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Offen im Denken

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The appearance of mass in discrete analog models:

graphene and optical lattices

Bad Honnef, Feb. 2012

Curvature, Defects, Geometry in *Graphene and Optical Lattices*

ANALOG MODELS

Crystalline structures

Particles in periodic potentials

↓

Effective description:
hopping on the lattice

(Quantum) Field Theory

Partial differential equations

↓

Approximate description:
discretization on the lattice

Discrete Hubbard-type model:

$$\hat{H} = -\sum_{<i,j>} T_{i,j}\, \hat{a}_i^+ \hat{a}_j + \sum_i V_i\, \hat{a}_i^+ \hat{a}_i + \frac{U}{2} \sum_i \left(\hat{a}_i^+ \hat{a}_i \right)^2$$

Lattice — *Fundamental parameters*

Lattice symmetries, ***internal structure*** ⟵ *Analogy* ⟶ ***Mass*** *and spin*

*(**supercells**)*

Lattice — *Gauge field theory*

Internal degrees of freedom, holonomy of hopping coefficients ⟵ *Analogy* ⟶ *Gauge fields*

(complex phases)

Lattice — *Differential Geometry*

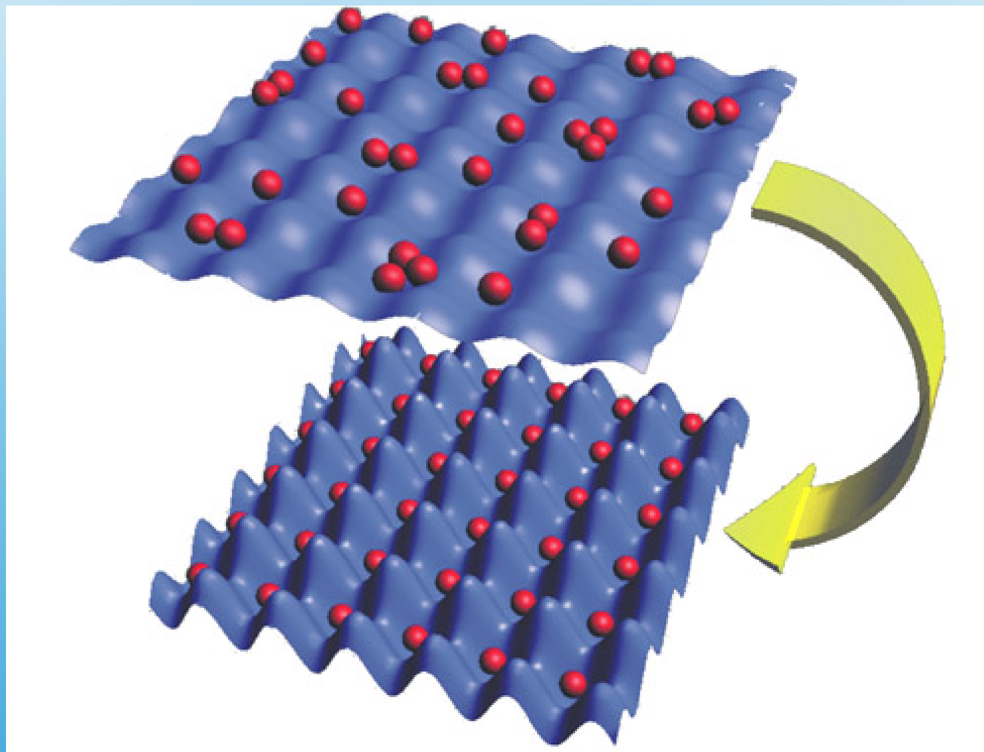
Distances and angles ⟵ *Analogy* ⟶ *Curvature and Torsion*

(metric and connection)

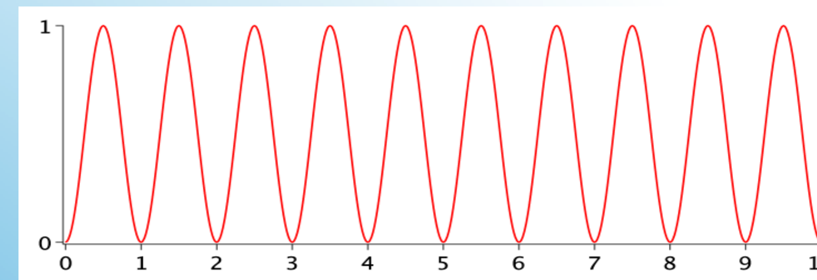
Analog concepts

Optical lattices

Ultra-cold atoms moving in a periodic optical potential: $V(x,y) = V_0 \cdot \left[\cos^2(x) + \cos^2(y)\right]$



In one dimension:



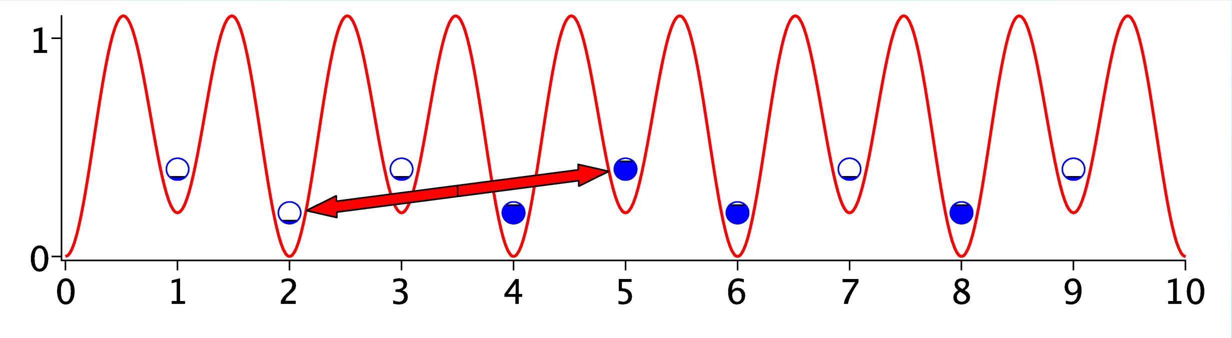
Hopping Hamiltonian:

$$\hat{H} = -J \cdot \sum_n \left(\hat{a}_n^+ \, \hat{a}_{n+1} + \hat{a}_{n+1}^+ \, \hat{a}_n \right)$$

Dispersion relation: $E(k) = -J \cdot \cos k$

Appearance of mass

Bi-chromatic optical potential → **periodic perturbations** of the lattice



Periodic super-structure

→ formation of **supercells** (double size)

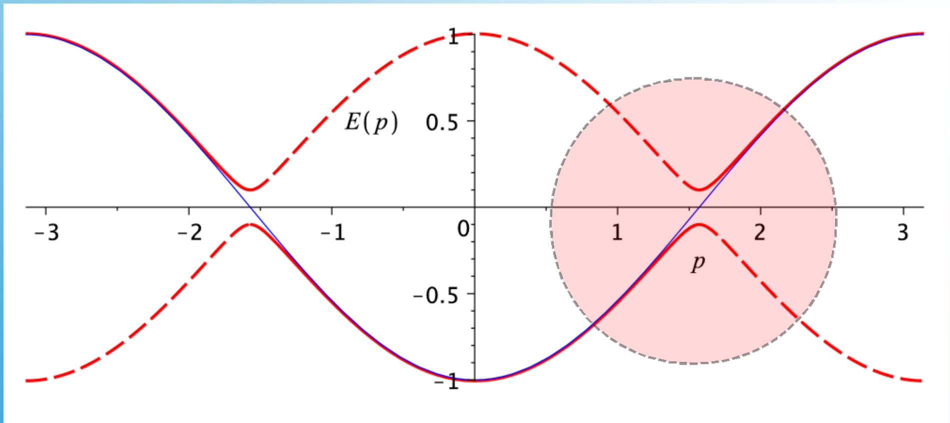
Hopping Hamiltonian:

$$\hat{H} = -J \cdot \sum_n \left(\hat{a}_n^+ \hat{a}_{n+1} + \hat{a}_{n+1}^+ \hat{a}_n \right) + \sum_n V_n \hat{a}_n^+ \hat{a}_n$$

$$V_n = (-1)^n M$$

→ Satisfies **discretized Dirac equation** in 1D

Band splitting:



Dispersion relation: $E(k) \approx \pm\sqrt{M^2 + J^2 \cos^2 k}$

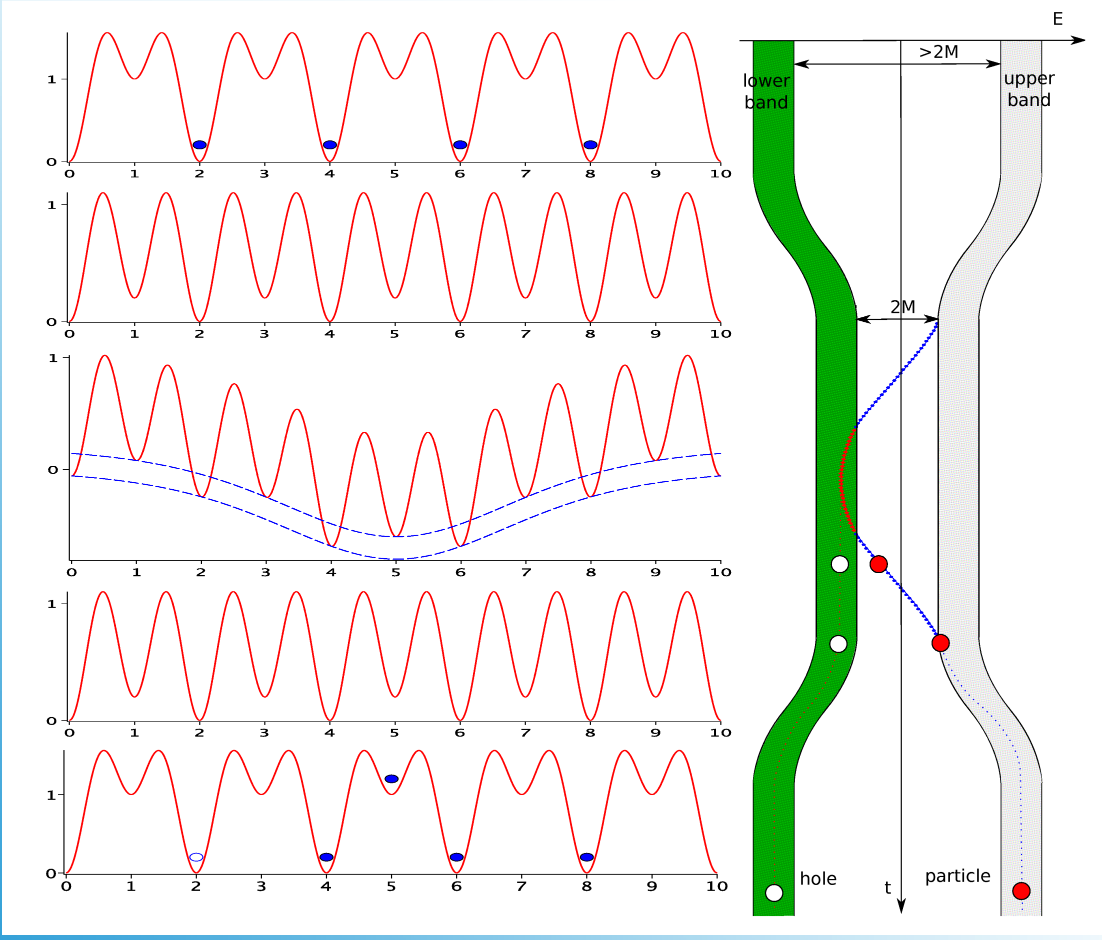
Analog QED in external fields *(Sauter–Schwinger effect)*

Optical lattice analogue of e**-** e**+** creation in strong electric fields

Initial ground state →
(vacuum)

Supercritical external potential →
(particle-hole pair creation via tunneling)

Final state →
(particle-hole pair)



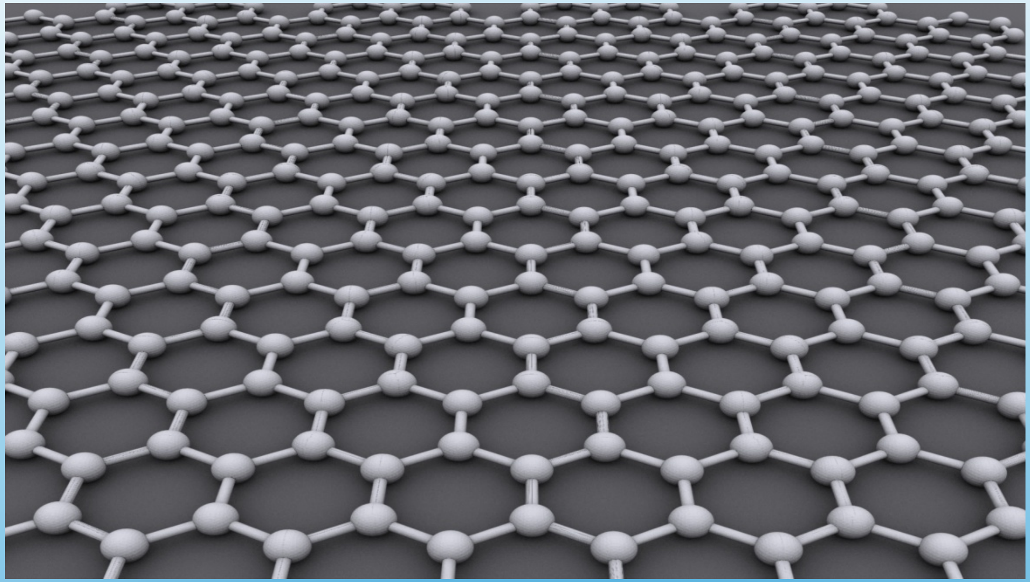
Optical lattice based ***Simulator***

Pair creation in strong external field

Time-dependent (band) spectrum

Graphene

2-dimensional "crystal"



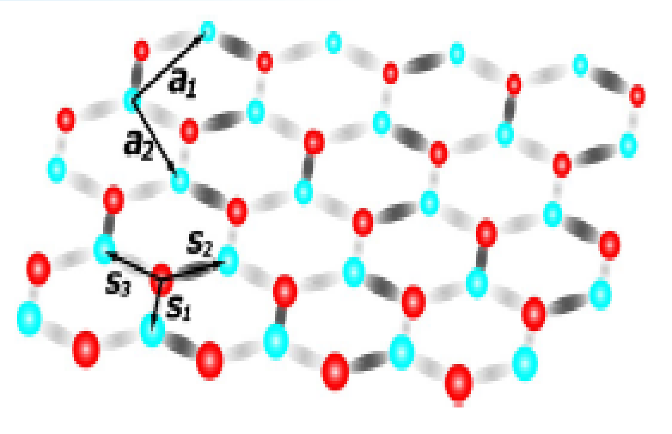
Nobel Prize 2010 – Geim und Novoselov

→ spinor-like wave function (2 degrees of freedom)

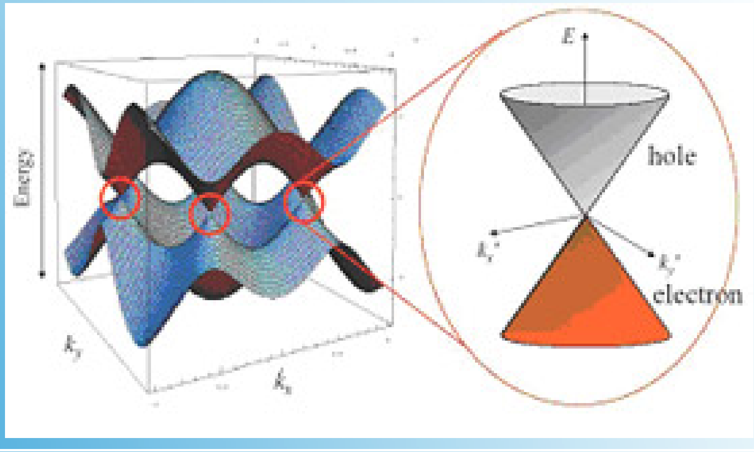
$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

→ Satisfies **discretized Dirac equation** in 2D

Honeycomb lattice → 2x triangular lattices:



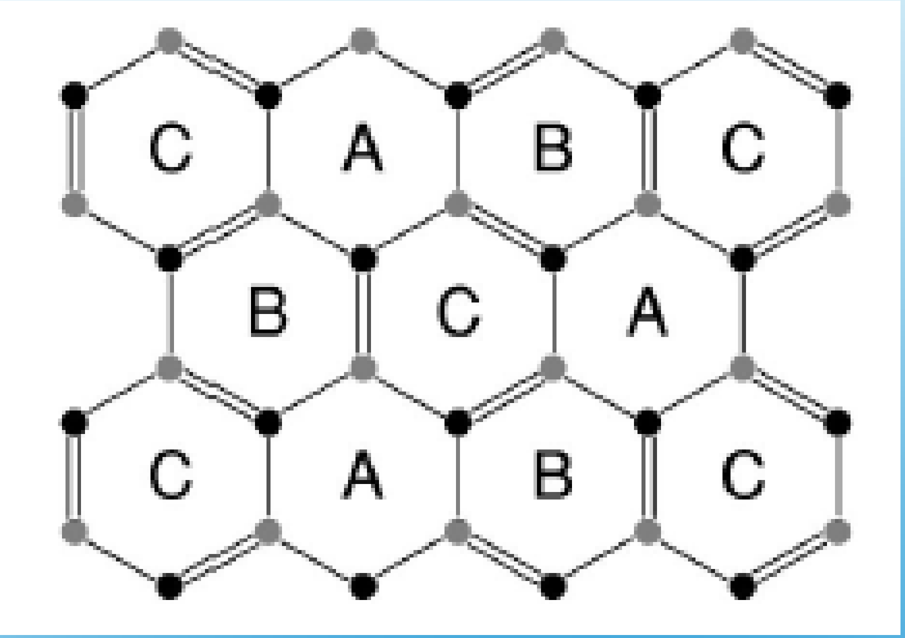
C.-Y. Hou, C. Chamon and C. Mudry, PRL **98**, 186809 (2007)



Dispersion relation: $E(k) \approx \pm |k|$

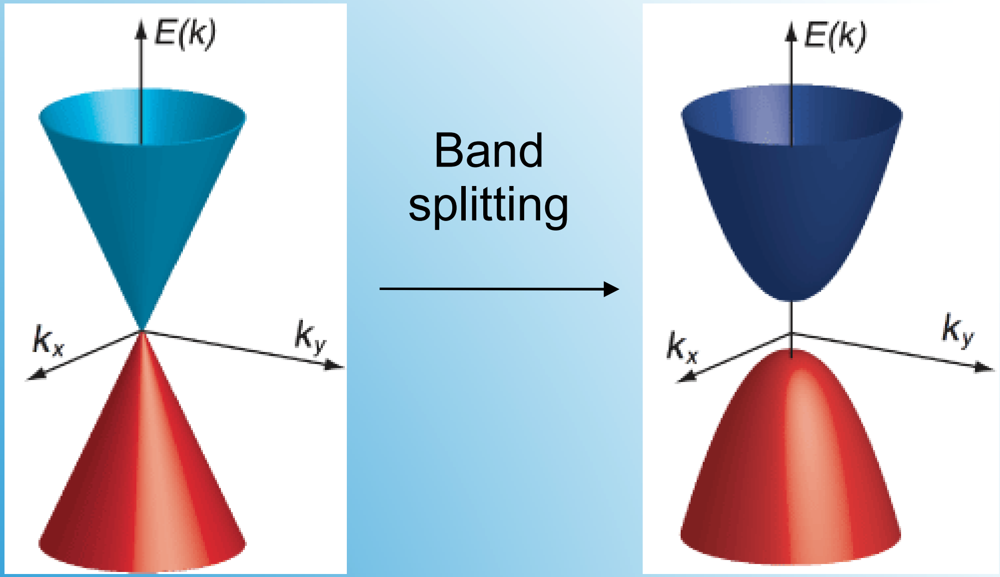
Appearance of mass

Kekule **periodic perturbations** of lattice links



Periodic super-structure

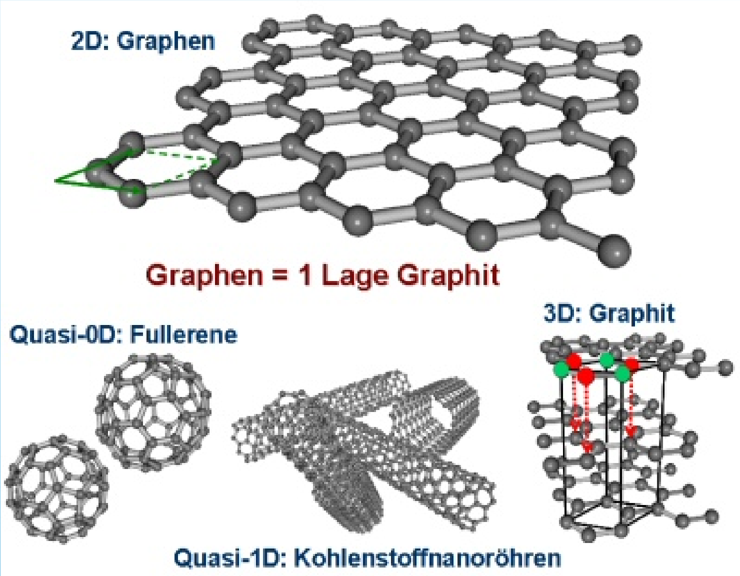
→ formation of **supercells** (A-B-C)



Dispersion relation:

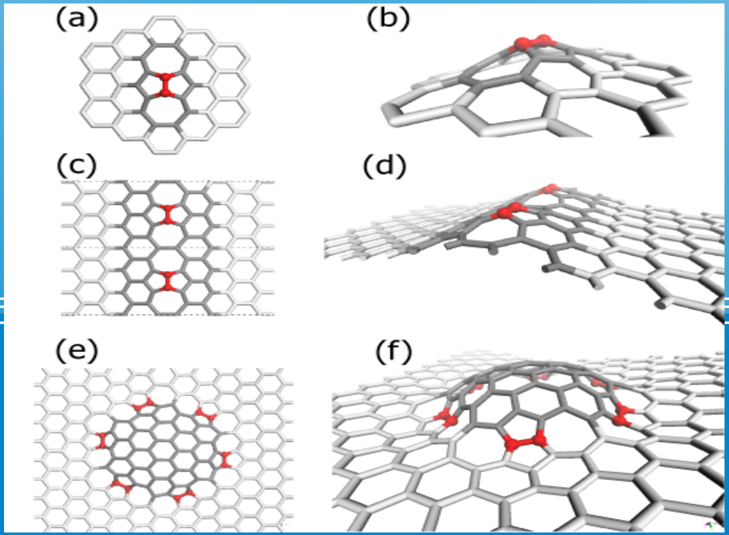
$$E(k) \approx \pm\sqrt{M^2 + J^2(k_x^2 + k_y^2)}$$

Curvature & Topology



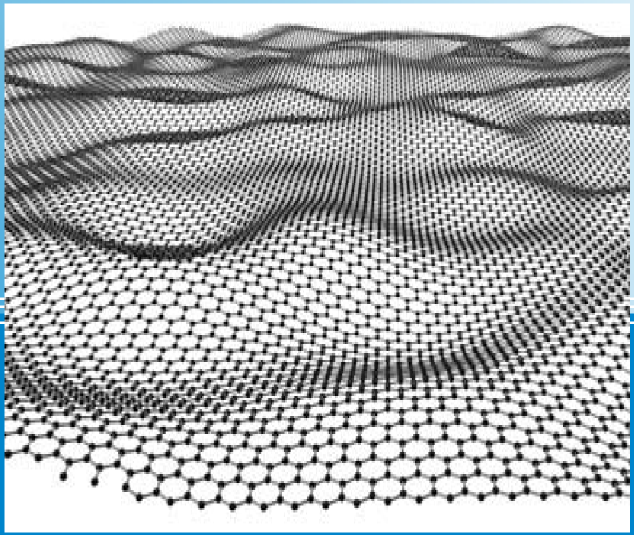
Graphene can be bent and curved → non-trivial topology and geometry

Curvature & Defects



D.J. Appelhans, L.D. Carr and M.T. Lusk, New Journal of Physics **12** (2010) 125006

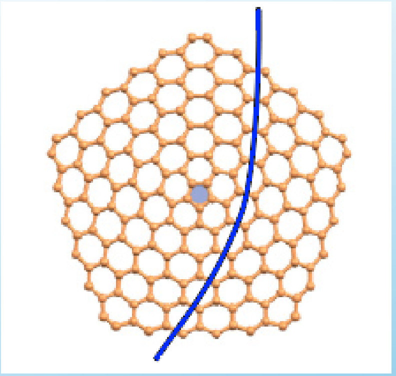
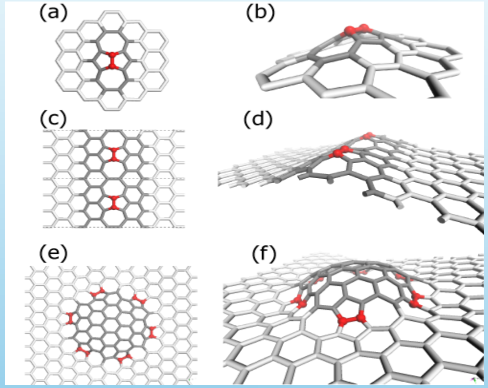
Ripples

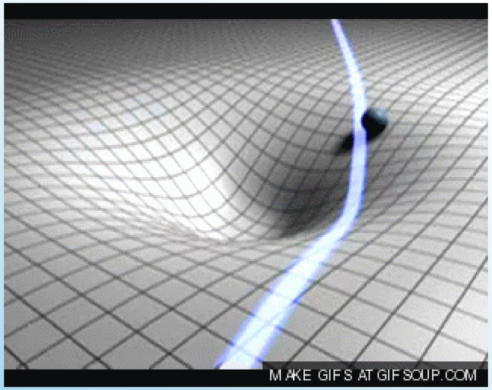


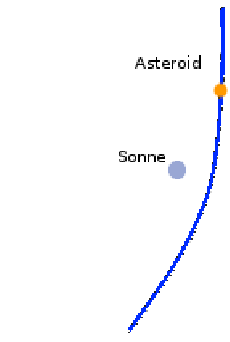
J.C. Meyer *et al*, Nature **446**, 60-63 (2007)

Graphene

General Relativity



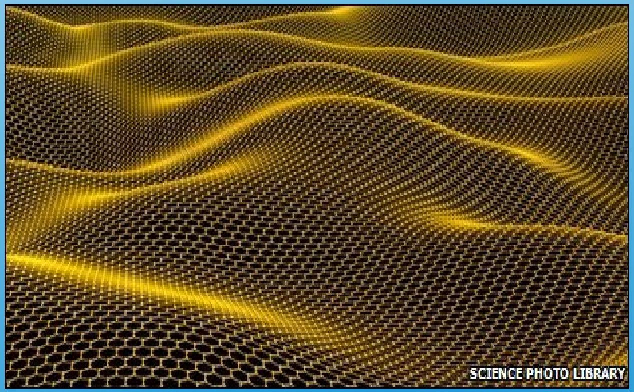


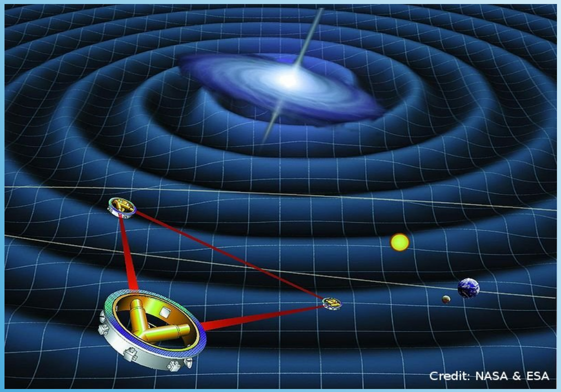
E. Cockayne *et al*, Phys. Rev. B **83**, 195425 (2011)

Forces

Defects

→ bending of "straight" lines (forces)





Ripples in graphene

Gravitational Waves *!*

Description ⇄ *Simulator*

→ language from differential geometry

→ astrophysics in laboratory