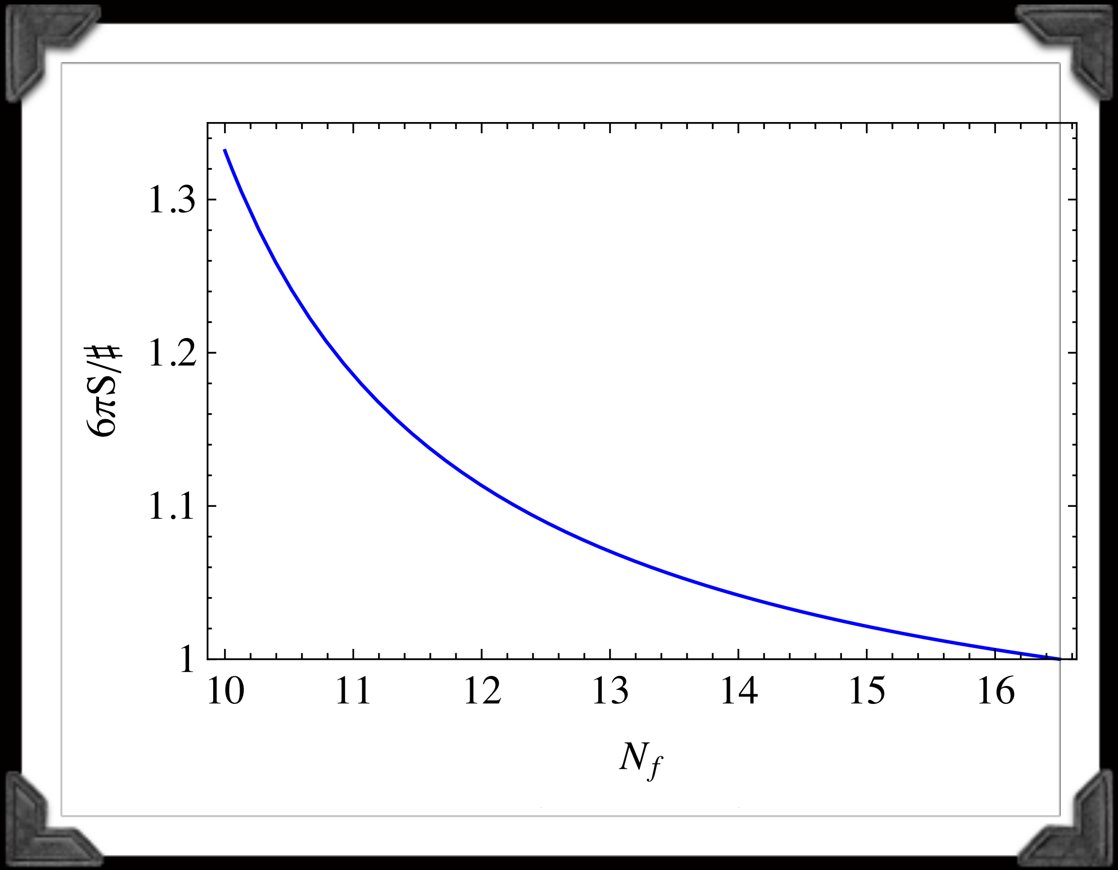

$$> \frac{\sharp}{6\pi}$$

WSR + vector dominance + Large N rescaling

$$S(0) \simeq 1.57 \frac{\sharp}{6\pi} > \frac{\sharp}{6\pi}$$

WSR + more sophisticated approx + Large N rescaling

$$S(0) \simeq 1.88 \frac{\sharp}{6\pi} > \frac{\sharp}{6\pi}$$

Peskin, Takeuchi, Phys. Rev. D 46, 381 (1992)

Calculation

For a degenerate technifermion doublet: $\Pi_{3Y} = \frac{1}{2}\Pi_{LR}$

1 loop

$$\Pi^{\mu\nu}_{LH} = T \sum_{l=-\infty}^{\infty} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \mathrm{Tr}\left[\gamma^\mu P_L \frac{\not{p} + m}{p^2 - m^2} \gamma^\nu P_H \frac{\not{q} + \not{p} + m}{(q+p)^2 - m^2}\right]$$

$$p^0 = i(2l+1)\pi T + \mu$$

$$S = -8\pi \frac{\Pi_{LR}(x, q^2, T, \mu) - \Pi_{LR}(x, 0, T, \mu)}{q^2}$$

Results

Cold (βm >> 1):

$$S_+ = -\frac{\sharp}{6\pi}\frac{\cosh(\beta\mu)\,\mathrm{sech}^4(\eta)}{1-\left(\frac{q/m}{2\cosh\eta}\right)^2}\frac{3\sqrt{2\pi}}{(\beta m)^{3/2}}\,e^{-\beta m}\cdot\left(1+\mathcal{O}\left(\frac{1}{\beta m}\right)\right)$$

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Hot ($\beta m \ll 1$): Result factorizes

$$S_+ = -S_0 + \mathrm{sech}^2\left(\tfrac{1}{2}\beta\mu\right)\cosh(\eta)\, S_+^{(1)}(q^2/m^2)\,(\beta m) + \mathcal{O}(\beta^3 m^3)$$

$$S_+^{(1)}(q^2/m^2) = \frac{3\pi}{q^2/m^2}\left(1 + i\sqrt{\frac{1}{4}\frac{q^2}{m^2} - 1}\right)$$

No remnant of T=0 result!

Results

Cold (βm >> 1):

$$S_+ = -\frac{\sharp}{6\pi}\frac{\cosh(\beta\mu)\,\mathrm{sech}^4(\eta)}{1-\left(\frac{q/m}{2\cosh\eta}\right)^2}\,\boxed{\frac{3\sqrt{2\pi}}{(\beta m)^{3/2}}\,e^{-\beta m}}\cdot\left(1+\mathcal{O}\left(\frac{1}{\beta m}\right)\right)$$

Hot (βm << 1): Result factorizes

$$S_+ = \boxed{-S_0} + \mathrm{sech}^2\left(\tfrac{1}{2}\beta\mu\right)\cosh(\eta)\,S_+^{(1)}(q^2/m^2)\,(\beta m) + \mathcal{O}(\beta^3 m^3)$$

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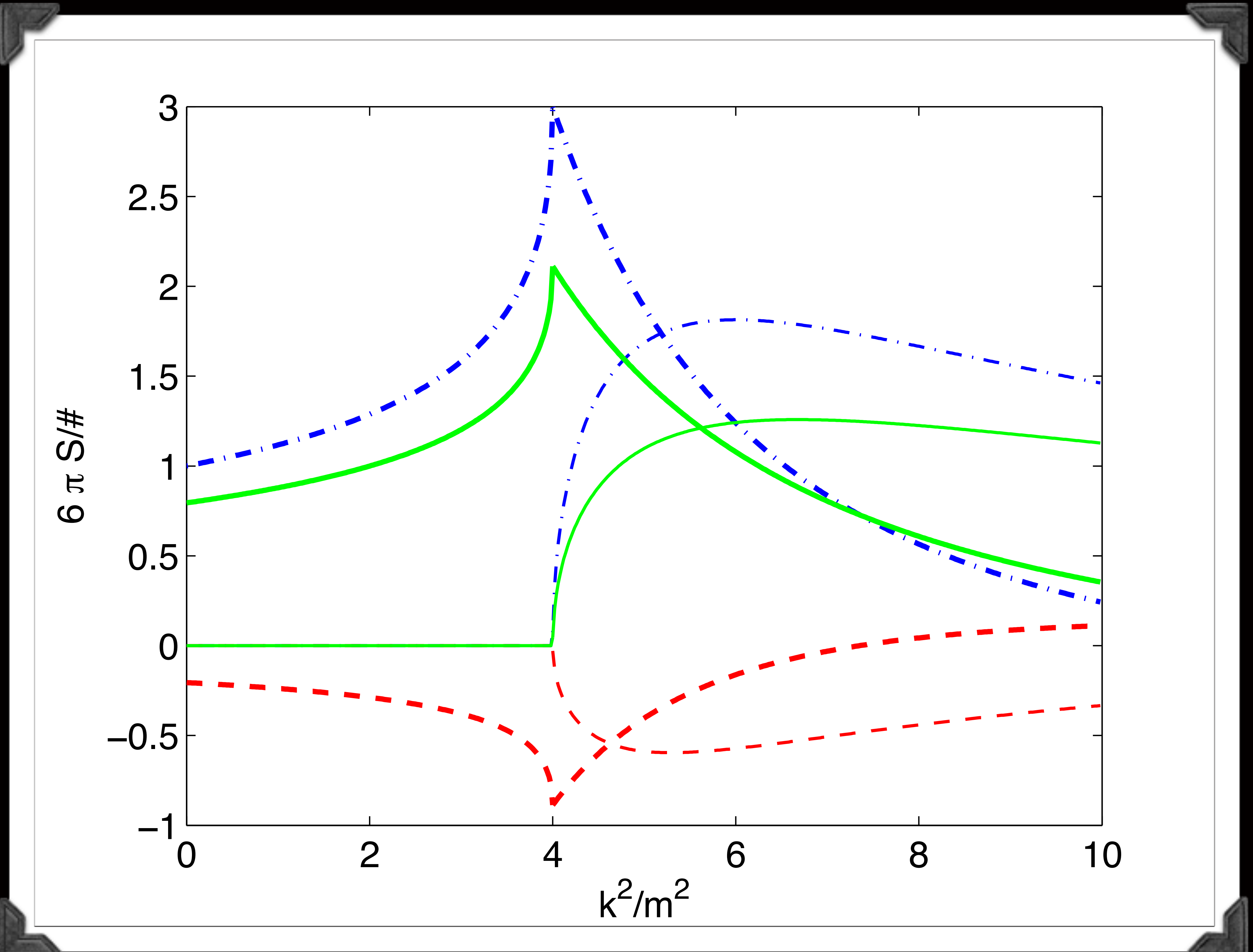
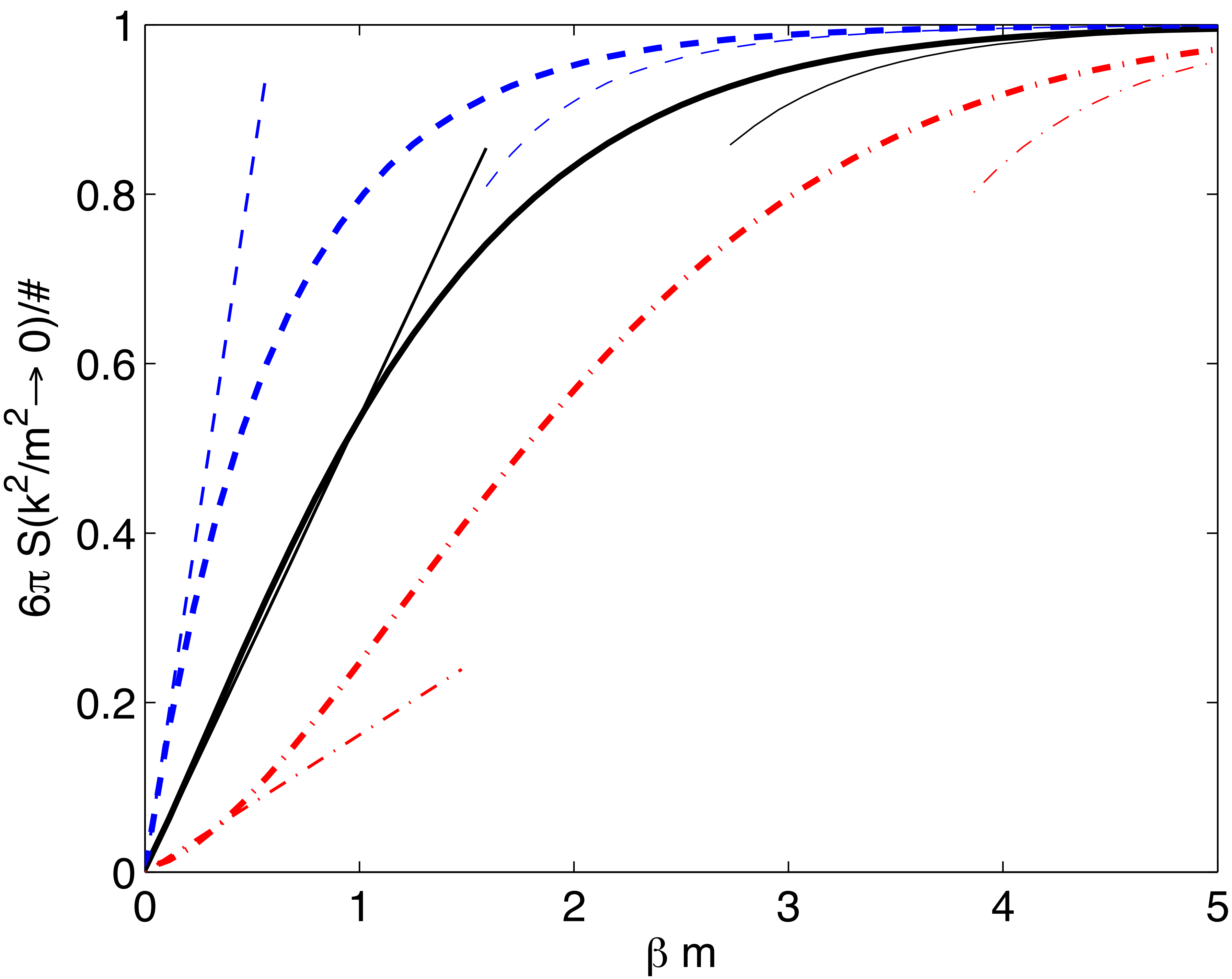
$$\boxed{S(k^2/m^2\to 0) = \frac{\sharp}{16}\,\mathrm{sech}^2\left(\tfrac{1}{2}\beta\mu\right)\cosh(\eta)\,(\beta m) + \mathcal{O}(\beta^3 m^3)}$$



$\beta m=1,\ x=\sqrt{2}$



βμ=0, 2.33, 4.33, x=√2

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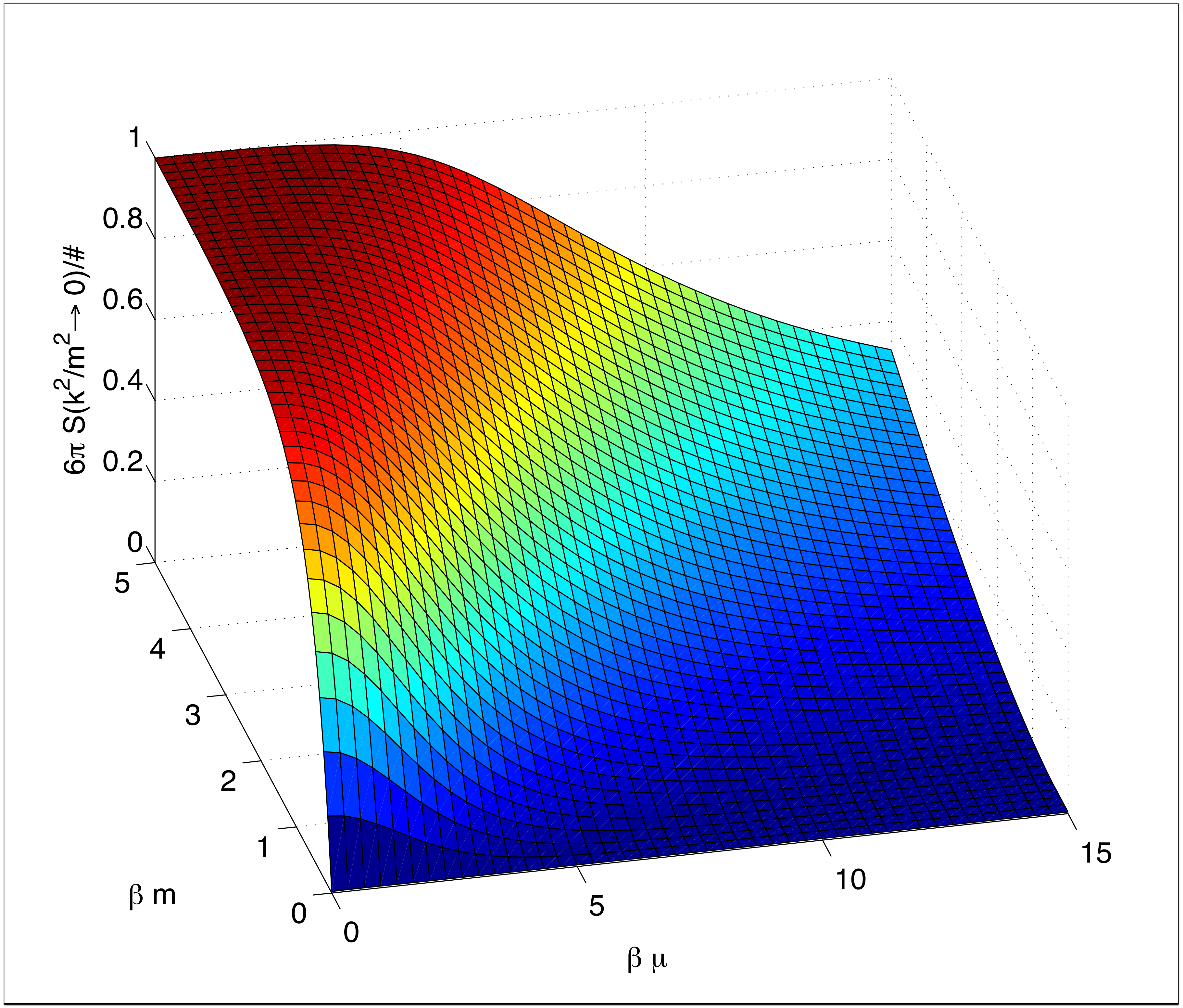
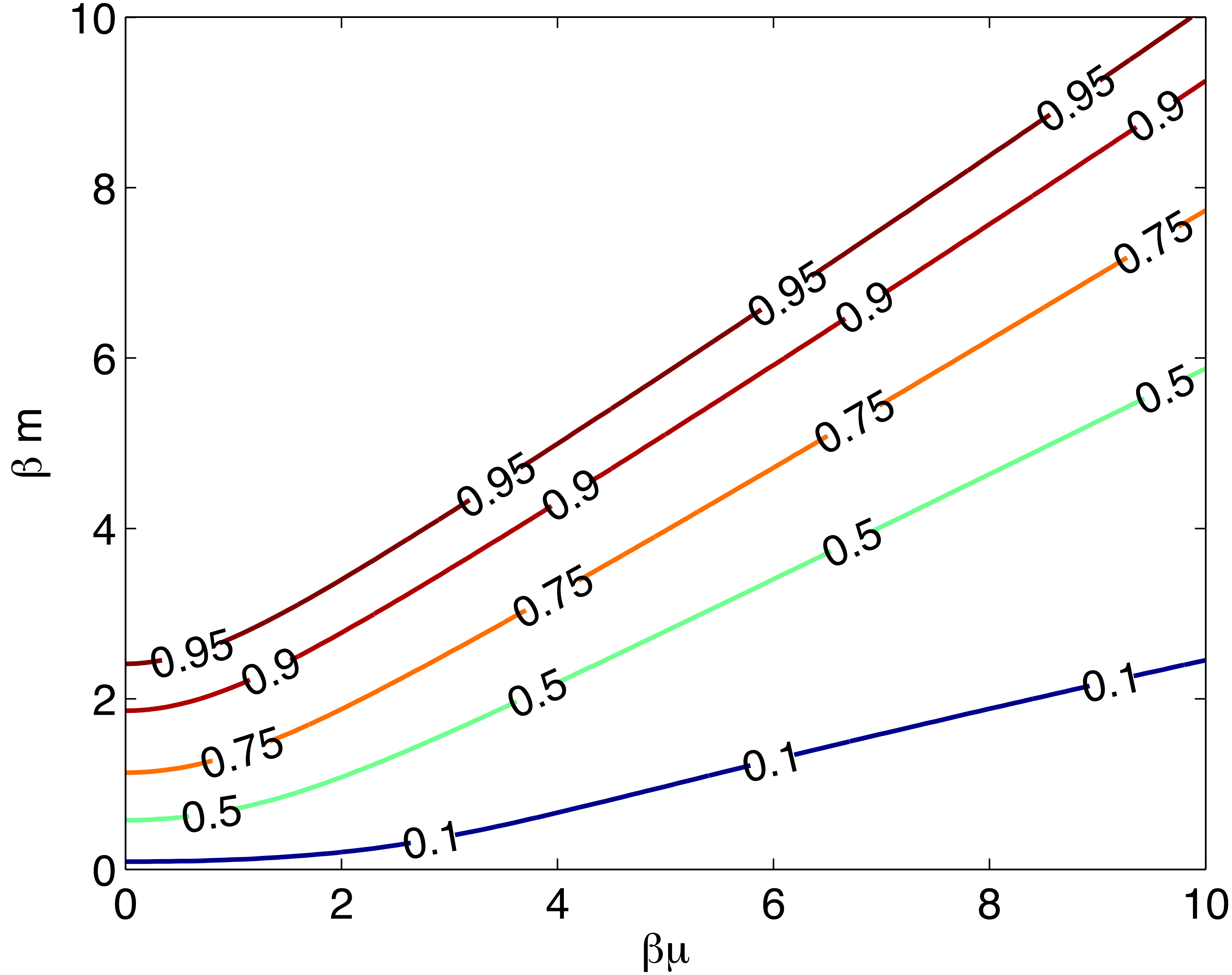




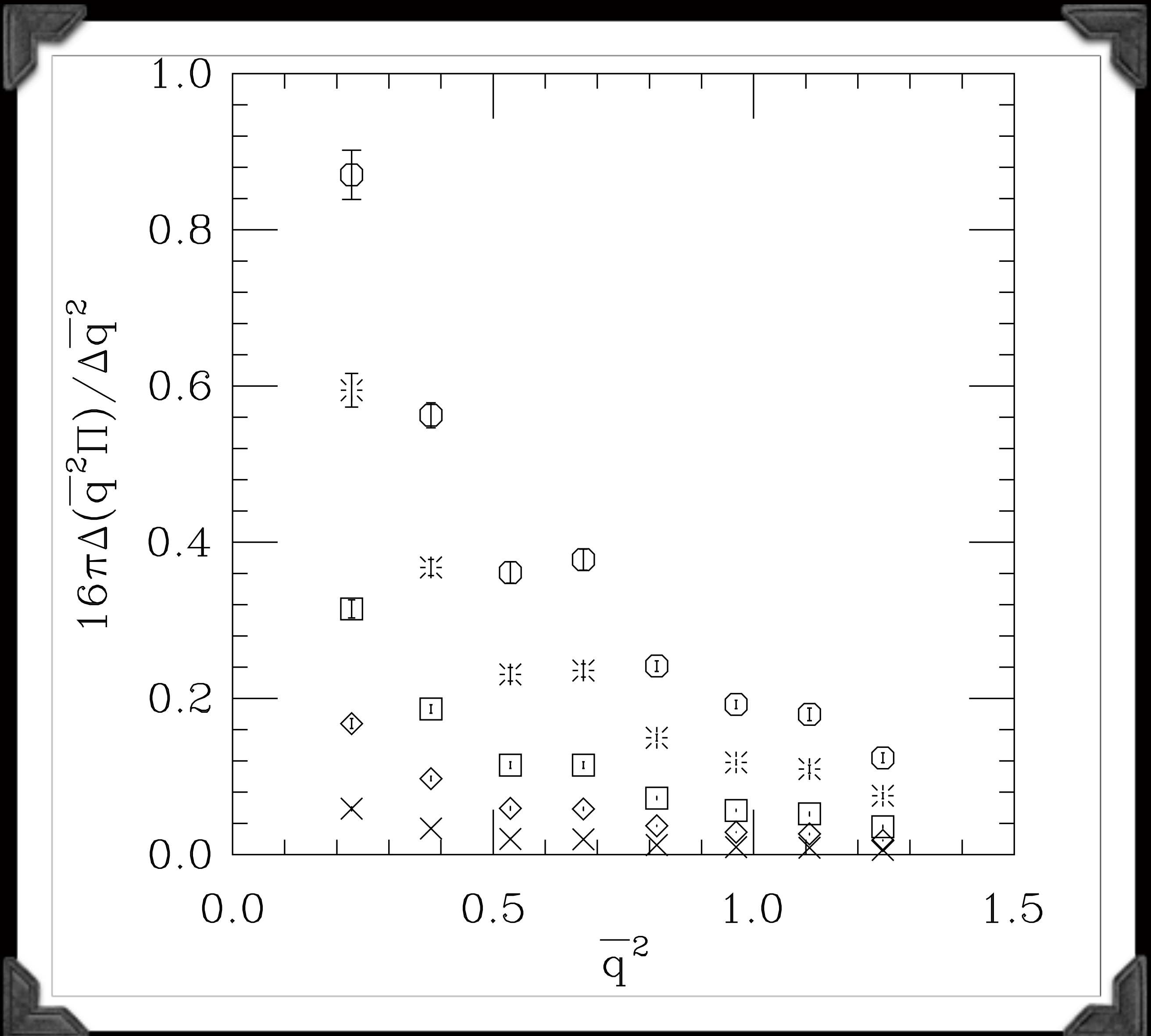
10
8
6
4
2
0
β m
0
2
4
6
8
10
βμ
0.95
0.9
0.75
0.5
0.1

Summary & Outlook

- S parameter measures the size of TC sector
- Conjecture:
  Perturbative calculation provides lower bound on S (throughout the phase diagram)
- Lattice calculations can corroborate/falsify, but finite size effects can be significant (especially in conformal theories)
- Future work: Compactify all dimensions?

Past lattice studies: Example

SU(3) sextet representation with 2 flavors

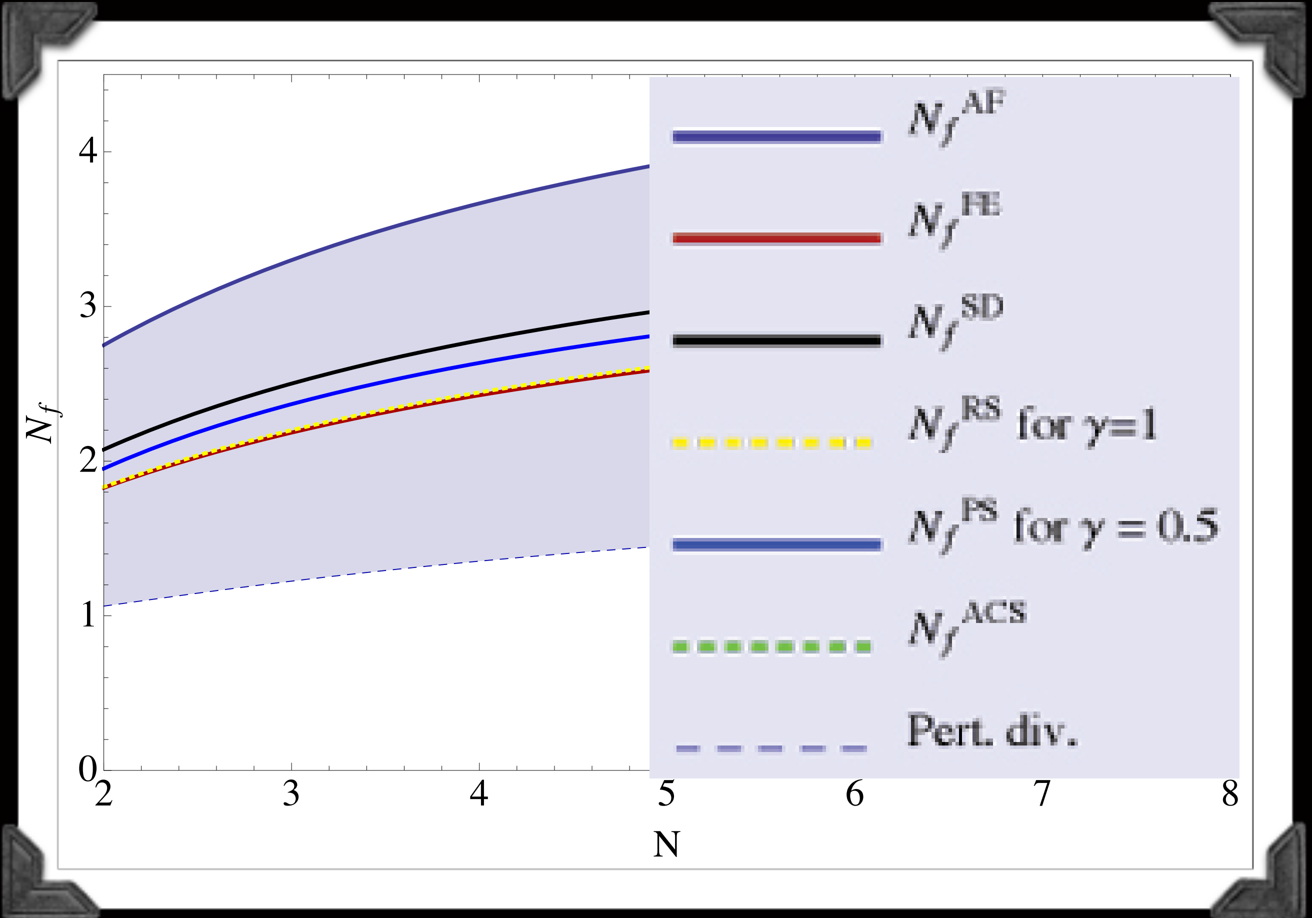


T. DeGrand, arXiv:1006.3777 [hep-lat]

From top to bottom:
am = 0.10 (m_ρ=0.48)
am = 0.75
am = 0.05
am = 0.035
am = 0.02 (m_ρ=0.33)

Lattice size = 12^4
$\beta m = amL_0$

Ranges between 1.2 and 0.24
(if conformal !)



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Neutral Current Matrix Elements

= 0 at tree-level

$$\mathcal{M}_{AA} = e^2 Q_1 G_{AA} Q_2$$

$$\mathcal{M}_{ZA} = \frac{e^2}{sc}\left[(I_3 - s^2 Q)_1 G_{ZA} Q_2 + Q_1 G_{ZA} (I_3 - s^2 Q)_2\right]$$

$$\mathcal{M}_{ZZ} = \frac{e^2}{s^2 c^2}(I_3 - s^2 Q)_1 G_{ZZ} (I_3 - s^2 Q)_2$$

Charged Current Matrix Element

$$\mathcal{M}_{WW} = \frac{e^2}{2s^2} I_+ G_{WW} I_-$$