Hyperscaling relation for the conformal window

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15.2.12, strong-BSM workshop Bad Honnef

Overview

- Intro: Motivation & introduction conformal window studies [4 slides]

- Part I: mass-deformed conformal gauge theories (observables) [8 slides]
    - hyperscaling laws of hadronic observables e.g. $f[0^{++}] \sim m^{\eta(\gamma_*)}$

- Part II: the quark condensate -- various appraoches [4 slides]

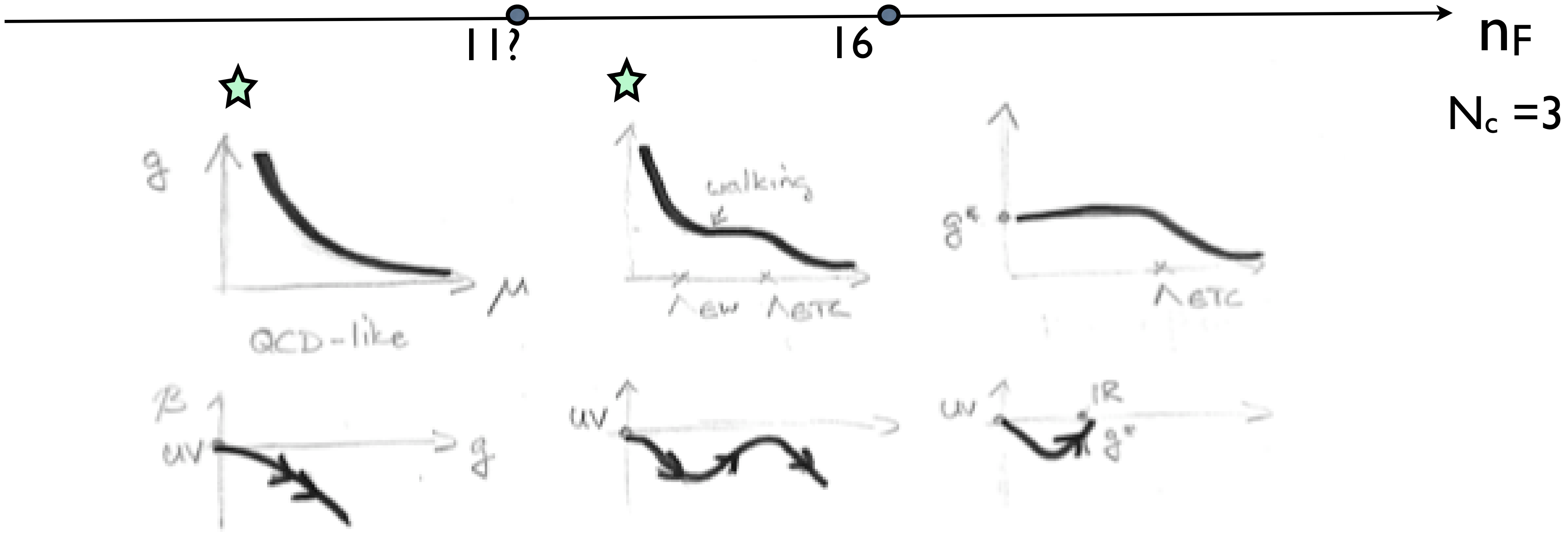
lattice material (Del Debbio's talks)
walking technicolour (Shrock, Sannino, others)

Del Debbio & RZ
PRD'10 & PLB'11

types of gauge theories

☆ Adjustable: gauge group SU(N_c) -- N_f (massless) fermions -- fermion irrep

☆ Focus on asymptotically free theories (not many representations)
2) well-defined on lattice 2) chance for unification in TC


11? 16 n_F

$N_c = 3$

TC-models: *QCD-type* *walking-type* *IR-conformal*

Susskind-Weinberg ‘79

Holdom ‘84

Dietrich-Sannino’04
Luty-Okui’04

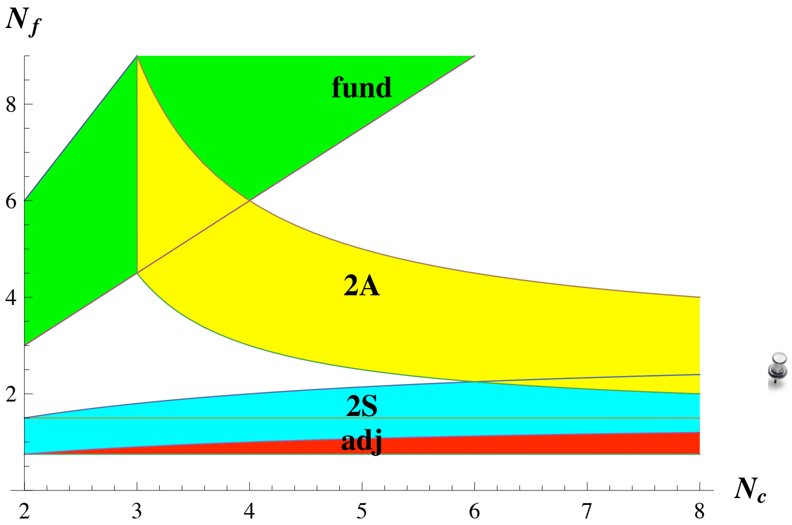
☆confinement & chiral symmetry breaking $SU_L(n_F) \times SU_R(n_F) \rightarrow SU_V(n_F)$
the latter is dynamical eletroweak symmetry breaking $M_W = g f_\pi^{(TC)}$

Conformal window (the picture)

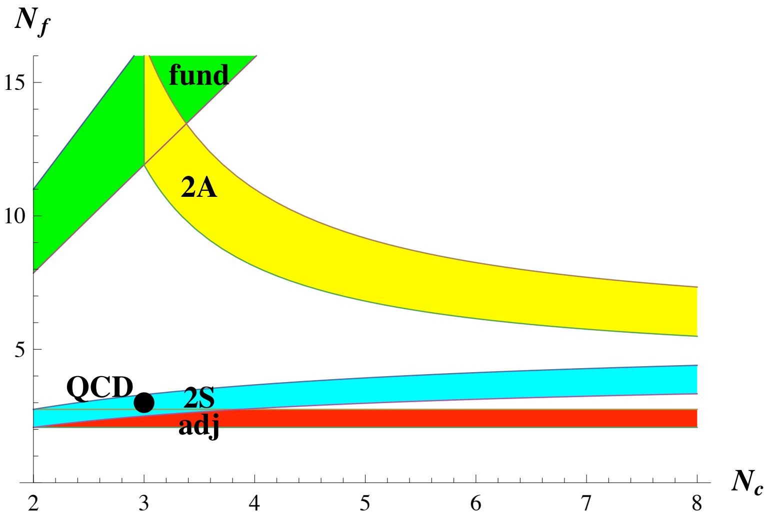
“SU(N)”

N=1 SUSY

non-SUSY



- upper line AF



- just below pert. BZ/BM fixed pt:

weak coupling

- lower line BZ/BM fixed pt “electromagnetic dual”

strong coupling

- assume in between conformal use $\beta_{NSVZ}(\gamma^*) = 0$ to get γ^*

- $\gamma^*|_{strong} = 1$ (unitarity bound QQ state)

$$\beta_0 \text{ tuned small } \frac{\alpha_s^*}{2\pi} = \frac{\beta_0}{-\beta_1} \ll 1$$

- lower line Dyson-Schwinger eqs predict chiral symmetry breaking (lattice results later ...)

- $\gamma^* \, |_{strong} \approx 1$ DS eqs ladder

anomalous dimension

canonical dimension d: (classical)

e.g. fermion field q: $d_q = \frac{D-1}{2}$ composite: $d_{\bar{q}q} = D-1$

anomalous dimension γ: (quantum corrections)

physical

scheme-dependent

$$\gamma = \gamma_* + \gamma_0(g-g_*) + \mathcal{O}((g-g_*)^2)\ldots$$

- **significance: change of renormalization scale μ → μ′ :**

scaling

leading correction

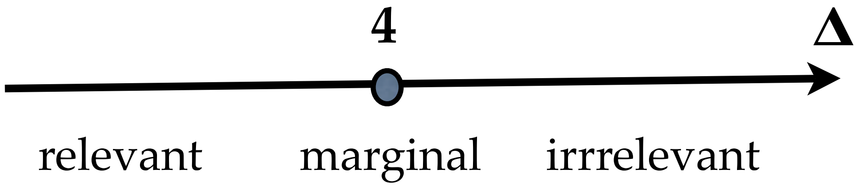
$$\mathcal{O}(\mu) = \mathcal{O}(\mu')\left(\frac{\mu}{\mu'}\right)^{\gamma_*}(1+\mathcal{O}(\ln\mu, /\mu'))$$

- **QCD:** UV-fixed point (asymptotic freedom) - γ_* $(g_* = 0) = 0$ (trivial)
  - correction RGE (logarithmic)
- **our interest:** IR fixed point non-trivial - $\gamma_* \neq 0$ (large?)

scaling dimension

scaling dimension: $\mathbf{\Delta_{scaling} = d_{canonical} + \gamma_{anomalous}}$

$$\mathcal{L} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$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Δ

relevant marginal irrrelevant

- Value of Δ_O is a dynamical problem (= d_O at trivial fixed-point, g_*=0)
  - unitarity bounds $\mathbf{\Delta_{Scalar} \geq 1}$ etc **Mack'77**
  - $\Delta_{OO'} \neq \Delta_O \, \Delta_{O'}$ generally (except SUSY and large-N_c)

- gauge theory: expect $\bar{q}q$ to be most relevant operator

  $\mathbf{\Delta_{qq} = 3 + \gamma_{qq} = 3 - \gamma_m}$ ($\gamma_m = \gamma_*$ at fixed point)

  γ_* is a very important parameter for model building

  Ward-identity

Part I:

Observables for Monte Carlo (lattice) for conformal gauge theories

-- parametric control --

Part I: Observables in a CFT?

or how to identify a CFT (on lattice)

pure-CFT:

Vanishing β-function & correlators (form 2 & 3pt correlators known)

e.g. $<O(x)O(0)> \sim (x^2)^{-\Delta}$

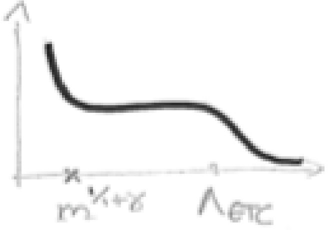
deformed-CFT:

Lattice: quarks massive / finite volume
⇒ consider <u>mass-deformed conformal gauge theories</u> (mCGT)*

$$\boxed{\mathcal{L} = \mathcal{L}_{\mathrm{CGT}} - m\bar{q}q}$$

* hardly related to 2D CFT mass deformation a part of algebra and 'therefore' integrability is maintained

- if mass-deformation relevant $\Delta_{qq} = 3 - \gamma_* < 4$
  theory flows away from fixed-point (likely)



physical picture: (*Miransky* ’98) finite m_q ; quarks decouple ➪ pure YM confines (string tension confirmed lattice) ➪ hadronic spectrum

signature: hadronic observables (masses, decay constants)

hypothesis: hadronic observables → 0 as m_q → 0 (conformality restored)

$$\mathcal{O} \sim m^{\eta_{\mathcal{O}}}(1 + ..)\ , \qquad \eta_{\mathcal{O}}(\gamma_*) > 0$$

If fct η_O known: a) way to measure γ_*
b) consistency test through many observable

Mass scaling from trace anomaly & Feynman-Hellman thm

trace/scale anomaly:

Adler et al, Collins et al
N.Nielsen '77 Fujikawa '81

$$\theta_\alpha{}^\alpha|_{\rm on-shell} = \tfrac{1}{2}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$$

$$\beta = 0 \quad \& \quad \langle H(p)|H(k)\rangle = 2E_p\delta^{(3)}(p-k) \Rightarrow$$

$$\boxed{2M_h^2 = N_f(1+\gamma_*)m\langle H|\bar{q}q|H\rangle}$$

reminiscent GMOR-relation

Feynman-Hellman thm:

$$\frac{\partial E_\lambda}{\partial\lambda} = \langle\psi(\lambda)|\frac{\partial\hat{H}(\lambda)}{\partial\lambda}|\psi(\lambda)\rangle$$

idea: $\frac{\partial\langle\psi(\lambda)|\psi(\lambda)\rangle}{\partial\lambda} = 0$

- ⋆ applied to our case (λ ➢ m) : $$m\frac{\partial M_H^2}{\partial m} = N_f m\langle H|\bar{q}q|H\rangle$$
- ⋆ combined with GMOR-like: $$m\frac{\partial M_H}{\partial m} = \frac{1}{1+\gamma_*}M_H$$

$$\boxed{M_H \sim m^{\frac{1}{1+\gamma_*}}}$$

scaling law
for all masses

Brief comparison with QCD

QCD-spectrum

mCGT-spectrum

- $m_\varrho = O(\Lambda_{QCD}) + m_q$
  $m_B = m_b + O(\Lambda_{QCD})$
  $m_\pi = O(\,(m_q\,\Lambda_{QCD})^{1/2}\,)$

$\longleftrightarrow$

- $m_{ALL} = \; m^{1/(1+\gamma^*)}\, O(\Lambda_{ETC}{}^{\gamma^*/(1+\gamma^*)}\,)$

- Breaking global flavour symmetry : $SU_L(n_F) \times SU_R(n_F) \rightarrow SU_V(n_F)$
  (chiral symmetry)

	QCD:	*mCGT:*	*CGT:*
spontaneous	yes*	no	no
explicit (mass term)	yes	yes	no
confinement	yes	yes	no

$\Rightarrow$ no chiral perturbation theory in mCGT

(pion not singled out -- Weingarten-inequality still applies)

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* $F_\pi \neq 0$ $m \rightarrow 0$ order parameter

Hyperscaling laws from RG

- local matrix element: $\mathcal{O}_{12}(g, \hat{m} \equiv \frac{m}{\mu}, \mu) \equiv \langle \varphi_2 | \mathcal{O} | \varphi_1 \rangle$

physical states
no anomalous dim.

1. $\mathcal{O}_{12}(g, \hat{m}, \mu) = b^{-\gamma_{\mathcal{O}}} \mathcal{O}_{12}(g', \hat{m}', \mu') \, ,$

RG -trafo O_{12}
$\mu = b\mu'$

$$g' = b^{0+\gamma_g} g \quad \hat{m}' = b^{1+\gamma_*} \hat{m} \, , \quad y_m = 1 + \gamma_* \, , \quad \gamma_g < 0 \text{ (irrelevant)}$$

2. $\mathcal{O}_{12}(\hat{m}', \mu') = b^{-(d_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})} \mathcal{O}_{12}(\hat{m}', \mu)$

change
physical units

3. Choose b s.t. $\hat{m}' = 1 \Rightarrow$ trade b for m

"master equation"

Hyperscaling
relations

$$\Rightarrow \quad \boxed{\mathcal{O}_{12}(\hat{m}, \mu) \sim (\hat{m})^{(\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/(1+\gamma_*)}}$$

* From Weinberg-like RNG eqs on correlation functions (widely used in critical phenomena)

Applications:

- master formula (local matrix element): $\langle \varphi_1 | \mathcal{O} | \varphi_2 \rangle \sim m^{(\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/(1+\gamma_*)}$

alternative derivation ...

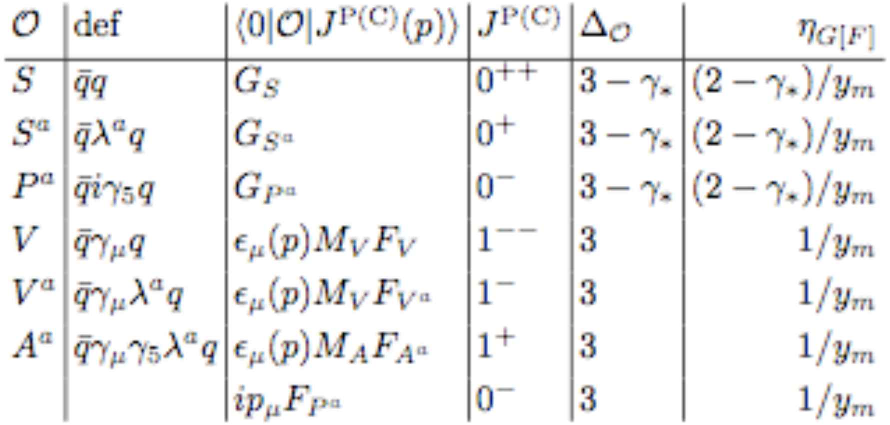
1. hadronic masses: $$2M_h^2 = N_f(1+\gamma_*) m \langle H | \bar{q} q | H \rangle \sim m^{\frac{2}{1+\gamma^*}}$$

2. vacuum condensates: $$\langle G^2 \rangle \sim m^{\frac{4}{1+\gamma_*}} \;, \qquad \langle \bar{q} q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$$

more later on...

3. decay constants:

$|\phi\rangle = |H(\text{adronic})\rangle$
N.B. ($\Delta_H = d_H = -1$ choice)



$\mathcal{O}$	def	$\langle 0 \vert \mathcal{O} \vert J^{\mathrm{P(C)}}(p) \rangle$	$J^{\mathrm{P(C)}}$	$\Delta_{\mathcal{O}}$	$\eta_{G[F]}$
S	$\bar{q}q$	G_S	0^{++}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
S^a	$\bar{q}\lambda^a q$	G_{S^a}	0^{+}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
P^a	$\bar{q}i\gamma_5 q$	G_{P^a}	0^{-}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
V	$\bar{q}\gamma_\mu q$	$\epsilon_\mu(p) M_V F_V$	1^{--}	3	$1/y_m$
V^a	$\bar{q}\gamma_\mu \lambda^a q$	$\epsilon_\mu(p) M_V F_{V^a}$	1^{-}	3	$1/y_m$
A^a	$\bar{q}\gamma_\mu \gamma_5 \lambda^a q$	$\epsilon_\mu(p) M_A F_{A^a}$	1^{+}	3	$1/y_m$
		$ip_\mu F_{P^a}$	0^{-}	3	$1/y_m$

Remarks S-parameter: $S = 4\pi\Pi_{V-A}(0) - [\mathrm{pion-pole}]$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\delta_{ab}\Pi_{V-A}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\left(V_a^\mu(x)V_b^\nu(0) - (V \leftrightarrow A)\right)|0\rangle$$

hadronic representation: $\Pi_{V-A}(q^2) \simeq \frac{f_V^2}{m_V^2 - q^2} - \frac{f_A^2}{m_A^2 - q^2} - \frac{f_P^2}{m_P^2 - q^2} + \dots .$

- difficulties: a) <u>*non-local*</u> b) difference (density <u>*not positive*</u> definite)

modulo
(conspiracy) cancellations
improve on non-
perturbatve computations
(lattice, FRG, DSE...)

$$\boxed{\begin{aligned} &\Pi^{\mathrm{W-TC}}_{V-A}(0) \sim O(m^{-1}) \\ &\Pi^{\mathrm{mCGT}}_{V-A}(0) \sim O(m^{0}) \\ &\Pi^{\mathrm{mCGT}}_{V-A}(q^2) \sim \frac{m^{2/y_m}}{q^2} \end{aligned}}$$

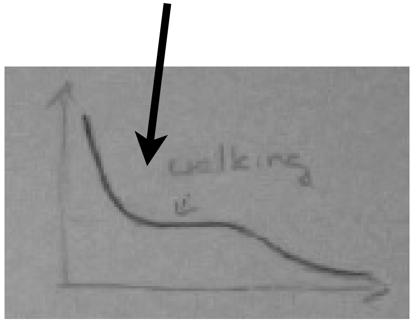
pion-pole

for $-q^2 \gg (\Lambda_{\mathrm{ETC}})^2$

Sannino'10 free theory

Summary - Transition

- low energy (hadronic) observables carry *memory of scaling phase*



$$m_H \sim m^{\frac{1}{1+\gamma_*}}$$

$$f_{H(0^-)} \sim m^{\frac{2-\gamma_*}{1+\gamma_*}} \qquad \langle \bar{q}q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$$

- “all” quantities scale with one parameter -- witness relations between the zoo critical exponents α, β, γ, ν .. = hyperscaling

- clarify: heavy quark phase and mCGT are parametrically from

similar: $m_{B(0^-)} \sim m_b \quad \neq \quad m_{H(0^-)} \sim m^{\frac{1}{1+\gamma_*}}$

distinct: $f_{B(0^-)} \sim m^{-1/2} \quad \neq \quad f_{H(0^-)} \sim m^{\frac{2-\gamma_*}{1+\gamma_*}}$

Part 2:

story quark condensate <qq>
the most relevant operator

- important for conformal TC/ partially gauged TC models

How does CFT react to a perturbation

Unparticle area

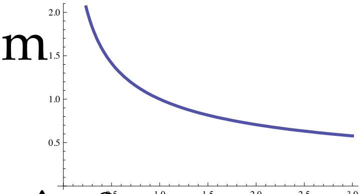
I. couple CFT Higgs-sector: $\mathcal{L}_{\text{eff}} \sim C\,\mathcal{O}|H|^2 \overset{\text{VEV}}{\to} C\,\mathcal{O}v^2$

criteria breaking (<u>NDA</u>): $\Lambda_B^4 \simeq Cv^2\Lambda_B^{\Delta_\mathcal{O}} \quad \Rightarrow \quad \Lambda_B \sim (Cv^2)^{\frac{1}{4-\Delta_\mathcal{O}}}$

Fox, Rajaraman, Shirman '07

II. Heuristics: <u>deconstruct</u> the continuous spectrum of a 2-function.
Infinite sum of adjusted particles can mimick continuous spectrum

Stephanov'07



$$\mathcal{O}(x) \sim \sum_n f_n \varphi_n(x)\,; \qquad \langle\varphi_n|\mathcal{O}|0\rangle \sim f_n\ , \qquad \begin{cases} f_n^2 = \delta^2\,(M_n^2)^{\Delta-2} \\ M_n^2 = n\delta^2 \end{cases}$$

➪ tadpole & mass term as potential ⇒ find new minimum

$$V_{\text{eff}} = -m\sum_n f_n\varphi_n - 1/2\sum_n M_n^2\varphi_n^2$$

Delgado, Espinoso, Quiros'07

minimise - solve - reinsert:

$$\delta_{\varphi_n} V_{\rm eff} = 0 \quad \Rightarrow \quad m f_n + M_n^2 \varphi_n = 0 \quad \Rightarrow \quad \langle \varphi_n \rangle = -m f_n / M_n^2$$

$$\langle \mathcal{O} \rangle \sim \sum_n f_n \langle \varphi_n \rangle - m \sum_n \frac{f_n^2}{M_n^2} \overset{\delta \to 0}{\to} -m \int_{\Lambda_{\rm IR}^2}^{\Lambda_{\rm UV}^2} s^{\Delta_{\mathcal{O}}-3} ds$$

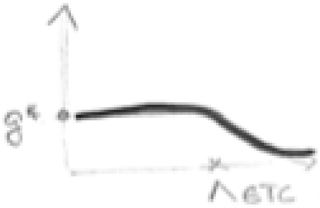
result depends on IR and UV physics ➪ need model(s)

III. Within conformal gauge theory

Sannino RZ ’08

generic 0^{++}-operator: $\mathcal{O} \to \bar{q}q$ within gauge theories

N.B. 4D only ”known” CFT gauge theories. why?



- $\Lambda_{\rm UV} \to \Lambda_{\rm ETC}$ where $\Delta_{qq} \to 3 \Rightarrow (\Lambda_{\rm ETC})^2$-effect
- $\Lambda_{\rm IR}$: $(m_{\rm constituent})^{\Delta qq} \sim <qq>$ generalising Politzer OPE

  Solve self-consistency condensate eqn

$$\frac{1}{A\not{q} - B} = \frac{1}{\not{q}} + \frac{4g^2}{q^4}$$

OPE extension

of pole mass

$$m = \frac{B}{A}$$

- ’10: $\Lambda_{\rm IR}$: $= m_H \; m^{1/(1+\gamma^*)} O(\Lambda_{\rm ETC}{}^{\gamma^*/(1+\gamma^*)})$ agrees depending on value γ^*

Del Debbio, RZ Sep’10

IV. Generalized Banks-Casher relation

- Banks & Casher ’80 à la Leutwyler & Smilga 92’:

Green’s function: $\langle q(x)\bar{q}(y)\rangle = \sum_n \frac{u_n(x)u_n^\dagger(y)}{m-i\lambda_n}$, where $\not{D} u_n = \lambda_n u_n$

$$\langle \bar{q}q\rangle_V = \int \frac{dx}{V}\langle \bar{q}(x)q(x)\rangle \overset{\lambda_n \to -\lambda_n}{=} -\frac{2m}{V}\sum_{\lambda_n>0}\frac{1}{m^2+\lambda_n^2} \overset{V\to\infty}{\to} -2m\int_0^\infty \frac{d\lambda\rho(\lambda)}{m^2+\lambda^2}$$

$$= \underbrace{-2m\int_0^{\mu_F} d\lambda\frac{\rho(\lambda)}{m^2+\lambda^2}}_{\text{IR}-\text{part}} \underbrace{-2m^5\int_{\mu_F}^\infty \frac{d\lambda}{\lambda^4}\frac{\rho(\lambda)}{m^2+\lambda^2}}_{\text{twice subtracted}} + \underbrace{\gamma_1}_{\Lambda_{\text{UV}}^2} m + \underbrace{\gamma_2}_{\ln\Lambda_{\text{UV}}} m^3$$

- IR-part: change of variable: $\rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \langle\bar{q}q\rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}}$
  - *QCD*: $\eta_{\bar{q}q} = 0 \Rightarrow \rho(0) = -\pi\langle\bar{q}q\rangle$
  - *mCGT*: another way to measure γ_*

Banks, Casher’80
DeGrand’09
DelDebbio RZ’10 May

- UV-part: known from perturbation theory (scheme dependent)

Summary

I. NDA $\Lambda_{\mathrm{IR}} \sim m^{1/(1+\gamma_*)}$ ok

II. Deconstruction: $\langle \mathcal{O} \rangle \sim \int_{\Lambda_{\mathrm{IR}}^2}^{\Lambda_{\mathrm{UV}}^2} s^{\Delta_{\mathcal{O}}-3} ds$ model dependence

III. Model mCGT & deconstruction

- Λ_{UV} properly identified thanks asymptotic freedom
- Λ_{IR} $m_{\mathrm{constituent}}$ generalized QCD (Polizter OPE) ok dep value γ_*

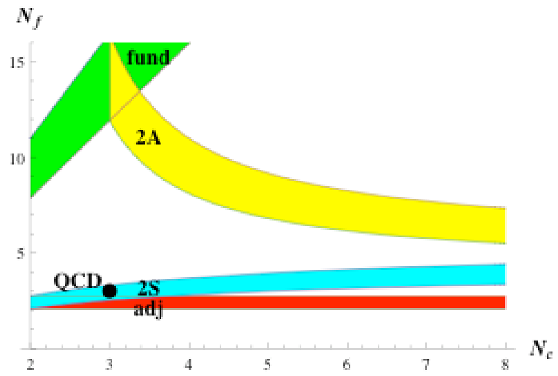
IV. Model mCGT & generalized Banks-Casher

- clean seperation of IR & UV everything consistent
  e.g. can use $m_H \sim m^{1/(1+\gamma_*)}$ in deconstruction as well

Epilogue

- Identified "universal" hyperscaling laws in mass deformation valid for any conformal theory in the vicinity of the fixed point (small mass)

- One thought:
  1) CGT likely phase diagram as compared to walking theory
  2) CGT instable m-deformation. Any quark that receives mass can be expected to decouple and finally drive the theory into a confining phase and the remaining quarks can undergo chiral symmetry breaking and thus dynamical electroweak symmetry breaking.



Contrasts: "Dynamical stability of local gauge symmetry..."

Forster, Nielsen & Ninomiya'80

Danke für die Aufmerskamkeit!

Backup slides …

Some relevant/useful references

- ☆ Miransky hep-ph/9812350 spectrum (with mass) as signal of conformal window works with pole mass -- weak coupling regime $\Lambda_{YM} \cong m \,\mathrm{Exp}[-1/b_{YM}\, \alpha^*]$
  ➪ glueballs lighter than mesons
- ☆ Luty Okui JHEP'96 conformal technicolor propose spectrum as signal of cw
- ☆ Dietrich/Sannino PRD'07 conformal window SU(N) higher representation using Dyson-Schwinger techniques known from WTC
- ☆ Sannino/RZ PRD'08 <qq> done heuristically IR and UV effects understood 0905
- ☆ DelDebbio et al ArXiv 0907 Mass lowest state from RGE equation
- ☆ DeGrand scaling <qq> stated ArXiv 0910
- ☆ DelDebbio RZ ArXiv 0905 scaling of vacuum condensates, all lowest lying states
- ☆ DelDebbio RZ ArXiv 0909 scaling extended to entire spectrum and all local matrix elements

Del Debbio, RZ Sep'10

Mass & decay constant trajectory

- At large-N$_c$ neglect width

$$g_{H_n} \equiv \langle 0|\mathcal{O}|H_n\rangle \text{ (decay constant)}$$

$$\Delta(q^2) \sim \int_x e^{ixq}\langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle = \sum_n \frac{|g_{H_n}|^2}{q^2+M_{H_n}^2}$$

- In limit m → 0 (scale invariant correlator)

$$\Delta(q^2) = \int_0^\infty \frac{ds\, s^{1-\gamma_*}}{q^2+s} + \text{s.t} \propto (q^2)^{1-\gamma_*}$$

- Solution are given by:

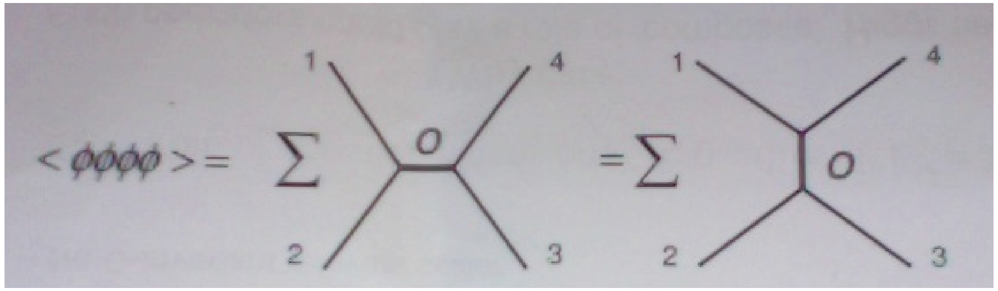
$$\boxed{M_{H_n}^2 \sim \alpha_n m^{\frac{2}{1+\gamma_*}} \;, \quad g_{H_n}^2 \sim \alpha'_n(\alpha_n)^{1-\gamma_*} m^{\frac{2(2-\gamma_*)}{1+\gamma_*}}}$$

where α_n arbitrary function (corresponds freedom change of variables in $\int$)

- QCD expect $\alpha_n \sim$ n (linear radial Regge-trajectory) (few more words)
- For those who know: resembles deconstruction Stephanov'07
difference physical interpretation of spacing due to scaling spectrum

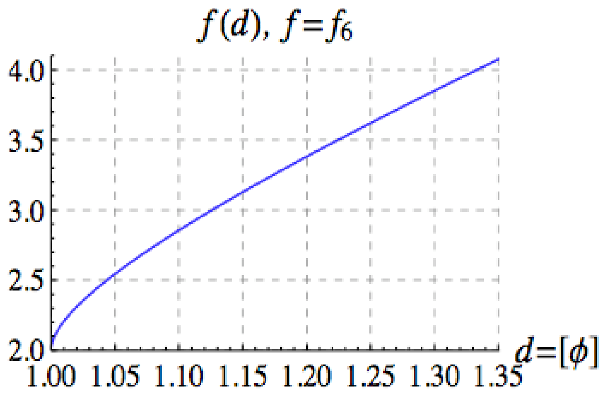
Addendum (bounds scaling dimension)

- assume add L = mqq (N.B. not a scalar under global flavour symmetry!)
- using bootstrap ('associative' OPE on 4pt function) possible to obtain <u>upper</u>-bound on scaling dimension Δ of lowest operator in OPE



non-singlet

Rattazzi, Rychkov Tonni & Vichi'08

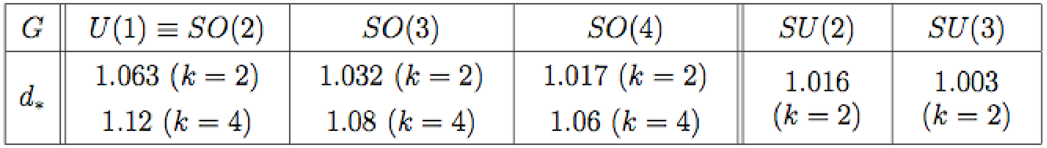


1.35 still rather close to unitarity bound

singlet $\Delta \leq 4$

allows for Δ_{qq} to be:

Rattazzi, Rychkov & Vichi '10



G	$U(1) \equiv SO(2)$	$SO(3)$	$SO(4)$	$SU(2)$	$SU(3)$
d_*	1.063 $(k = 2)$ 1.12 $(k = 4)$	1.032 $(k = 2)$ 1.08 $(k = 4)$	1.017 $(k = 2)$ 1.06 $(k = 4)$	1.016 $(k = 2)$	1.003 $(k = 2)$

very close to unitarity bound!

good news for Luty's conformal TC

QCD

observable

Gell-Mann Oakes Renner:

$$f_\pi^2 m_\pi^2 = -2m\langle\bar{q}q\rangle$$

mCGT

not so directly observable

very important for Technicolour

$$\mathcal{L}^{\text{eff}} = \alpha_{ab}\frac{\bar{Q}T^aQ\bar{\psi}T^b\psi}{\Lambda^2_{ETC}} + \beta_{ab}\frac{\bar{Q}T^aQ\bar{Q}T^bQ}{\Lambda^2_{ETC}} + \gamma_{ab}\frac{\bar{\psi}T^a\psi\bar{\psi}T^b\psi}{\Lambda^2_{ETC}} + \dots$$

↑ $fermion\ masses$

↑ $FCNC$