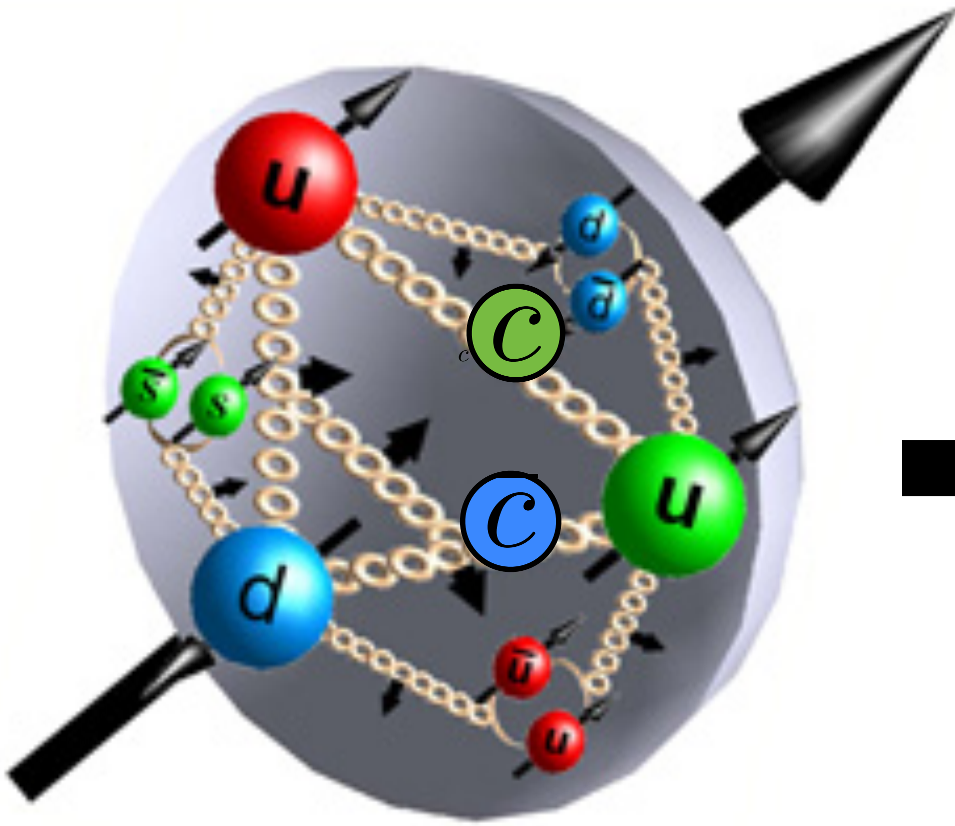
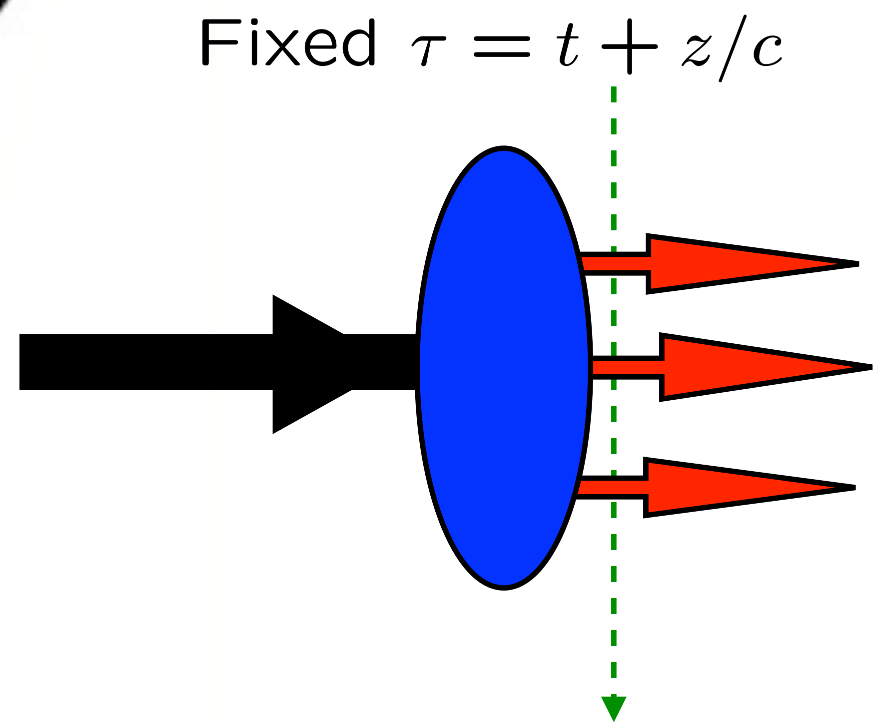
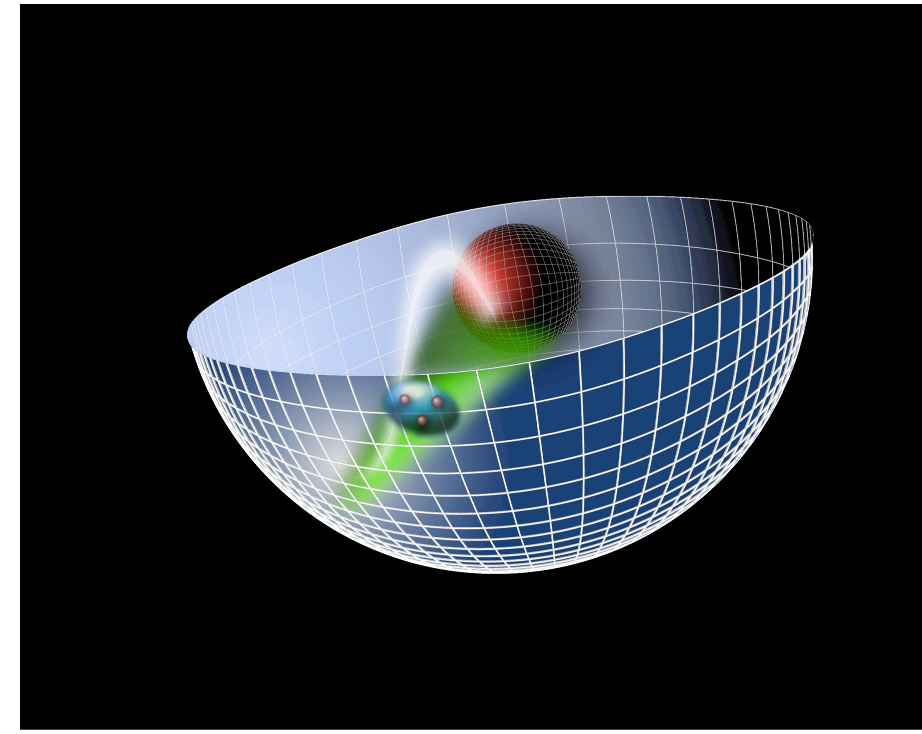
The Light-Front Schrödinger Equation: A New Approach to Color Confinement and Non-Perturbative QCD







Strong Interactions in the LHC Era

Wilhelm und Else Heraeus Stiftung

Bad Honnef
November 13, 2014

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY

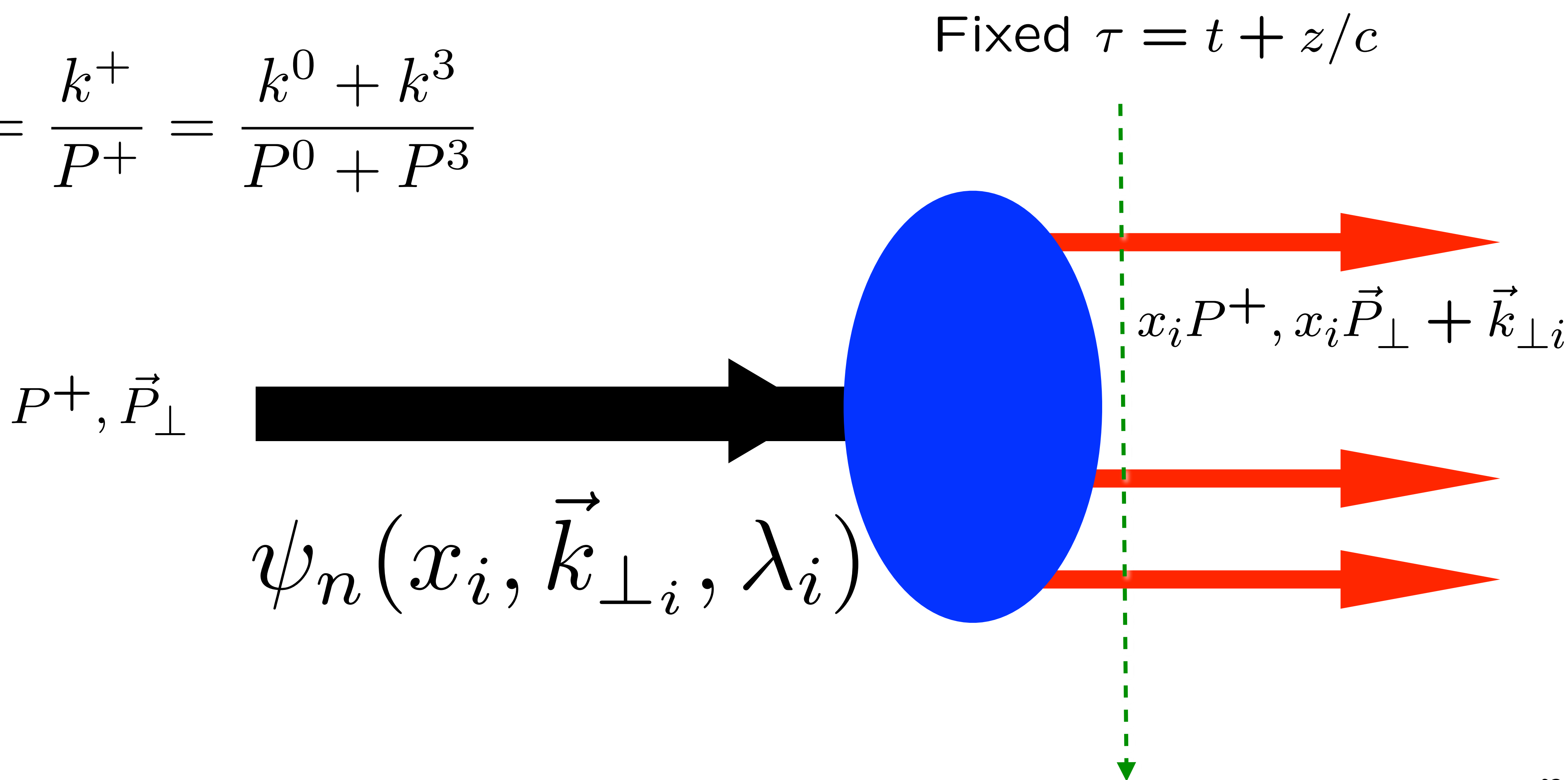
Goal: an analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What sets the QCD mass scale?**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**
- **Chiral Symmetry**
- **Systematically improvable**

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

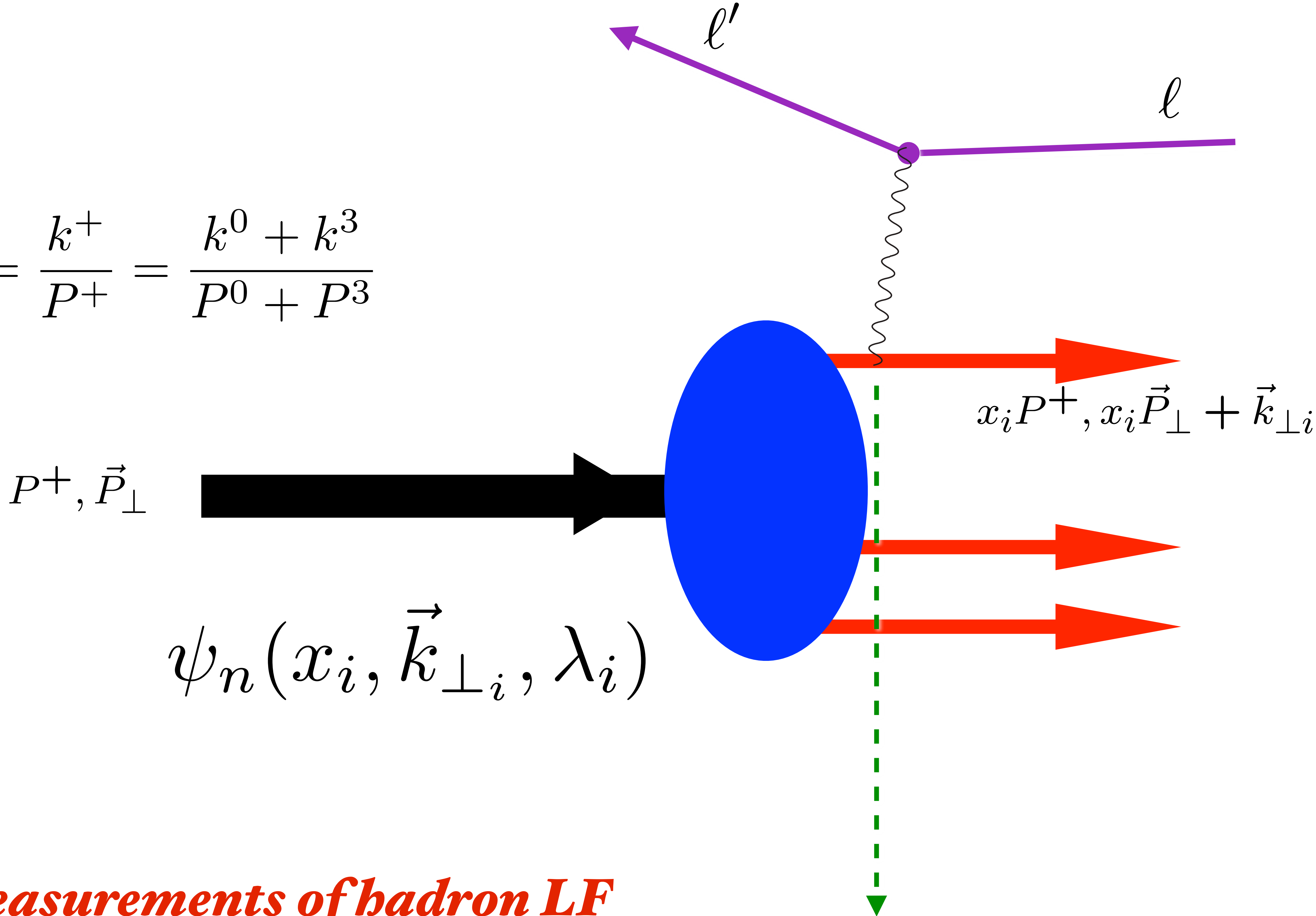
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

$$x_{bj} = x = \frac{k^+}{P^+}$$

Like a flash photograph

Each element of flash photograph illuminated along the light front

at a fixed

$$\tau = t + z/c$$

Evolve in LF time

$$P^{-} = i\frac{d}{d\tau}$$

Eigenvalue

$$P^{-} = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

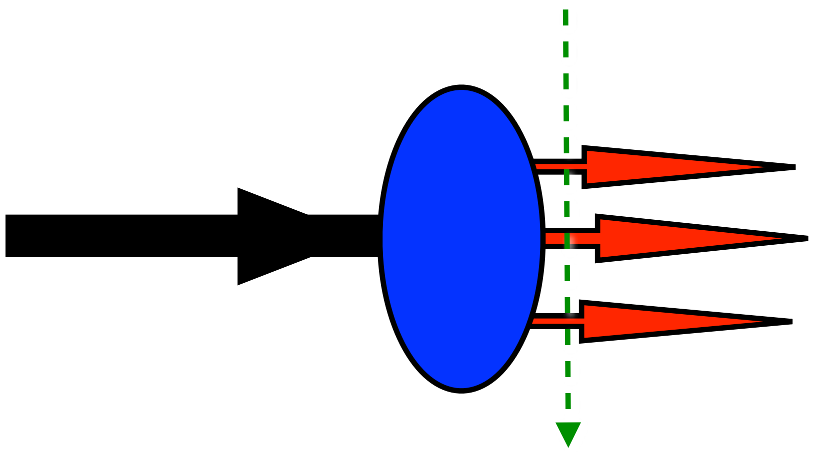
Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts!
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics — quark kinetic energy + quark-gluon dynamics — quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

QCD Mass Scale from Confinement not Explicit

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

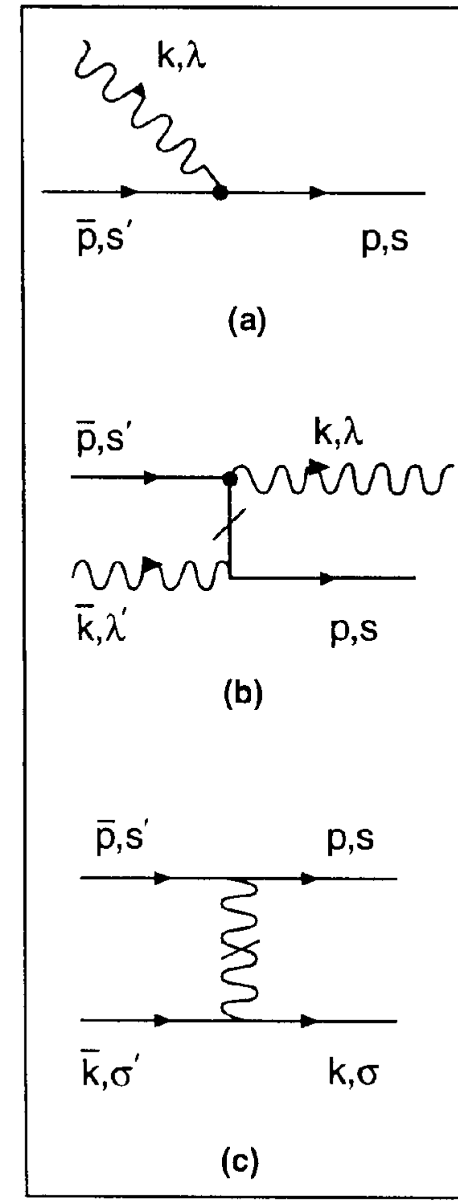
H^{int}_{LF}: Matrix in Fock Space

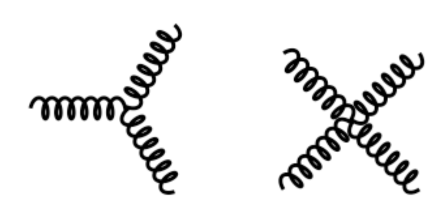
$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass

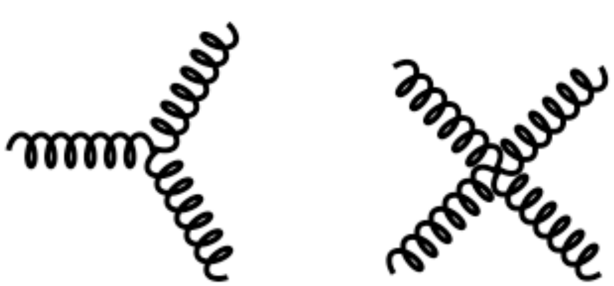


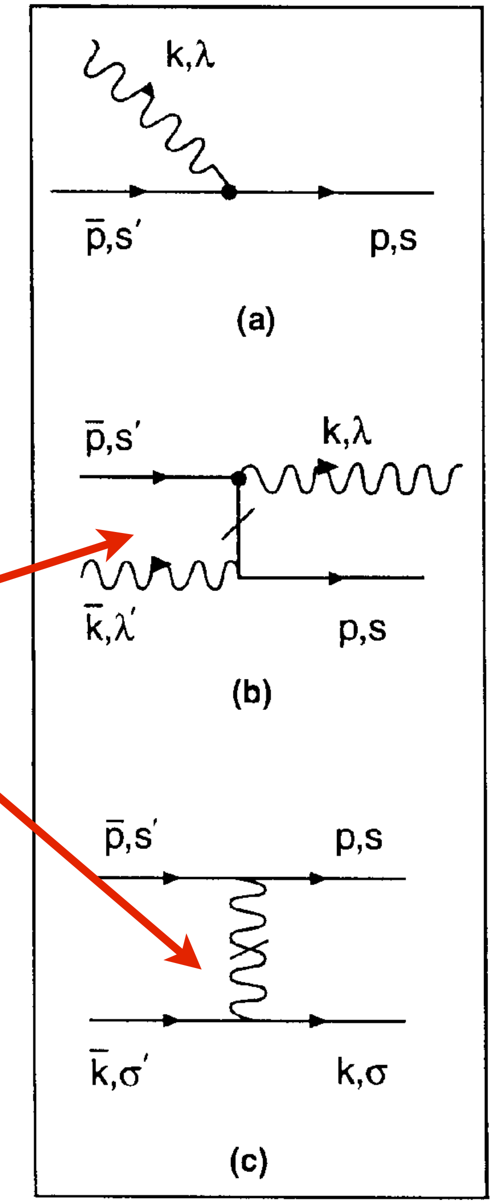


H^{int}_{LF}

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$
\begin{aligned}
H^{LF}_{QCD} &= \frac{1}{2}\int d^3x \bar{\tilde{\psi}}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\tilde{\psi} - A^i_a (i\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(i\partial^+)^2} \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\
&- g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ \left(\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\
&+ g^2 \int d^3x \mathbf{Tr}\left(\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2 \int d^3x \bar{\tilde{\psi}}\tilde{\not{A}}\frac{\gamma^+}{i\partial^+}\tilde{\not{A}}\tilde{\psi} \\
&+ g \int d^3x \bar{\tilde{\psi}}\tilde{\not{A}}\tilde{\psi} \\
&+ 2g \int d^3x \mathbf{Tr}\left(i\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right)
\end{aligned}
$$





Physical gauge: $A^+ = 0$

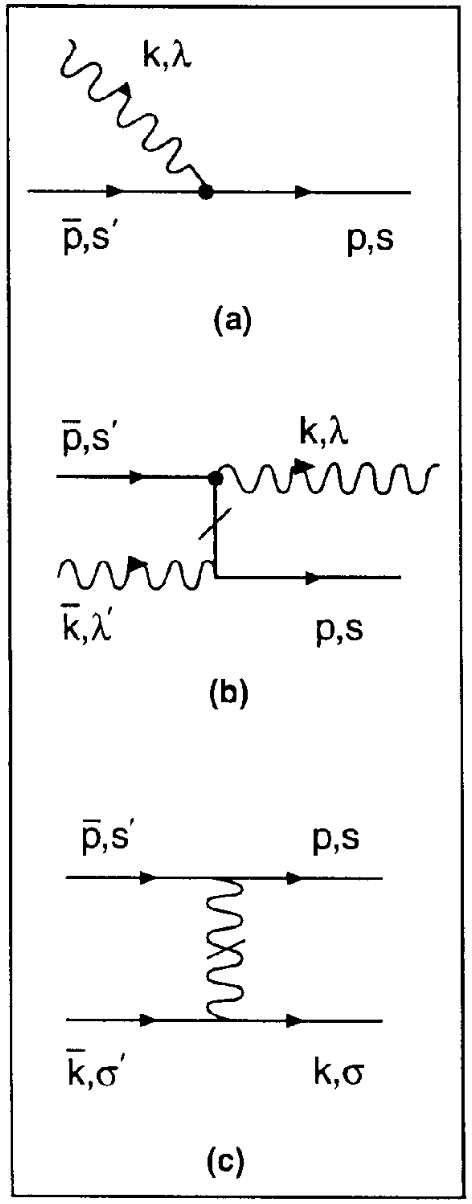
Light-Front QCD

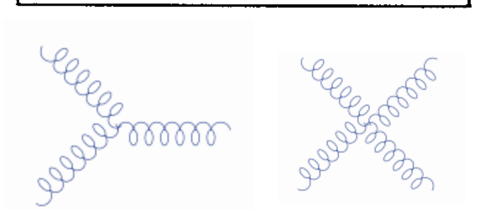
Heisenberg Equation

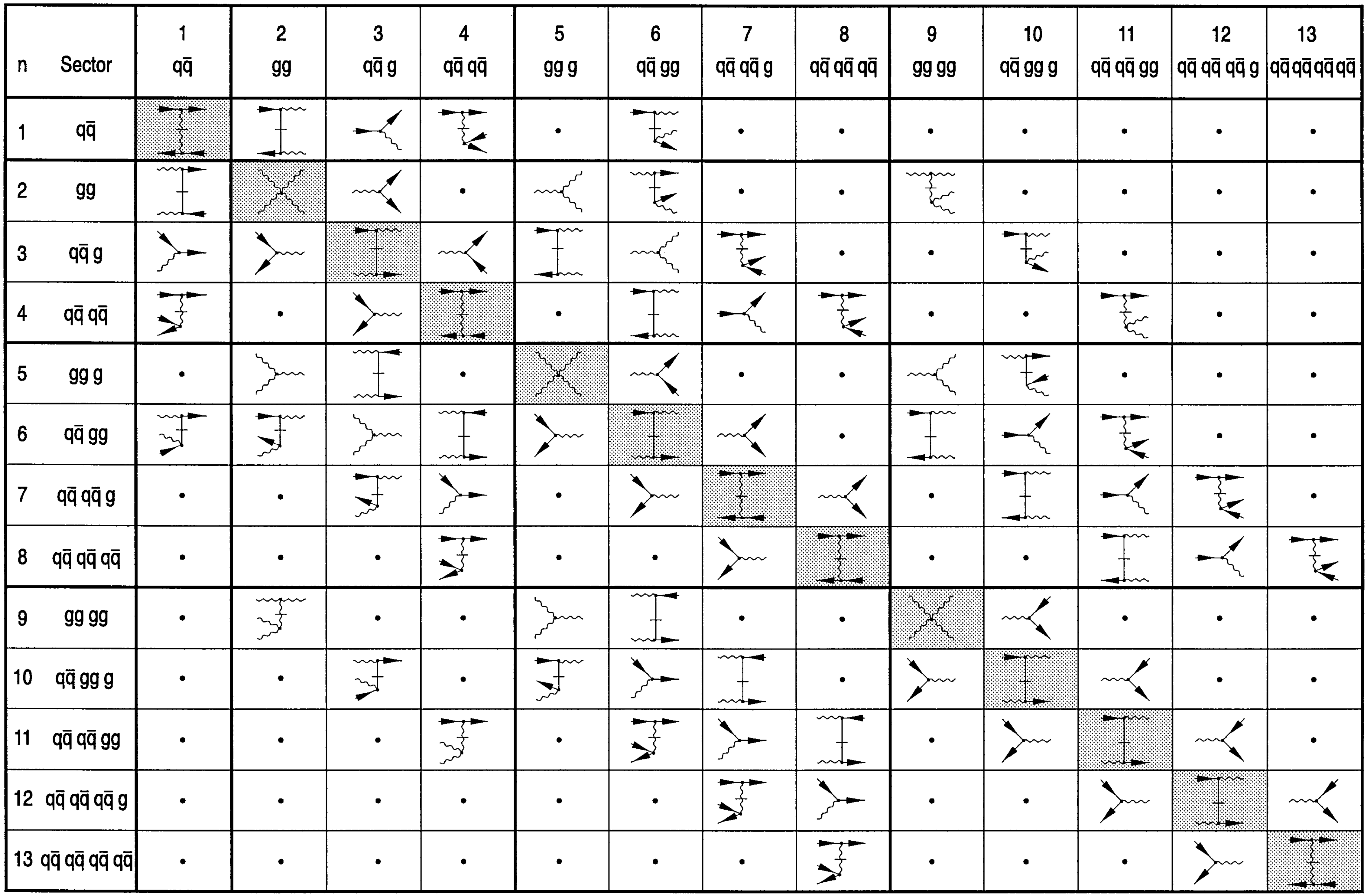
$$\boxed{H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\,|\Psi_h\rangle}$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb

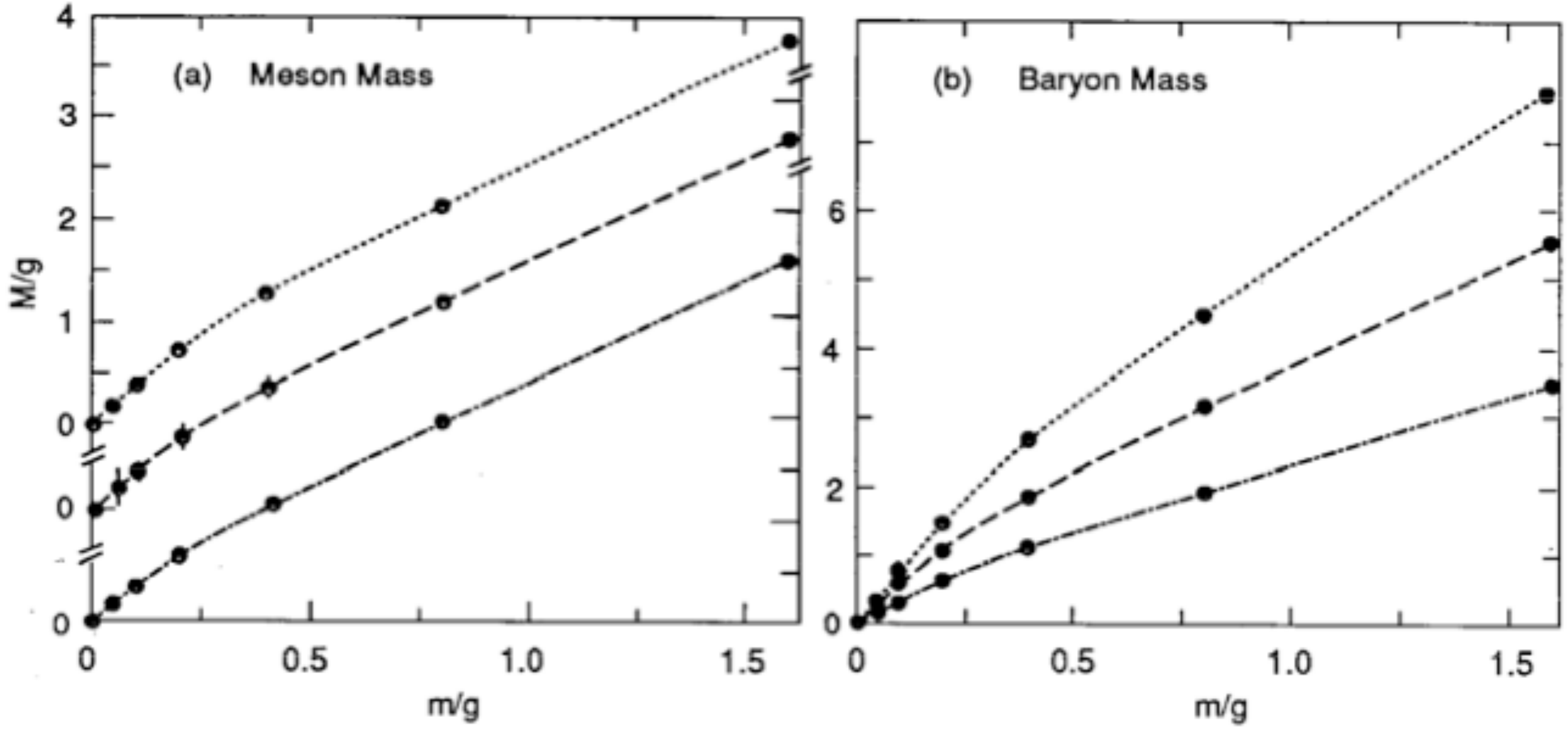




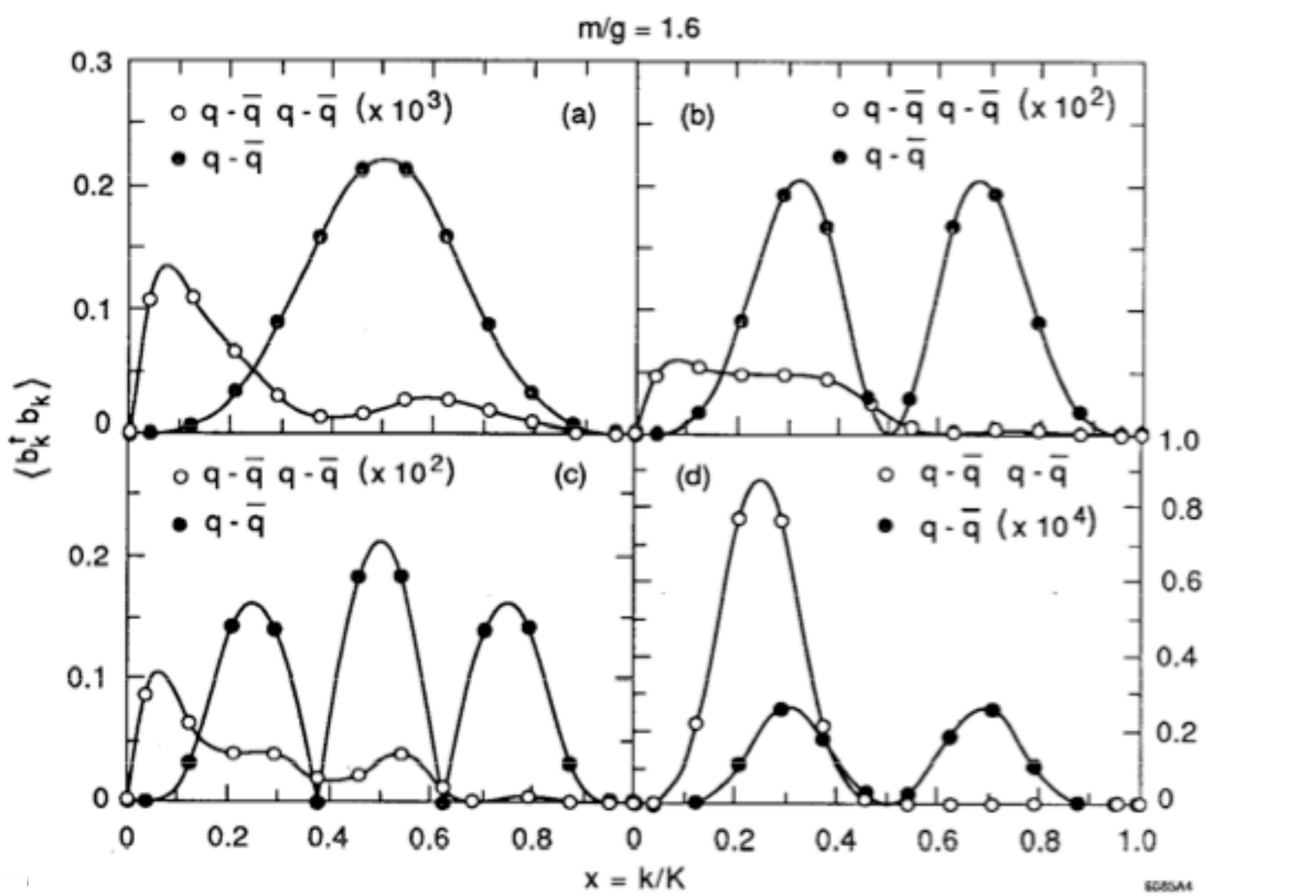


Minkowski space; frame-independent; no fermion doubling; no ghosts
trivial vacuum

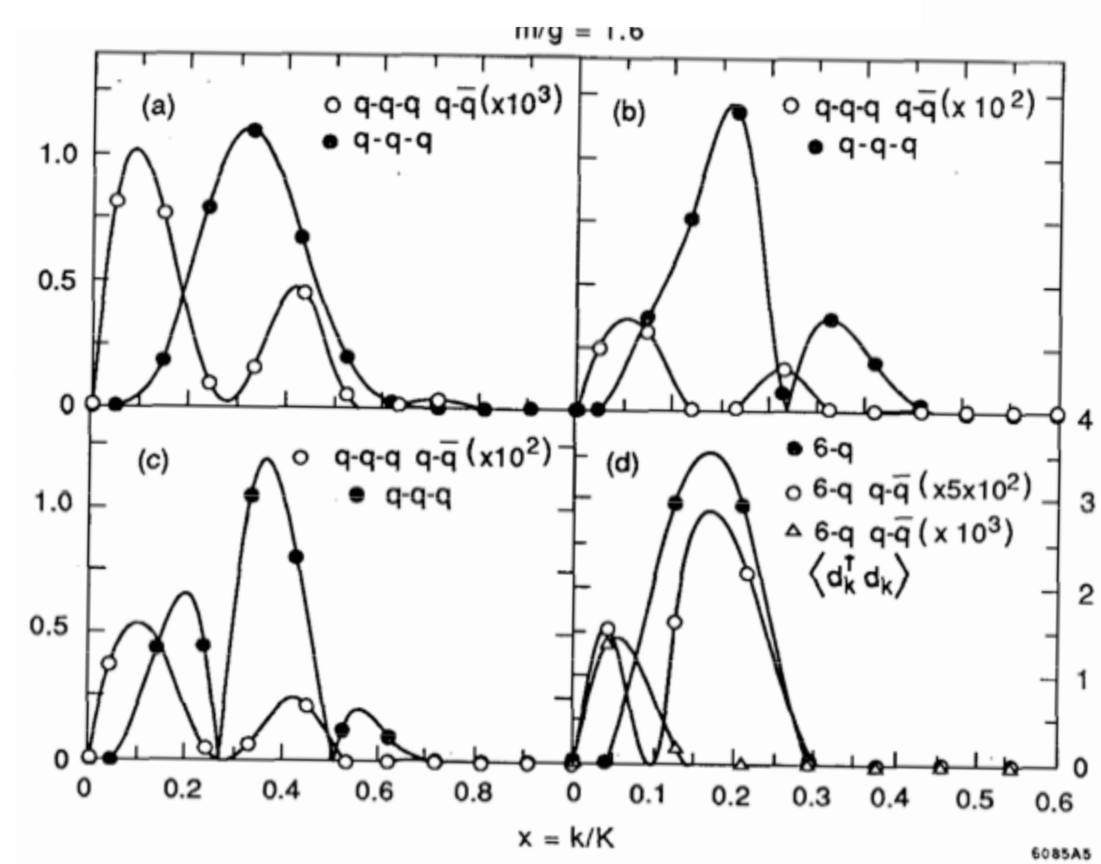
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i >$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

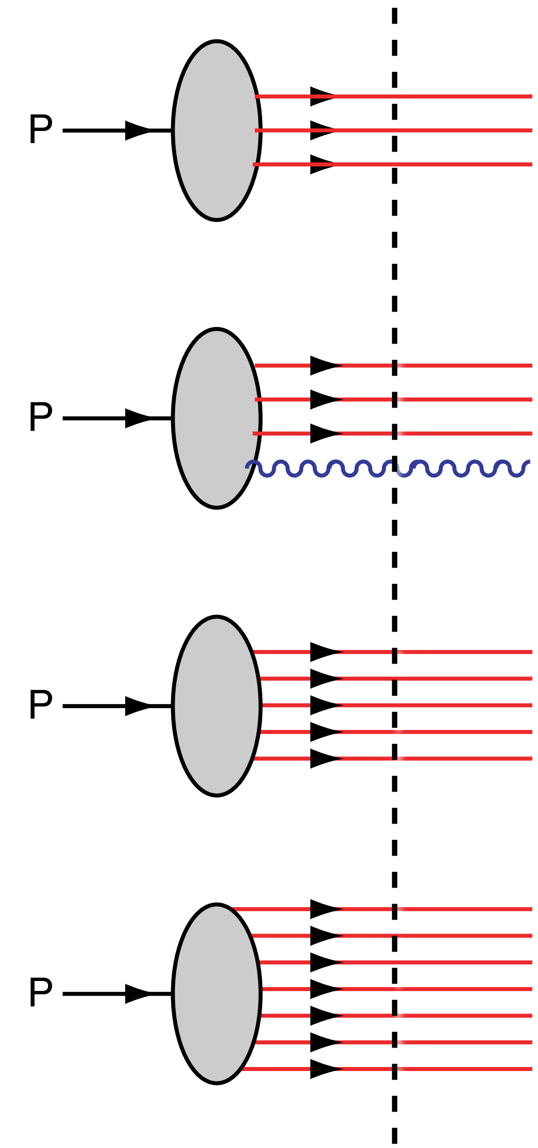
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

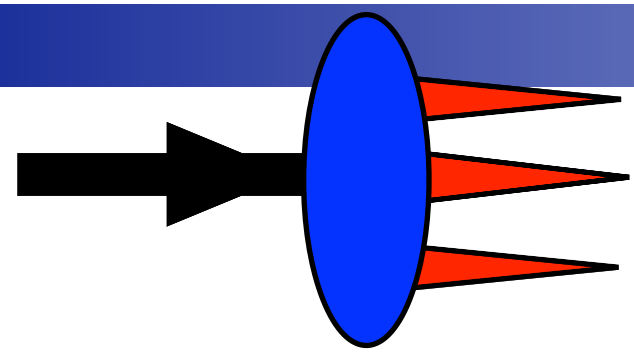
Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states **BFKL Pomeron**

Hidden Color

- *Light Front Wavefunctions:*



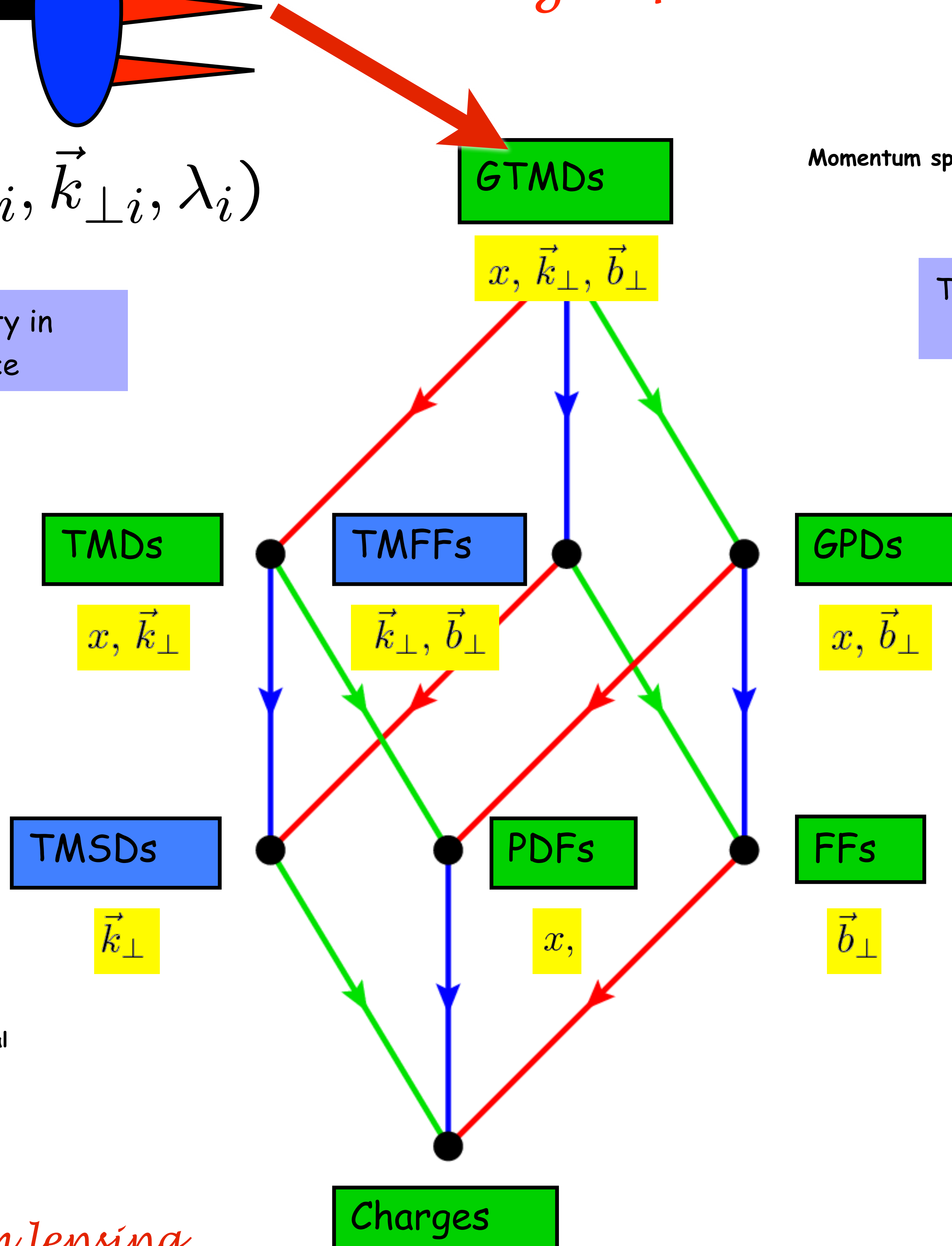
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

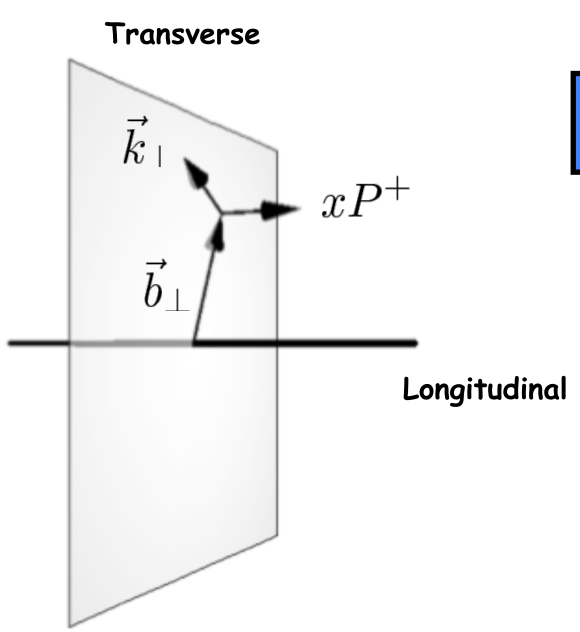
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



Lorce, Pasquini



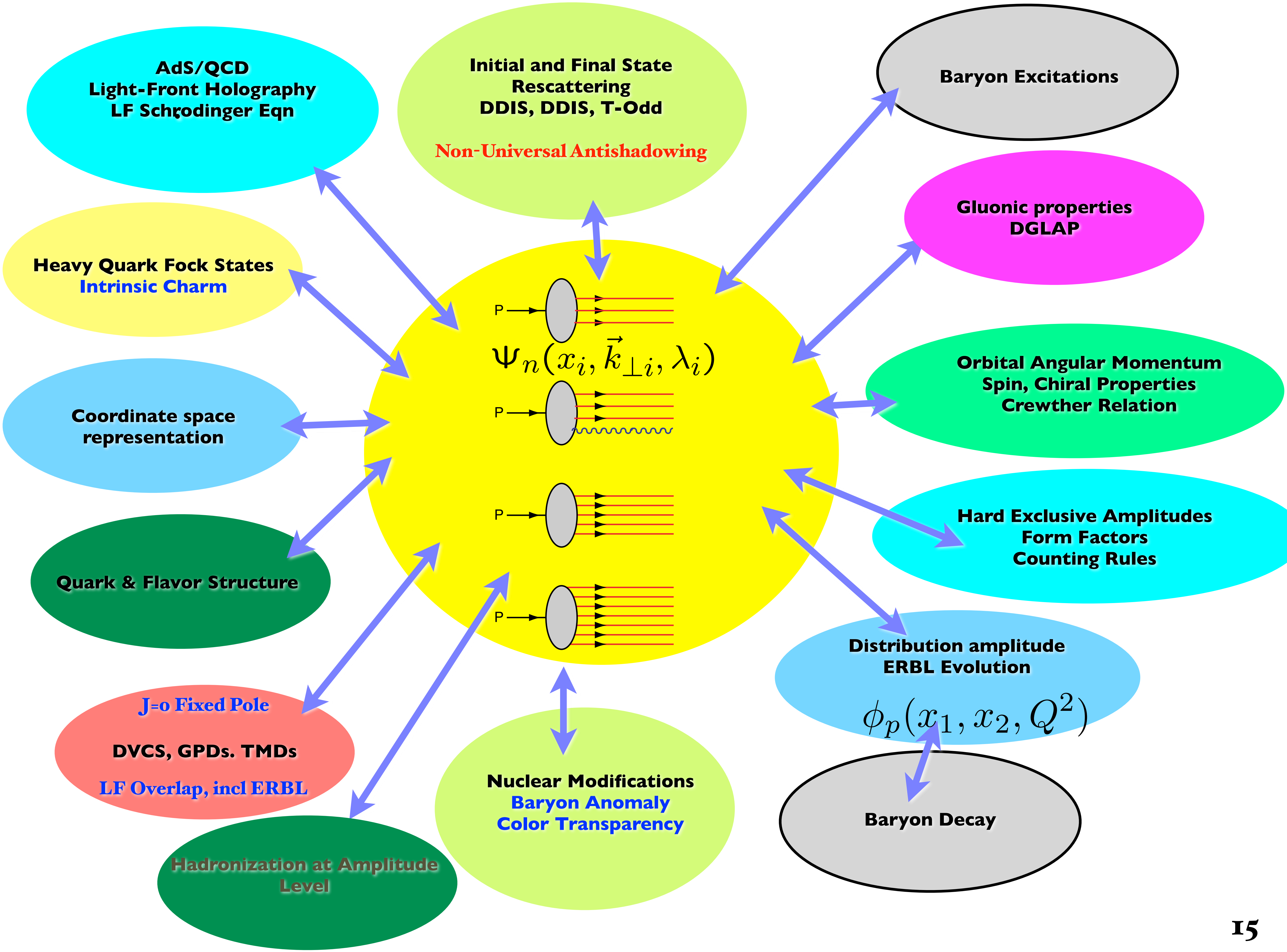
Sivers, T-odd from lensing

$\int d^2 b_\perp$

$\int dx$

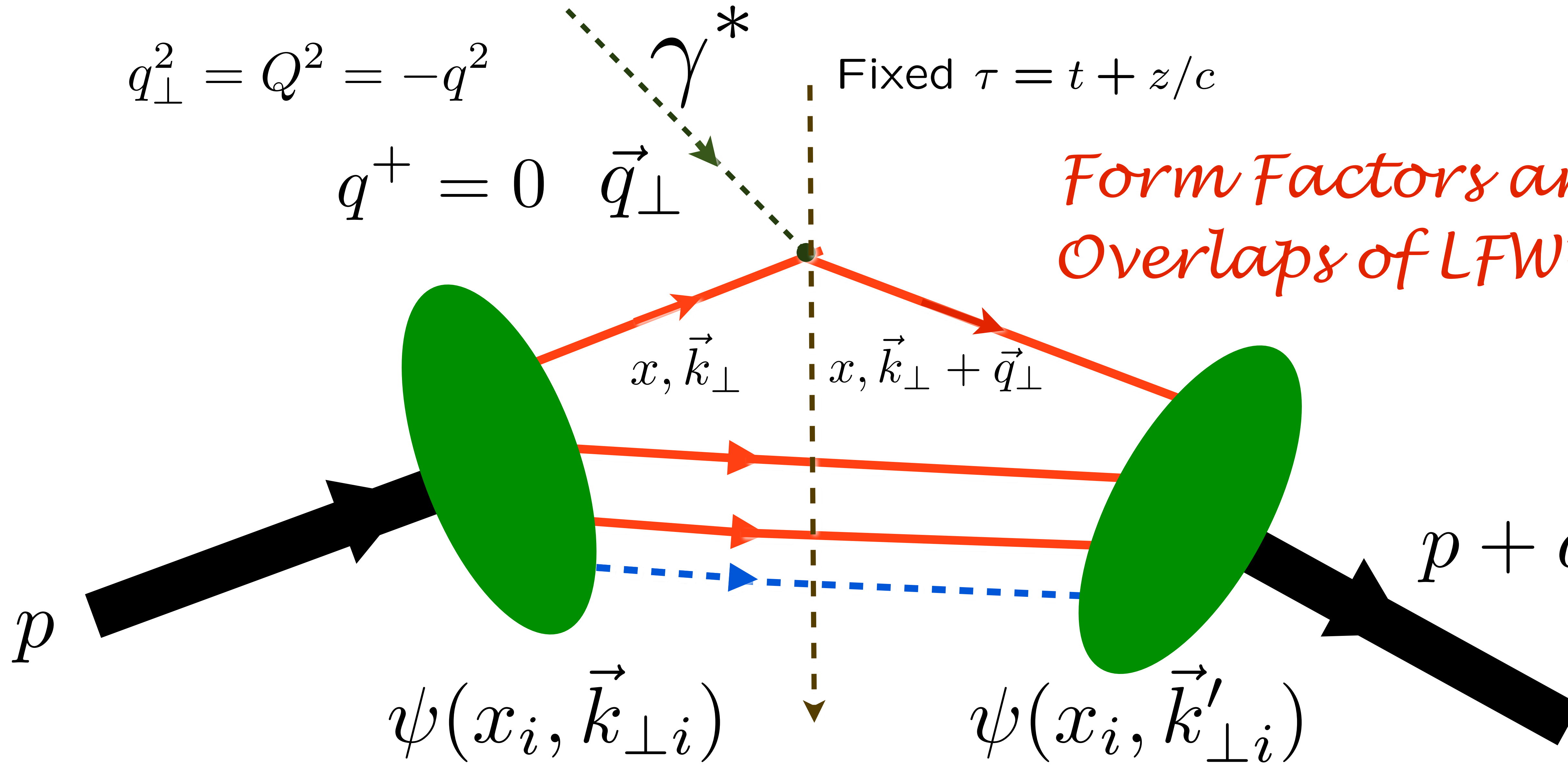
$\int d^2 k_\perp$

QCD and the LF Hadron Wavefunctions



$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

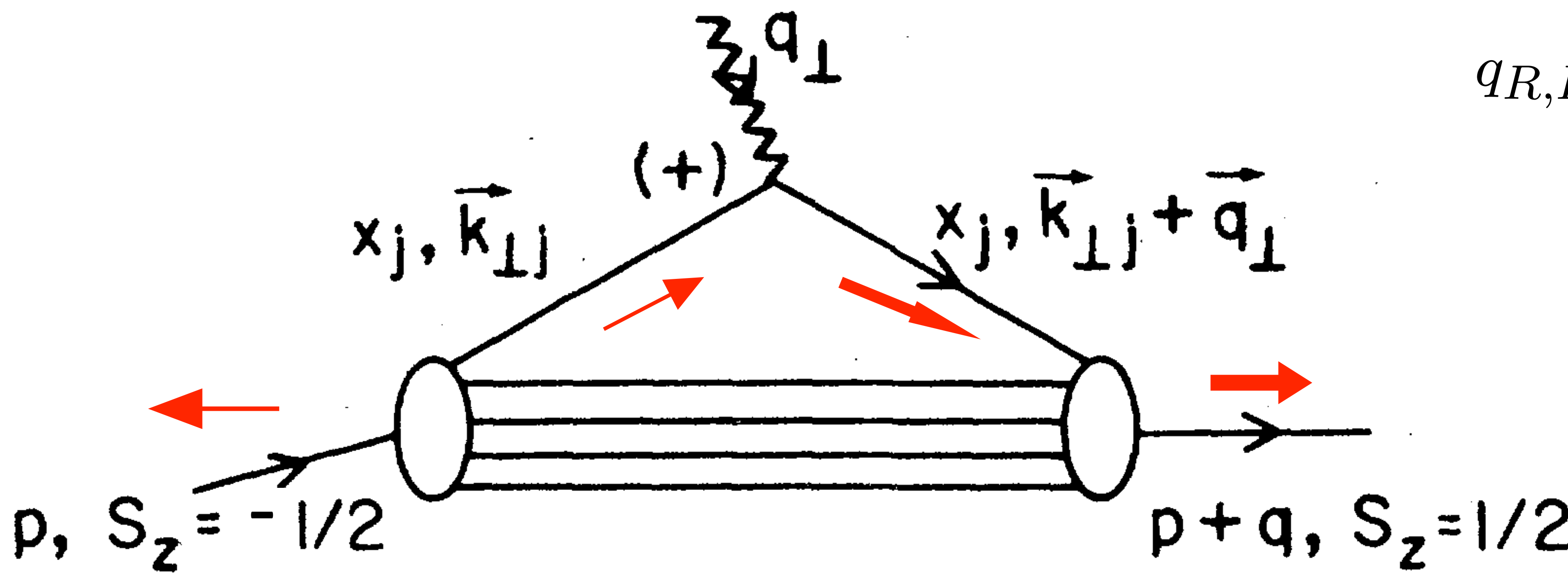
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \, \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

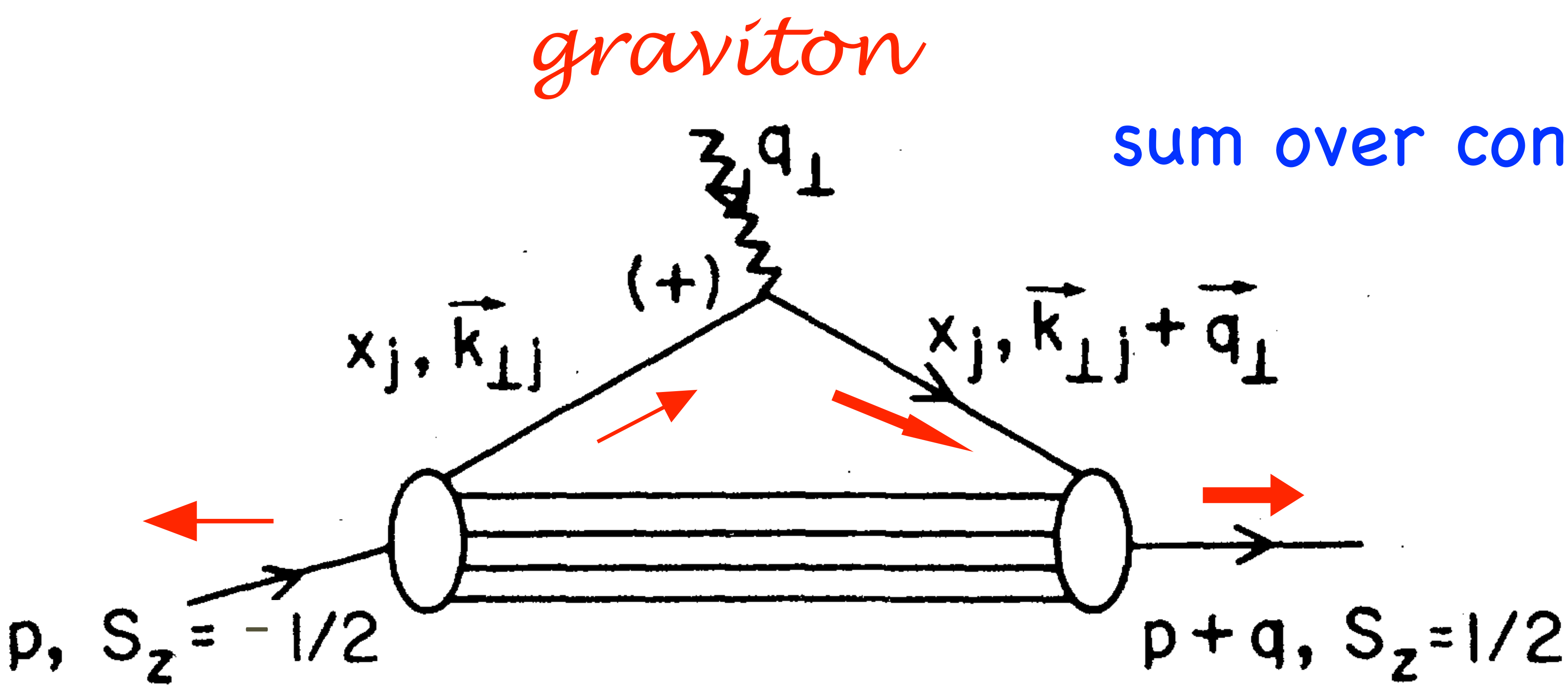


$$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*



sum over constituents

Hwang, Schmidt, sjb; Holstein et al

$\boxed{B(0) = 0}$

Each Fock State

Single-spin asymmetries

Leading Twist Sivers Effect

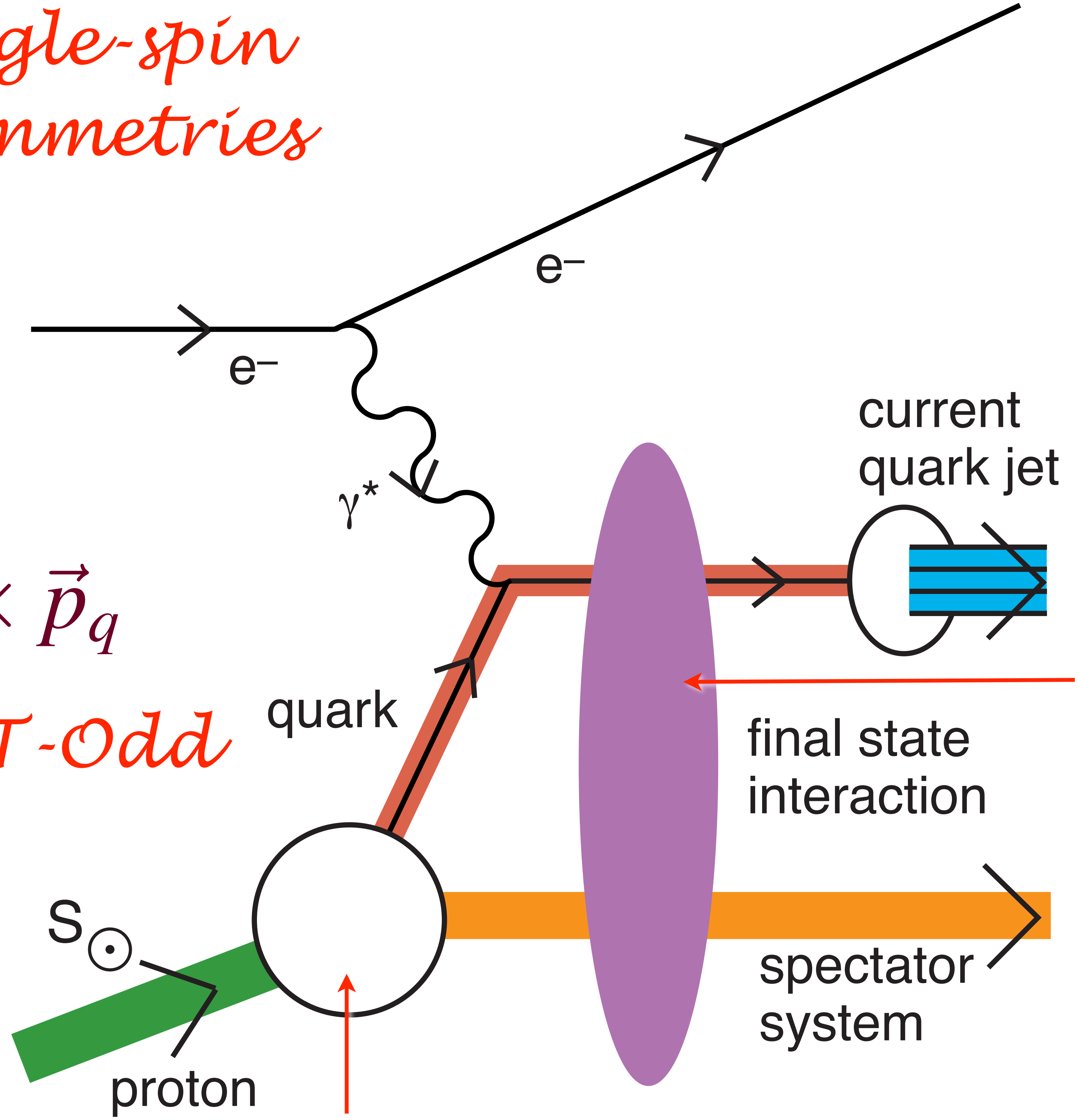
Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo- T-Odd

"Lensing" involves soft scales



Light-Front Wavefunction S and P- Waves!

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

Sign reversal in DY!

Leading-Twist Rescattering Violates pQCD Factorization!

Static	Dynamic
• Square of Target LFWFs	Modified by Rescattering: ISI & FSI
• No Wilson Line	Contains Wilson Line, Phases
• Probability Distributions	No Probabilistic Interpretation
• Process-Independent	Process-Dependent - From Collision
• T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
• No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
• Sum Rules: Momentum and J^z	Sum Rules Not Proven
• DGLAP Evolution; mod. at large x	DGLAP Evolution
• No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

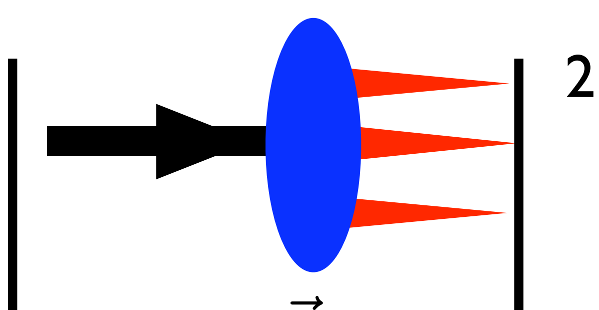
Hwang, Schmidt, sjb,

Mulders, Boer

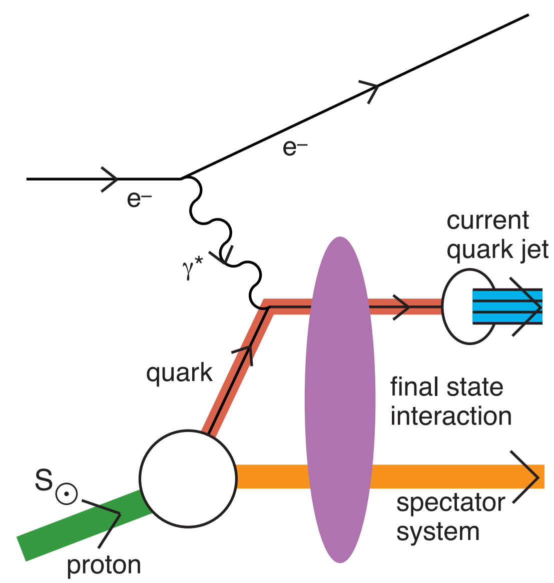
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Atomic Physics from First Principles

$\mathcal{L}_{QED} \rightarrow$

$$\boxed{H_{QED}}$$

QED atoms: positronium and muonium

$\downarrow$

$$(H_0 + H_{int}) \; |\Psi >= E \; |\Psi >$$

Coupled Fock states

$\downarrow$

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

$\downarrow$

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

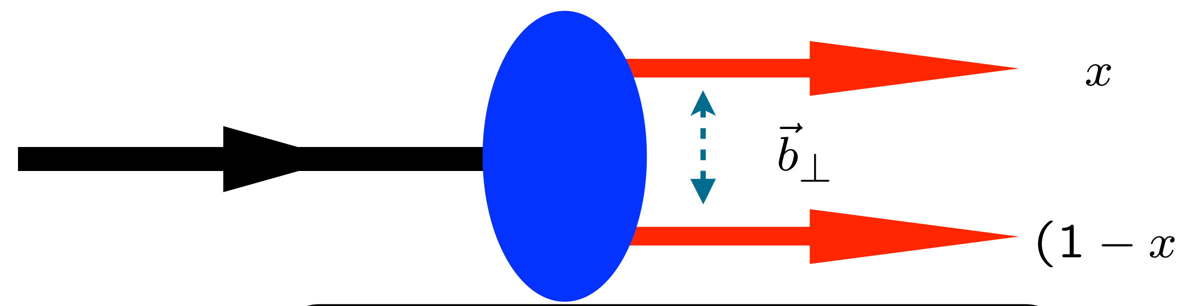
$$\boxed{V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}}$$

Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

Coupled Fock states

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Eliminate higher Fock states and retarded interactions

Effective two-particle equation

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Azimuthal Basis

ζ, ϕ

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Light-Front Schrödinger Equation

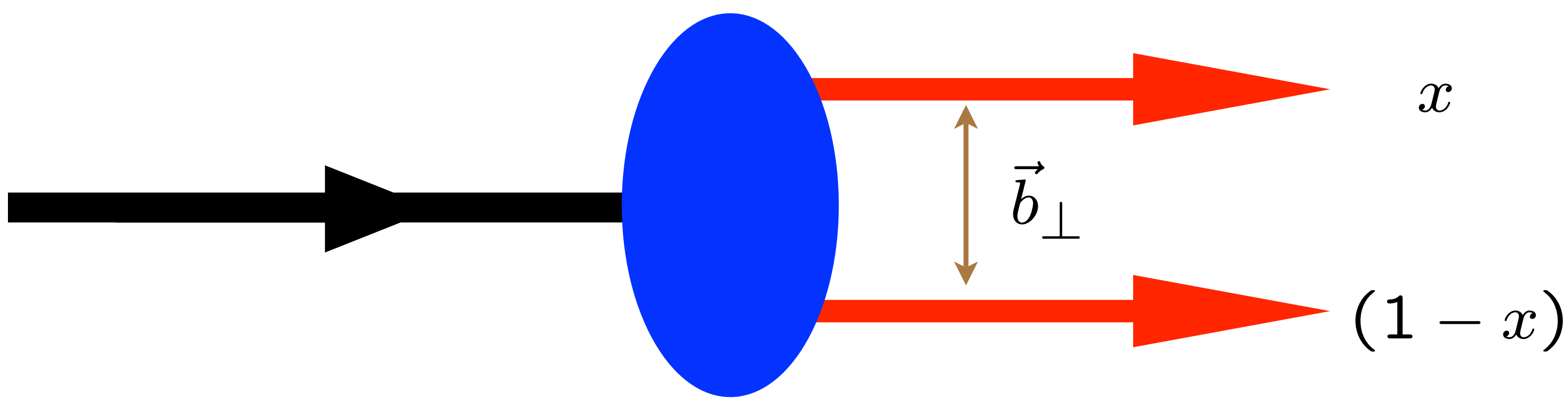
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

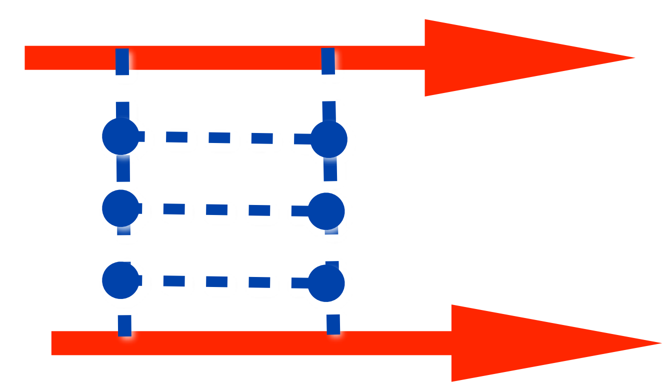
Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$



U is the confining QCD potential
Conjecture: 'H'-diagrams generate

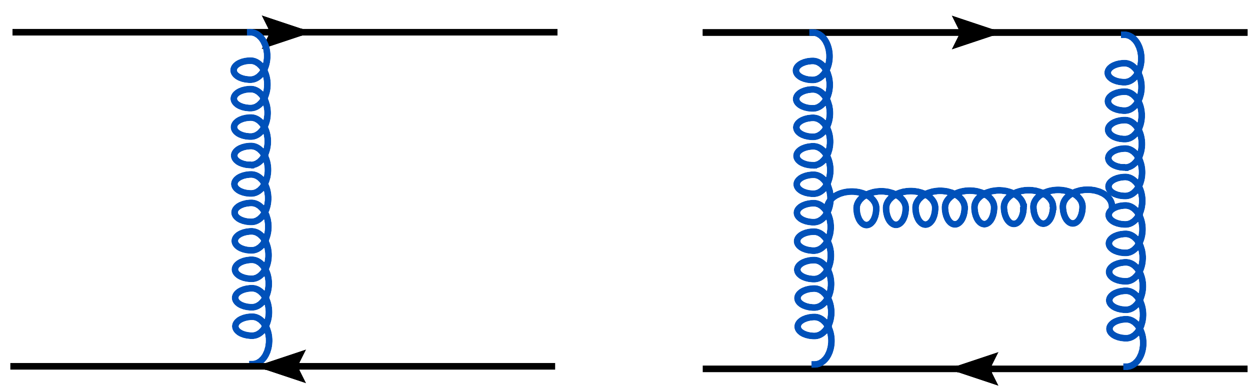


$$\boxed{U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)}$$

Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$
$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + \boxed{8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3} \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

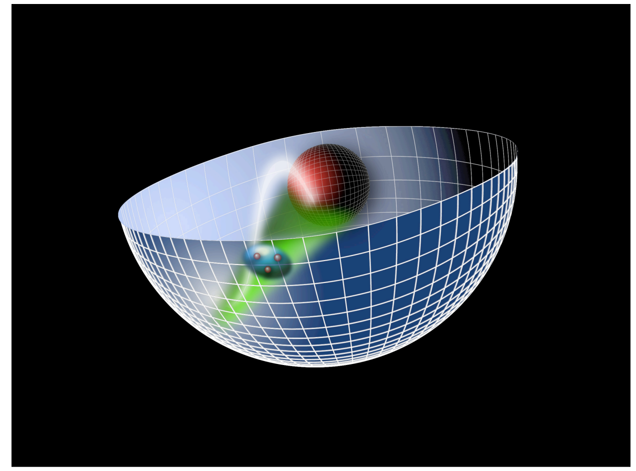
Summation of H graphs: confining potential

Confinement eliminates IR divergences

Self-consistent mass scale κ

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

$$\boxed{\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)}$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\boxed{\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)}$$

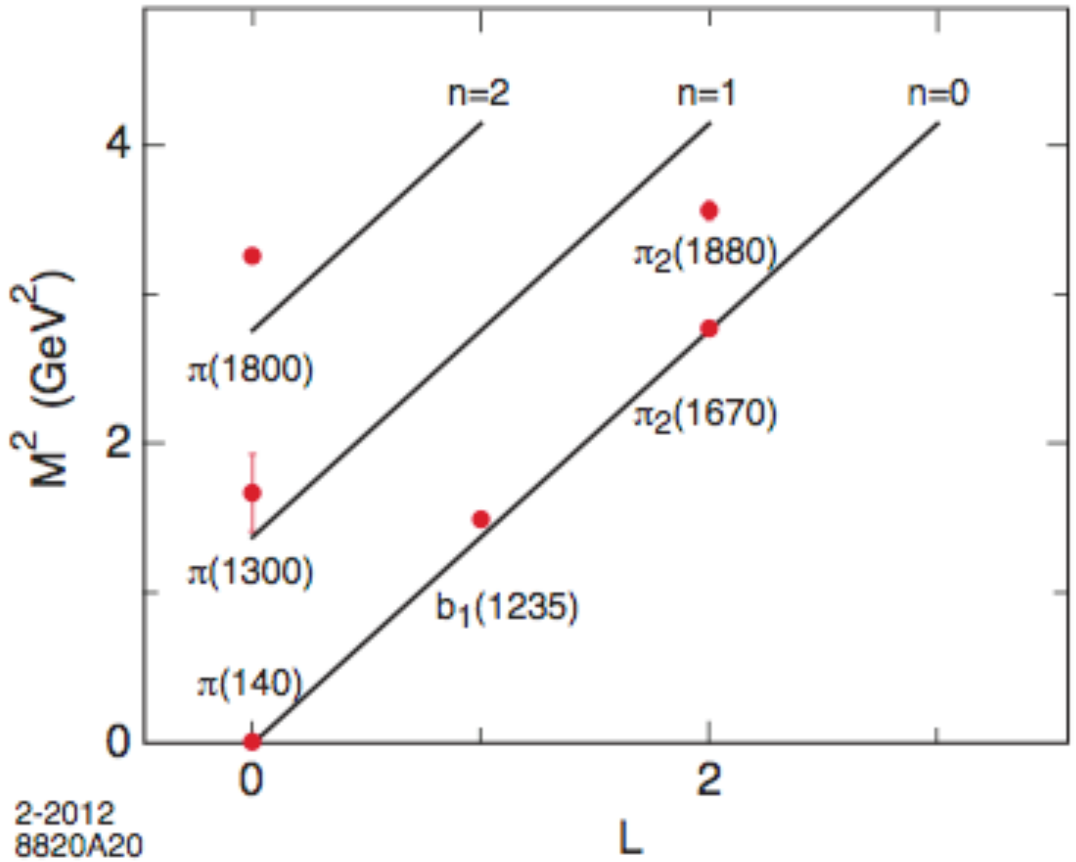
G. de Teramond, H. G. Dosch, sjb

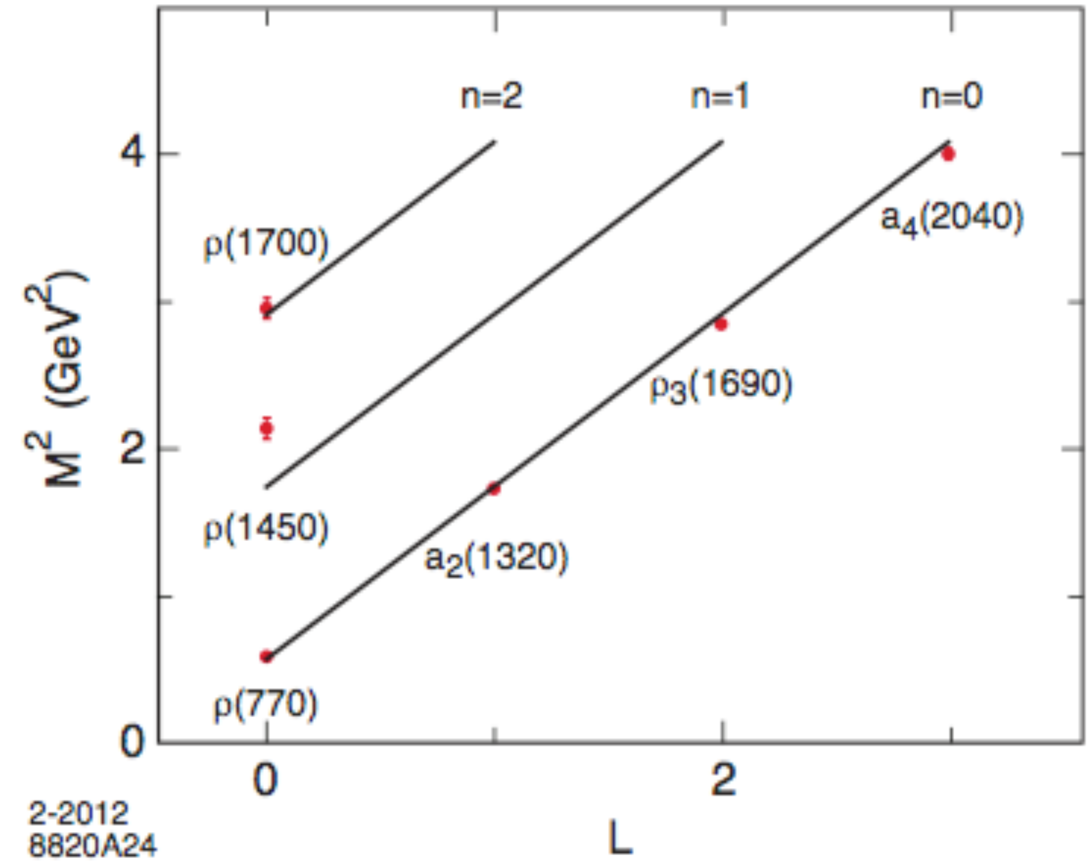
- $J = L + S, I = 1$ meson families $\boxed{\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)}$

$\boxed{m_q = 0}$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Massless pion in Chiral Limit! ***Same slope in n and L!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

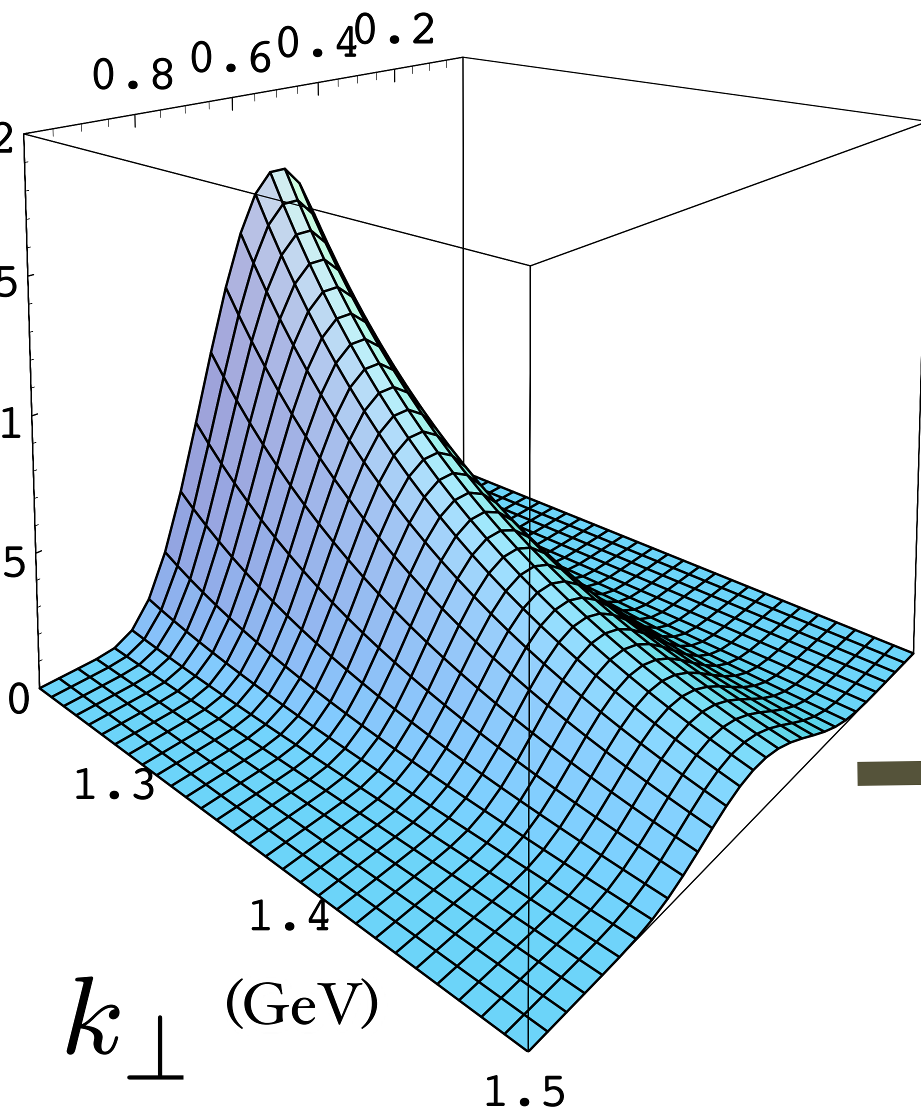
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

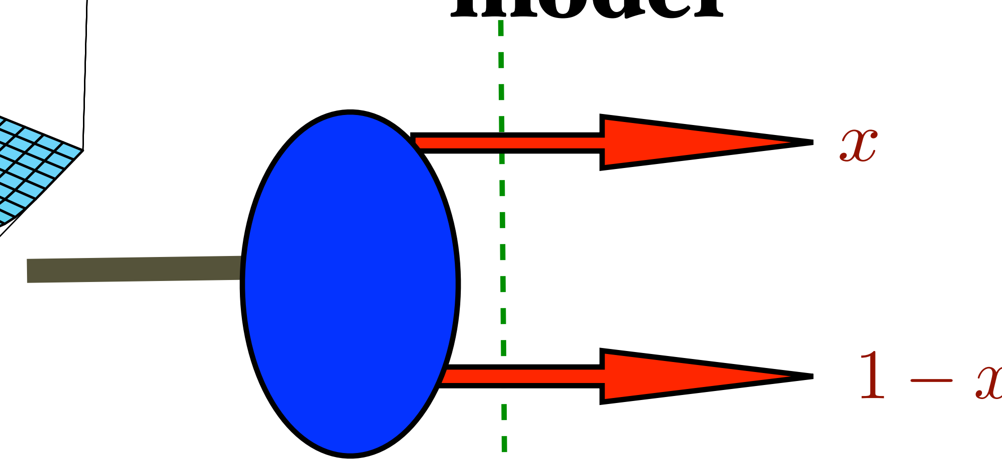
Prediction from AdS/QCD: Meson LFWF

$\psi_M(x, k_\perp^2)$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle \psi | \sum_a m_a^2 / x_a | \psi \rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x, \zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x} + \frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda \zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M_{n,L,S}^2 = M_{K^\pm}^2 + 4\lambda \left(n + \frac{J+L}{2}\right)$$

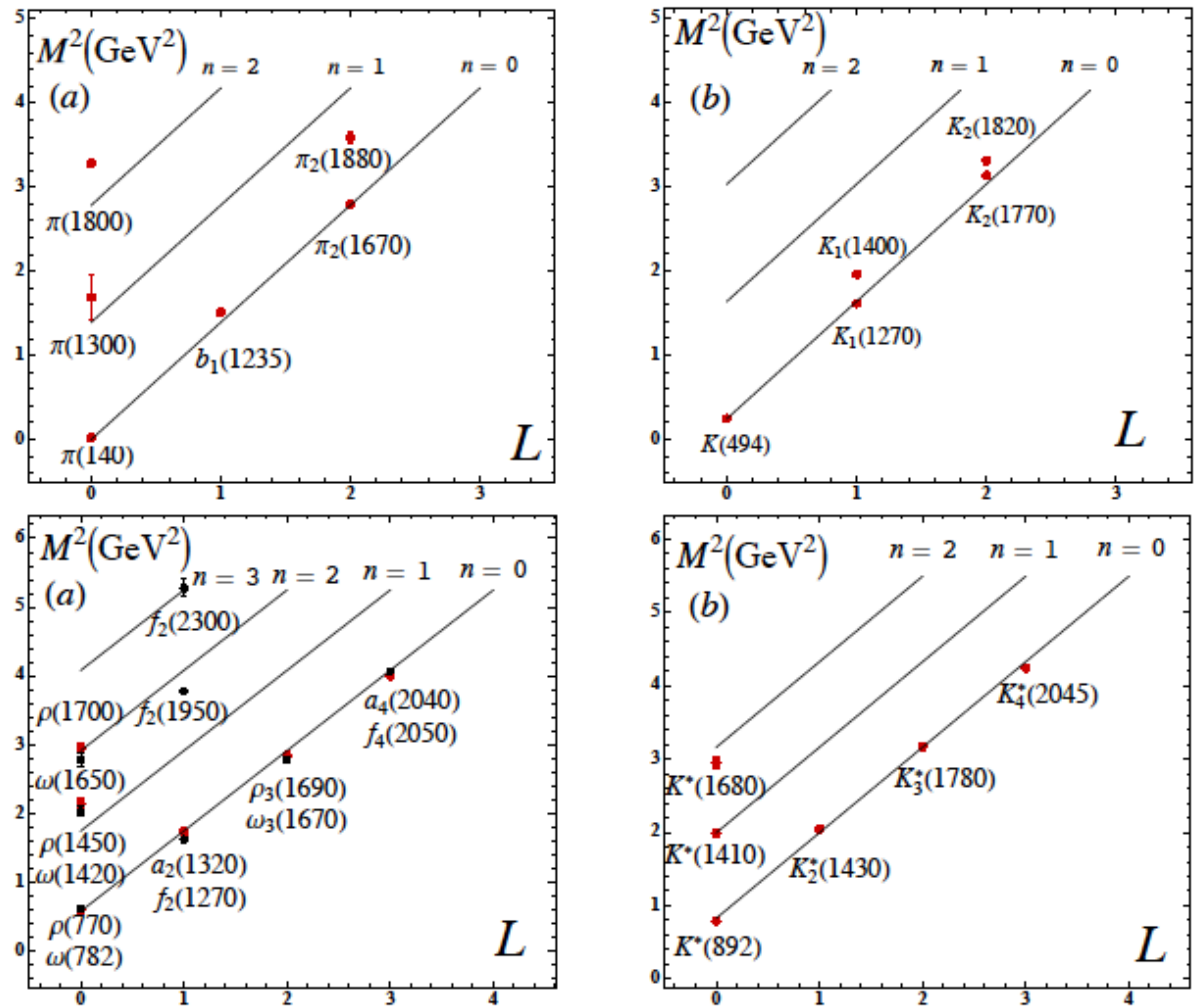
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

De Teramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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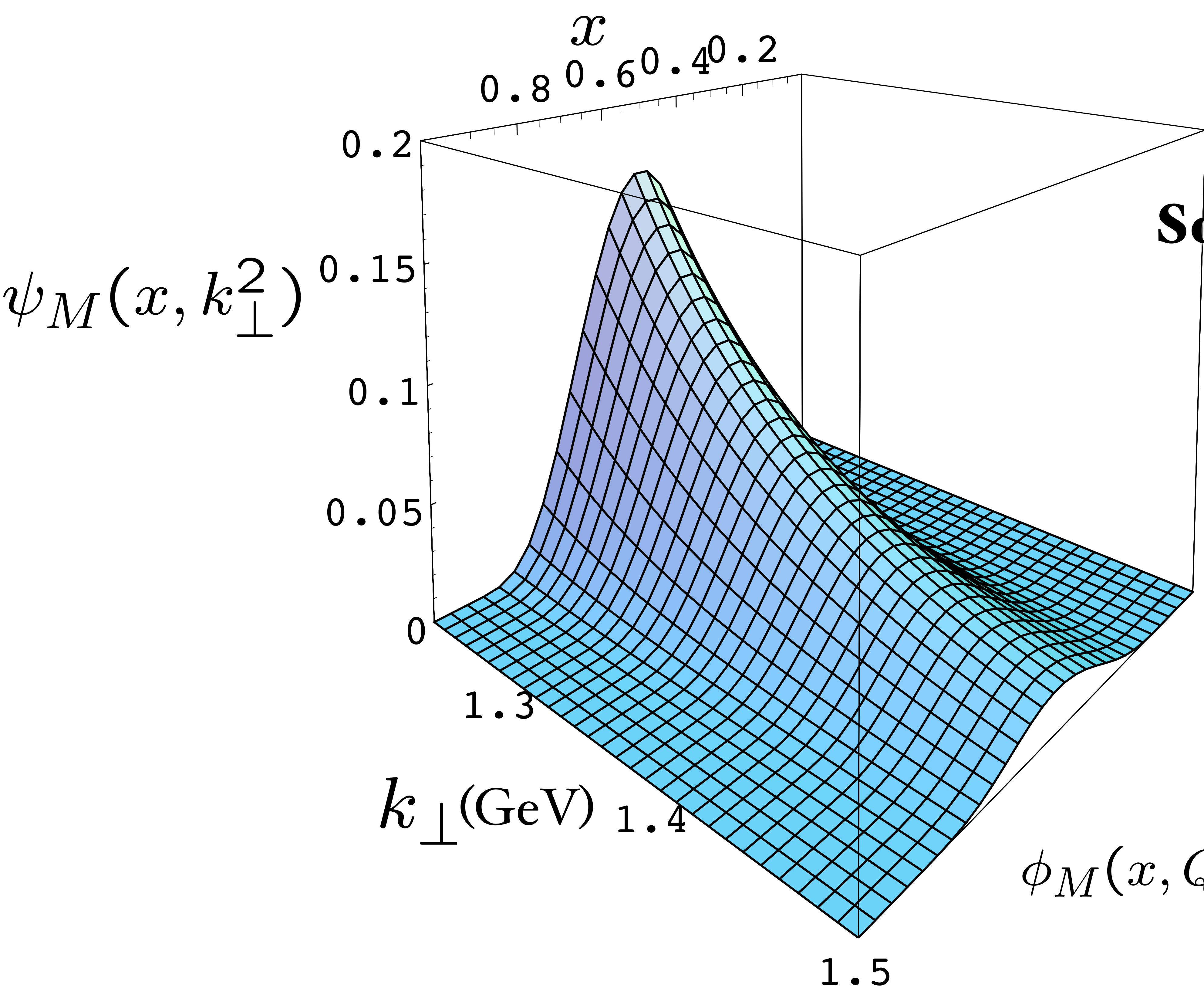
(Received 5 April 2012; published 20 August 2012)

We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Prediction from AdS/CFT: Meson LFWF



Soft Wall Model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

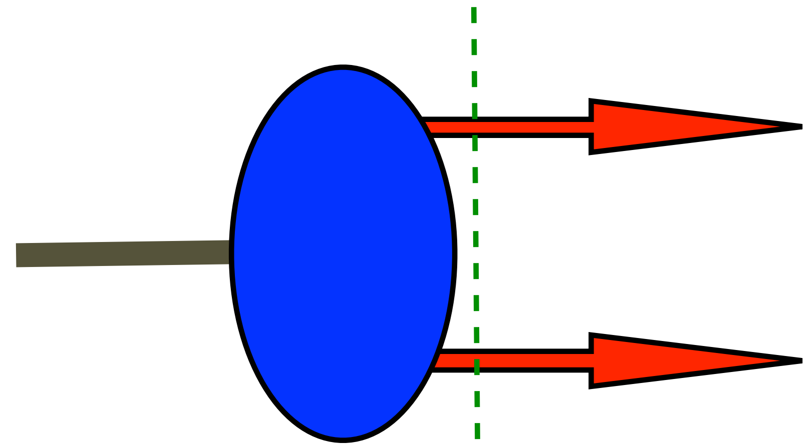
Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

See: Roberts et al.

Hadron Distribution Amplitudes

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

$$\sum_i x_i = 1$$



$k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

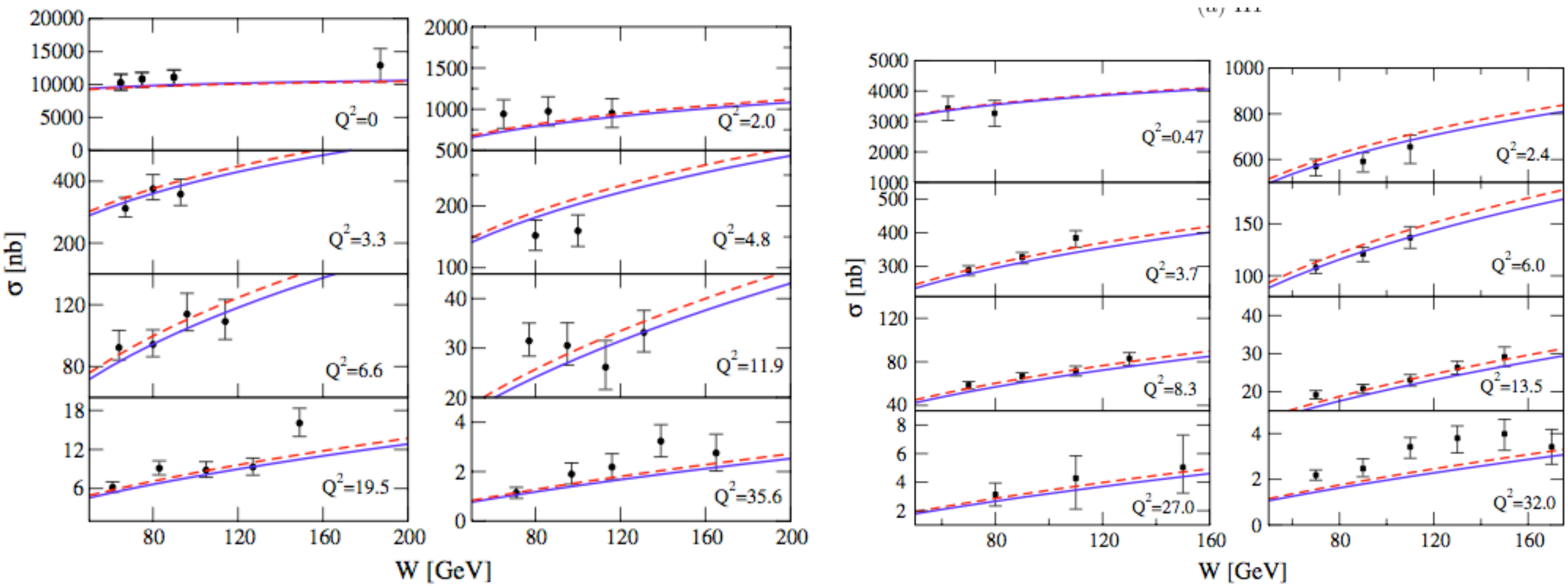
Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

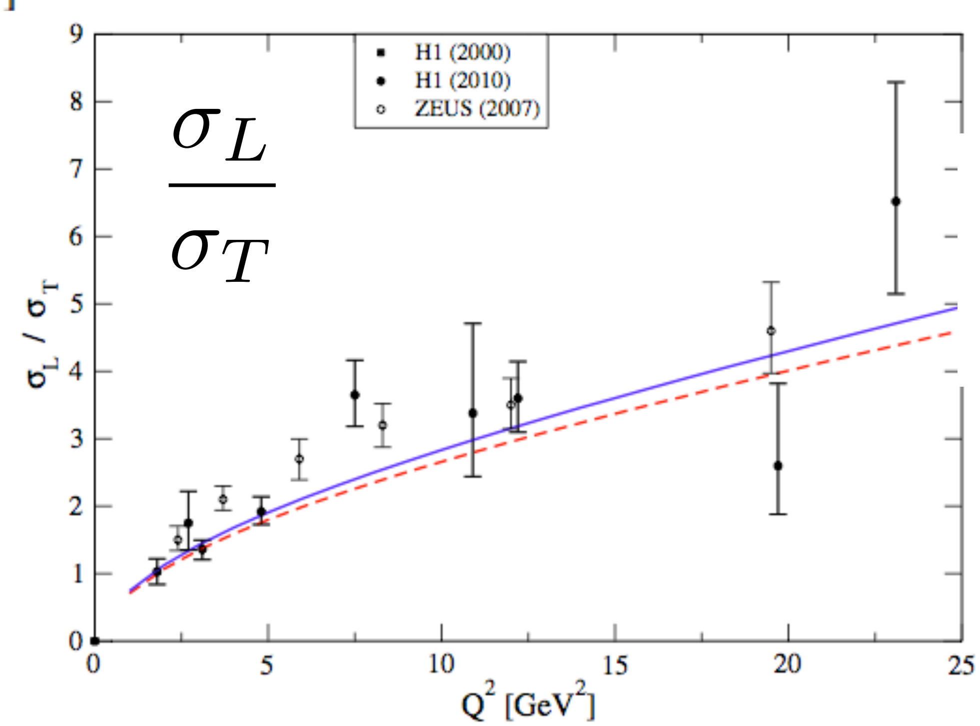


(a) H

(b) ZEUS

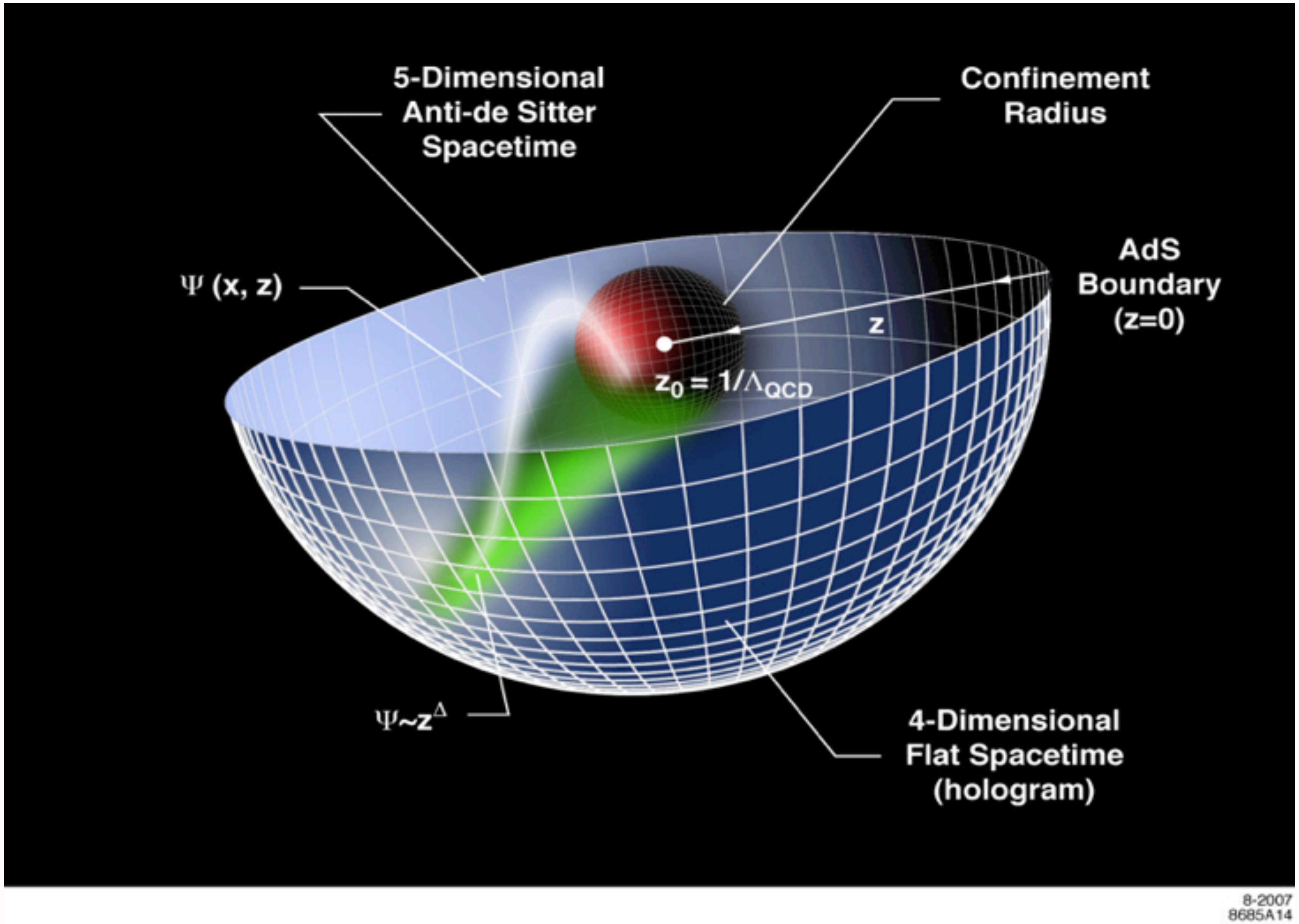
J. R. Forshaw,
R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\frac{\sigma_L}{\sigma_T}$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

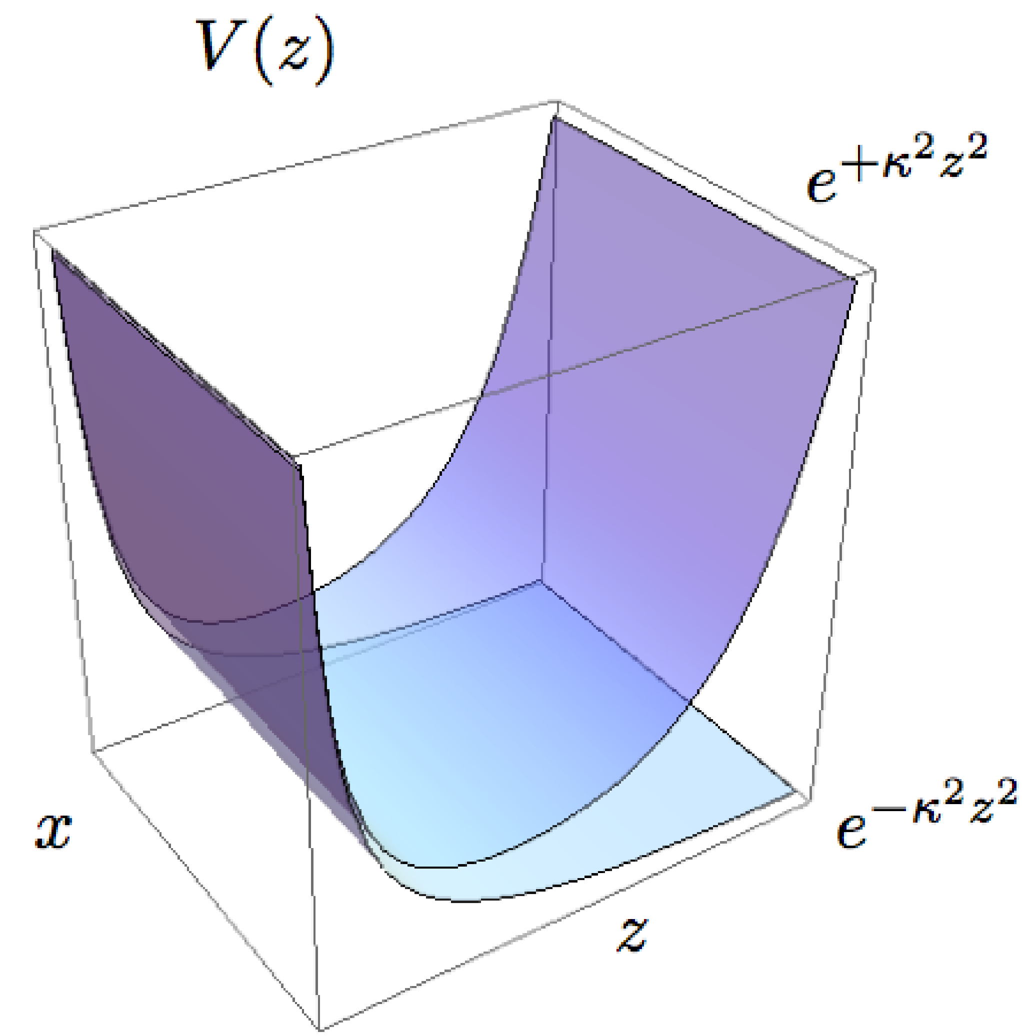
$$ds^2 = \frac{R^2}{z^2} \boxed{e^{\varphi(z)}} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z}}$$

Positive-sign dilaton

- de Teramond, sjb

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Quark separation increases with L

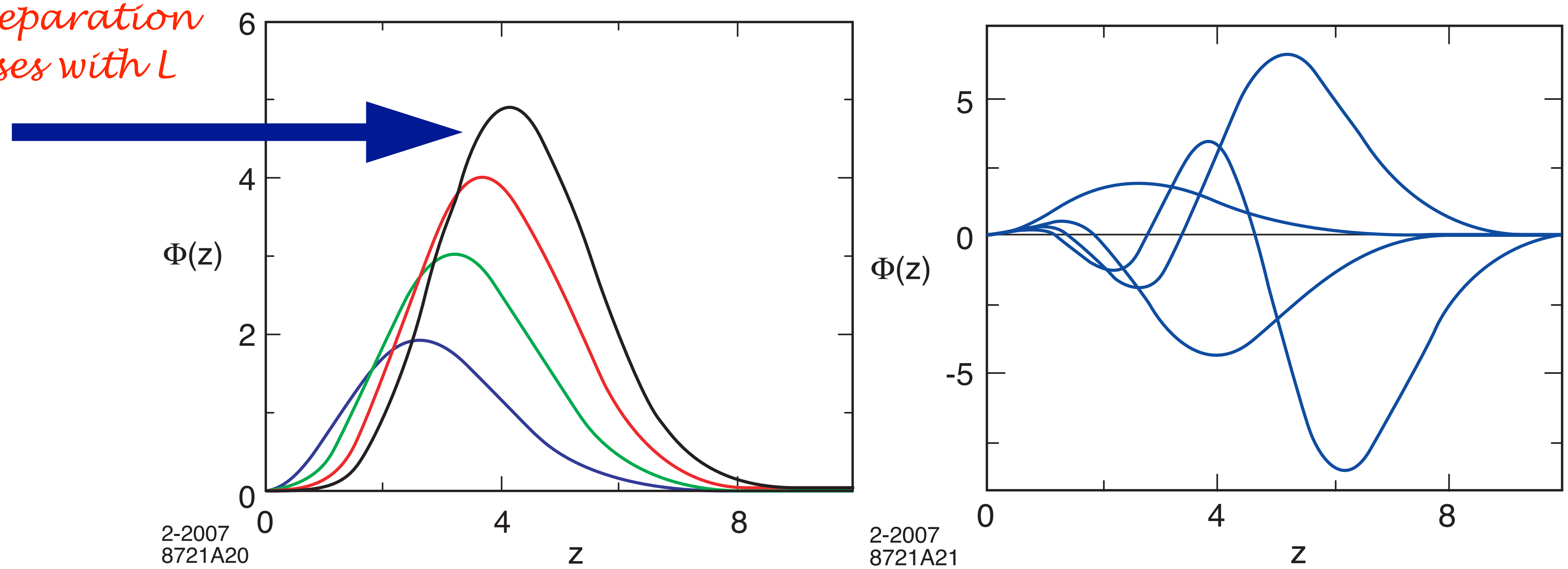
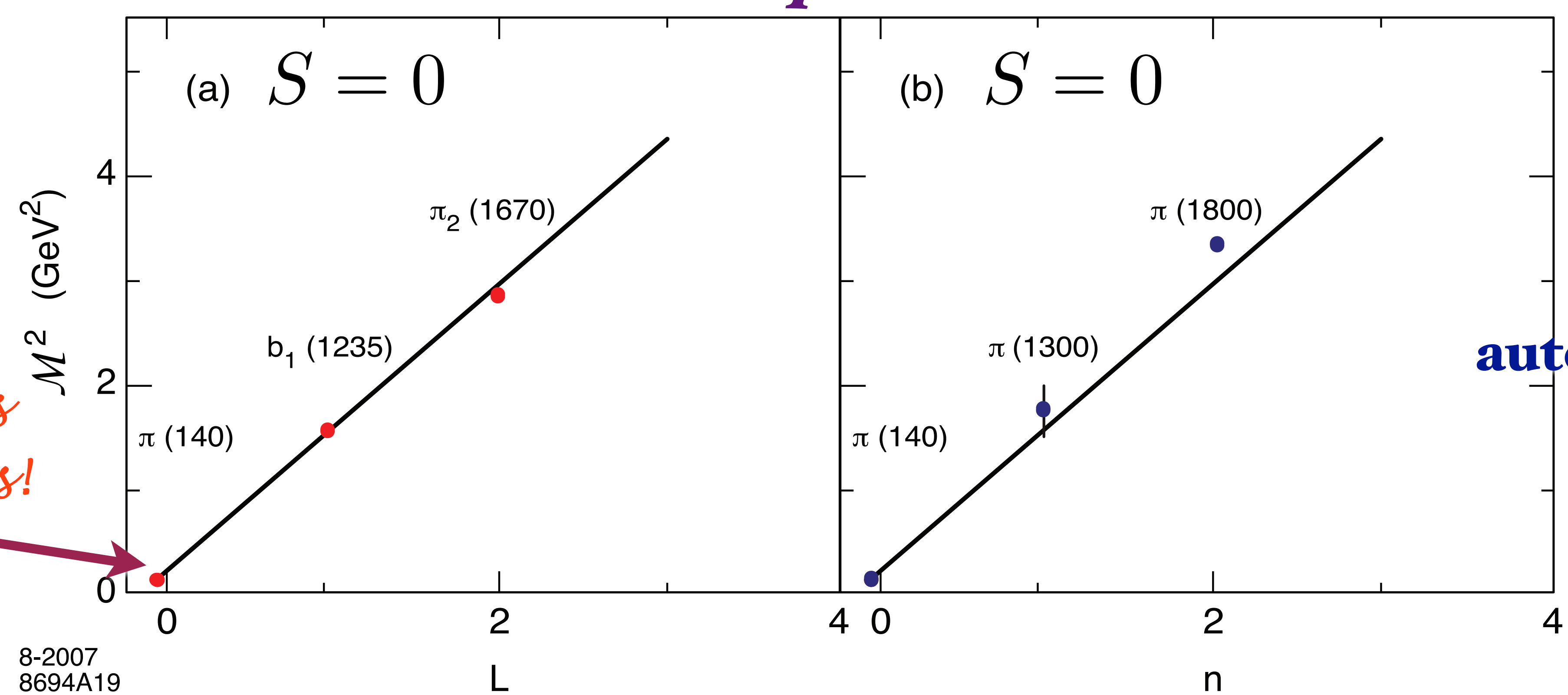


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!

Pion has zero mass!



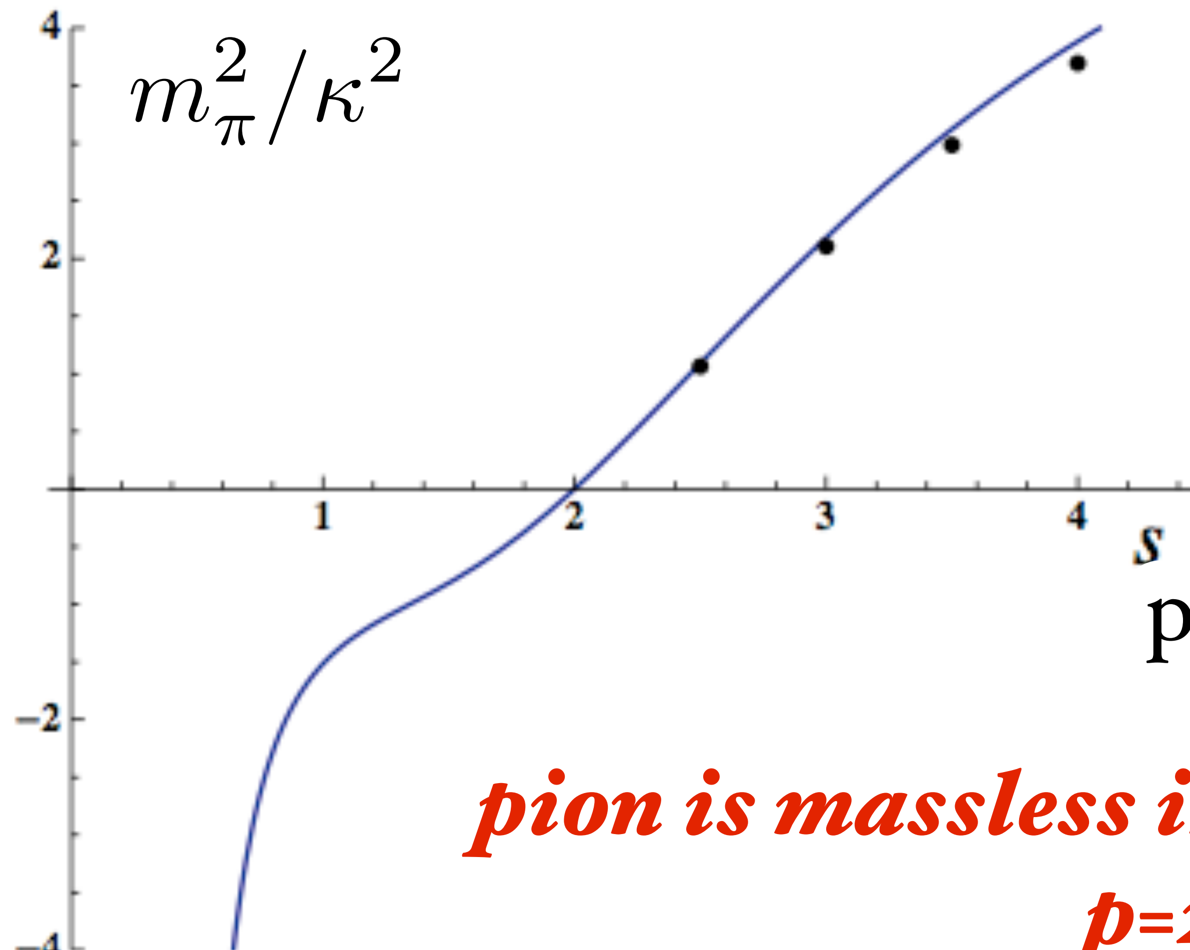
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

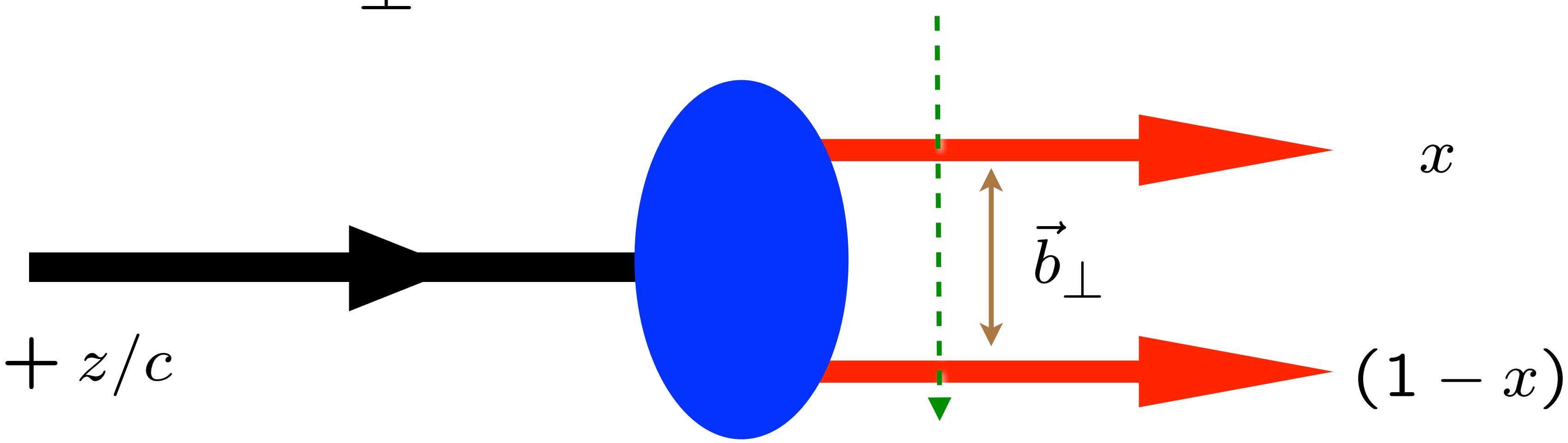
- **Dosch, de Teramond, sjb**

de Teramond, sjb

$LF(3+1) \qquad \longleftrightarrow \qquad AdS_5$

$$\psi(x, \vec{b}_\perp) \qquad \longleftrightarrow \qquad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \qquad \longleftrightarrow \qquad z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Teramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$

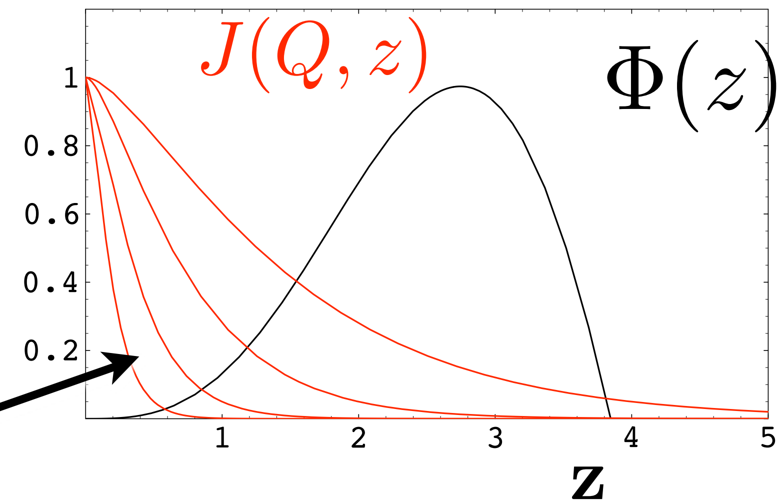
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



high Q^2

Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

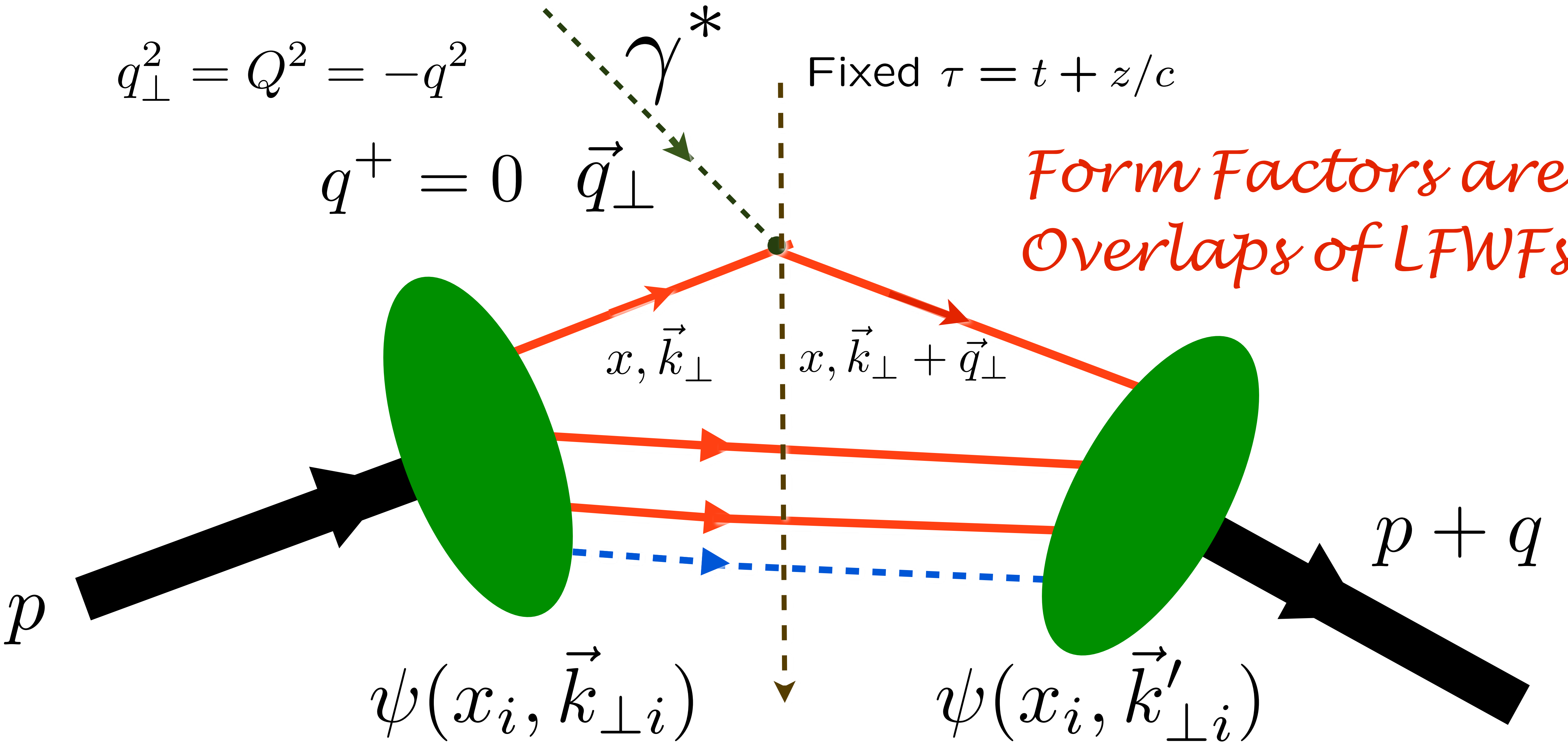
Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$.

$$\text{Twist } \tau = n + L$$

$$< p + q | j^+(0) | p > = 2p^+ F(q^2)$$

Interaction picture



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Soper: DYW: Product of LFWFs in transverse space

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

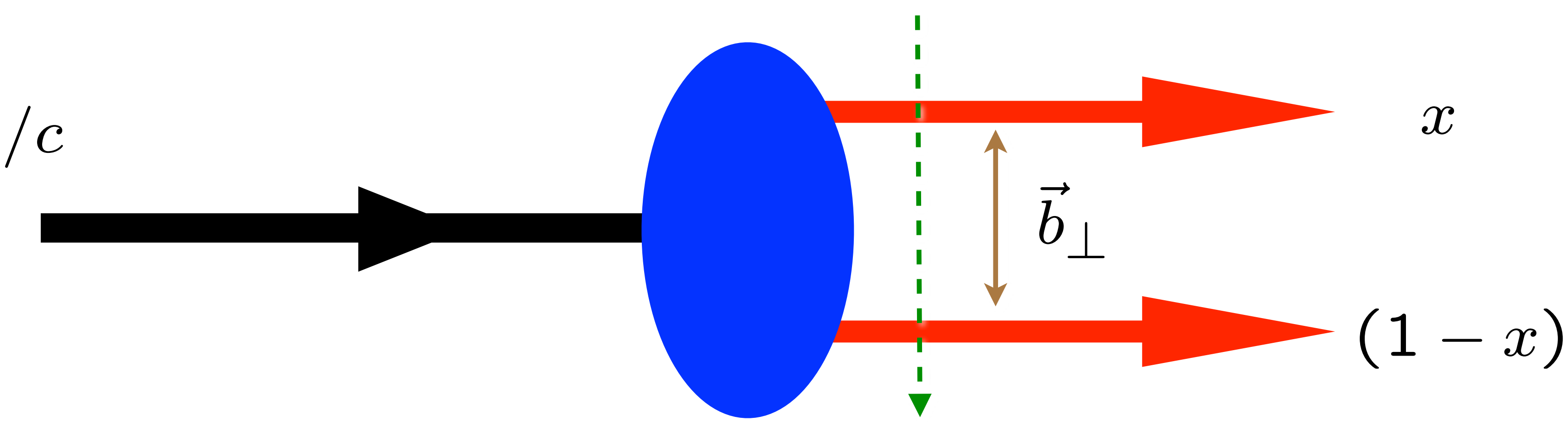
Identical to Polchinski-Strassler Convolution of AdS Amplitudes de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

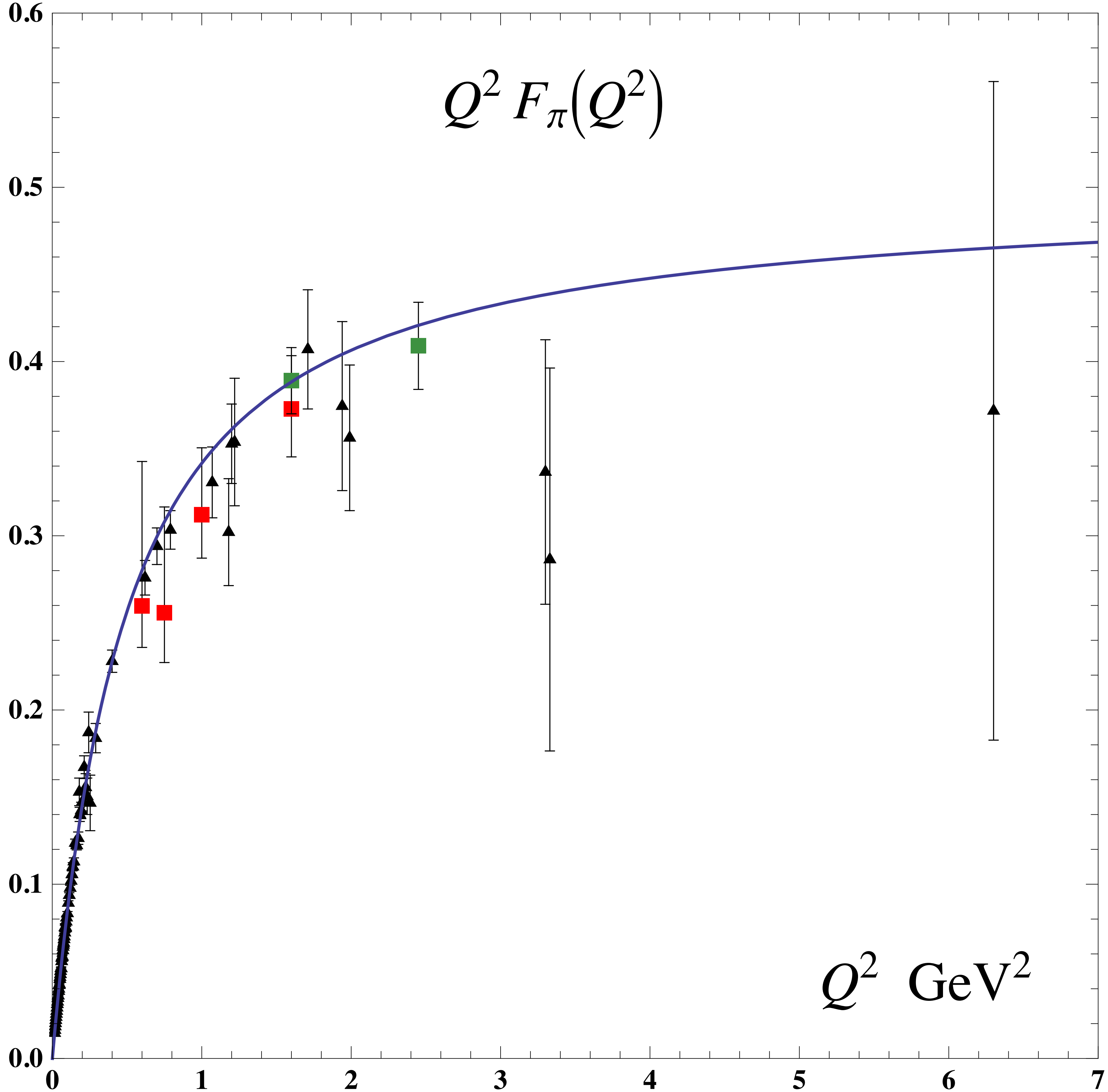
$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

Fixed $\tau = t + z/c$



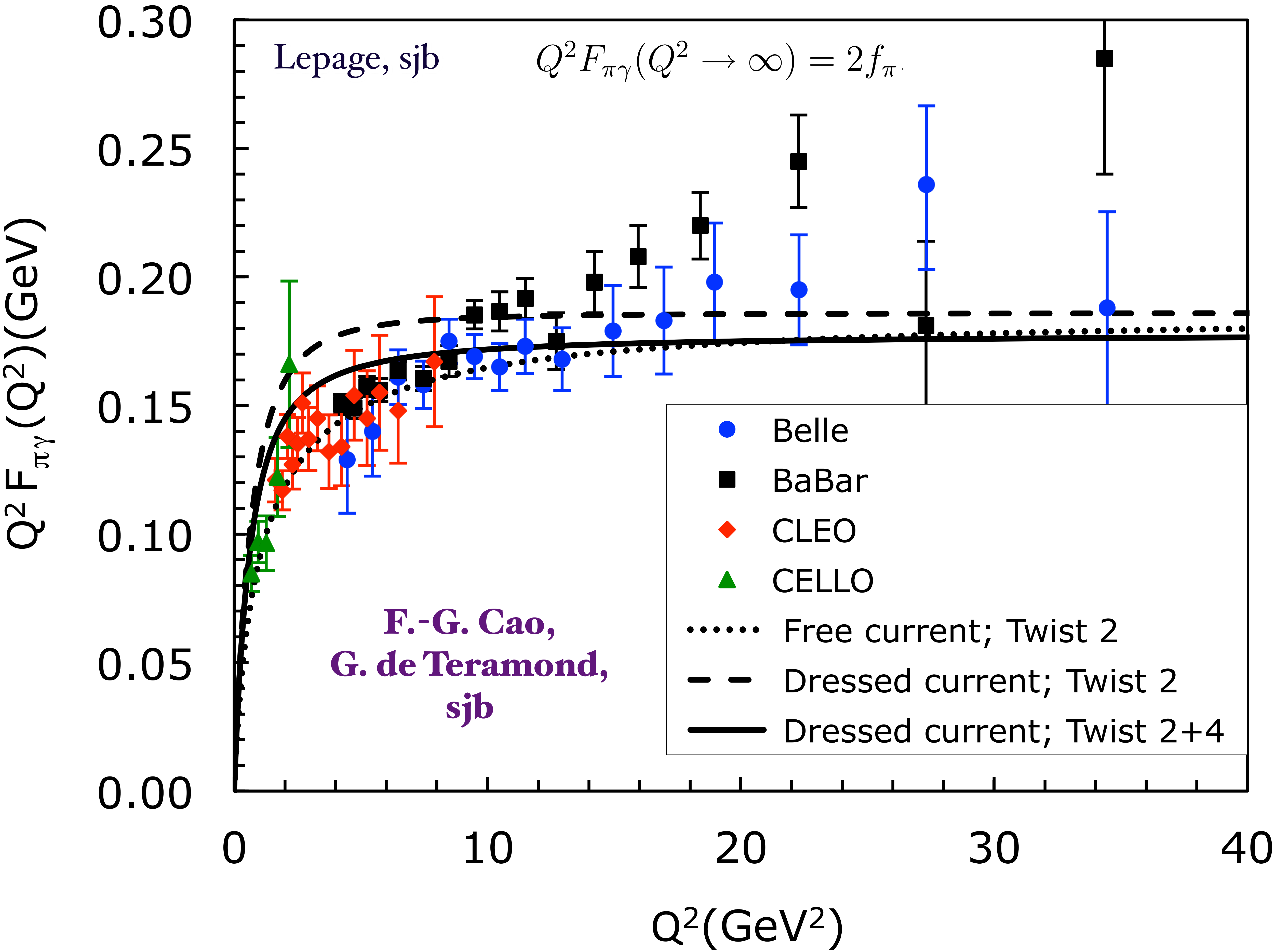
$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Photon-to-pion transition form factor



Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

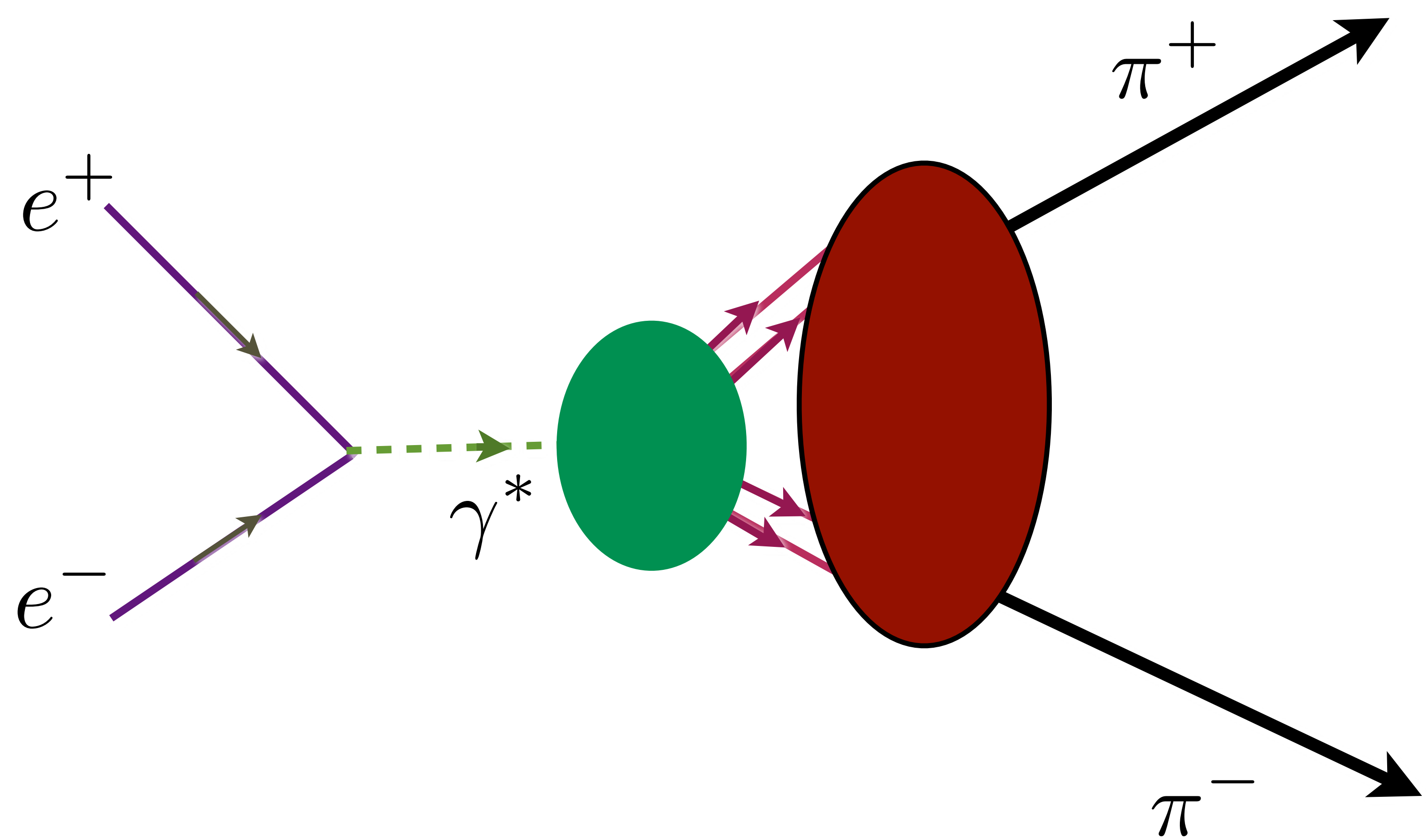
$$F(Q^2) = R^3 \int \frac{dz}{z^3} \, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

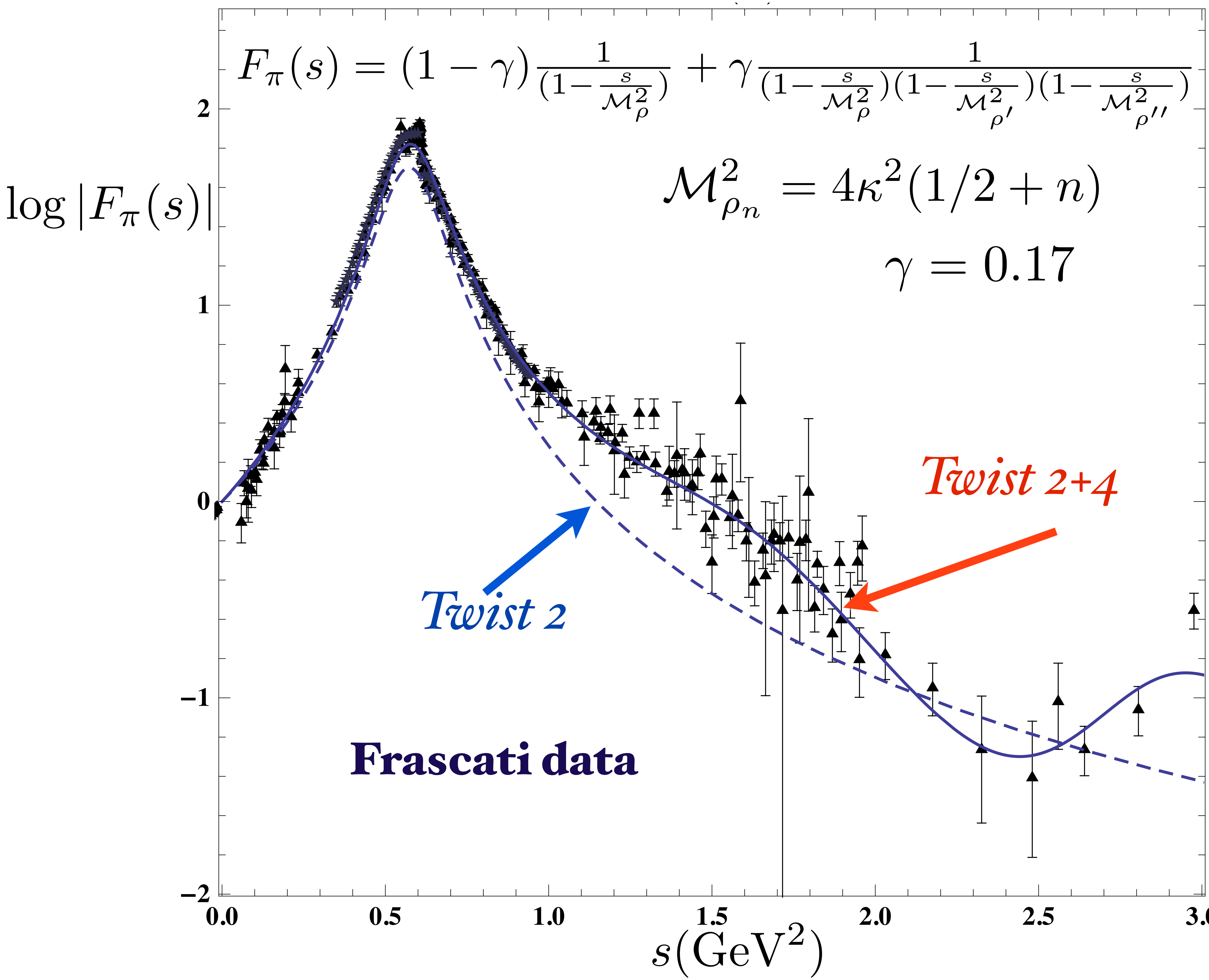
$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



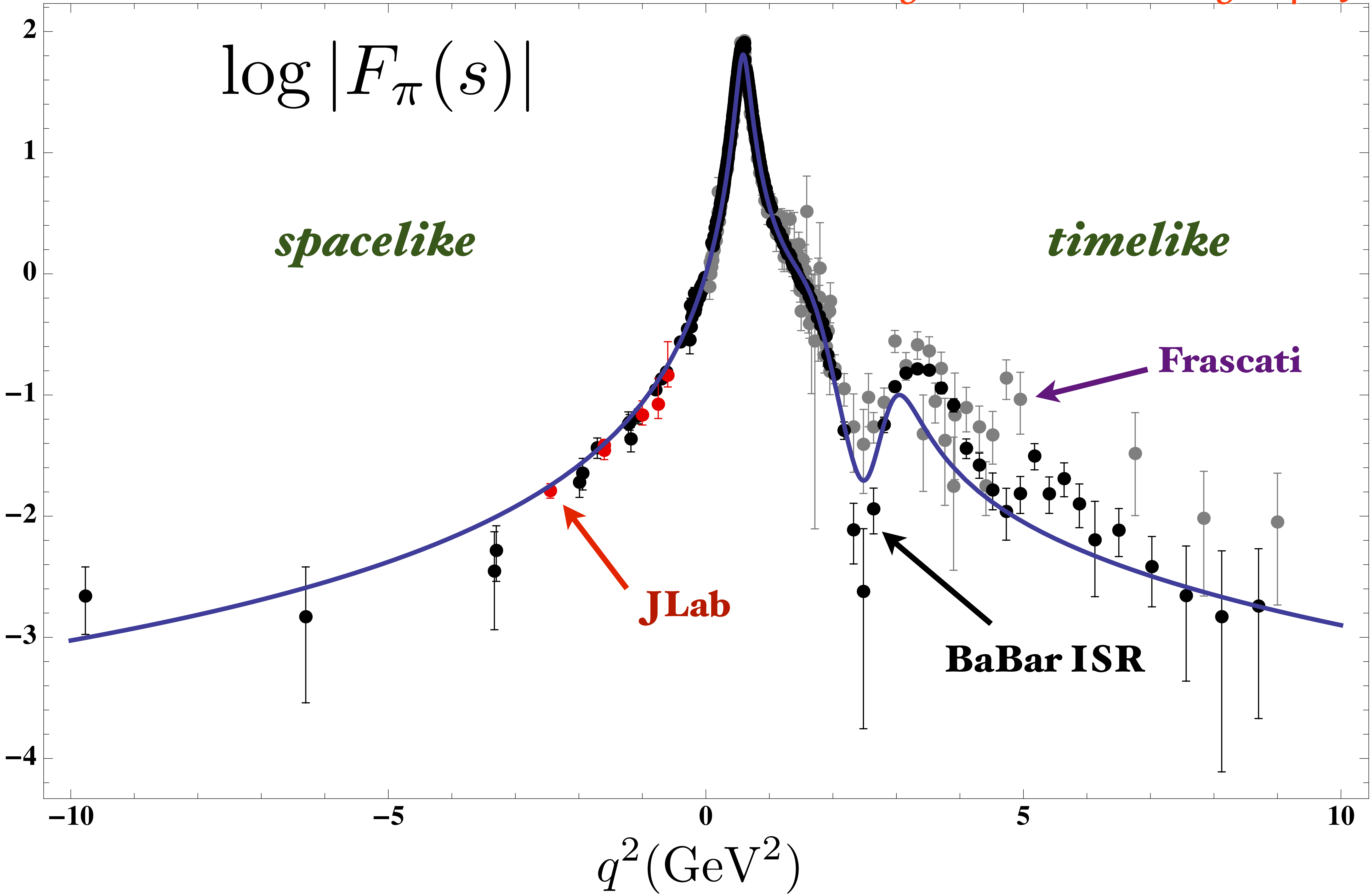
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

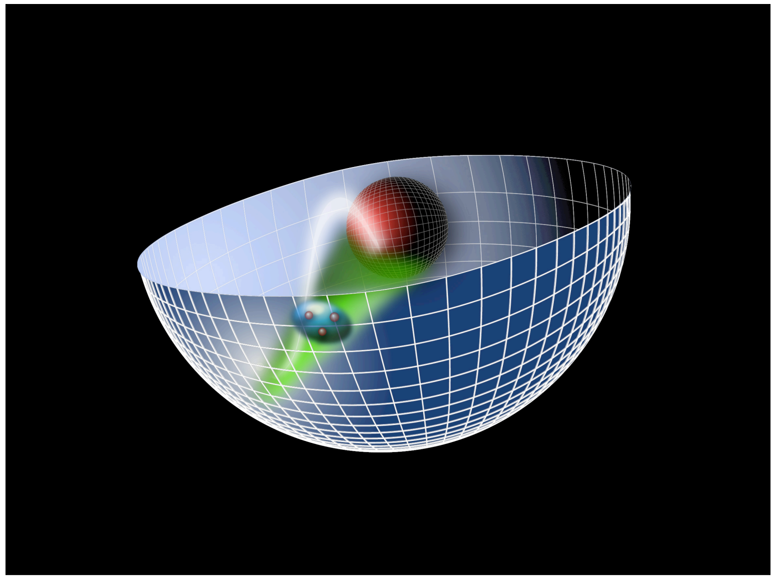
G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



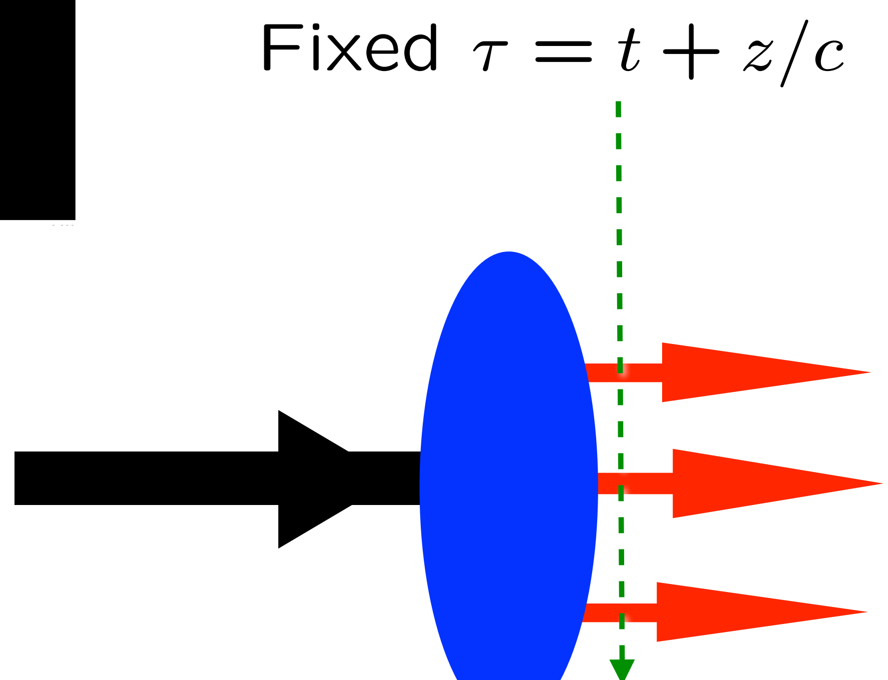
AdS5: Conformal Template for QCD

$\phi(z)$

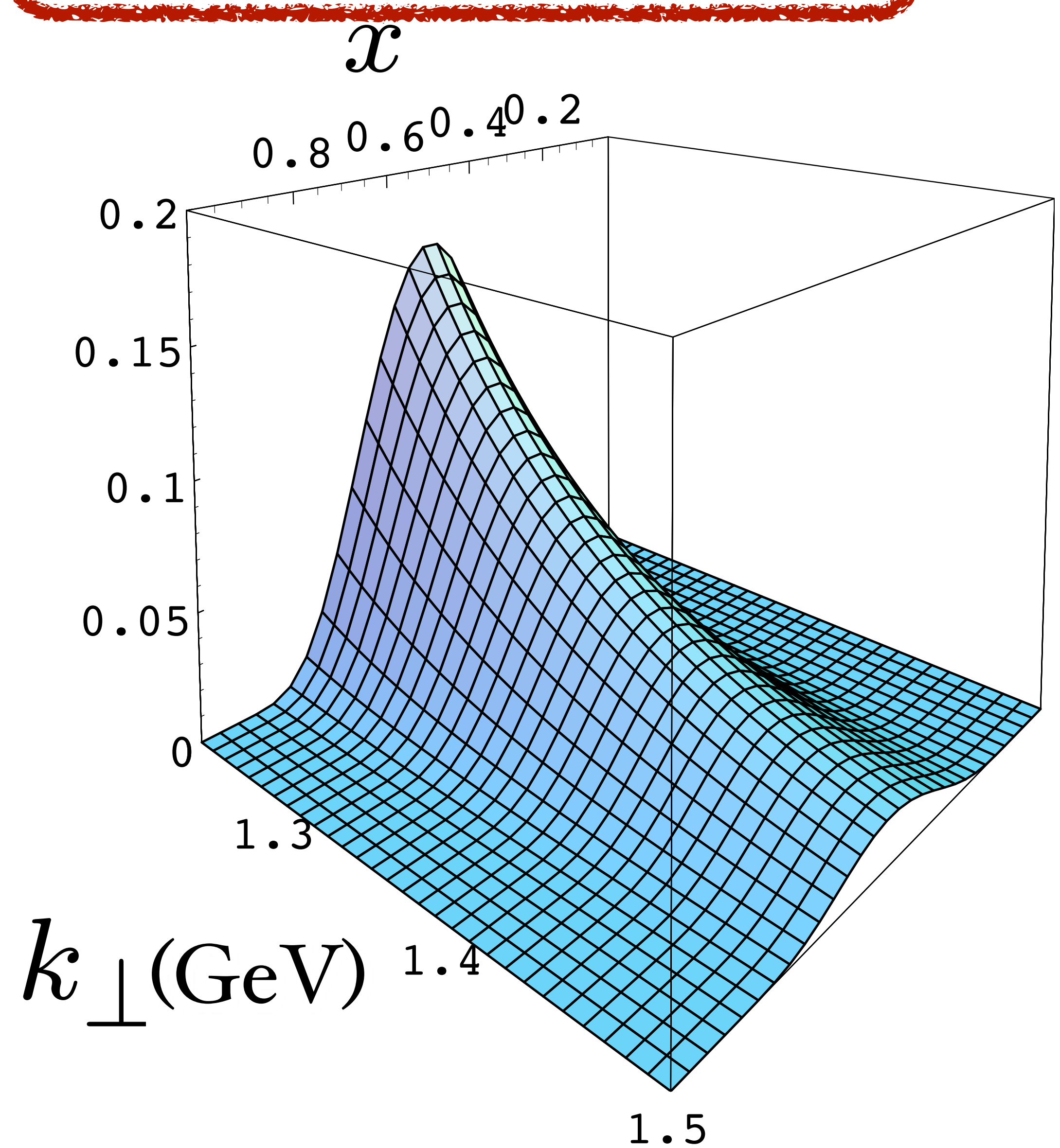


- *Light-Front Holography*

Duality of AdS$_5$ with LF Hamiltonian Theory



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



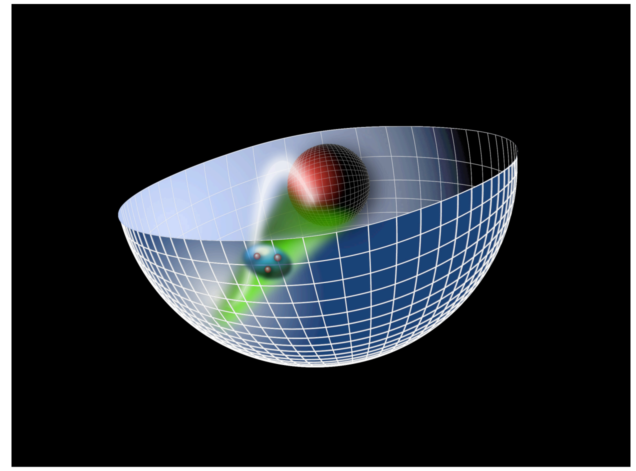
- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

Single scheme-independent fundamental mass scale κ



$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

(m_q=0)

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass   -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$\boxed{U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)}$$

QCD Lagrangian

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** *Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!*

Unique confinement potential!

Uniqueness

de Teramond, Dosch, sjb

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L: same slope!**
- **Massless pion in chiral limit!   No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)

- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$

- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$

- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ

- Only dimensionless mass ratios (and M times R) predicted

- Mass and time units [GeV] and [sec] from physics external to QCD

- New feature: bounded frame-independent relative time between constituents

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double Parton Processes**

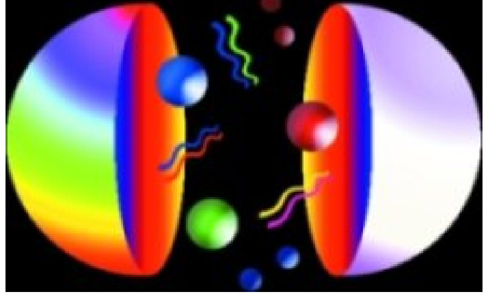
Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)]

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g}\, e^{\varphi(z)} \left(i\overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z) \overline{\Psi}\Psi - \mu \overline{\Psi}\Psi \right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x}\Psi(z)$

$$\boxed{\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.}$$

- Solution $(\nu = \mu R - \frac{1}{2},\ \nu = L + 1)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$ positive parity

- Obtain spin-J mode $\Phi_{\mu_1\cdots\mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\,\Lambda_{\rm QCD}$

Dirac Equation for Nucleons in Soft-Wall AdS/QCD

- We write the Dirac equation

$$\left(\alpha\Pi(\zeta) - \mathcal{M}\right)\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

$$\nu = L + 1$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \to \zeta$ and

$$\Psi_J(x,z) = e^{-iP\cdot x} z^2 \psi^J(z) u(P),$$

find LFWE for $d = 4$

$$\begin{aligned} \frac{d}{d\zeta}\psi_+^J + \frac{\nu+\frac{1}{2}}{\zeta}\psi_+^J + U(\zeta)\psi_+^J &= \mathcal{M}\psi_-^J, \\ -\frac{d}{d\zeta}\psi_-^J + \frac{\nu+\frac{1}{2}}{\zeta}\psi_-^J + U(\zeta)\psi_-^J &= \mathcal{M}\psi_+^J, \end{aligned}$$

where $U(\zeta) = \frac{R}{\zeta}V(\zeta)$

- Choose linear potential $U = \kappa^2\zeta$
- Eigenfunctions

$$\psi_+^J(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2), \qquad \psi_-^J(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)$$

- Eigenvalues

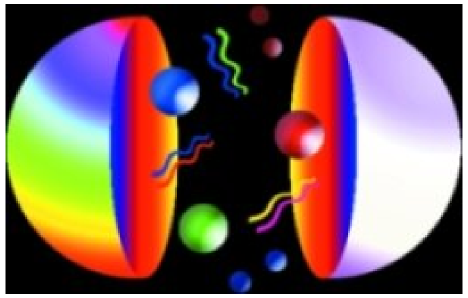
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1), \quad \nu = L+1 \quad (\tau = 3)$$

- Full $J - L$ degeneracy (different J for same L) for baryons along given trajectory !

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

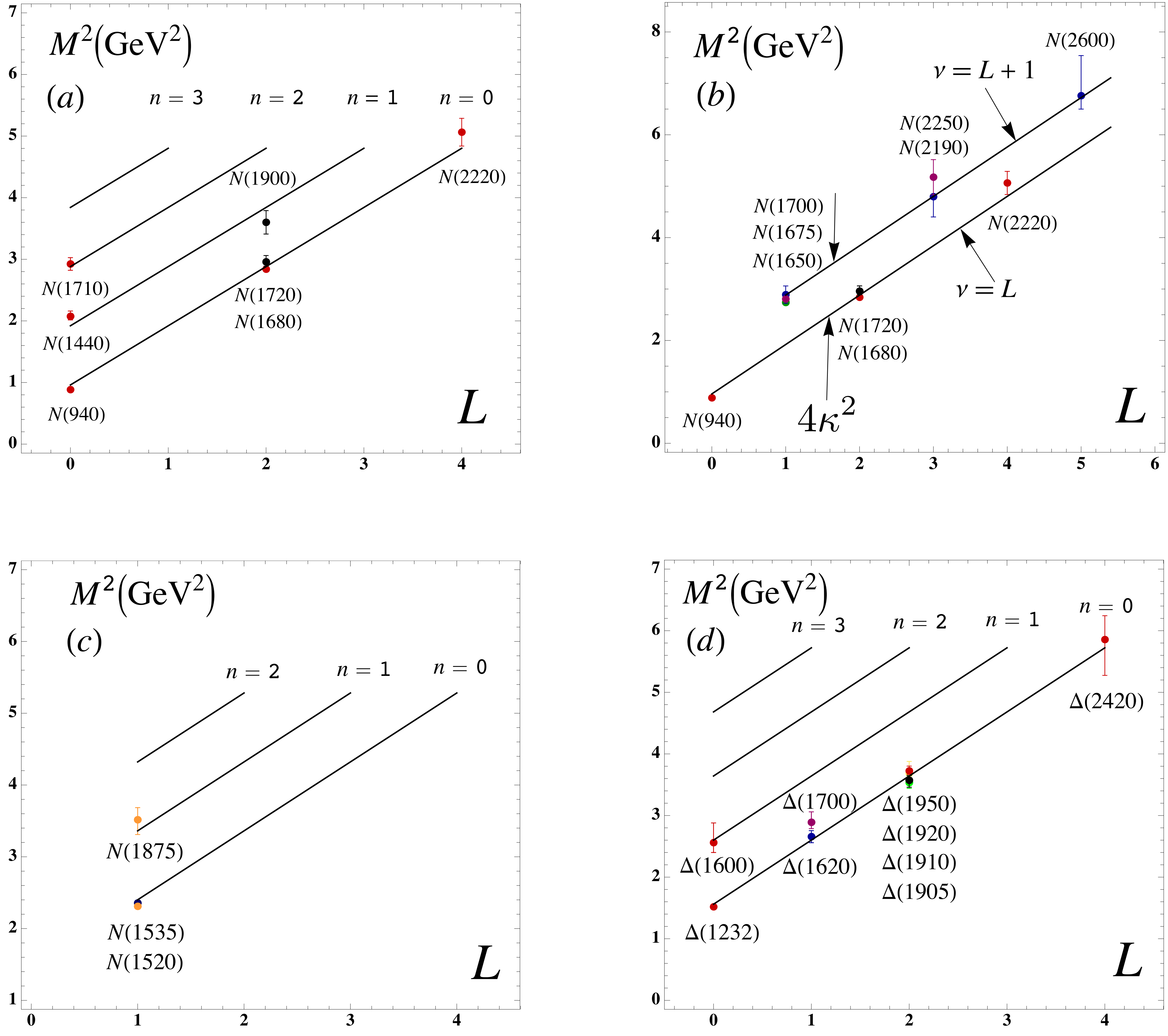
Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$\boxed{J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0}$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!



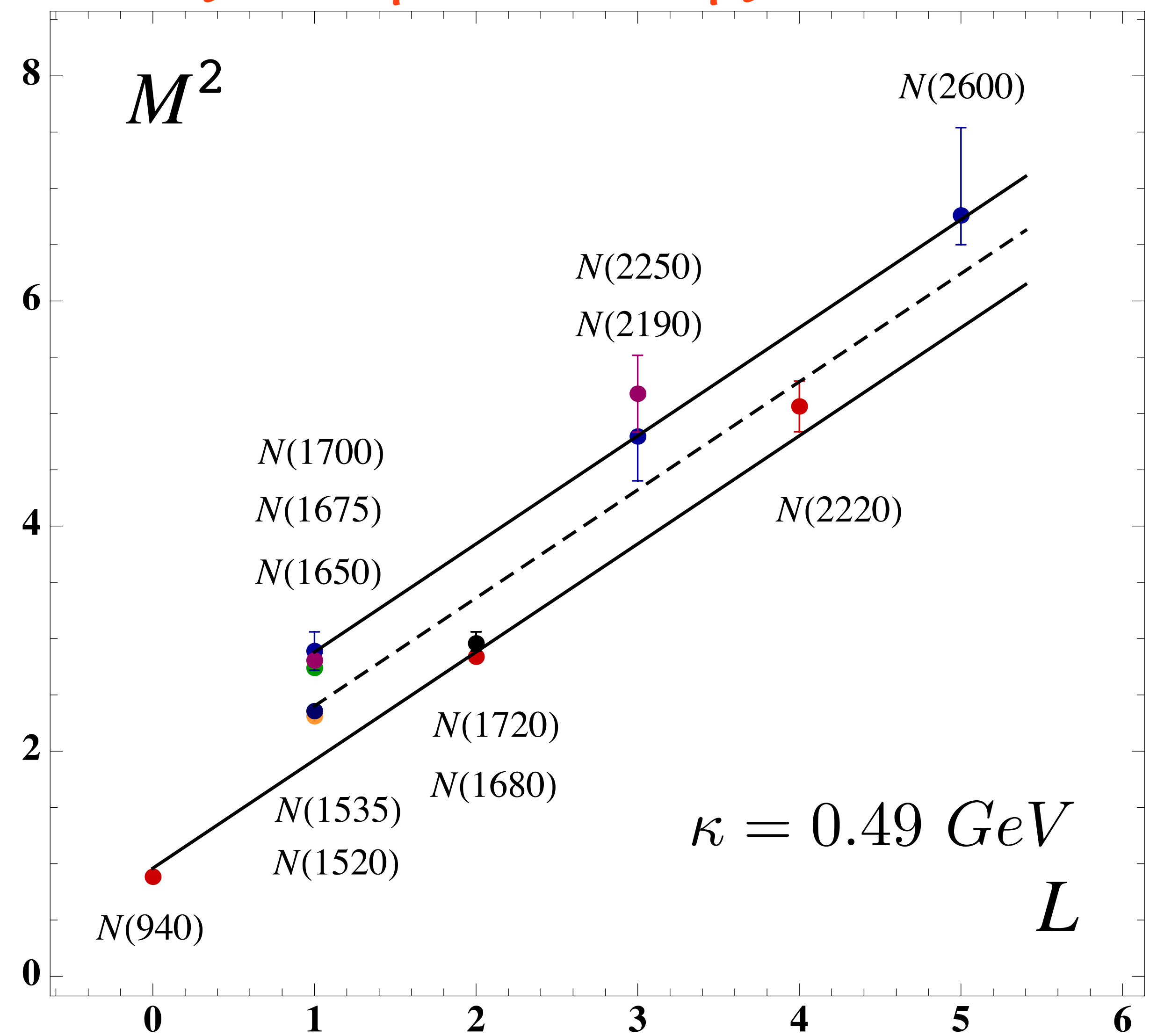
$M^2(\mathrm{GeV}^2)$
(a)
$n=3$
$n=2$
$n=1$
$n=0$
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
L
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L + 1$
N(2600)
N(2250)
N(2190)
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
L
$M^2(\mathrm{GeV}^2)$
(c)
$n=2$
$n=1$
$n=0$
N(1875)
N(1535)
N(1520)
L
$M^2(\mathrm{GeV}^2)$
(d)
$n=3$
$n=2$
$n=1$
$n=0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
L

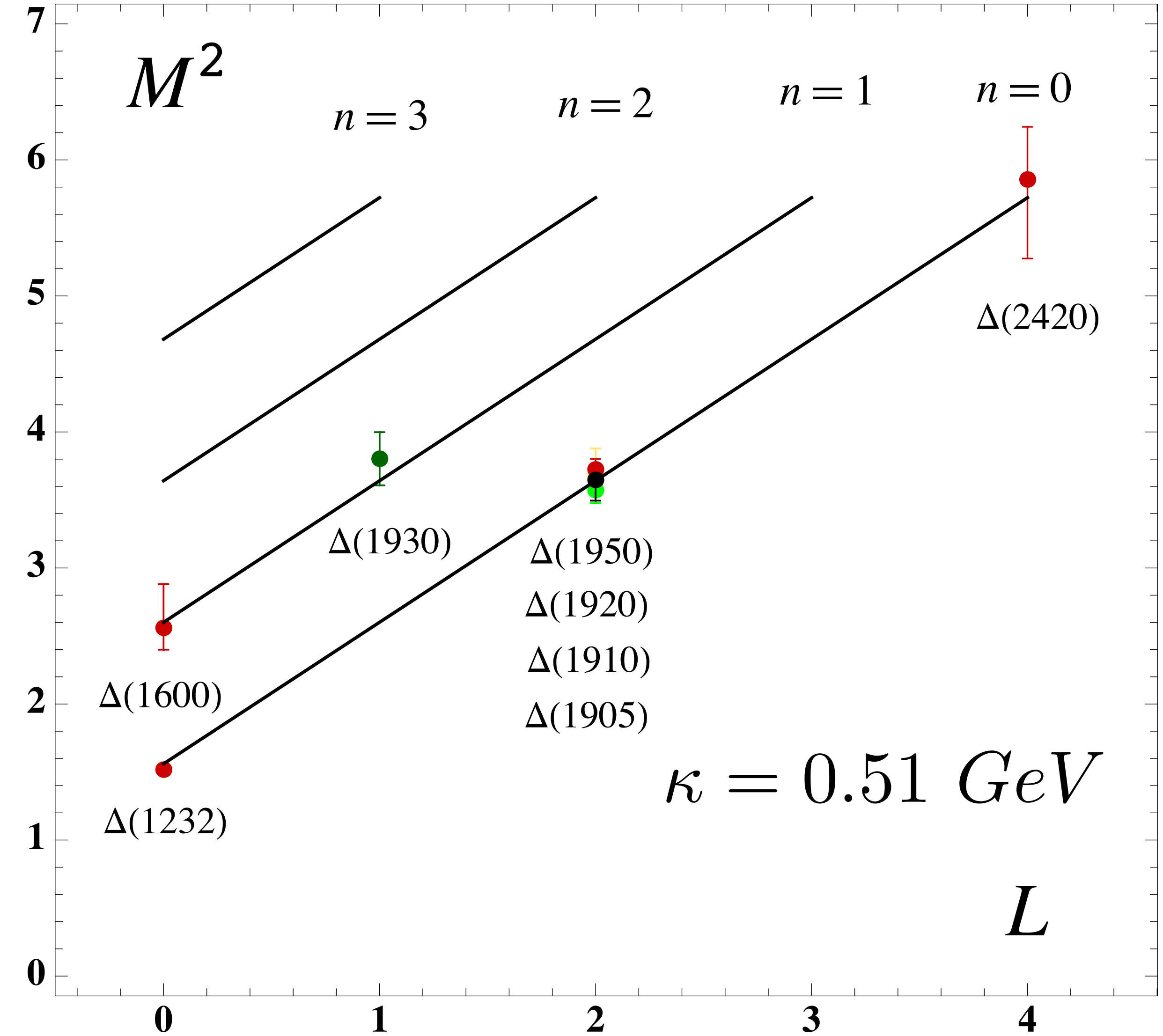
Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S, L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

PDG 2012

$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\frac{1}{2}^+(940)$
	$\frac{1}{2}$	0	1	$N\frac{1}{2}^+(1440)$
	$\frac{1}{2}$	0	2	$N\frac{1}{2}^+(1710)$
	$\frac{3}{2}$	0	0	$\Delta\frac{3}{2}^+(1232)$
	$\frac{3}{2}$	0	1	$\Delta\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{3}{2}$	1	1	$N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$
	$\frac{1}{2}$	1	0	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	0	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{1}{2}$	2	1	$N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$
	$\frac{3}{2}$	2	0	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	0	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

Baryon Spectroscopy from AdS/QCD and Light-Front Holography





de Teramond, sjb

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$ **positive parity**

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$ **negative parity**

All confirmed resonances from PDG 2012

See also Forkel, Beyer, Federico, Klempt

$$
\begin{aligned}
-\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - V(\zeta)\psi_- &= M\psi_+, \\
\frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - V(\zeta)\psi_+ &= M\psi_-,
\end{aligned}
$$

$$M^2 = 4\kappa^2(n+\nu+1)$$

Orbital assignment for baryon trajectories according to parity and internal spin.

$$\nu = |\mu R| - 1/2$$

	$S = \frac{1}{2}$	$S = \frac{3}{2}$
P = +	$\nu = L$	$\nu = L + \frac{1}{2}$
P = -	$\nu = L + \frac{1}{2}$	$\nu = L + 1$

$$M^{2\,(+)}_{n,L,S=\frac{3}{2}} = M^{2\,(-)}_{n,L,S=\frac{1}{2}}$$

No spin-orbit coupling

J=1/2 "Chiral partners", e.g. N(1535) and N(1400), with different L, non-degenerate

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

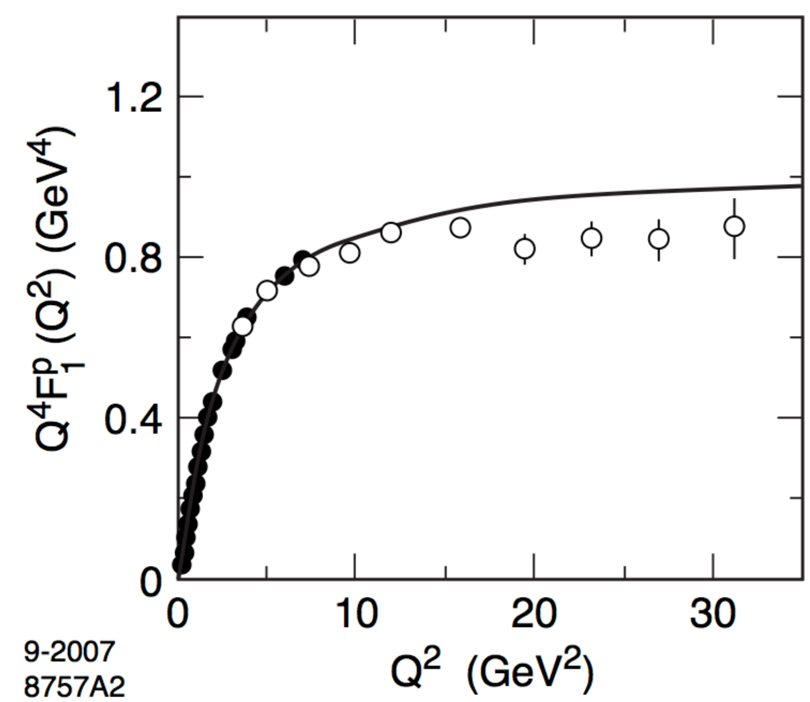
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$



Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

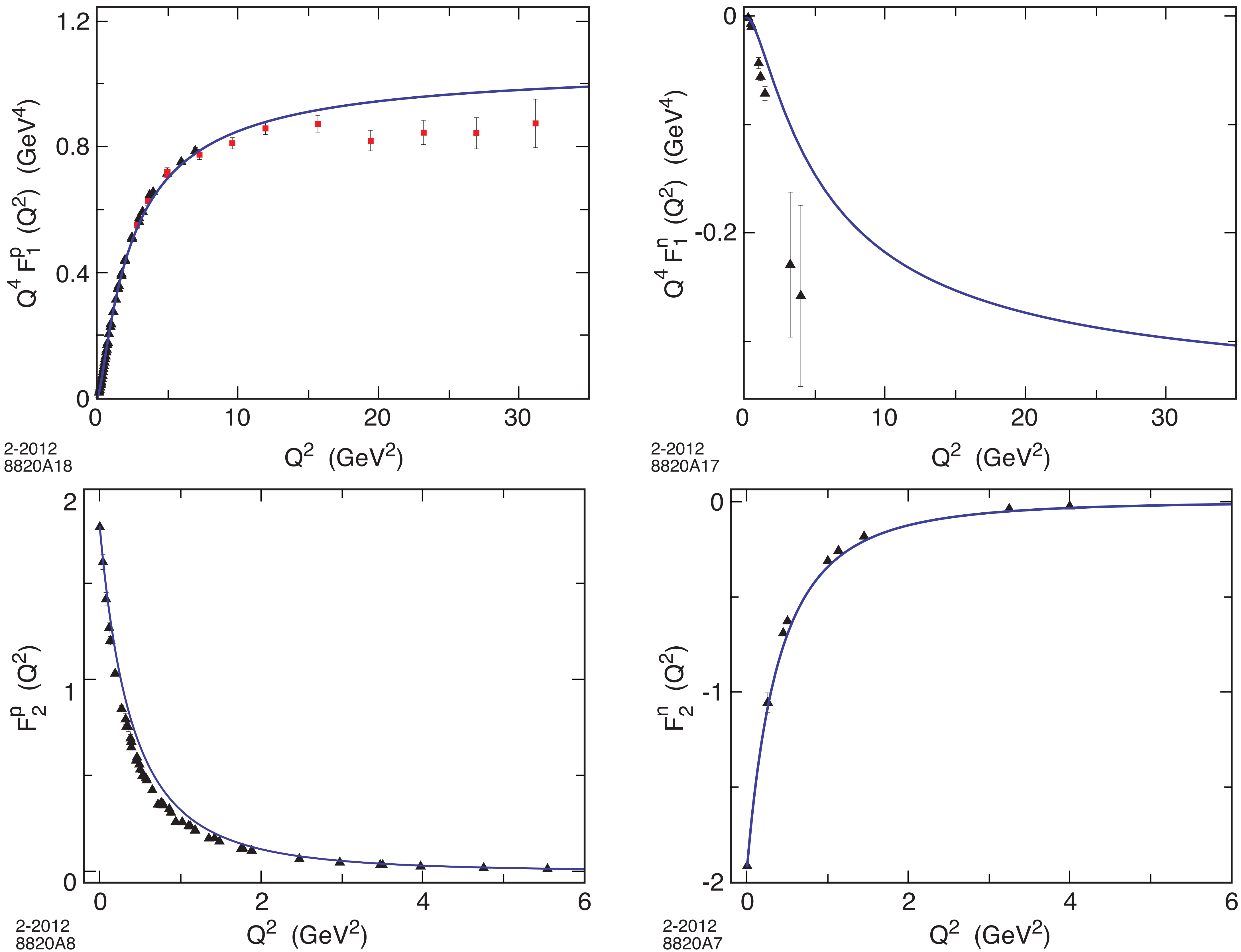
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

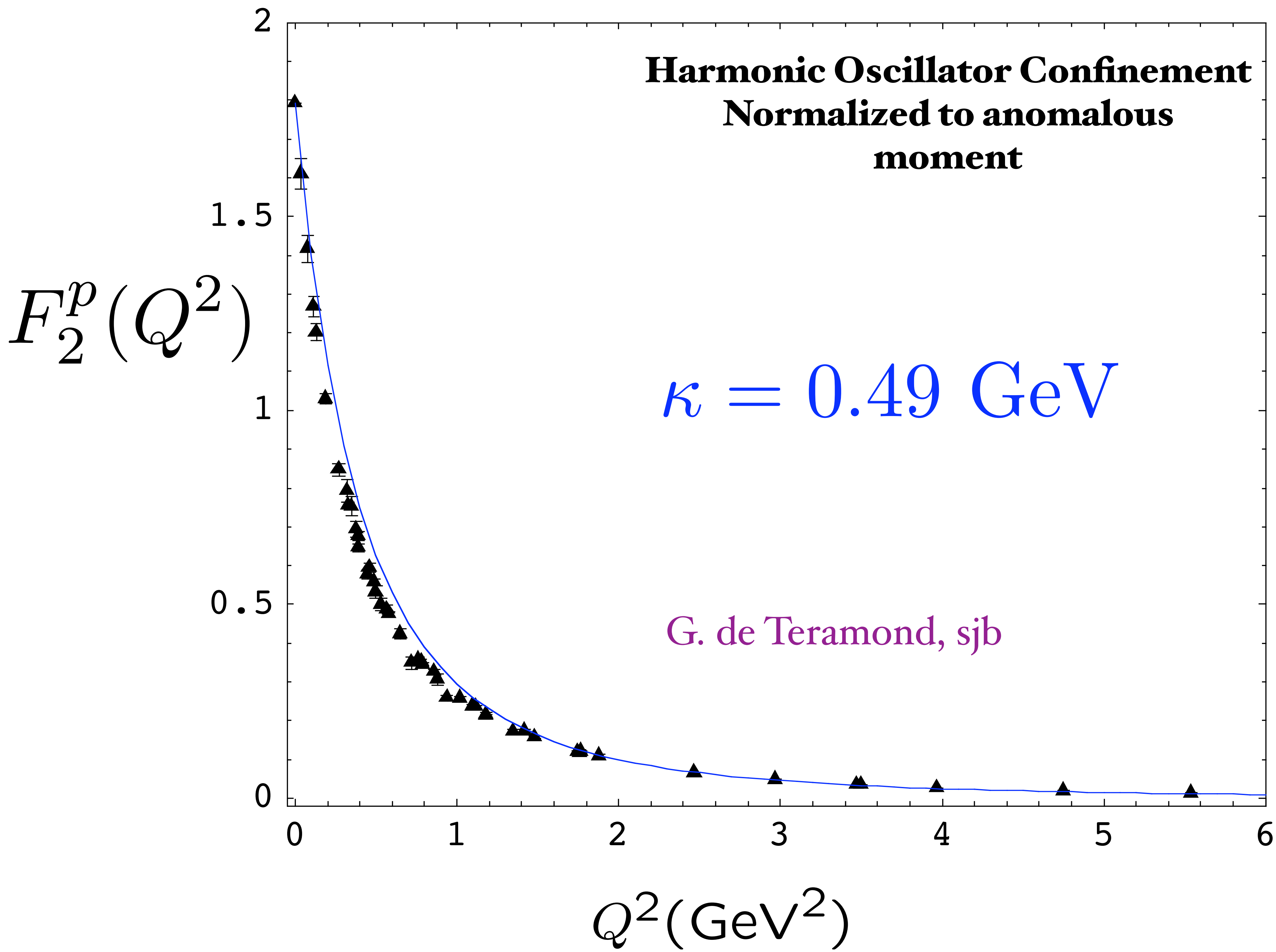
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

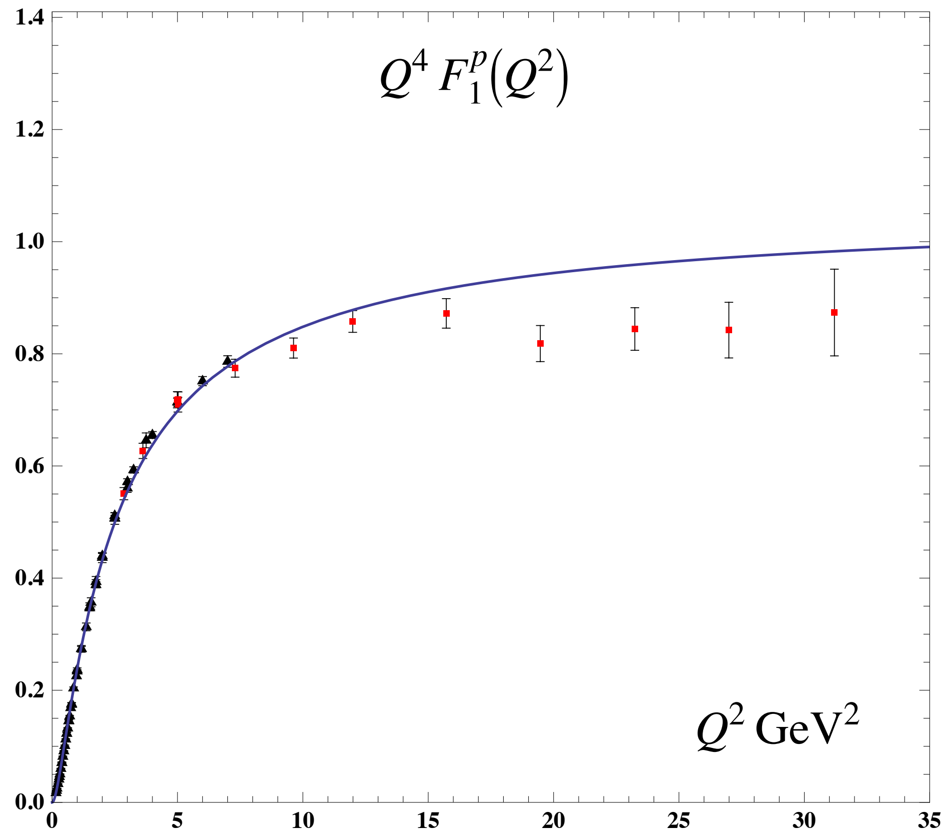
From overlap of $L = 1$ and $L = 0$ LFWFs

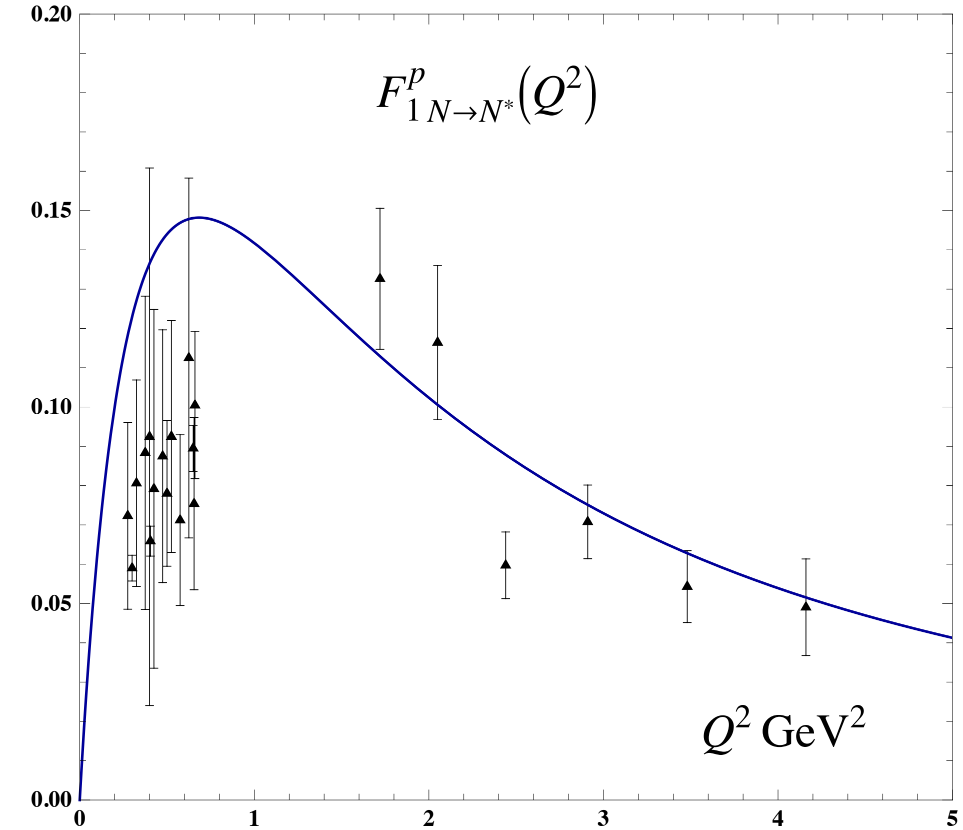


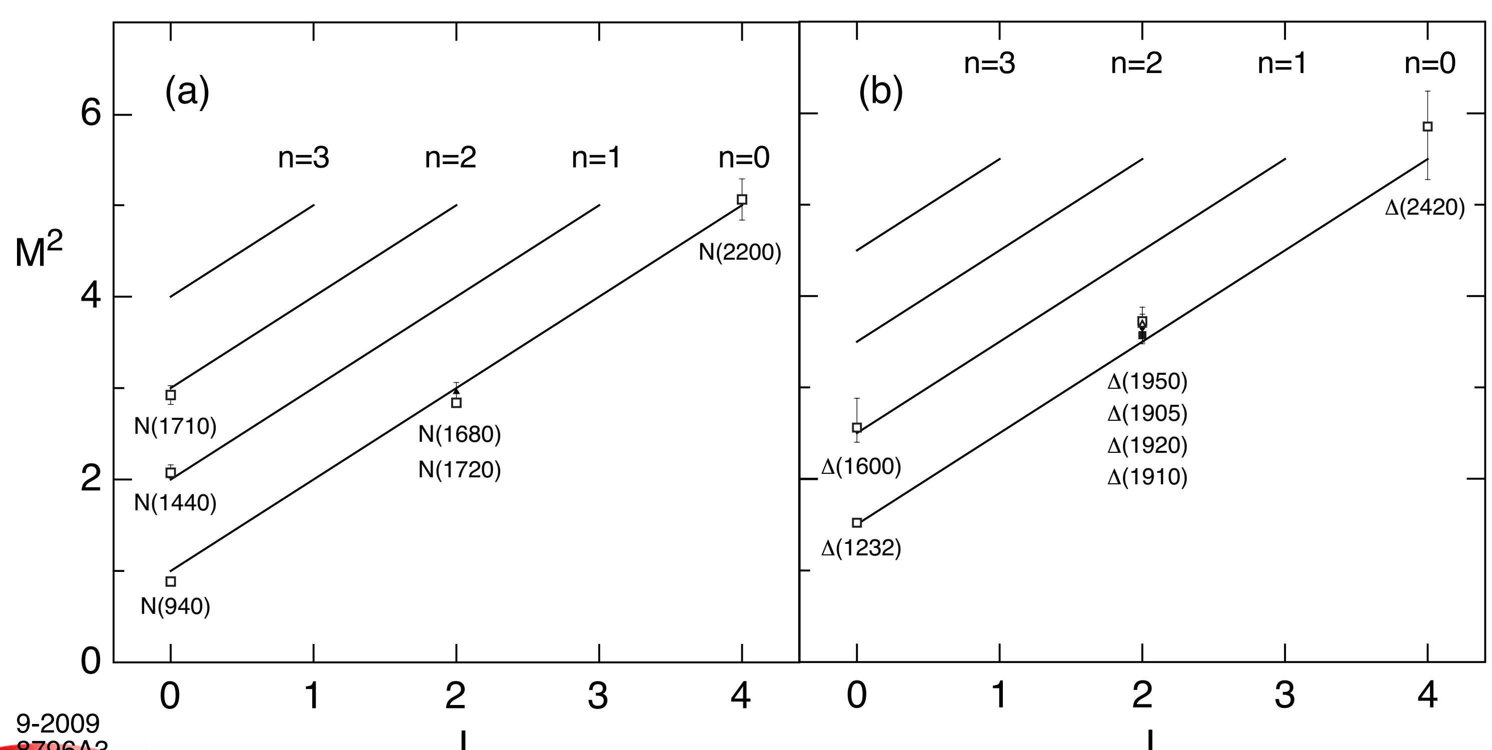
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb







Bad Honnef
November 2014

Light-Front Schrödinger Equation and QCD Confinement

Stan Brodsky
SLAC
NATIONAL ACCELERATOR LABORATORY

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

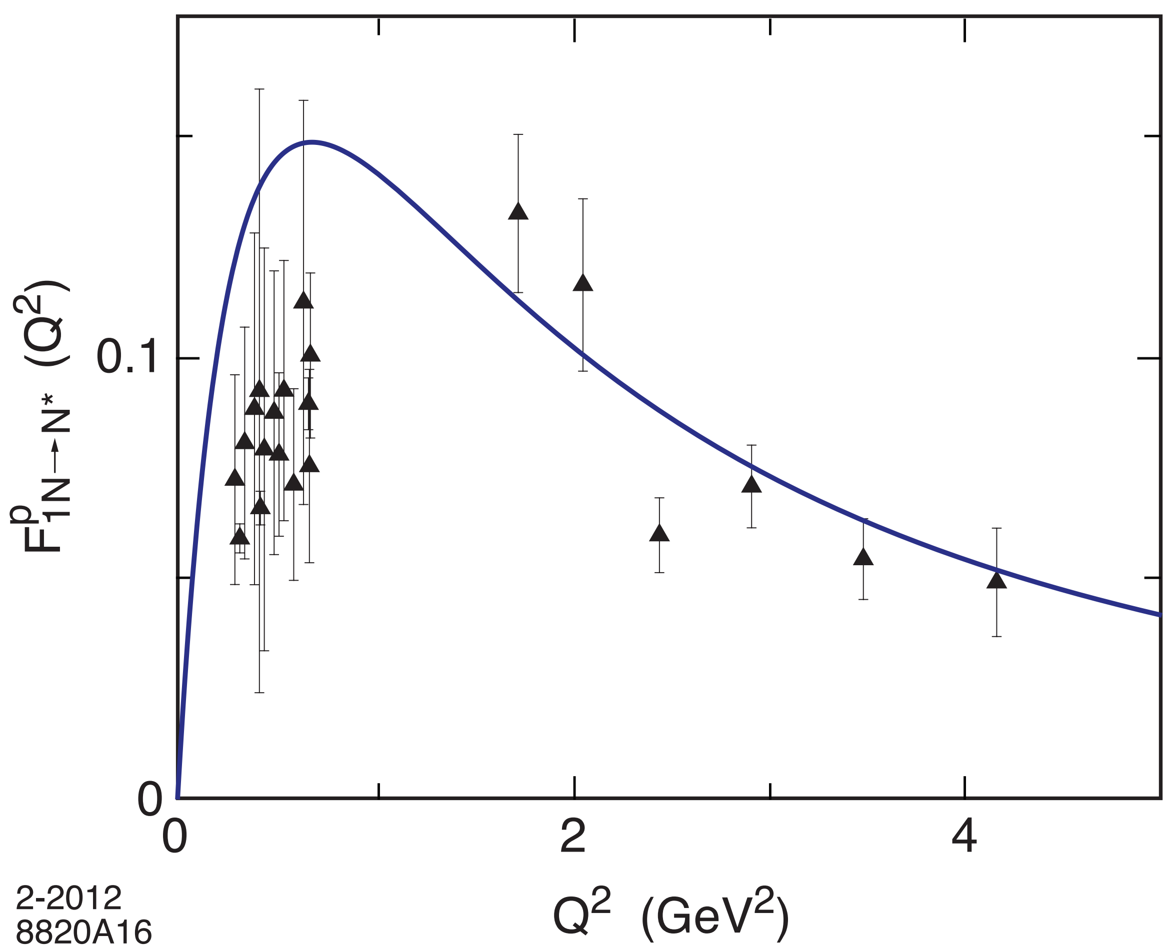
with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$

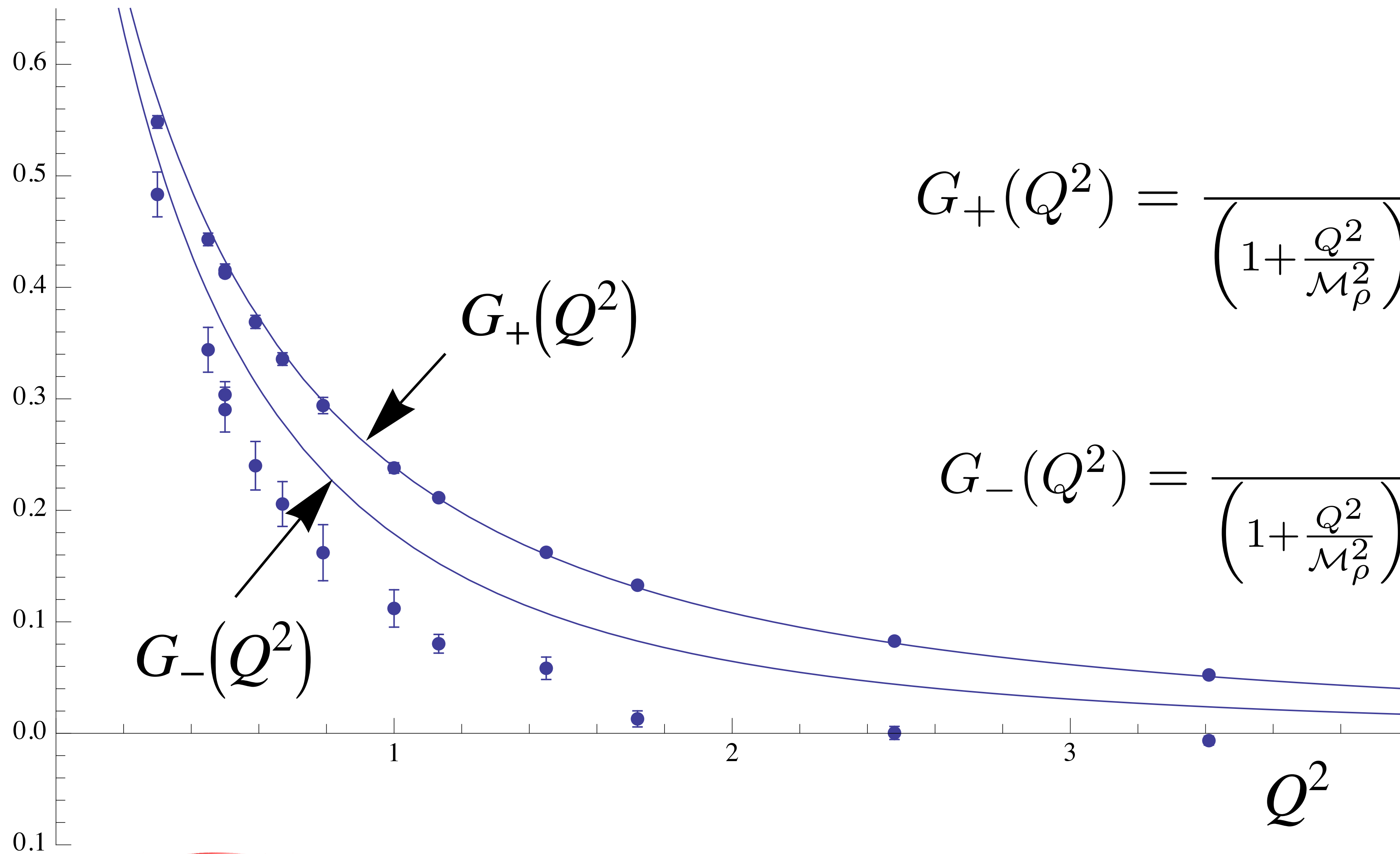


Proton transition form factor to the first radial excited state. Data from JLab

Flavor Decomposition of Elastic Nucleon Form Factors

G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F^p_{u,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-, \qquad F^p_{d,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F^n_{u,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-, \qquad F^n_{d,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$



$$G_+(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)}$$

$$G_-(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}$$

AdS/QCD and Light-Front Holography

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- **Zero mass pion for m_q =0 (n=J=L=0)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2: Dimensional counting** $[Q^2]^{n-1} F(Q^2) \to \mathrm{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

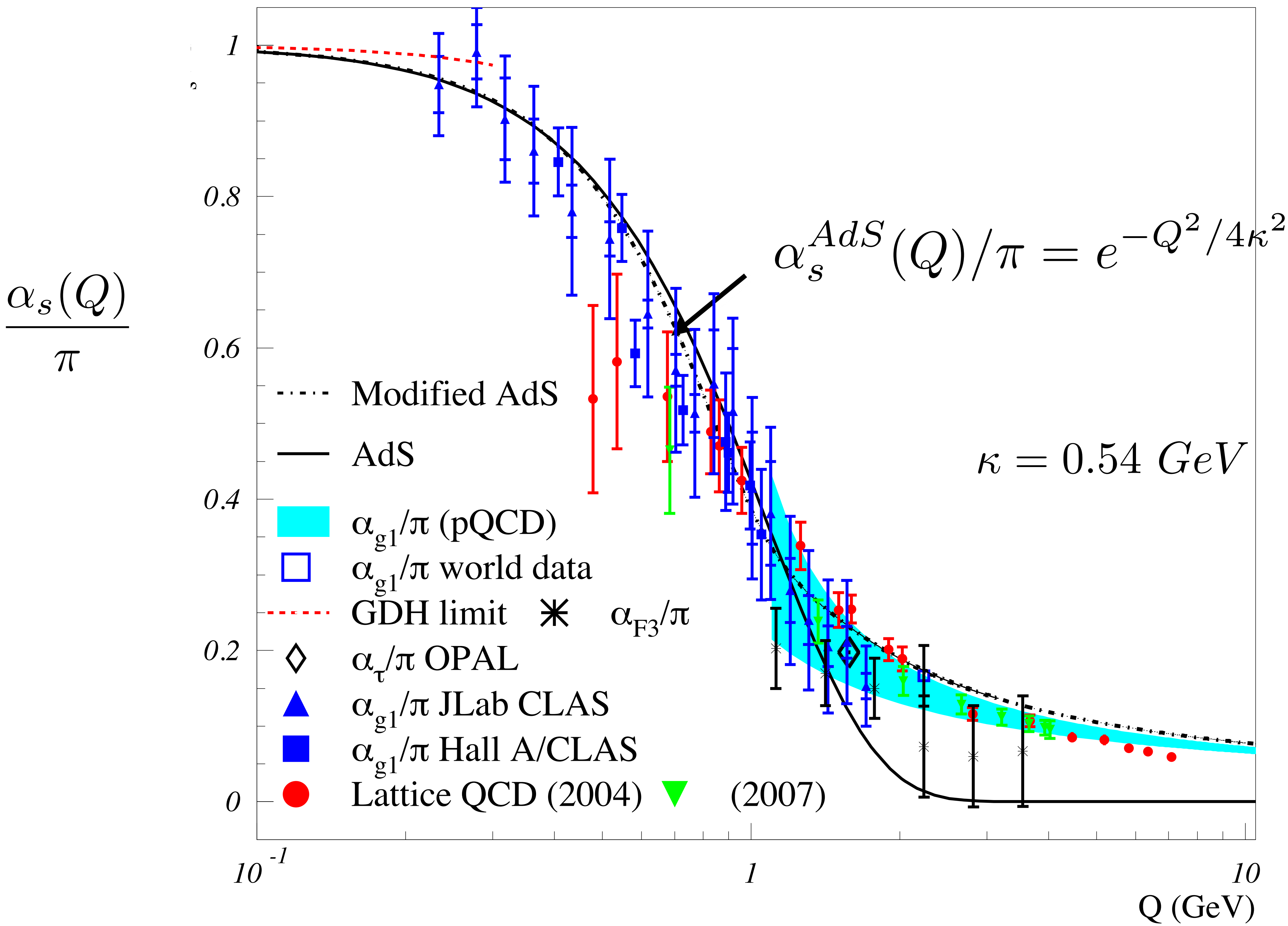
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



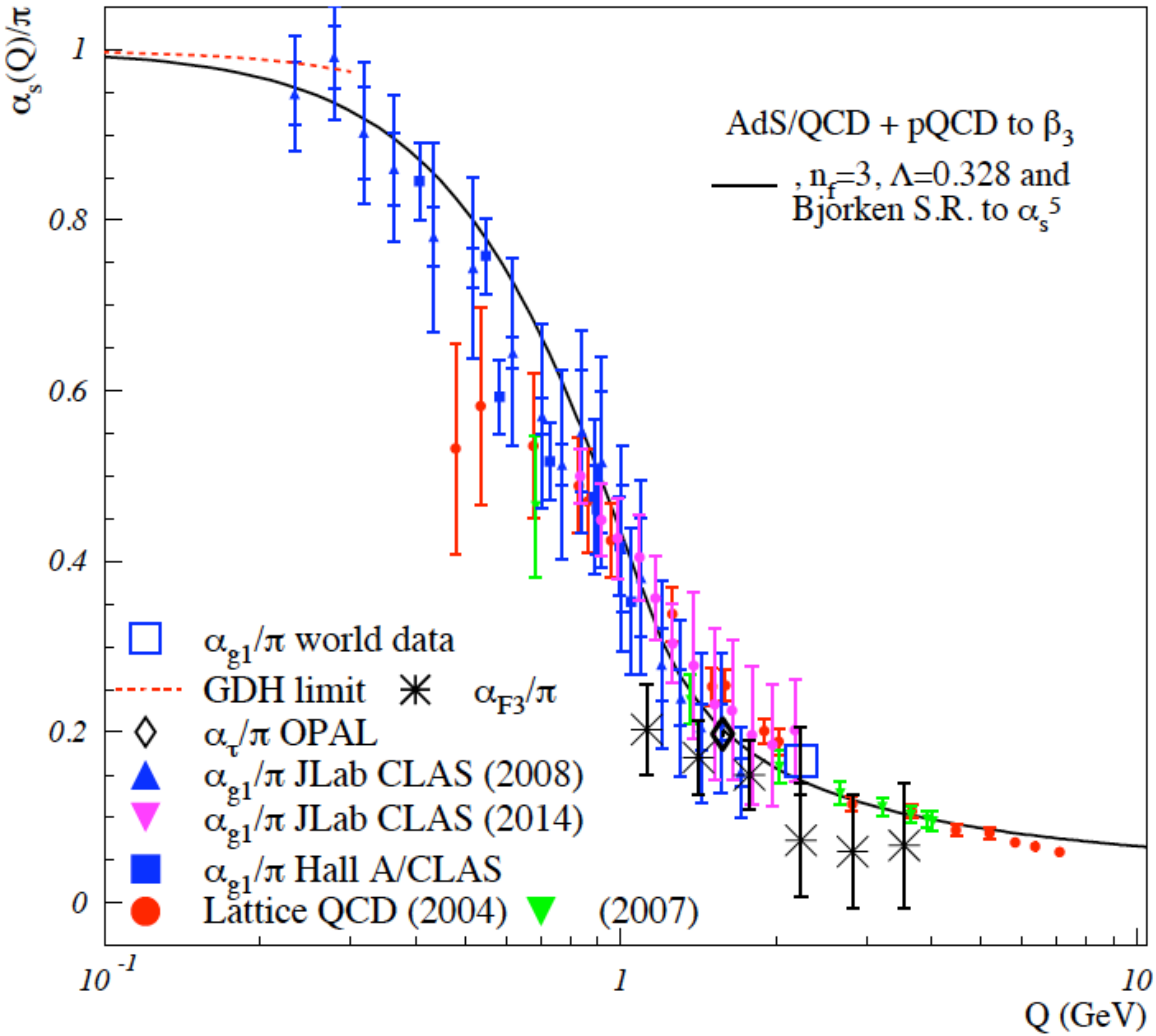
AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

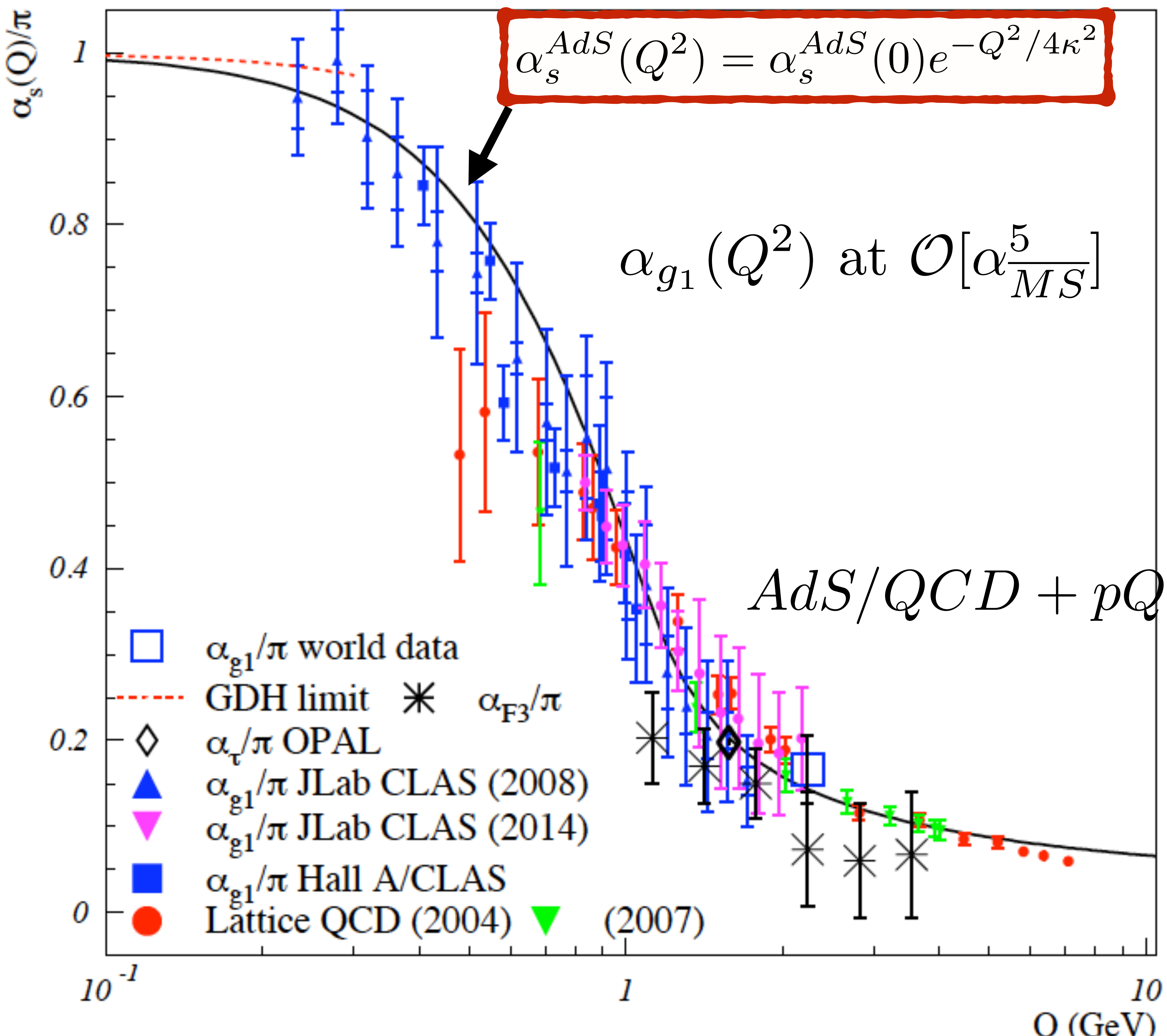
Deur, de Teramond, sjb

Deur, de Teramond, sjb

Derive $\Lambda_{\overline{MS}} = 0.33\ GeV$ from κ or hadron mass scale



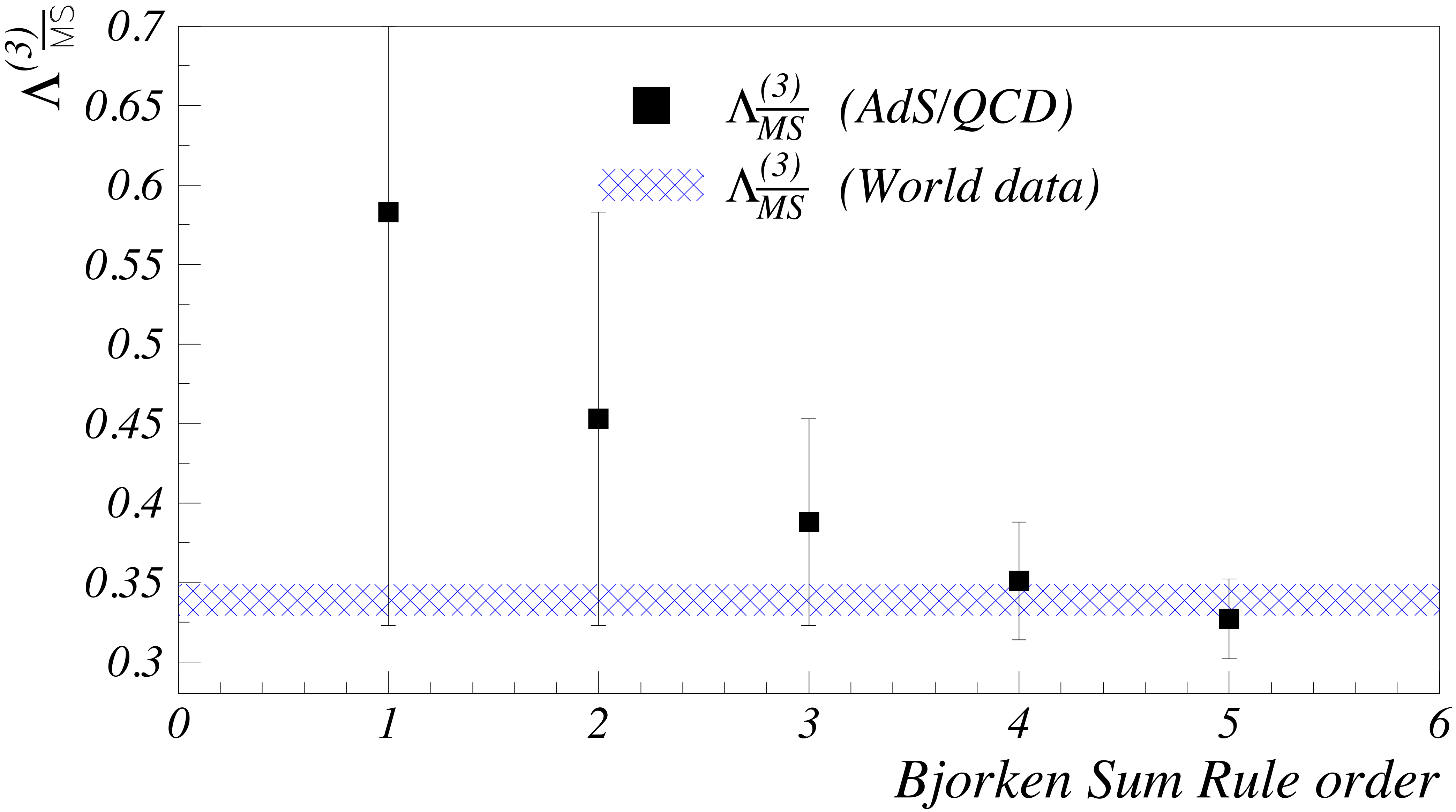
$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$



Deur,
de Teramond, sjb

Deur, de Teramond, sjb

See also A. Peters et al.

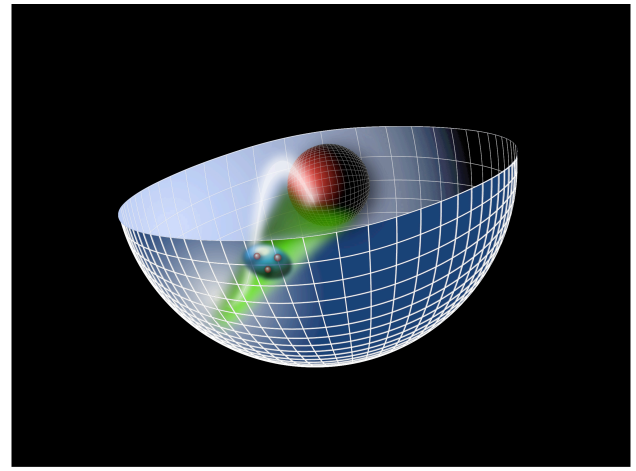


$$\Lambda_{\overline{MS}} = 0.5983\kappa = 0.5983\frac{m_\rho}{\sqrt{2}} = 0.4231 m_\rho = 0.328\ GeV$$

Connect $\Lambda_{\overline{MS}}$ to hadron masses!

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

AdS/QCD and Light-Front Holography

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- **Zero mass pion for $m_q = 0$ (n=J=L=0)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2: Dimensional counting** $[Q^2]^{n-1}F(Q^2) \to \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi\sqrt{x(1-x)}$

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

$\Updownarrow$

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Connection to the Linear Instant-Form Potential

- Compare invariant mass in the instant-form in the hadron center-of-mass system $\mathbf{P} = 0$,

$$M^2_{q\bar{q}} = 4\, m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame, $\mathbf{k}_q + \mathbf{k}_{\bar{q}} = 0$

$$M^2_{q\bar{q}} = \frac{\mathbf{k}_\perp^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2}\, V + 2\, V\sqrt{\mathbf{p}^2 + m_q^2}$$

where $\mathbf{p}_\perp^2 = \frac{\mathbf{k}_\perp^2}{4x(1-x)}$, $\;p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and V is the effective potential in the instant-form

- For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Extensions of AdS/QCD LF Holography

- ***Massive quarks***
- ***Broken Chiral Symmetry***
- ***Structure Functions***
- ***Counting Rules at x ~1, Duality***
- ***Nucleon GPDs***

Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega
Ian Cloet, C. D. Roberts
Ruben Sandapen, Jeff Forshaw
Burkardt, Schmidt, Lepage, sjb

- **Conformal template:**
- **Use isometries of AdS5**

String Theory

↓

Goal: First Approximant to QCD

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

AdS/CFT

↓

Counting rules for Hard Exclusive Scattering
Regge Trajectories

Conformal behavior at short distances

AdS/QCD

↓

QCD at the Amplitude Level

Confinement at large distance
Unique!

Semi-Classical QCD / Wave Equations

↓

Holography

Boost Invariant 3+1 Light-Front Wave Equations

↓

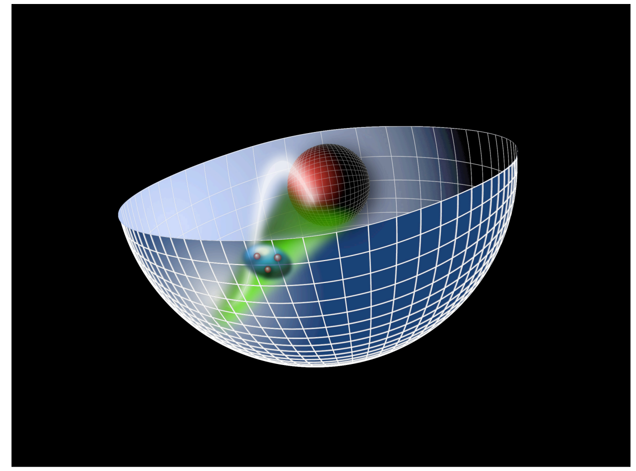
J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model



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Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable ζ conjugate to invariant mass squared
- Relativistic, Frame-Independent, Color-Confining
- Unique confining potential!
- QCD Coupling at all scales: Essential for Gauge Link phenomena
- Hadron Spectroscopy and Dynamics from one parameter
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates: Zero cosmological constant!
- Systematically improvable with DLCQ-BLFQ Methods

Features of BLM/PMC

- ***Predictions are scheme-independent***
- ***Matches conformal series***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (*Kataev, Lu, Rathsman, sjb*)***
- ***No n! Renormalon growth***
- ***New scale at each order; n_F determined at each order***
- ***Multiple Physical Scales Incorporated***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Eliminates unnecessary theory error***

What is PMC ?

Principle of Maximum Conformality

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

↓

Choose μ_R^{init}*; arbitrary initial renormalization scale*

↓

Identify $\{\beta_i^R\}$ *− terms using* n_f *− terms through the PMC − BLM correspondence principle*

↓ order-by-order

Shift scale of α_s *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\}$ *− terms*

↓

Conformal Series

Result is independent of $\mu_R^{\rm init}$ *and scheme at fixed order*

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

PMC−BLM – one

Phys. Rev. Lett. **109**, 042002 (2012)

R_δ−scheme – two

Phys. Rev. Lett. **110**, 192001 (2013)

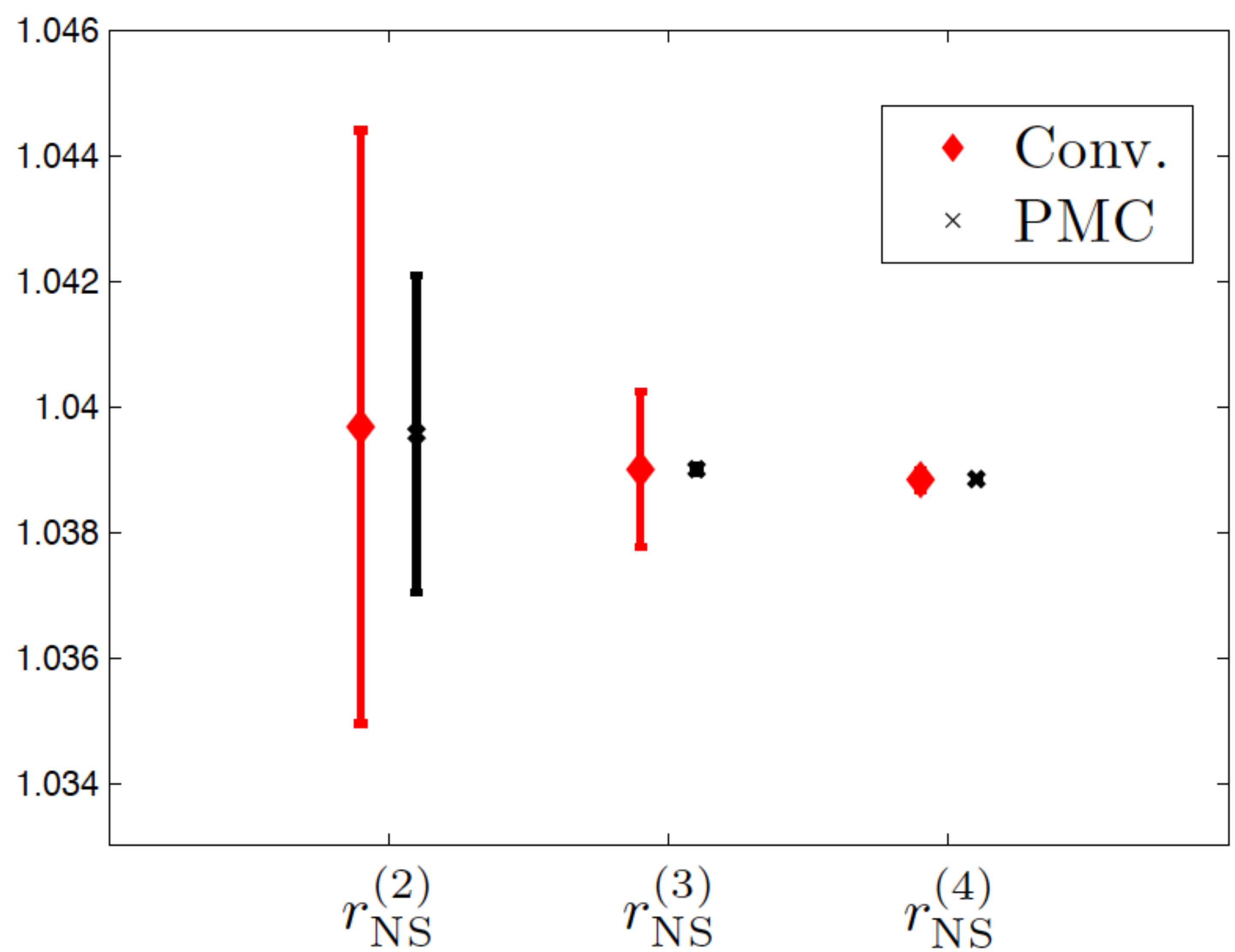
Eliminate β−terms

n_f dependence of pQCD series does not uniquely identify the β terms

Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\rm NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\rm NS} a_s^i$ and their errors $\pm|C_n^{\rm NS} a_s^n|_{\rm MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\rm init} = M_Z$.

Principle of Maximum Conformality (PMC)

- **Sets pQCD renormalization scale correctly at every finite order**
- **Predictions are scheme-independent**
- **Satisfies all principles of the renormalization group**
- **Agrees with Gell Mann-Low procedure for pQED in Abelian limit**
- **Shifts all β terms into α_s, leaving conformal series**
- **Automatic procedure: R_δ scheme**
- **Number of flavors n_f set**
- **Eliminates n! renormalon growth**
- **Choice of initial scale irrelevant**
- **Eliminates unnecessary systematic error -- conventional guess is scheme-dependent, disagrees with QED**
- **Reduces disagreement with pQCD for top/anti-top asymmetry at Tevatron from 3σ to 1σ**

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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Kavil Institute for Theoretical Physics, University of California,
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zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

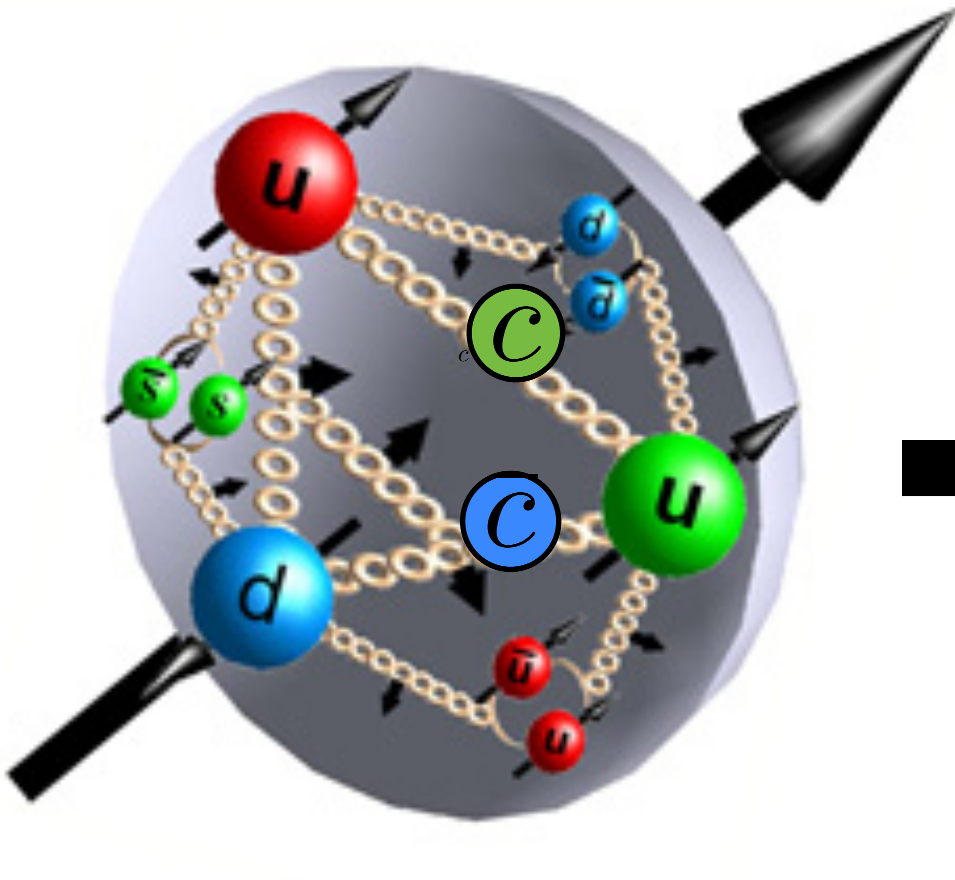
Elements of the solution:
(A) Light-Front Quantization: causal, frame-independent vacuum
(B) New understanding of QCD "Condensates"
(C) Higgs Light-Front Zero Mode

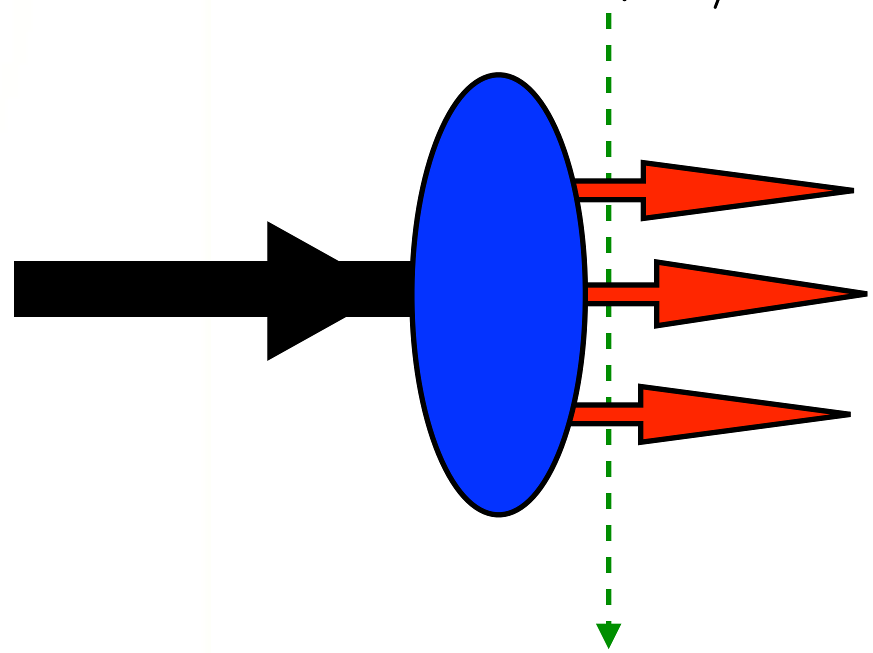
Light-Front vacuum can simulate empty universe

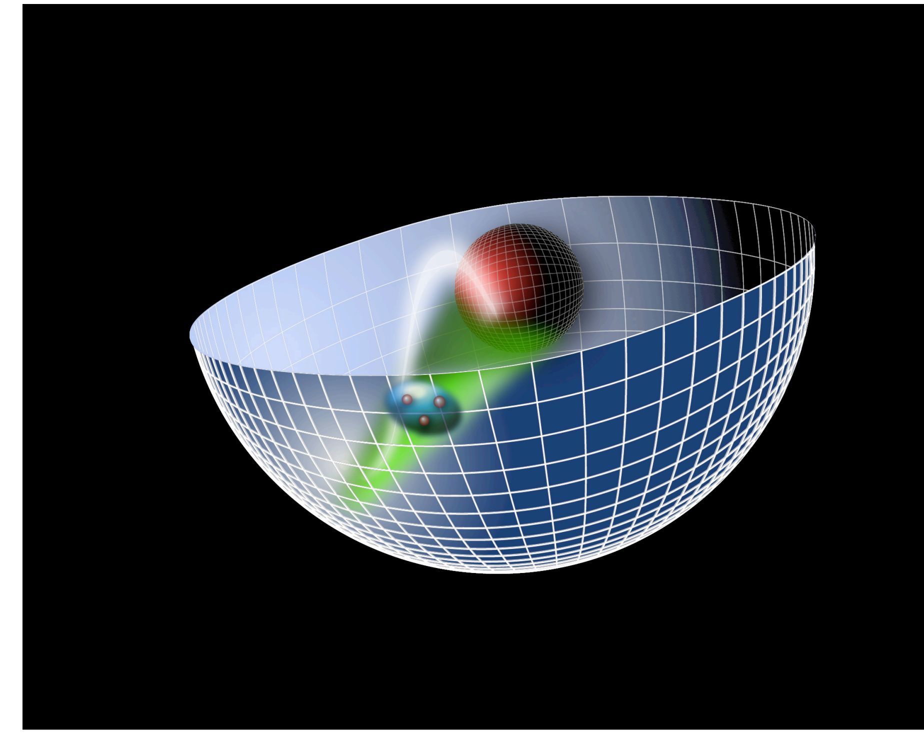
Shrock, Tandy, Roberts, sjb

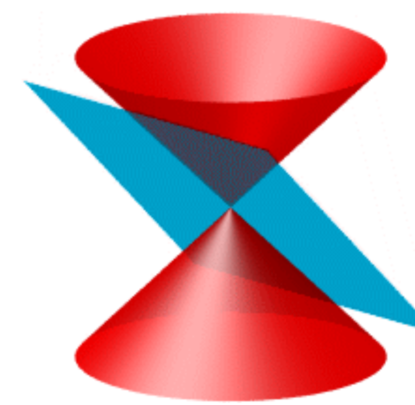
- **Independent of observer frame**
- **Causal**
- **Lowest invariant mass state M= 0.**
- **Trivial up to k^+=0 zero modes-- already normal-ordering**
- **Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)**
- **QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.**
- **QED vacuum; no loops**
- **Zero cosmological constant from QED, QCD**

The Light-Front Schrödinger Equation
A New Approach to Color Confinement and Non-Perturbative QCD









Strong Interactions in the LHC Era

Wilhelm und Else Heraeus Stiftung

Bad Honnef
November 13, 2014

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Leland Stanford Junior University

PHYSICAL REVIEW D **66**, 045019 (2002)

Light-front formulation of the standard model

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Theoretical Physics Department, Fermilab, Batavia, Illinois 60510,
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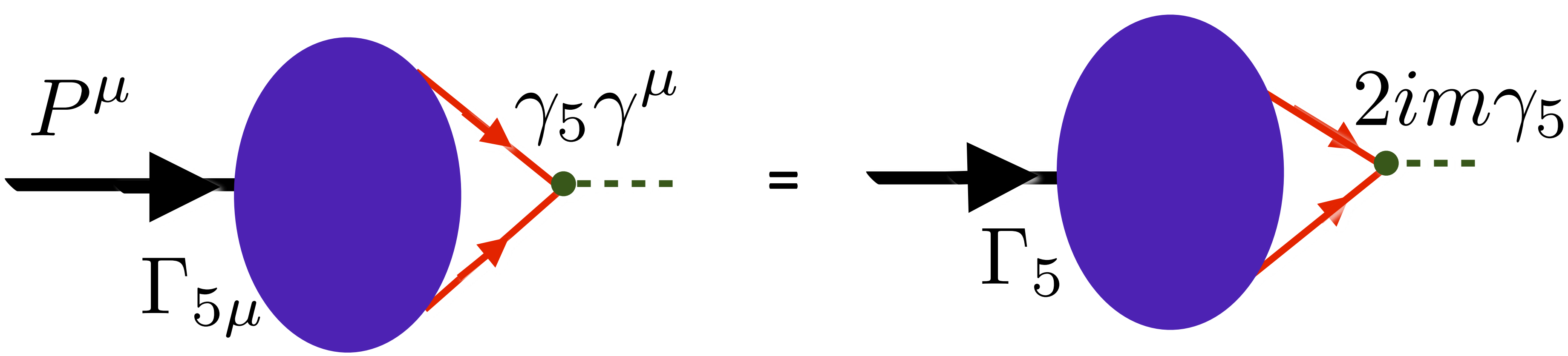
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 20 February 2002; published 20 August 2002)

Light-front (LF) quantization in the light-cone (LC) gauge is used to construct a renormalizable theory of the standard model. The framework derived earlier for QCD is extended to the Glashow-Weinberg-Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The nontransverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is unitary and ghost free (except for the ghosts at $k^+=0$ associated with the light-cone gauge prescription). The massive gauge field propagator has well-behaved asymptotic behavior. The interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, plus additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone boson (or electroweak) equivalence theorem, as the illustrations show.

Ward-Takahashi Identity for axial current

$$P^\mu \Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

Identify pion pole at $P^2 = m_\pi^2$

$$P^\mu < 0|\bar{q}\gamma_5\gamma^\mu q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u + m_d)\rho_\pi$$

Revised Gell Mann-Oakes-Renner Formula in QCD

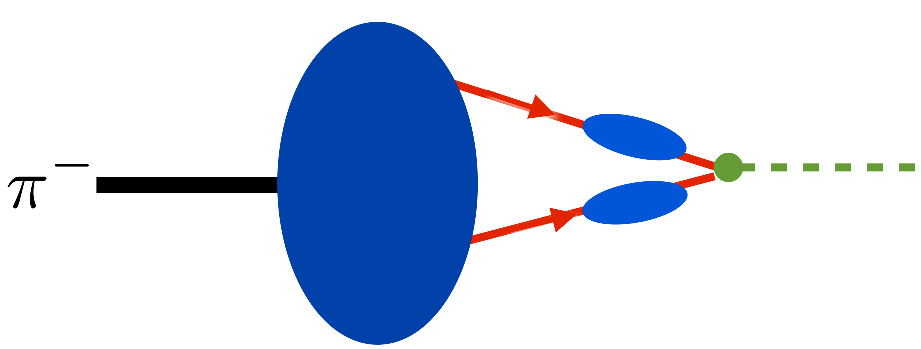
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

P. Srivastava, sjb

Standard Model on the Light-Front

- **Same phenomenological predictions**
- **Higgs field has three components**
- **Real part creates Higgs particle**
- **Imaginary part (Goldstone) become longitudinal components of W, Z**
- ***Higgs VEV of instant form becomes $k^+=0$ LF zero mode!***
- **Analogous to a background static classical Zeeman or Stark Fields**
- **Zero contribution to $T^\mu{}_\mu$; zero coupling to gravity**

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0 >= E_0|\psi_0 >, E_0 = min\{E_i\}$$

Eigenstate defined at one time t over all space; Acausal! Frame-Dependent

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0 >_{LF}= M_0^2|\psi_0 >_{LF}, M_0^2 = 0.$$

Frame-independent eigenstate at fixed LF time τ = t+z/c within causal horizon

Frame-independent description of the causal physical universe!

We view the universe as light reaches us along the light-front at fixed

$$\tau = t + z/c$$

Front Form Vacuum Describes the Empty, Causal Universe

Front Form Vacuum Describes the Empty, Causal Universe

- $P^+ = \sum_i p_i^+,\ p_i^+ > 0$: LF vacuum is the state with $P^+ = 0$ and contains no particles: all other states have $P^+ > 0$ (usual vacuum bubbles are kinematically forbidden in the front form !)

- Frame independent definition of the vacuum within the causal horizon

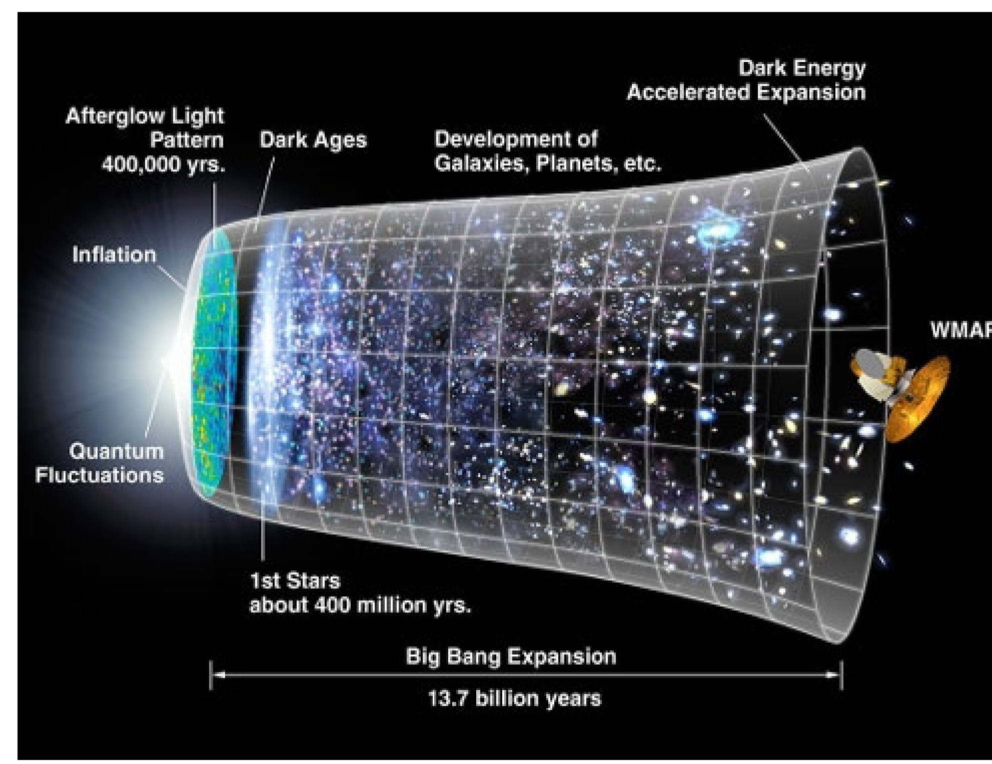
$$\boxed{P^2|0\rangle = 0}$$

(LF vacuum also has zero quantum numbers and $P^+ = 0$)

- LF vacuum is defined at fixed LF time $x^+ = x^0 + x^3$ over all $x^- = x^0 - x^3$ and $\mathbf{x}_\perp$, the expanse of space that can be observed within the speed of light

- Causality is maintained since LF vacuum only requires information within the causal horizon

- The front form is a natural basis for cosmology: universe observed along the front of a light wave



P. Srivastava, sjb

Abelian U(1) LF Model with Spontaneous Symmetry Breaking

$$\mathcal{L} = \partial_+\phi^\dagger\partial_-\phi + \partial_-\phi^\dagger\partial_+\phi - \partial_\perp\phi^\dagger\partial_\perp\phi - \mathcal{V}(\phi^\dagger\phi)$$

$$\text{where } V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \qquad \text{with } \lambda > 0,\ \mu^2 < 0$$

$$\text{Constraint equation: } \int d^2x_\perp dx^- \left[\partial_\perp\partial_\perp\phi - \frac{\delta V}{\delta\phi^\dagger}\right] = 0$$

$$\phi(\tau, x^-, x_\perp) = \omega(\tau, x_\perp) + \varphi(\tau, x^-, x_\perp)$$

$$\omega(\tau, x_\perp) \text{ is a } k^+ = 0 \text{ zero mode}$$

$$\omega = v/\sqrt{2} \text{ where } v = \sqrt{-\mu^2/\lambda}$$

Thus a c-number in LF replaces conventional Higgs VEV

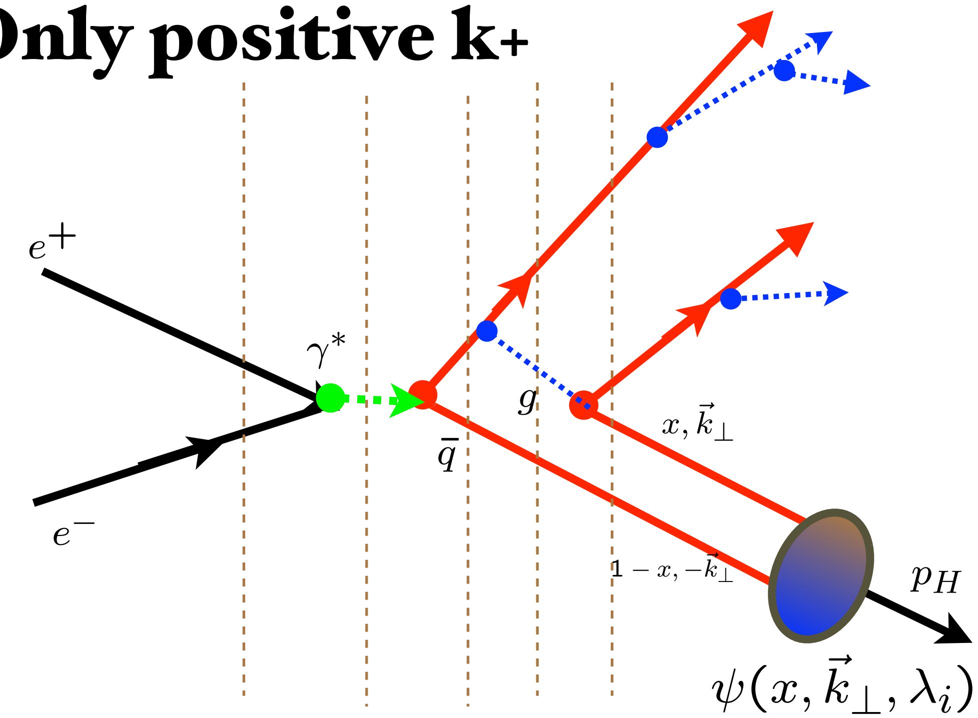
No coupling to gravity!

$$\text{Possibility: } \partial_\perp\omega \neq 0$$

Off -Shell T-Matrix

Event amplitude generator

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **J^z Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** Chueng Ji, sjb
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: g-2 to three loops**



Roskies, Suaya, sjb

Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS_5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite $N_c = 3$: Baryons built on 3 quarks -- Large N_c limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent for spacelike observables
- Origin of Linear and HO potentials: Stochastic arguments (Travinski, Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)

Advantages of the Front Form

- **Light-Front Time-Ordered Perturbation Theory: Elegant, Physical**
- **Frame-Independent**
- **Few LF Time-Ordered Diagrams (not n!) -- all k^+ must be positive**
- **J^z conserved at each vertex**
- **Cluster Decomposition -- only proof for relativistic theory**
- **Automatically normal-ordered; LF Vacuum trivial up to zero modes**
- **Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED**
- **Reproduces Parke-Taylor Rules and Amplitudes (Stasto-Cruz)**
- **Hadronization at the Amplitude Level with Confinement**

Basis LF Quantization

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,

  G.F. de Teramond, P. Sternberg, X. Zhao, E.G. Ng, C. Yang, sjb

Light-Front Holographic QCD and Emerging Confinement

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(Submitted to Physics Reports)

Applications to Collider Physics

- **Non-Perturbative Structure Functions**
- **Fundamental understanding of angular momentum**
- **Higher Fock States: Intrinsic Heavy Quarks**
- **Higgs at High x_F**
- **Hadronization at the Amplitude Level**
- **Direct Higher-Twist Processes: Violation of leading twist scaling**
- **Collisions of Flux-Tubes: Ridge effect in p-p scattering**
- **Multiparton amplitudes: Cluster decomposition, Jz conservation, Parke-Taylor**
- **Multi-gluon initiated processes: Novel nuclear effects**
- **Non-Universal Anti-shadowing**
- **Hadronization from first principles -- at the Amplitude Level**
- **Principle of Maximum Conformality**
- **Connection to Pomeron**

Goals

- **Test QCD to maximum precision at the LHC**
- **Maximize sensitivity to new physics**
- **High precision determination of fundamental parameters**
- **Determine renormalizations scales without ambiguity**
- **Eliminate scheme dependence**

Renormalization Group Invariance:

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme

Myths concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess $\mu_R = Q$ with an arbitrary range $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

Clearly heuristic. Wrong in QED. Scheme dependent!

Electron-Electron Scattering in QED

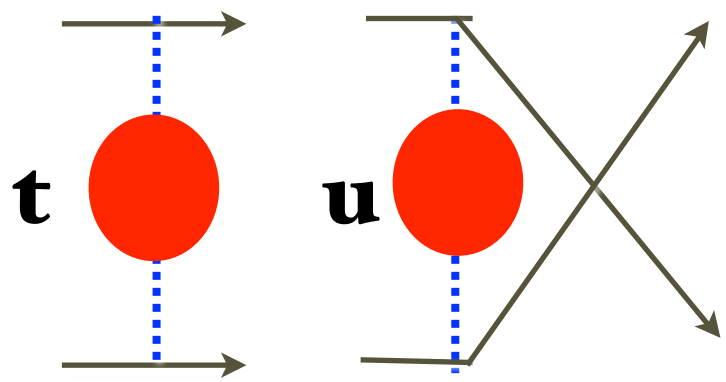
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\;\alpha(t) + \frac{8\pi s}{u}\;\alpha(u)$$

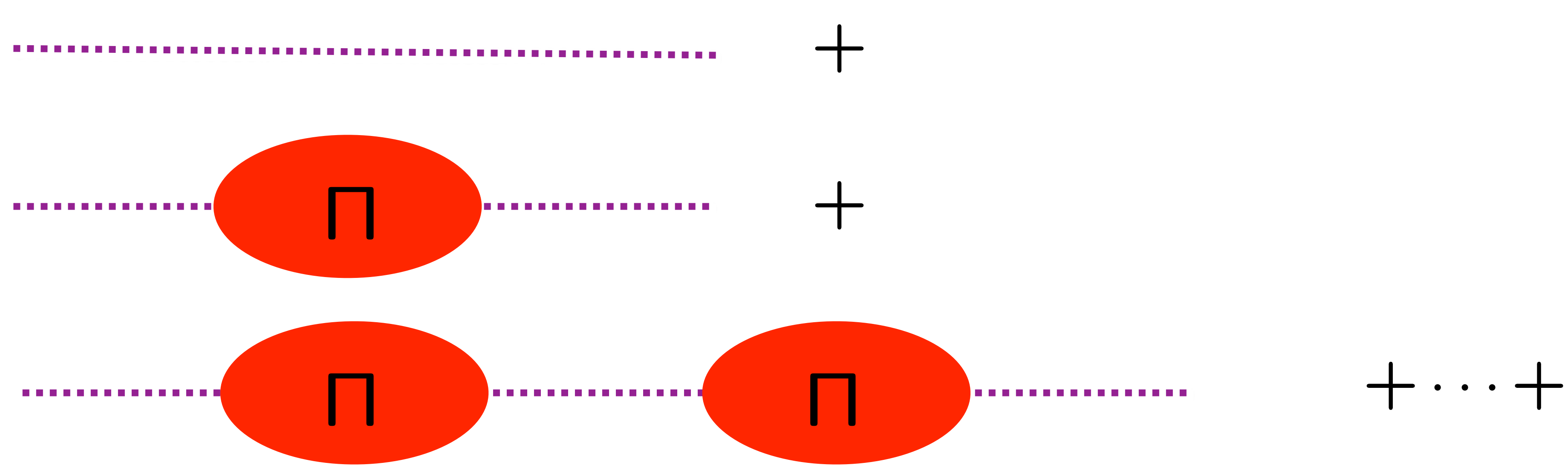


- **Two separate physical scales: t, u = photon virtuality**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

QED Running Coupling

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

All-orders lepton-loop corrections to dressed photon propagator

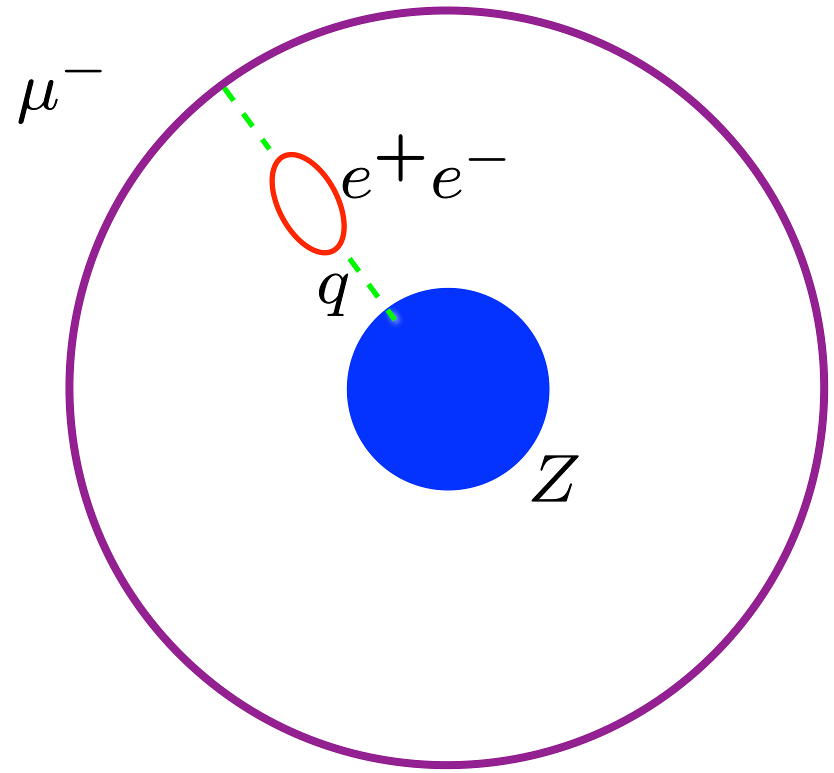


$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_0)}{1-\Pi(t_0)}$$

Initial **scale t_o is arbitrary -- Variation gives RGE Equations**

Physical renormalization scale t not arbitrary!

Example in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1 - \Pi(q^2)}$$

Scale is unique: Tested to ppm

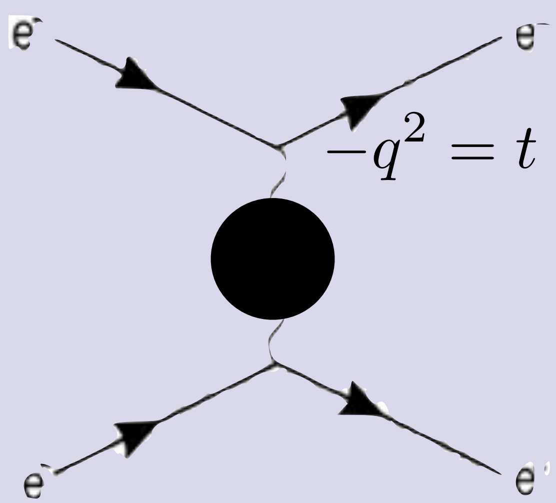
Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

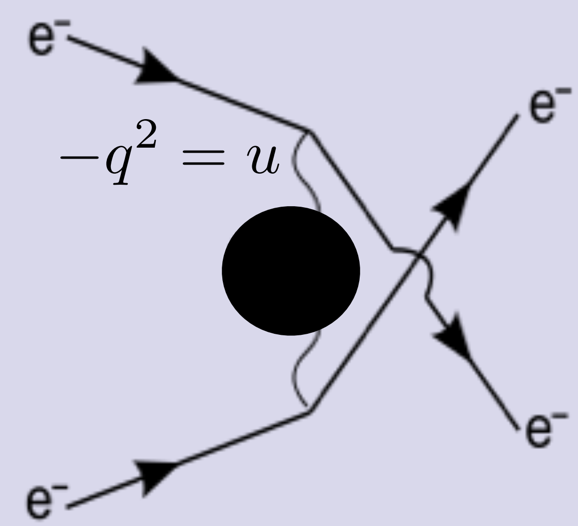
$$\alpha(t) = \frac{\alpha(t_0)}{1-\Pi(t,t_0)} \qquad \Pi(t,t_0) = \frac{\Pi(t)-\Pi(t_0)}{1-\Pi(t_0)}$$

Example: ee-scattering



$$\mathcal{M}_{ee\to ee} = \frac{8\pi s}{t}\alpha(t) + \frac{8\pi s}{u}\alpha(u)$$

Two separate scales;
one for each skeleton graph.



For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

$$\log\frac{\mu^2_{\overline{MS}}}{m_\ell^2} = 6\int_0^1 dx\, x(1-x)\log\frac{m_\ell^2 + Q^2 x(1-x)}{m_\ell^2}, \quad \xrightarrow{Q^2 \gg m_\ell^2} \log\frac{Q^2}{m_\ell^2} - \frac{5}{3}$$

$$\boxed{\alpha_{\overline{MS}}(e^{-5/3}q^2) = \alpha_{GM-L}(q^2).}$$

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Coupling at Initial Scale

Running Coupling Effects

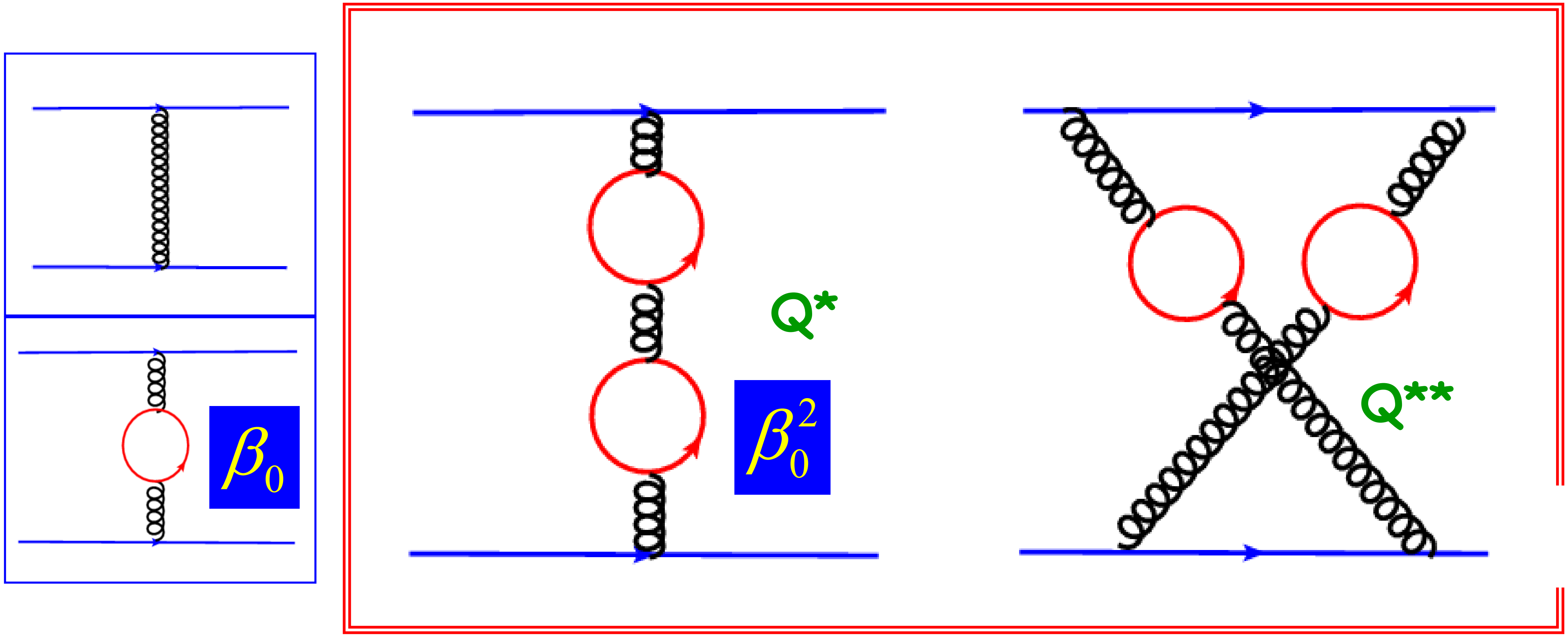
Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

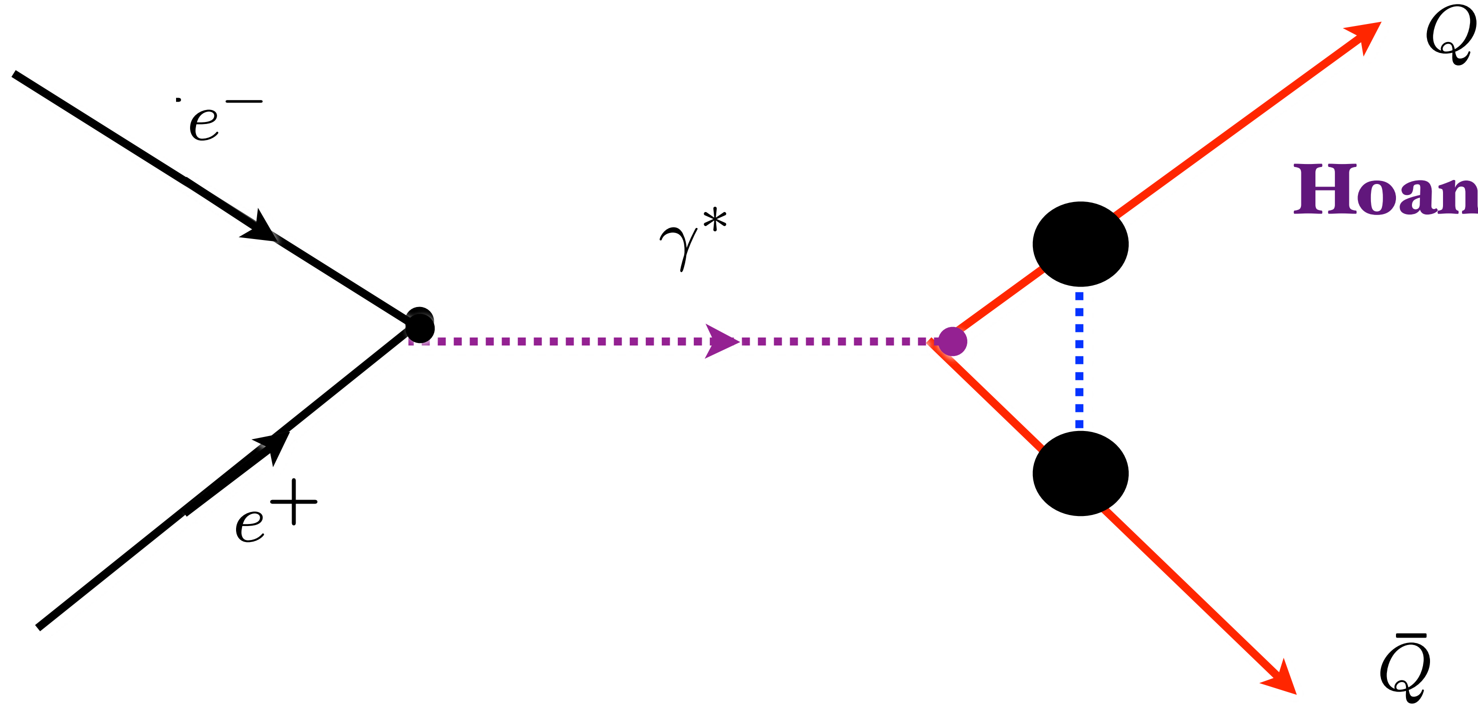
Light by Light Loops

BLM/PMC: Absorb β terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda^2_{QCD}}{Q^2}) + F(\frac{\Lambda^2_{QCD}}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$



New Scales Appear at Higher Order



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = [1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}] \times [1 + \frac{\pi\alpha_s(sv^2)}{4v}]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Angular distributions of massive quarks and leptons close to threshold.

BLM/PMC: Set Scales

such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$
\begin{aligned}
\rho(Q^2) = r_{0,0} & + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1} \\
& + (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3} \\
& + r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1} \\
& + \cdots
\end{aligned}
$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

How do we identify the β terms?

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln\frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\mathrm{MS}}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu^2_\delta = \mu^2_{\overline{\mathrm{MS}}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\text{MS}}\ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \qquad \mu^2 = \mu^2_{\overline{\text{MS}}} \exp(\ln 4\pi - \gamma_E)\ , \quad \mu^2_{\delta_2} = \mu^2_{\delta_1} \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \text{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

Special Degeneracy in PQCD

There is nothing special about a particular value for δ, thus for any δ

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 \underline{+ \cdots)r_{2,1}}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 \underline{+ \cdots)r_{3,2}} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 \underline{+ \cdots)r_{3,1}}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

$$a_{\mathcal{S}} = \frac{\alpha_{\mathcal{S}}}{4\pi} = \frac{g_{\mathcal{S}}^2}{16\pi^2}$$

$$\frac{\mathrm{d}a_{\mathcal{S}}}{\mathrm{d}\ln\mu^2} = \beta_{\mathcal{S}}(a) = -a^2[\beta_0 + \beta_1 a + \beta_2^{\mathcal{S}} a^2 + \beta_3^{\mathcal{S}} a^3 + \cdots]$$

$$\beta_0 = \frac{11}{3}N_C - \frac{2}{3}n_F$$

QED: N_C=0

Relating different renormalization scales:

Taylor expanding $a(\mu)$ around $\ln(\mu_0)$:

$$a(\mu) = a(\mu_0) - \beta_0 a(\mu_0)^2 \ln\frac{\mu^2}{\mu_0^2} - \left[\beta_1 - \beta_0^2 \ln\frac{\mu^2}{\mu_0^2}\right] a(\mu_0)^3 \ln\frac{\mu^2}{\mu_0^2} + \cdots$$

M. Mojaza, Xing-Gang Wu, sjb

General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &\quad + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5)\end{aligned}$$

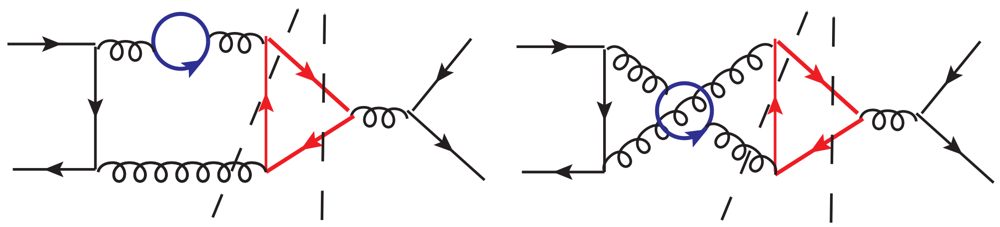
PMC scales thus satisfy

$$\begin{aligned} r_{1,0}a(Q_1) &= r_{1,0}a(Q) - \beta(a) r_{2,1} \\ r_{2,0}a(Q_2)^2 &= r_{2,0}a(Q)^2 - 2a(Q)\beta(a) r_{3,1} \\ r_{3,0}a(Q_3)^3 &= r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a) r_{4,1} \\ &\vdots \\ r_{k,0}a(Q_k)^k &= r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a) r_{k+1,1}\end{aligned}$$

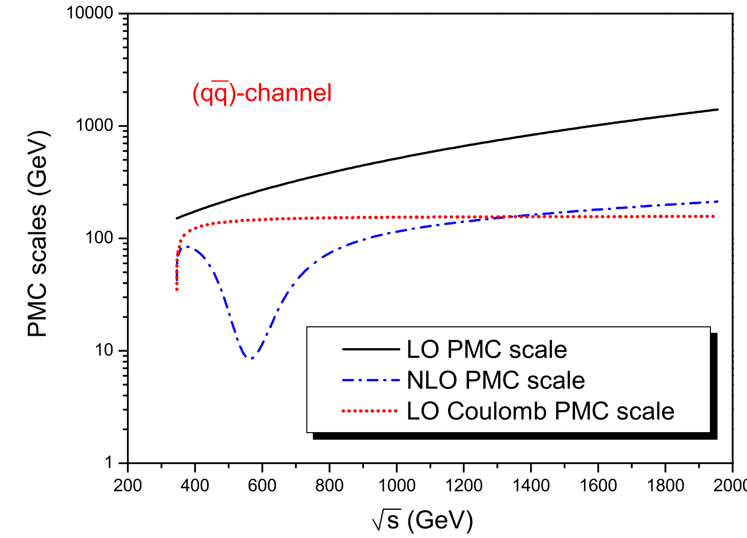
Important Example: Top-Quark FB Asymmetry

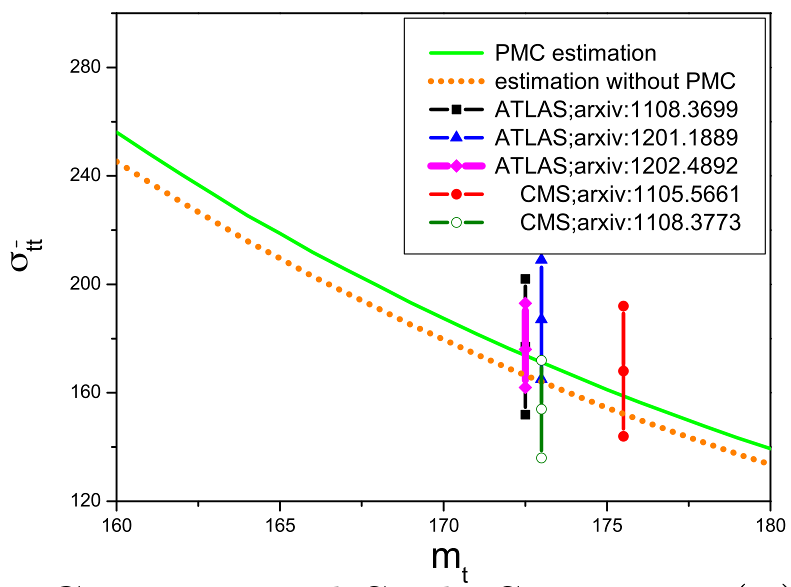
Brodsky, Wu, **Phys.Rev.Lett. 109**, [arXiv:1203.5312]

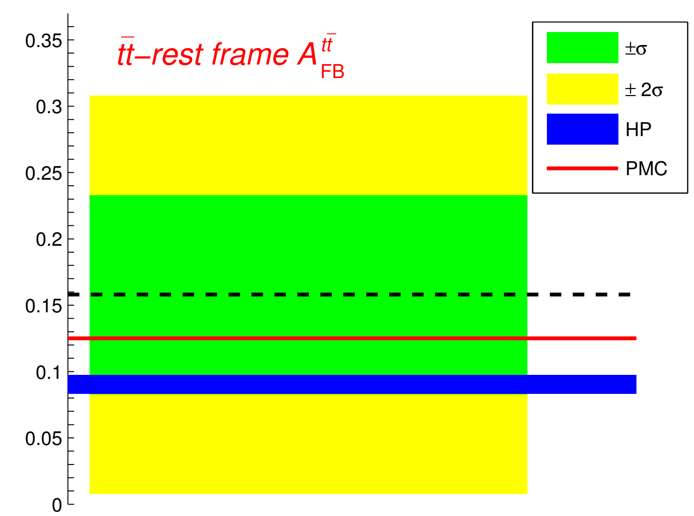
$$A_{FB}^{t\bar{t}} = \frac{\sigma(y_t^{t\bar{t}} > 0) - \sigma(y_t^{t\bar{t}} < 0)}{\sigma(y_t^{t\bar{t}} > 0) + \sigma(y_t^{t\bar{t}} < 0)}$$

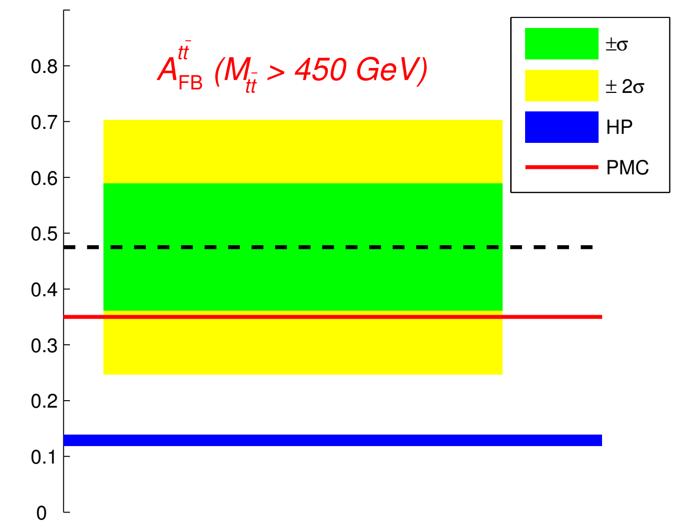


$$\mu_r \neq \mu_f \ (!)$$









Conventional Scale Setting: $\alpha(\mu) = \alpha_{\overline{MS}}(\mu)$ and $\mu = [\frac{1}{2}Q, 2Q]$

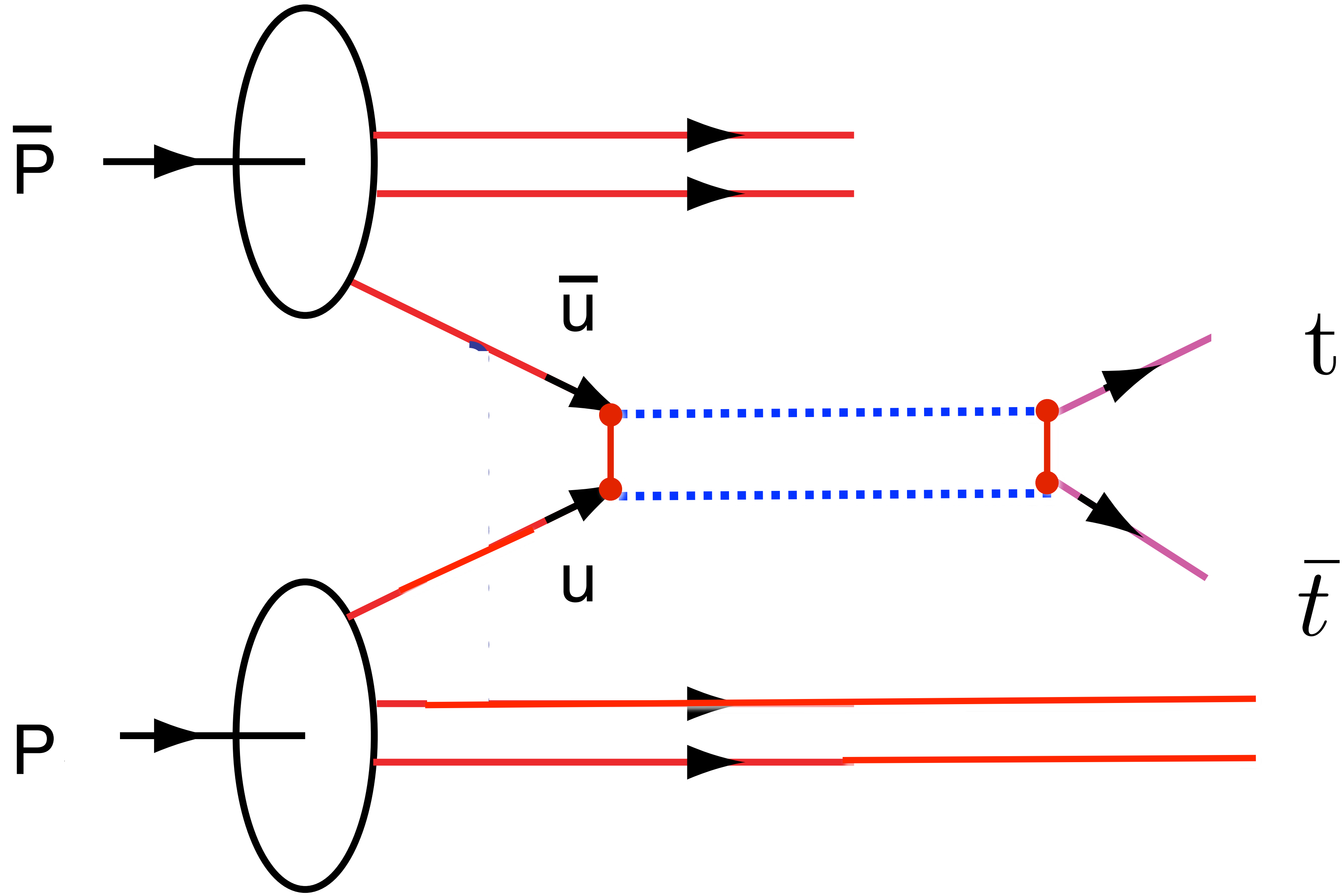
HP: Hollik, Pagani, **Phys.Rev. D84**(2011)

Conventional 'uncertainty estimate' can be misleading

(see also Blumlein & van Neerven, Phys.Lett. B450, 417[1999])

Improving pQCD precision important for exposing new physics correctly!

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron

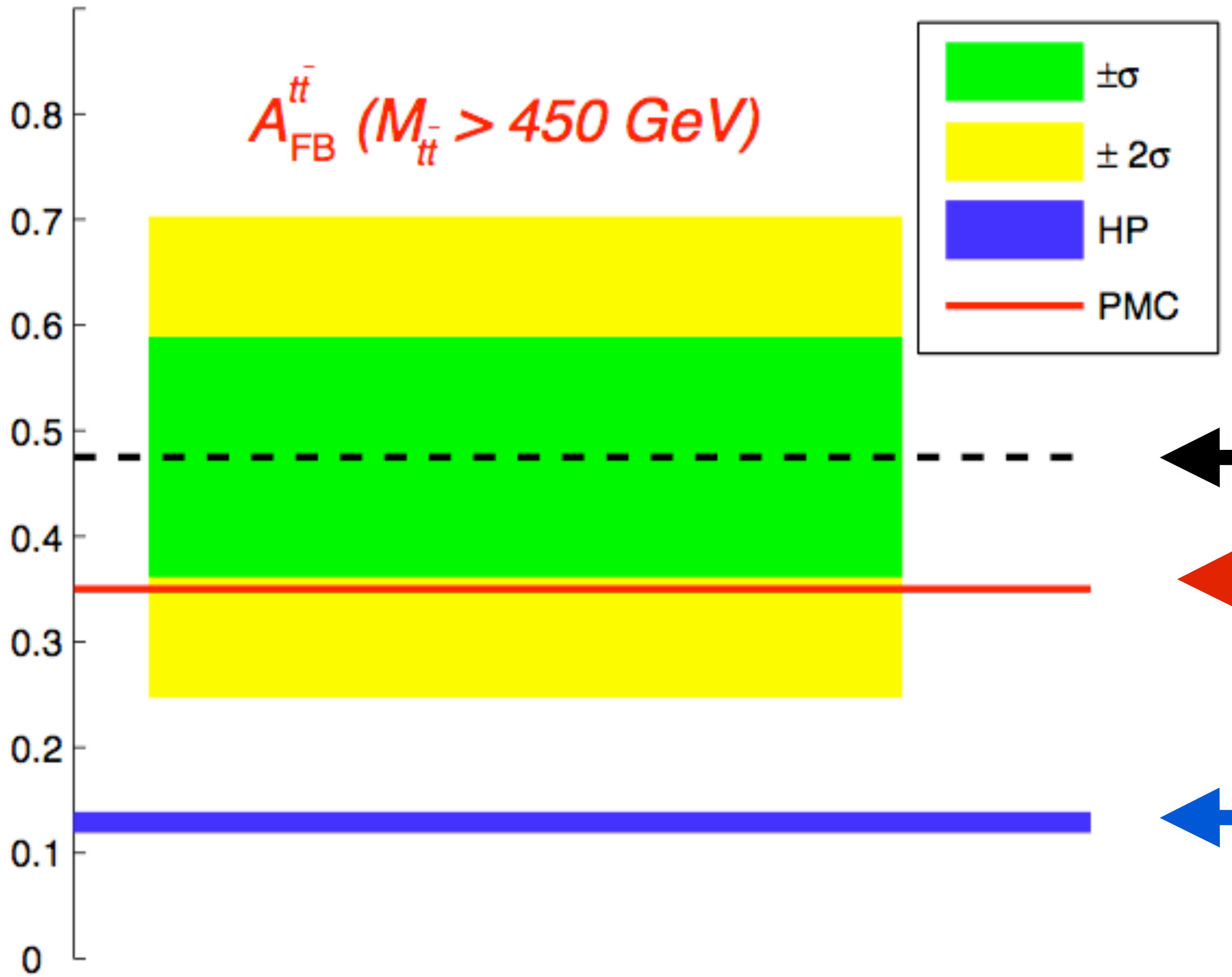


Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED

Xing-Gang Wu, sjb

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
SJB

Experimental asymmetry

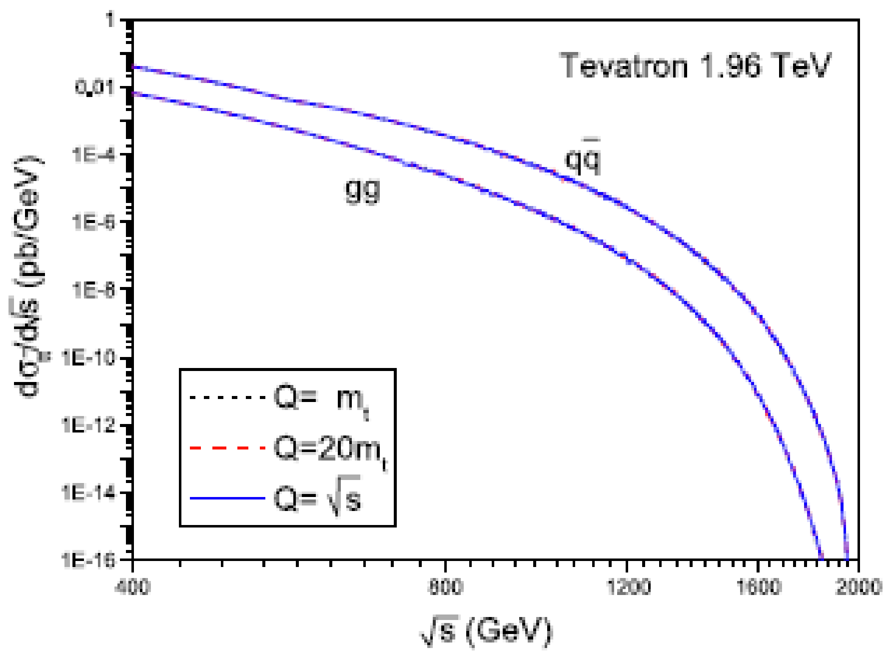
PMC Prediction

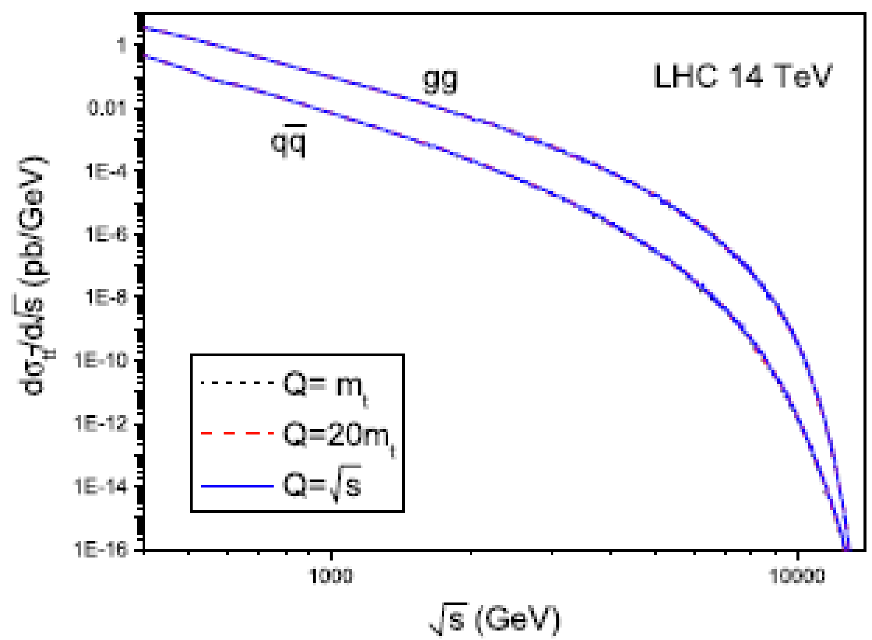
Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

total differential cross-section versus √s (very small initial scale-dependence)

$$\boxed{d\sigma_{t\bar{t}}/d\sqrt{s} \propto \left(2\sqrt{s}\mathcal{L}_{ij}\hat{\sigma}_{ij}\right)}$$





total cross-section almost unchanged !

	PMC scale-setting					Conventional scale-setting		
	$Q=m_t/4$	$Q=m_t/2$	$Q=m_t$	$Q=2m_t$	$Q=4m_t$	$\mu_r \equiv m_t/2$	$\mu_r \equiv m_t$	$\mu_r \equiv 2m_t$
Tevatron (1.96 TeV)	7.620(5)	7.622(5)	7.626(3)	7.622(6)	7.623(6)	7.742(5)	7.489(3)	7.199(5)
LHC (7 TeV)	171.6(1)	171.7(1)	171.8(1)	171.7(1)	171.7(1)	168.8(1)	164.6(1)	157.5(1)
LHC (14 TeV)	941.8(8)	941.9(8)	941.3(5)	941.4(8)	941.4(8)	923.8(7)	907.4(4)	870.9(6)

unchanged, within error 10^{-3}

3%-4%

$10m_t$ ($20m_t$) =>15%(19%)

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

$\downarrow$

Choose μ_R^{init}*; arbitrary initial renormalization scale*

$\downarrow$

Identify $\{\beta_i^R\}$ *– terms using* n_f *– terms*

through the PMC – BLM correspondence principle

$\downarrow$

Shift scale of α_s *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\}$ *– terms*

$\downarrow$

Conformal Series

Result is independent of $\mu_R^{\rm init}$ *and scheme at fixed order*

Principle of Maximum Conformality

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at N_C=0

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

δ-Scheme automatically identifies β-terms!

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

$$\bar{R}_{e^+e^-}(s) = \frac{1}{2\pi i}\int_{-s-i\epsilon}^{-s+i\epsilon}\frac{\bar{D}(Q^2)}{Q^2}dQ^2 \qquad \bar{D}(Q^2) = \gamma(a) - \beta(a)\frac{d}{da}\Pi(Q^2,a)$$

Initial expression

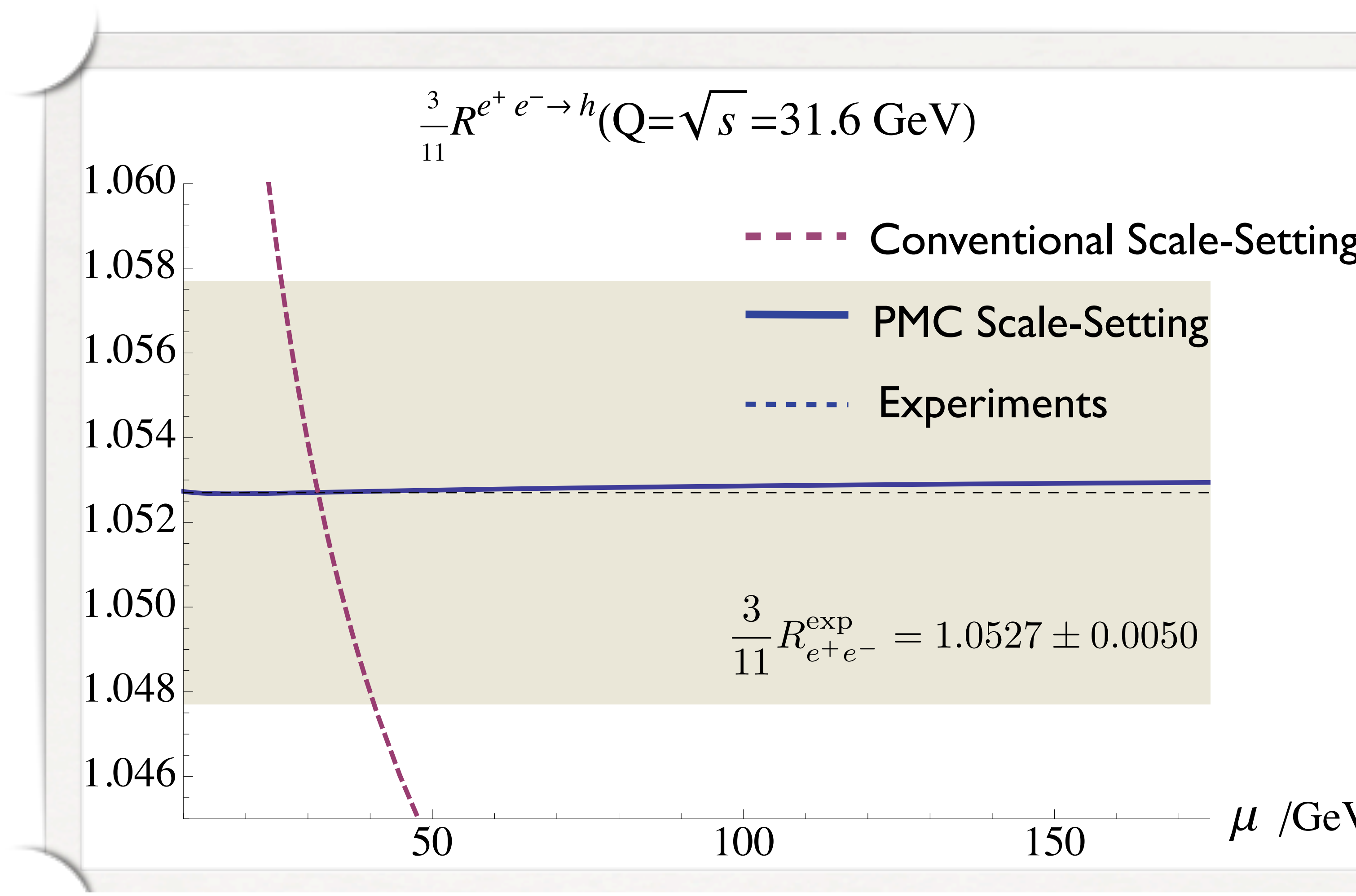
$$\bar{R}_{e^+e^-}(s) = \gamma_0 + \gamma_1 a(\mu) + [\gamma_2 + \beta_0 \underline{\Pi_1}]a(\mu)^2 + [\gamma_3 + \beta_1 \underline{\Pi_1} + 2\beta_0 \underline{\Pi_2} - \beta_0^2 \frac{\pi^2\gamma_1}{\underline{3}}]a(\mu)^3$$
$$+ [\gamma_4 + \beta_2 \underline{\Pi_1} + 2\beta_1 \underline{\Pi_2} + \beta_0\Pi_3 - \frac{5}{2}\beta_0\beta_1\frac{\pi^2\gamma_1}{\underline{3}} - 3\beta_0^2\frac{\pi^2\gamma_2}{3} - \beta_0^3\pi^2\Pi_1]a(\mu)$$

Final expression

$$\bar{R}_{e^+e^-}(Q) = \gamma_0 + \gamma_1 a(Q_1) + \gamma_2 a(Q_2)^2 + \gamma_3 a(Q_3)^3 + \gamma_4 a(Q_4)^4$$

Final PMC Scales

$$Q_1 = 1.3\ Q,\ Q_2 = 1.2\ Q,$$
$$Q_3 = 5.3\ Q,\ Q_4 \sim Q$$



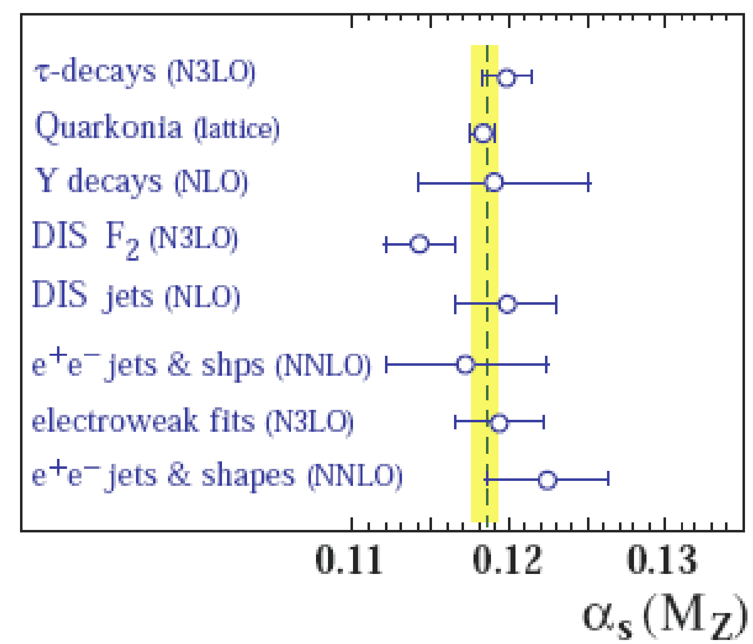
taking the experimental results for R(Q)

From the experimental value, $r_{e^+e^-}(31.6GeV) = \frac{3}{11}R_{e^+e^-}(31.6GeV) = 1.0527 \pm 0.0050$ [26], we obtain

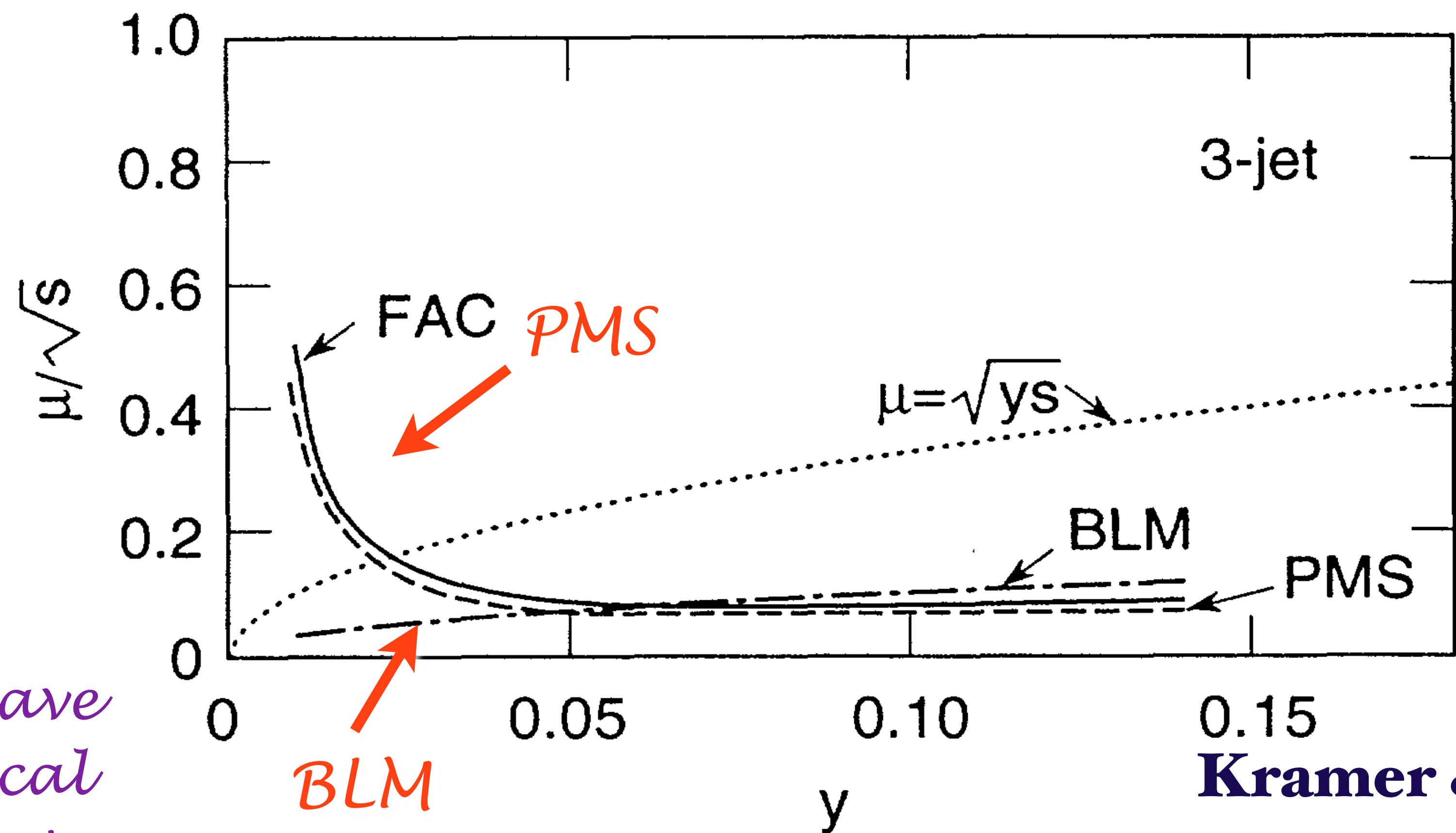
$$\Lambda^{'tH}_{\overline{MS}} = 412^{+206}_{-161}\mathrm{MeV}$$

$$\Lambda_{\overline{MS}} = 359^{+181}_{-140}\mathrm{MeV}$$

$$\alpha_s^{\overline{MS}}(M_Z) = 0.129^{+0.009}_{-0.010}$$



Three-jet production in electron-positron annihilation



Kramer & Lampe

Jet Invariant mass squared:

$$\mathcal{M}^2 = ys$$

BLM scale is gluon jet virtuality

PMS & FAC have wrong physical dependence!

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in e^+e^- annihilation, as computed by Kramer and Lampe Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low y. In particular, the latter two methods predict increasing values of μ as the jet invariant mass $\mathcal{M} < \sqrt{(ys)}$ decreases.

Other Jet Observables using BLM: Rathsman

PMS vs. PMC

- **PMS/FAC - incorrectly sums conformal terms -- even minimizes physical asymmetries!   PMS violates transitivity**
- **PMC/BLM: exposes conformal series - no renormalons**
- **Conformal series has new physics -- not associated with renormalization**
- **PMC: No need to analyze diagrams or codes -- simply identify non-conformal logarithms -- then shift scale**
- **PMC: Applies to subprocesses with multiple final particles- recursive procedure**
- **PMC/BLM: Agrees with QED in Abelian limit**
- **PMC/BLM: Result is independent of scheme and initial scale choice**

Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

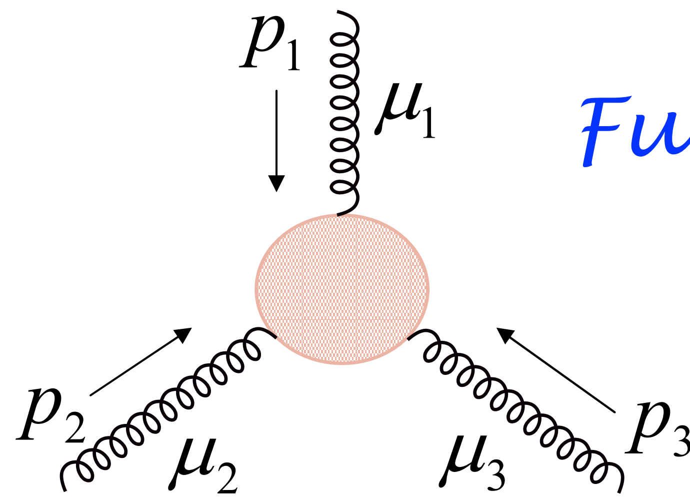
Lepage, Mackenzie, sjb
Phys.Rev.D28:228,1983

- **All terms associated with nonzero beta function summed into running coupling**
- **PMC/BLM Scale Q* sets the number of active flavors**
- **Only n_f dependence (associated with renormalization) required to determine renormalization scale at NLO**
- **Result is scheme independent! Q* has exactly the correct dependence to compensate for change of scheme**
- **Correct Abelian limit**
- **Resulting series identical to conformal series!**
- **Renormalon n! growth of PQCD coefficients from β function eliminated!**
- **In general, PMC/BLM scale depends on all invariants**

Binger, sjb
H. J. Lu

General Structure of the Three-Gluon Vertex

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



Full analytic calculation, general masses, spin Pinch Scheme

3 index tensor $\hat{\Gamma}_{\mu_1\mu_2\mu_3}$ built out of $g_{\mu\nu}$ and p_1, p_2, p_3

with $p_1 + p_2 + p_3 = 0$

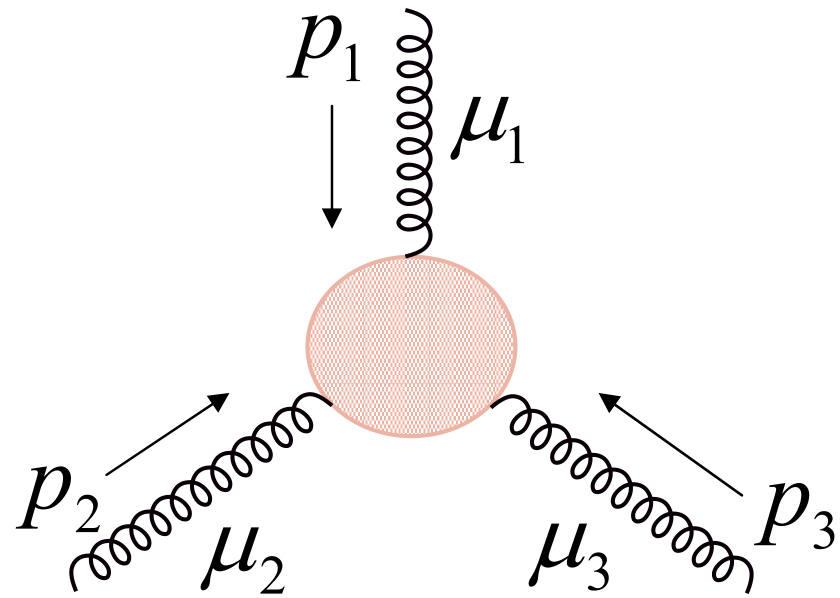
⇒ 14 basis tensors and form factors

PHYSICAL REVIEW D 74, 054016 (2006)

Form factors of the gauge-invariant three-gluon vertex

Michael Binger* and Stanley J. Brodsky†

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



H. J. Lu
Binger, sjb

$$\mu_R^2 \simeq \frac{p^2_{min} p^2_{med}}{p^2_{max}}$$

Scale also determines effective number of flavors

Grunberg

Effective Charges: analytic at quark mass thresholds, finite at small momenta

$$R_{e^+e^- \to X}(s) \equiv 3\Sigma_q e_q^2 \; [1 + \frac{\alpha_R(s)}{\pi}]$$

$$\Gamma(\tau \to X e \nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u \bar{d} e \nu) \times [1 + \frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Commensurate scale relations:

Relate observable to observable at commensurate scales

Lu, Kataev, Gabadadze, Sjb

Lu, Kataev, Gabadadze, Sjb

Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q{}^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right] .$$

$$\int_0^1 dx \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$
\begin{aligned}
\frac{\alpha_R(Q)}{\pi} = {} & \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3}\zeta_3\right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3}\zeta_3\right) f\right] \\
& + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \Bigg\{ \left(\frac{90445}{2592} - \frac{2737}{108}\zeta_3 - \frac{55}{18}\zeta_5 - \frac{121}{432}\pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12}\zeta_3 + \frac{55}{3}\zeta_5\right) C_A C_F - \frac{23}{32} C_F^2 \\
& + \left[\left(-\frac{970}{81} + \frac{224}{27}\zeta_3 + \frac{5}{9}\zeta_5 + \frac{11}{108}\pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6}\zeta_3 - \frac{10}{3}\zeta_5\right) C_F\right] f \\
& + \left(\frac{151}{162} - \frac{19}{27}\zeta_3 - \frac{1}{108}\pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6}\zeta_3\right) \frac{d^{abc} d^{abc}}{C_F d(R)} \frac{\left(\sum_f Q_f\right)^2}{\sum_f Q_f^2} \Bigg\}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\alpha_{g_1}(Q)}{\pi} = {} & \frac{\alpha_{\overline{\rm MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^2 \left[\frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f\right] \\
& + \left(\frac{\alpha_{\overline{\rm MS}}(Q)}{\pi}\right)^3 \Bigg\{ \left(\frac{5437}{648} - \frac{55}{18}\zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9}\zeta_3\right) C_A C_F + \frac{1}{32} C_F^2 \\
& + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta_3 + \frac{5}{9}\zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta_3\right) C_F\right] f + \frac{115}{648} f^2 \Bigg\}.
\end{aligned}
$$

Eliminate MS
Find Amazing Simplification

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q{}^2 \left[1 + \frac{\alpha_R(Q)}{\pi}\right].$$

$$\int_0^1 dx \left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right] \equiv \frac{1}{3}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_{g_1}(Q)}{\pi}\right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left(\frac{\alpha_R(Q^{**})}{\pi}\right)^2 + \left(\frac{\alpha_R(Q^{***})}{\pi}\right)^3$$

Geometric Series in Conformal QCD

Generalized Crewther Relation

Lu, Kataev, Gabadadze, Sjb

Lu, Kataev, Gabadadze, Sjb

Generalized Crewther Relation

$$[1+\frac{\alpha_R(s^*)}{\pi}][1-\frac{\alpha_{g_1}(q^2)}{\pi}]=1$$

$$\sqrt{s^*}\simeq 0.52Q$$

Conformal relation true to all orders in perturbation theory!

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

No renormalization scale ambiguity!

Both observables go through new quark thresholds at commensurate scales!

Essential Points

- ***Physical Results cannot depend on choice of scheme***
- ***Different PMC scales at each order***
- ***No scale ambiguity!***
- ***Series identical to conformal theory***
- ***Relation between observables scheme independent, transitive***
- ***Choice of initial scale irrelevant even at finite order***
- ***Identify β terms using R_δ method***

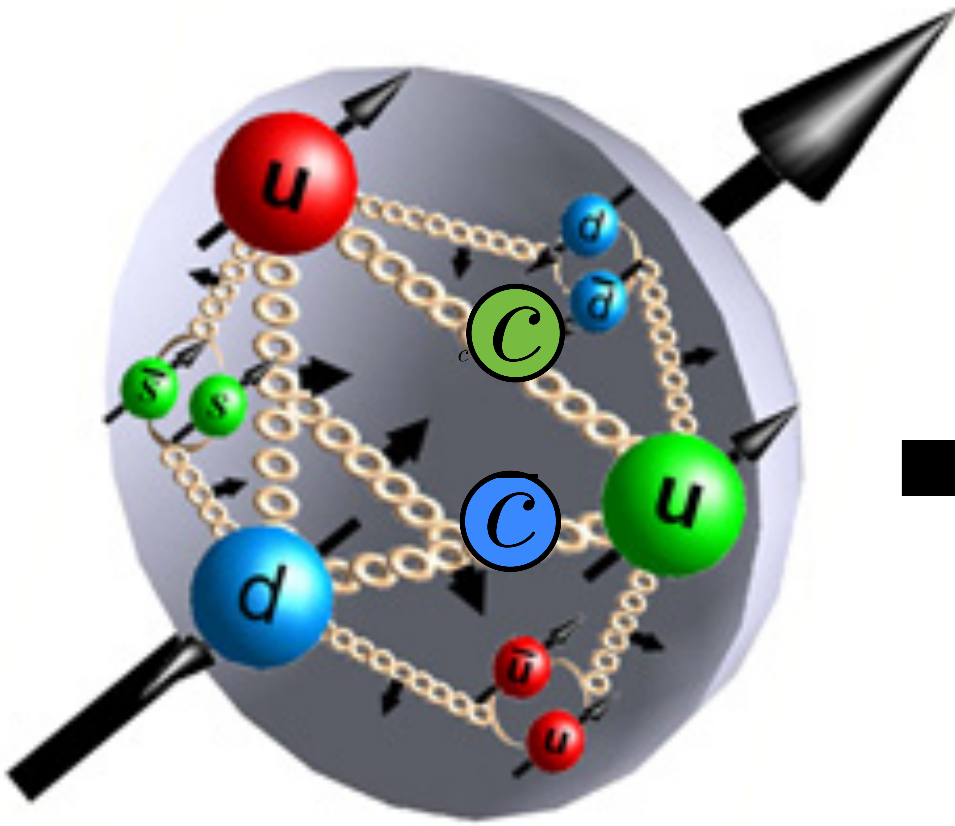
Scale Variations

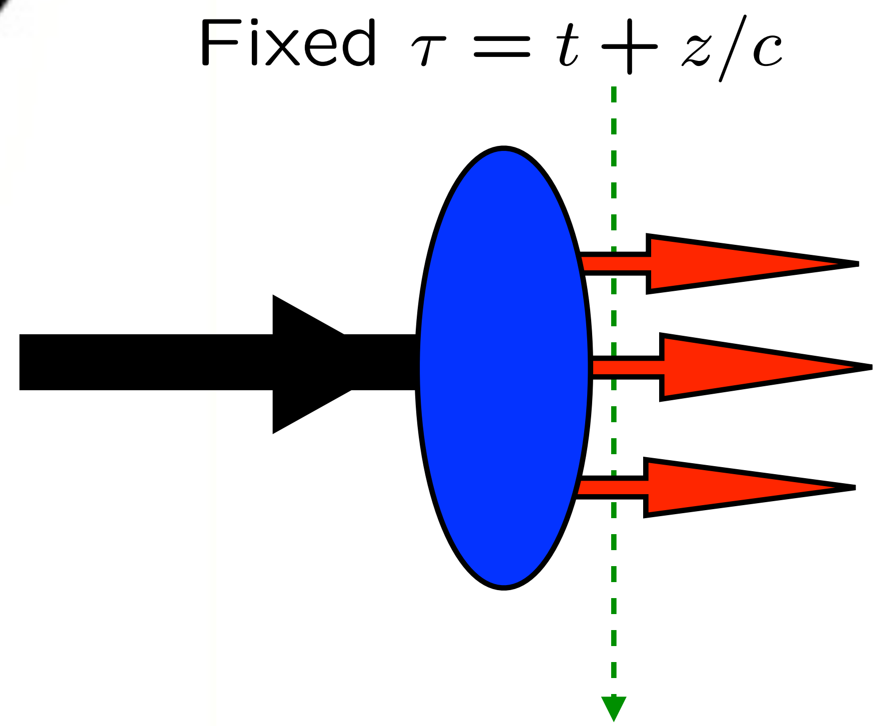
- Initial scale variation tests accuracy of PMC implementation
- Final scale variation away from PMC introduces nonconformal and divergent n! renormalon terms
- Destroys scheme independence of PMC
- Does not expose true conformal higher order effects
- Electron loop light-by-light scattering in muon g-2: huge sixth order conformal term not exposed by renormalization scale variation

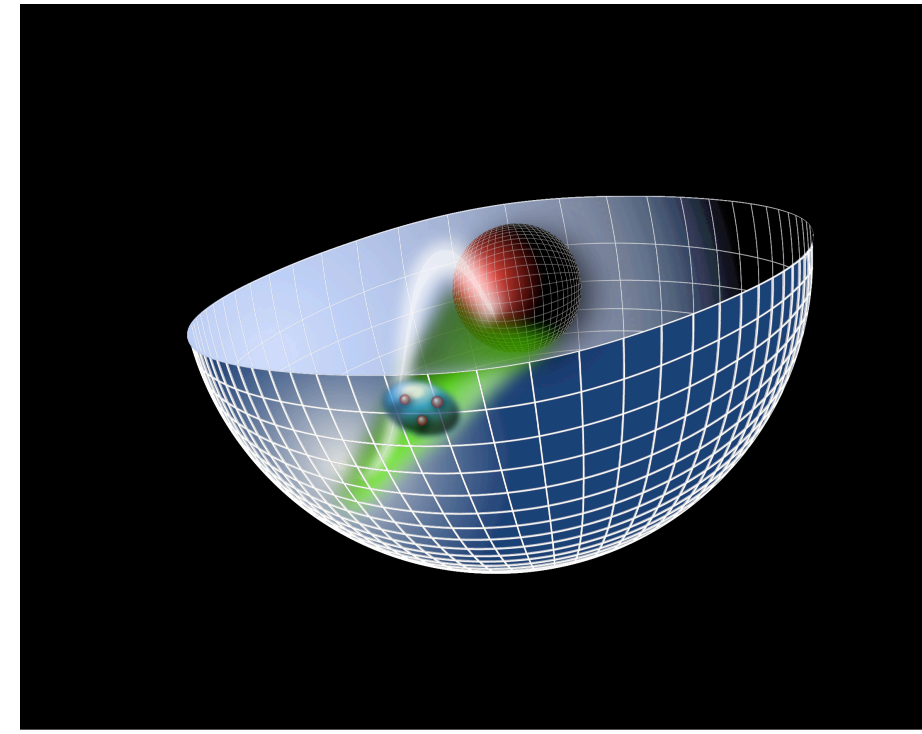
QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- QCD gives 10^{42} to the cosmological constant
- QCD Confinement and Mass Scale from $\Lambda_{\overline{MS}}$

The Light-Front Schrödinger Equation
A New Approach to Color Confinement and Non-Perturbative QCD







Strong Interactions in the LHC Era

Wilhelm und Else Heraeus Stiftung

Bad Honnef
November 13, 2014

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

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