Higgsing lattice gauge theories with strongly interacting fermions

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Motivations

- Gauge symmetry breaking plays a crucial role in
    - Standard Model - electroweak symmetry breaking
    - Grand Unified theories
    - BCS theory of superconductivity
- Green typically put in ``by hand" thru vev of scalar
- Problematic - A lot of fine tuning required to keep scalar sector insensitive to high energies (naturalness)

Long history of efforts to replace scalar by bound state of fermions or condensate - dynamical symmetry breaking

Dynamical symmetry breaking

- Problem: fermion condensates typically arise through non-perturbative effects in strongly coupled theories
- Difficult to study with analytical methods. But candidates for study using lattice gauge theory
- Lots of lattice work recently searching for models with near conformal dynamics - candidate walking technicolor theories as alternative to SM Higgs ...

Barriers to lattice approaches

- All lattice studies to date focus on the strong dynamics - and leave out the broken sector... (put in later using p theory ...)
- Why ? Several results seem to prohibit typical continuum symmetry breaking scenarios when moved over to the lattice
  - Can only gauge exact symmetries.
  - lattice constructions favor/require vector symmetries
  - Vafa-Witten theorem prohibits spontaneous symmetry breaking of vector symmetries
  - Forced to think of chiral/axial symmetries - but no lattice chiral fermion or exact lattice chiral symmetry

Nielson-Ninomyia

Punchline

- Possible to construct a model in which exact lattice symmetries are spontaneously broken due to strongly coupled fermion dynamics
- The broken symmetries which start out as vector-like transform into axial symmetries in continuum limit
- After gauging weak symmetries at non-zero lattice spacing yields dynamical Higgs mechanism.
- Higgs phase survive continuum limit ?

Staggered fermions

One popular lattice fermion used in QCD is staggered quark

$$S = \sum_{x,\mu} \eta_\mu(x)\overline{\chi}(x)\left[\chi(x+\mu) - \chi(x-\mu)\right] + m\overline{\chi}(x)\chi(x)$$

$$\eta_\mu(x) = (-1)^{\sum_{i=1}^{\mu} x_i}$$

Describes 4 Dirac fermions in continuum limit.

Decompose $\overline{\chi} = (\overline{\psi}_+, \overline{\lambda}_-)$ $\quad \chi = (\psi_-, \lambda_+)$

where eg. $\psi_-(x) = \frac{1}{2}\left(1 - \epsilon(x)\right)\chi(x)$ $\quad$ with $\epsilon(x) = (-1)^{\sum_{i=1}^{4} x_i}$

$$S = \sum_{x,\mu} \eta_\mu(x)\left[\overline{\psi}_+(x)D_\mu\psi_-(x) + \overline{\lambda}_-(x)D'_\mu\lambda_+(x)\right]$$

m=0

derivatives may be different

Internal symmetries can be different:

$$\overline{\psi}_+(x) \to \overline{\psi}_+(x)G^\dagger(x) \qquad \overline{\lambda}_-(x) \to \overline{\lambda}_-(x)H^\dagger(x)$$
$$\psi_-(x) \to G(x)\psi_-(x) \qquad \lambda_+(x) \to H(x)\lambda_+(x)$$

Additional comments ..

- Can throw away eg blue fields: reduced staggered fermion. Now just 2 Dirac fermions in continuum. No single site mass term possible $\overline{\chi}\chi = \overline{\psi}_+ \lambda_+$
- Instead use single link mass $\overline{\psi}_+(x) U_\mu(x) \psi_-(x+\mu)$
- Gauging staggered fermions easy:

$$D\psi = \frac{1}{2}\left(U_\mu(x)\psi(x+\mu) - U^\dagger_\mu(x-\mu)\psi(x-\mu)\right)$$

One example - technicolor-like model

Assume gauge group factors into strong and weak sectors:

$$SU(N)_{strong} \times [SU(M) \times SU(M)]_{weak}$$

$$\beta_S = \frac{2N}{g_S^2},\ \beta_W = \frac{2M}{g_W^2}\, and\ r = \frac{\beta_W}{\beta_S} >> 1$$

Fermions transforming as:

$$\psi : (\Box, \Box, 1) \qquad \lambda : (\Box, 1, \Box)$$

$$\Box \equiv \text{fundamental rep}$$

Condensates

First switch off weak gauge coupling.

For large enough strong coupling expect a condensate of form

$$< \overline{\psi}_+(x)\lambda_+(x) > + < \overline{\lambda}_-(x)\psi_-(x) > \neq 0$$

continuum

$\overline{\psi}_L\lambda_R + \ldots$

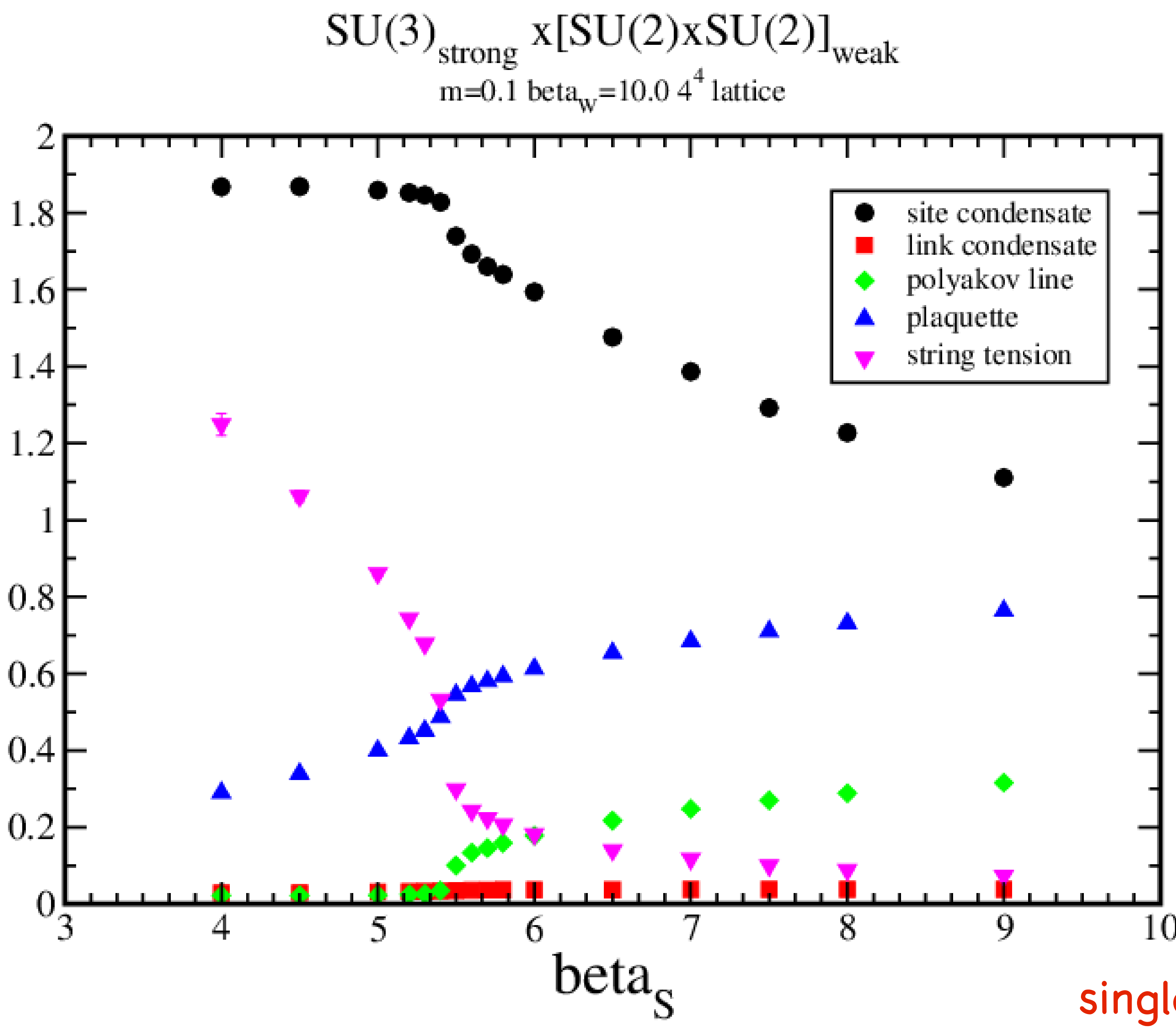
By construction singlet under strong force but will spontaneously break weak symmetries.

SU(M)xSU(M)->SU(M)$_{diag}$

Symmetry preserving condensate also possible

$$\epsilon(x)\xi_\mu(x) < \overline{\psi}_+(x)U_\mu(x)V_\mu(x)\psi_-(x+\mu) >$$

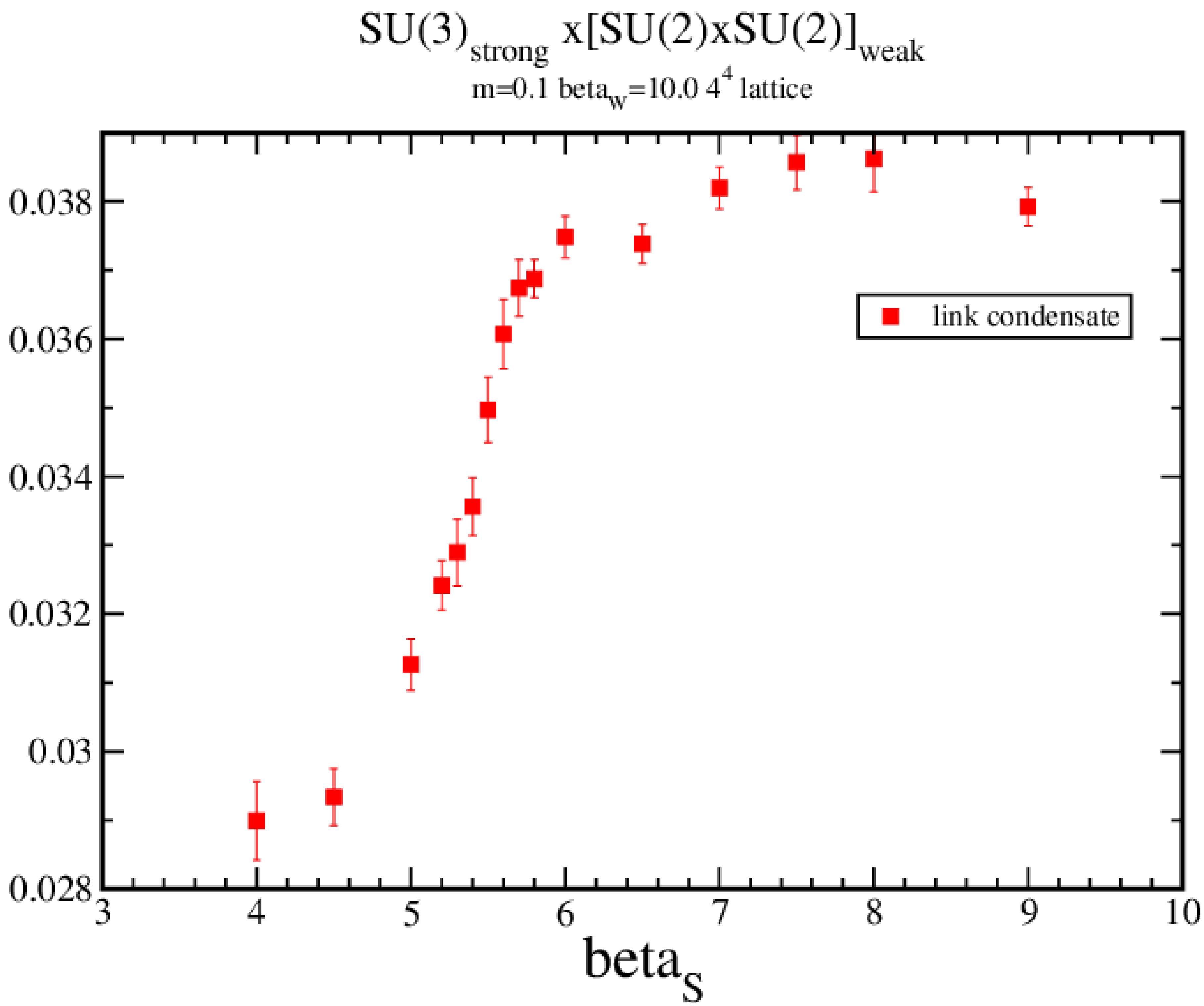
Phase structure vs β_S



SU(3) strong
SU(2) weak

single site condensate
strongly preferred
over symmetry preserving link
condensate

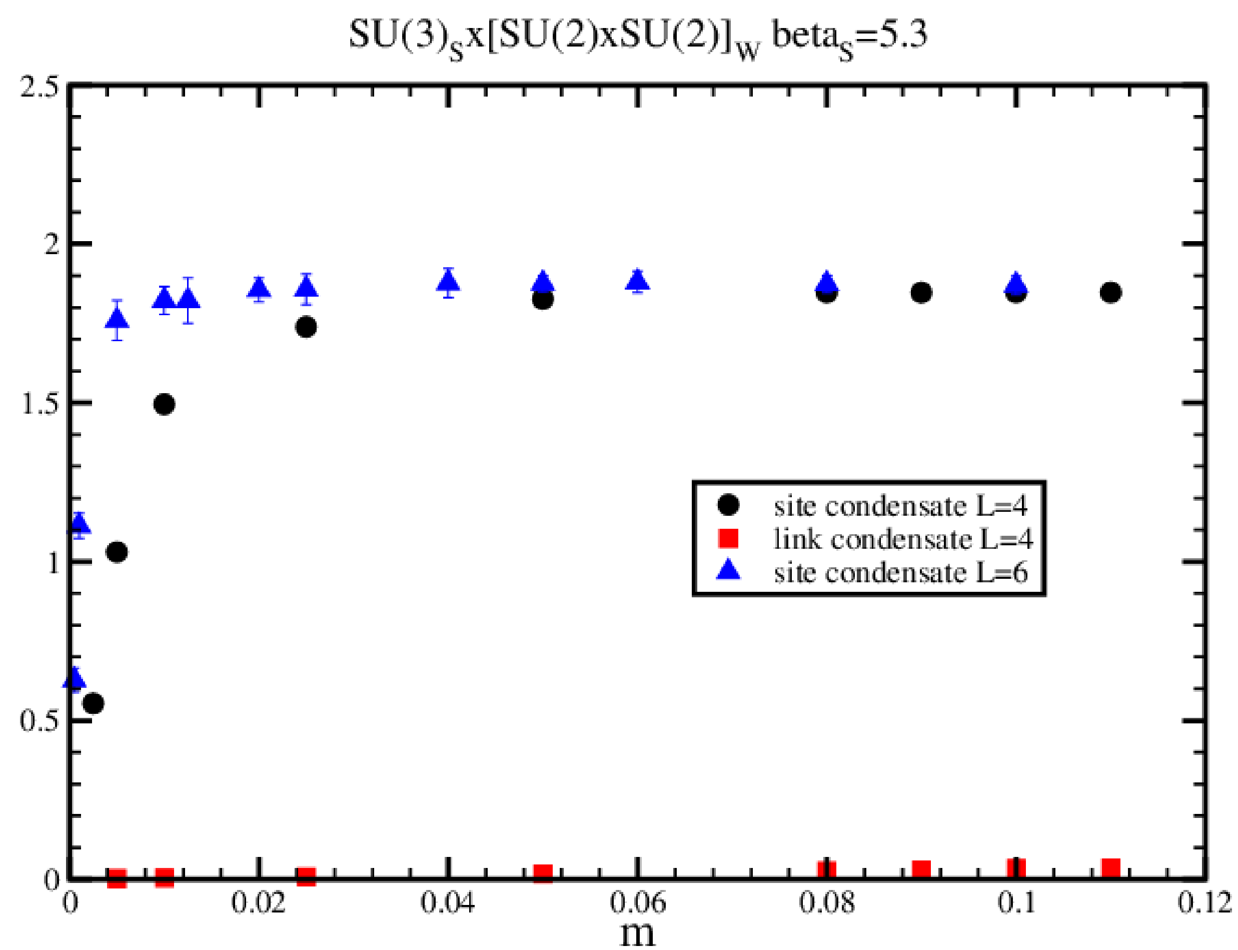
Single link condensate



Spontaneous symmetry breaking

Condensates computed in presence of source m
Interested in m->0 after thermodynamic limit

Stable plateau
for small m
Turns over for
m<1/V



Vafa Witten theorem

- Theorem prohibits spontaneous breaking of vector symmetries - continuum symmetries where

$$\psi \to G\psi, \qquad \overline{\psi} \to \overline{\psi} G^\dagger$$

- The symmetries here seem to be of this form .. can condensate survive continuum limit ?

- Yes ! In continuum replace staggered fields by Dirac $\Psi = (\psi, \lambda)$

$$\Psi^\alpha \Sigma^{\alpha\beta} \Psi^\beta = (\overline{\psi}\,\overline{\lambda}) \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} \psi \\ \lambda \end{pmatrix}$$

Chiral transform $\Sigma \to I$
broken generators pick up factor of γ_5

Broken symmetries
axial in background of
this condensate!

Gauging weak sector

When weak symmetry global expect M^2-1 massless GB after breaking

Once we gauge this symmetry these massless states should disappear from spectrum. Gauge bosons become massive - Higgs phase ...

What about Nielsen-Ninomiya, Karsten-Smit etc ? Continuum is vector-like - no anomalies.

Elitzur's theorem insists that no local order parameter in gauge theory. So fermion bilinear $\overline{\psi}\lambda$ is always zero

Instead Higgs phase corresponds to condensation of a (gauge invariant) four fermion operator

$$\Sigma \sim \overline{\psi}_+\lambda_+\overline{\lambda}_-\psi_- \longleftarrow \text{non-zero vev always}$$

Non-local order parameters allowed ...

Polyakov loops - non-local order parameter

Phases classified by behavior of Wilson line that wraps the lattice

$$P(x) = \frac{1}{N}\mathrm{Tr}\left(\prod_{t=1}^{T} U_t(x,t)\right)$$

Measures free energy of static source

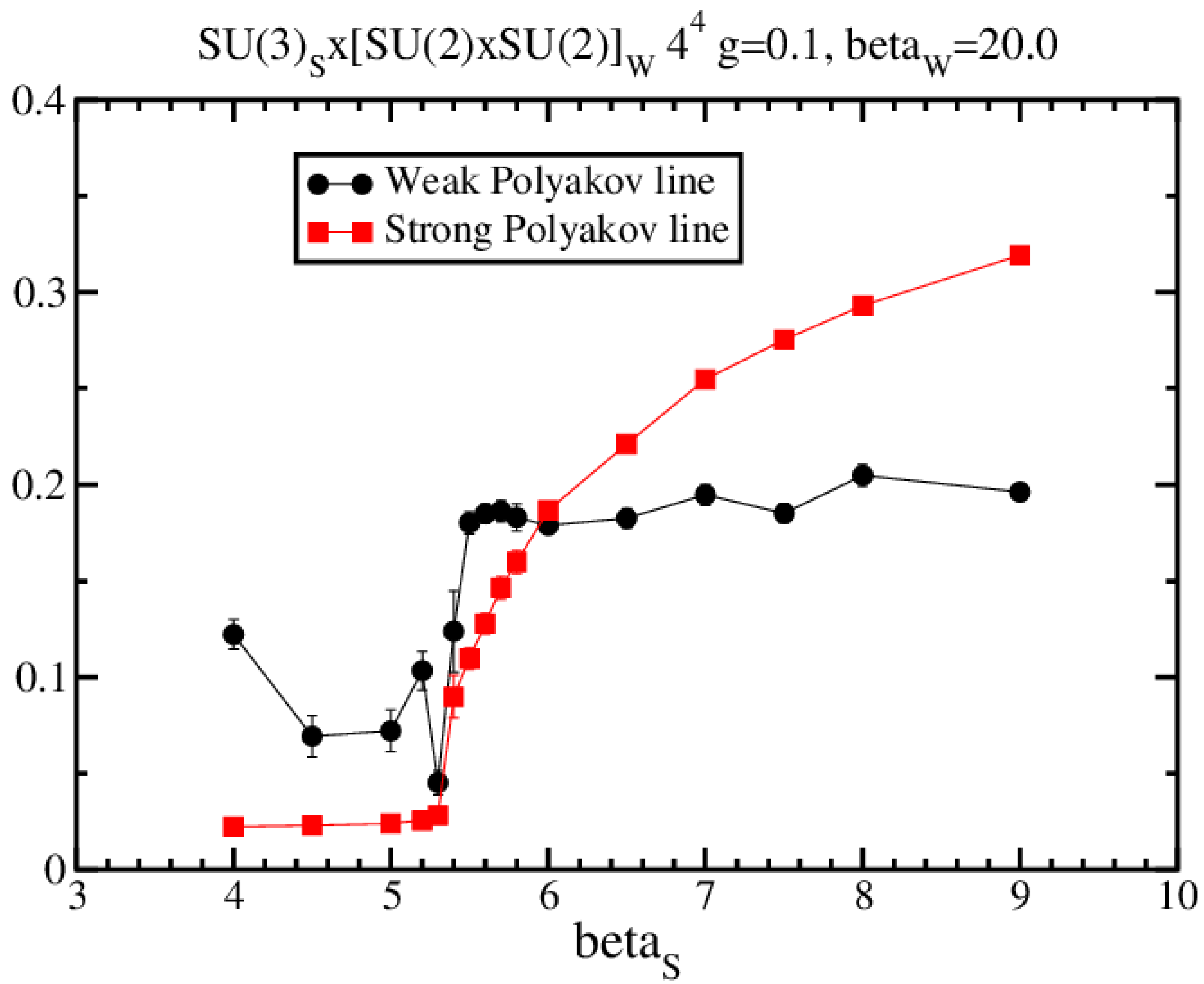
$$P \sim e^{-F_{q\overline{q}}T}$$

Weak sector source carries quantum numbers of $\overline{\psi}\lambda$

Hence corresponding observable:

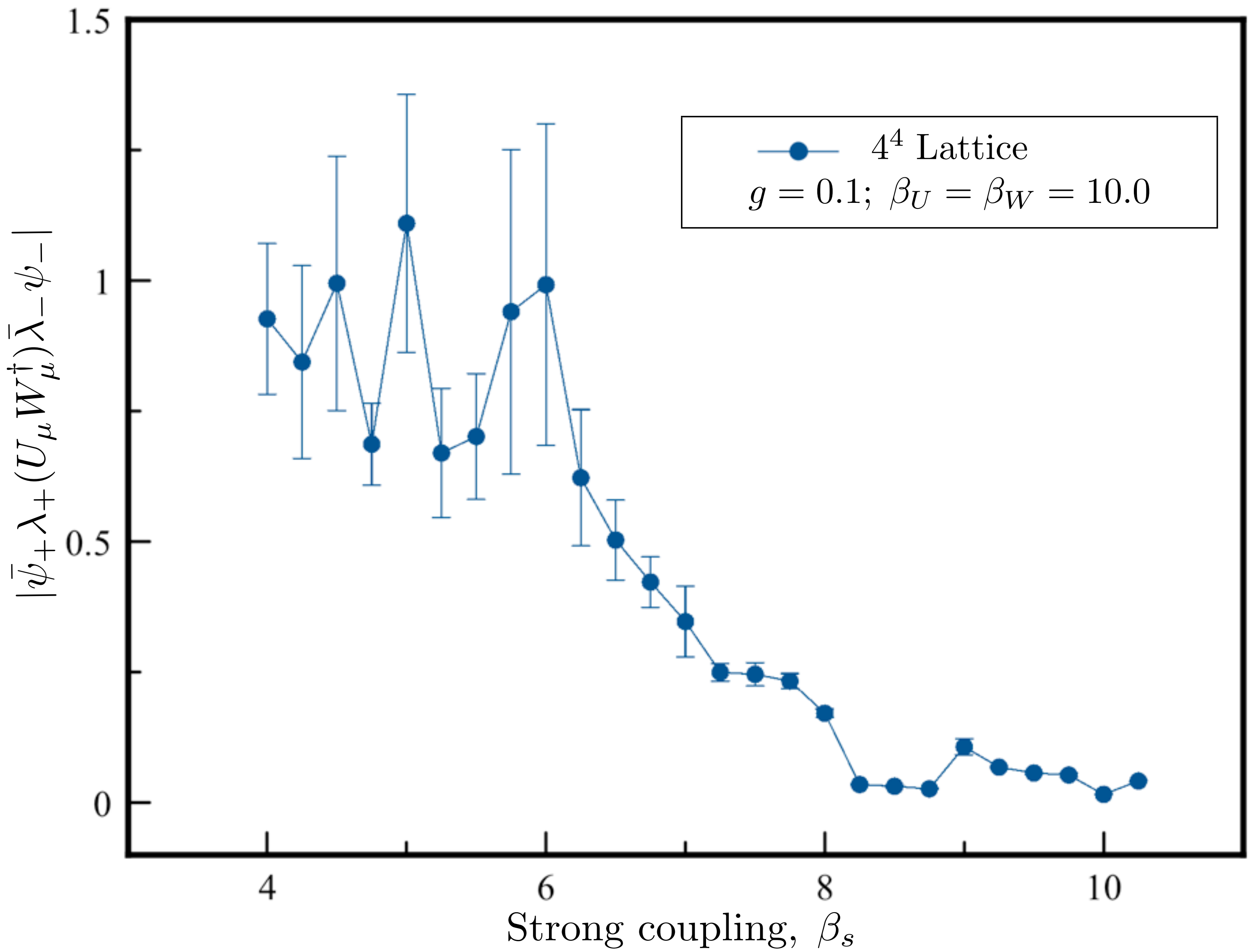
$$P_W = \left(\mathrm{Tr}\prod U_\mu\right)\left(\mathrm{Tr}\prod W^*\mu\right)$$

Polyakov lines



Once system confines discontinuous jump in free energy of weak source

Four fermion condensate



See enhanced 4 fermi condensate at strong coupling

Interpretation

- Evidence that system enters Higgs phase at strong coupling/coarse lattices.
- Does this phase survive continuum limit ?
- Naively it should not. At a=0 staggered fermions yield Dirac fermions - vector-like theory. Such a theory can always develop a fully gauge invariant condensate (eg single link).
- Vacuum alignment arguments in continuum would favor this symmetric condensate.
- Unravel by studying the theory in scaling region

$$\boxed{\beta_S, \beta_W, L \to \infty \text{ with } \frac{\beta_S}{\beta_W} = r \text{ fixed } << 1}$$

Vacuum alignment

- For zero weak coupling global symmetries are G=SU(8)xSU(8). Expect breaks to $SU_V(8)$
- Strong interaction vacuum highly degenerate corresponding to different embeddings of SU(2)xSU(2) into G. Broken SU(2) should be axial (VF)
- Switch on weak coupling. Unique vacuum picked out. Usual folklore is that this vacuum does not break (weakly coupled) gauge symmetries in vector-like theory ?
- In staggered lattice theory degeneracy of strong interaction vacuum is lifted and single site condensate breaking SU(2)xSU(2) dominates
- Strong interaction cut-off effects compete with weak interaction

At r=0 SU(2) breaks .. does this survive to finite r ?

Aside: Kahler-Dirac fermions

Consider 4 degenerate Dirac fermions. Global symmetry
$SO_{Lorentz}(4) \times SU_{flavor}(4)$

$$\gamma_\mu^{\alpha\beta}\partial_\mu\psi_\beta^a = 0,\; a = 1\ldots 4$$

Consider twisted Lorentz symmetry corresponding to
$SO_{twist}(4) = \mathrm{diag}(SO_{Lorentz}(4) \times SO_{flavor}(4))$ -- fermions become matrix!

$$S = \int \mathrm{Tr}\,\left(\overline{\psi}\gamma_\mu\partial_\mu\psi\right)$$ In massless case decomposes into 2 pieces

$$S = \int \mathrm{Tr}\left(\overline{\psi}_+\gamma_\mu\partial_\mu\psi_-\right) + \mathrm{Tr}\left(\overline{\psi}_-\gamma_\mu\partial_\mu\psi_+\right)$$

with $$\psi_\pm = \frac{1}{2}\left(\psi \pm \gamma_5\psi\gamma_5\right)$$

Like staggered story ...!

In fact can derive staggered action from this continuum KD action

Kahler-Dirac continued ...

Can gauge 2 pieces independently. Fermion bilinear invariant under twisted Lorentz symmetry is

$$\mathrm{Tr}\left(\overline{\psi}_+\psi_+\right) + \mathrm{Tr}\left(\overline{\psi_-}\psi_-\right)$$

Product yields G. I 4 fermion

Could break gauge symmetries as for staggered fermions

However in flat space KD fermions equivalent to 4xDirac.
Twisted symmetry enhances to usual SO(4)xSU(4)
one expects gauge invariant condensate ..

This is not true on curved space. Twisted symmetry is everything.
Spectrum of theory at scales O(curvature radius) is not Dirac
new symmetry breaking condensates possible ...

Bolder thoughts ....

Can one break lattice gauge symmetry with single gauge interaction ?

Imagine SU(4) and $\chi : 10$ $\psi : 4$ switch off weak coupling

$$< \epsilon_{\mu\nu\gamma\delta} \overline{\chi}_{+}^{\mu\nu} \psi_{+}^{\gamma} > \neq 0$$

Breaks SU(4)->SU(3)

Is single site condensate still formed ?

if so switch on weak coupling -> Higgs phase

continuum limit .....?

Summary

- Constructed a lattice model of (reduced) staggered fermions which exhibits a Higgs phase at non-zero lattice spacing
- Broken symmetry starts out as vector-like but reappears as an axial symmetry in continuum limit - consistent with VW theorem. Nevertheless UV theory is vector-like - no anomalies ..
- Correlated with formation of a gauge invariant four fermion condensate - consistent with Elitzur's theorem
- See abrupt transition in (weak) Polyakov line as system confines ...
- 60 million $ question: does this Higgs phase survive continuum limit ?
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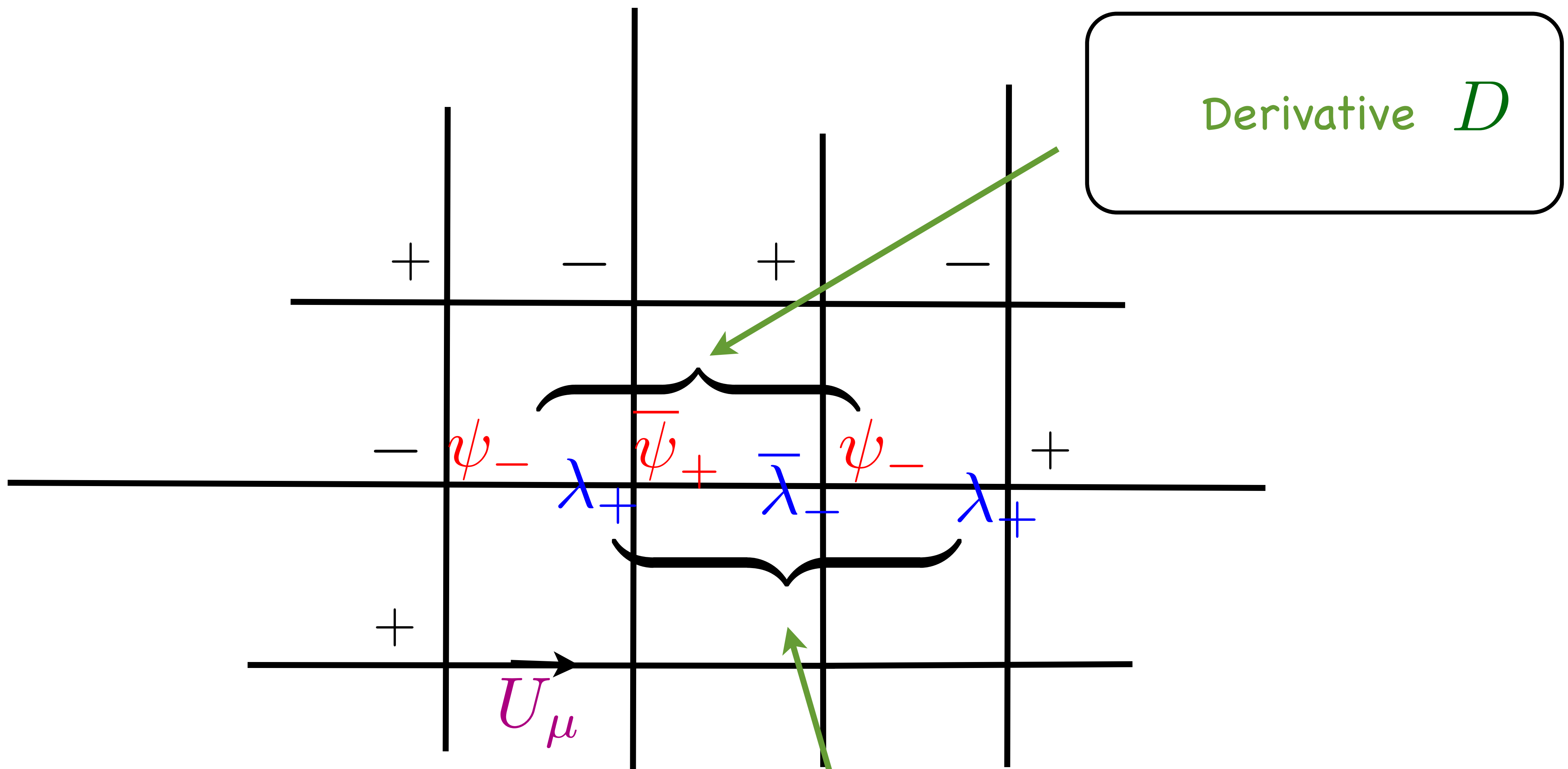
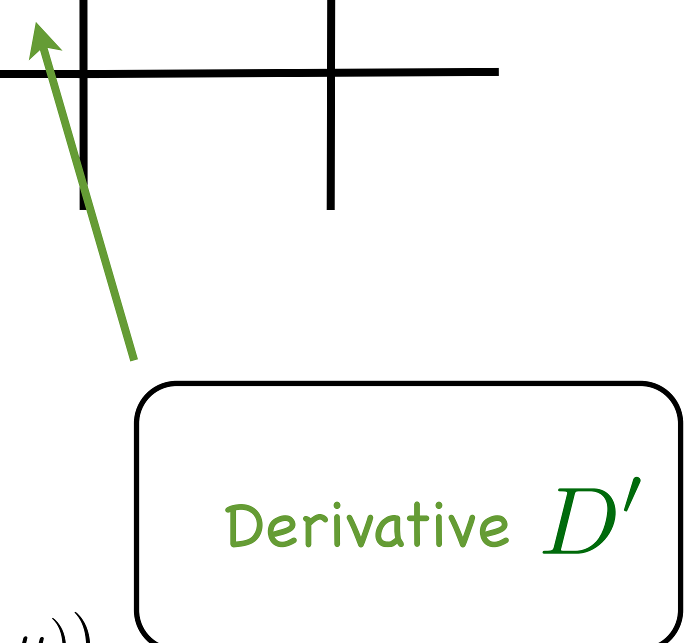
Need simulations on larger, finer lattices to see - in progress !





Parity of site

$$P = (-1)^{\sum_{\mu=1}^{4} x_\mu}$$

$$D_\mu \psi(x) = \frac{1}{2}\left(U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu)\right)$$