Confinement with perturbation theory, after all?

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Strong interactions in the LHC ERA

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Perturbation theory is powerful
- Poincaré invariance, analyticity & crossing symmetry, unitarity,...

The QCD coupling is likely to freeze: $\alpha_s(0) \approx 0.5$

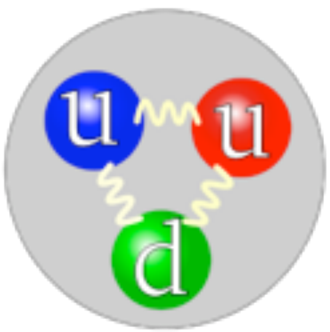
Hadron dynamics has simple features: Spectra, OZI, Duality,...

$\Rightarrow$ Could an α_s expansion be relevant even for soft physics?

Work done with: D. D. Dietrich and M. Järvinen

The two faces of hadrons

Quark model



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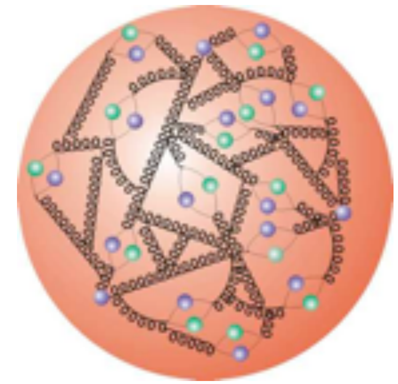
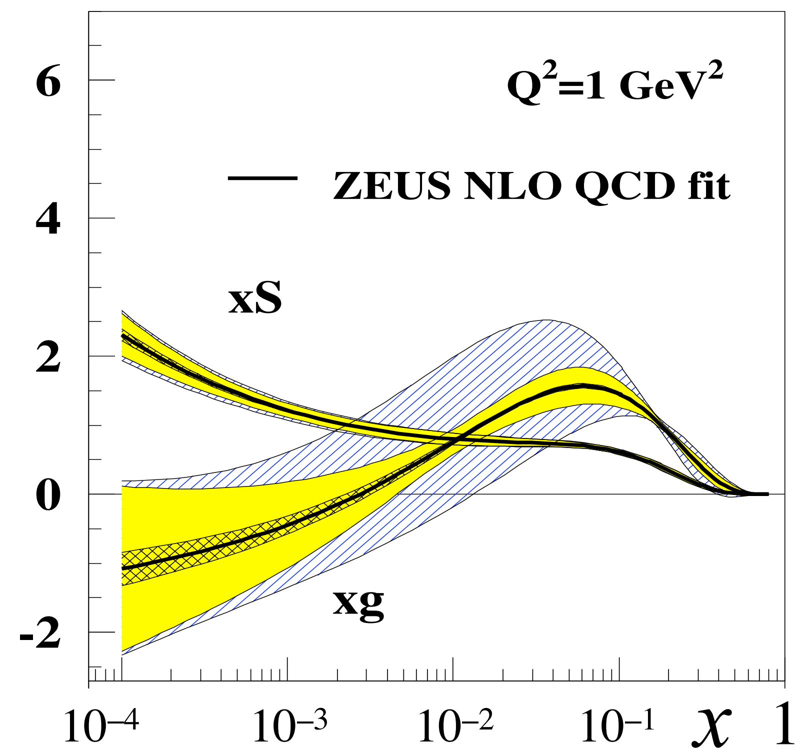
$\Leftrightarrow$

Parton picture



Sea partons are abundant

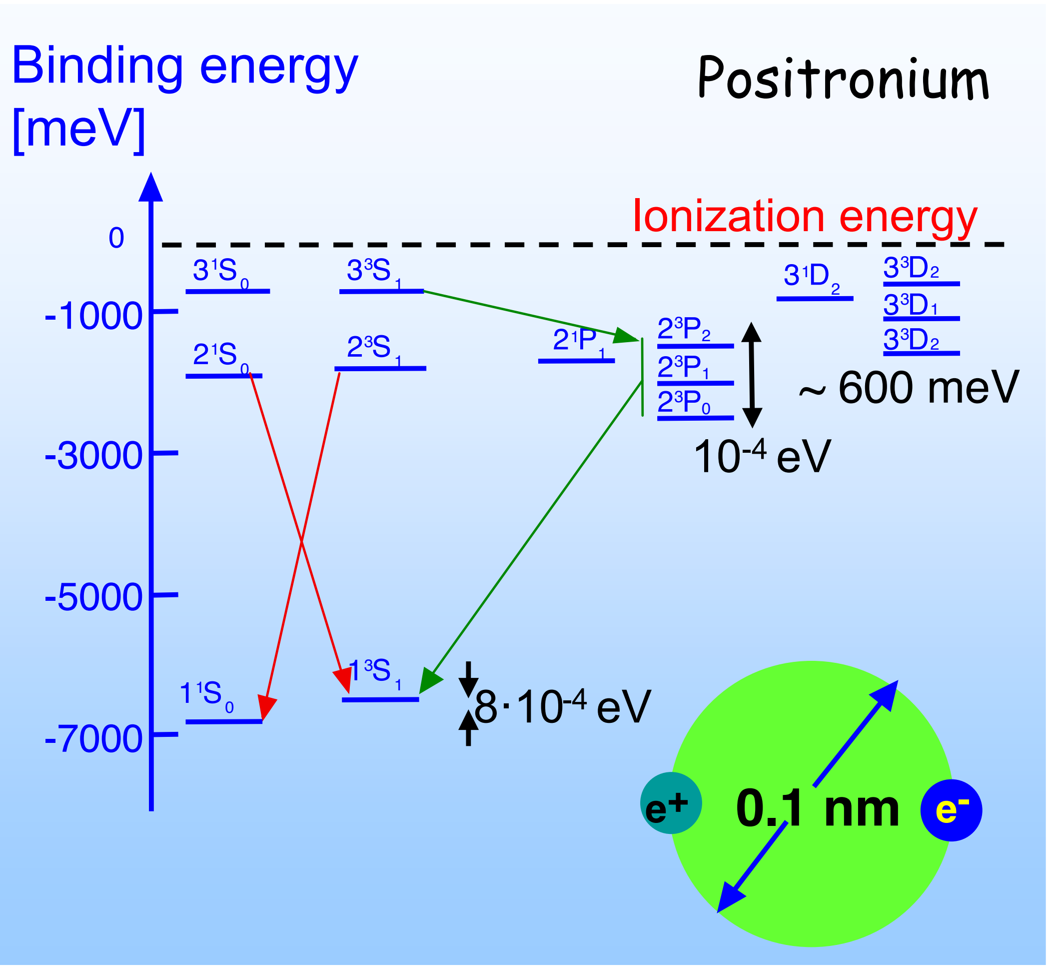


Quantum numbers reflect valence quark dof's

$$n\ {}^{2s+1}\ell_J$$

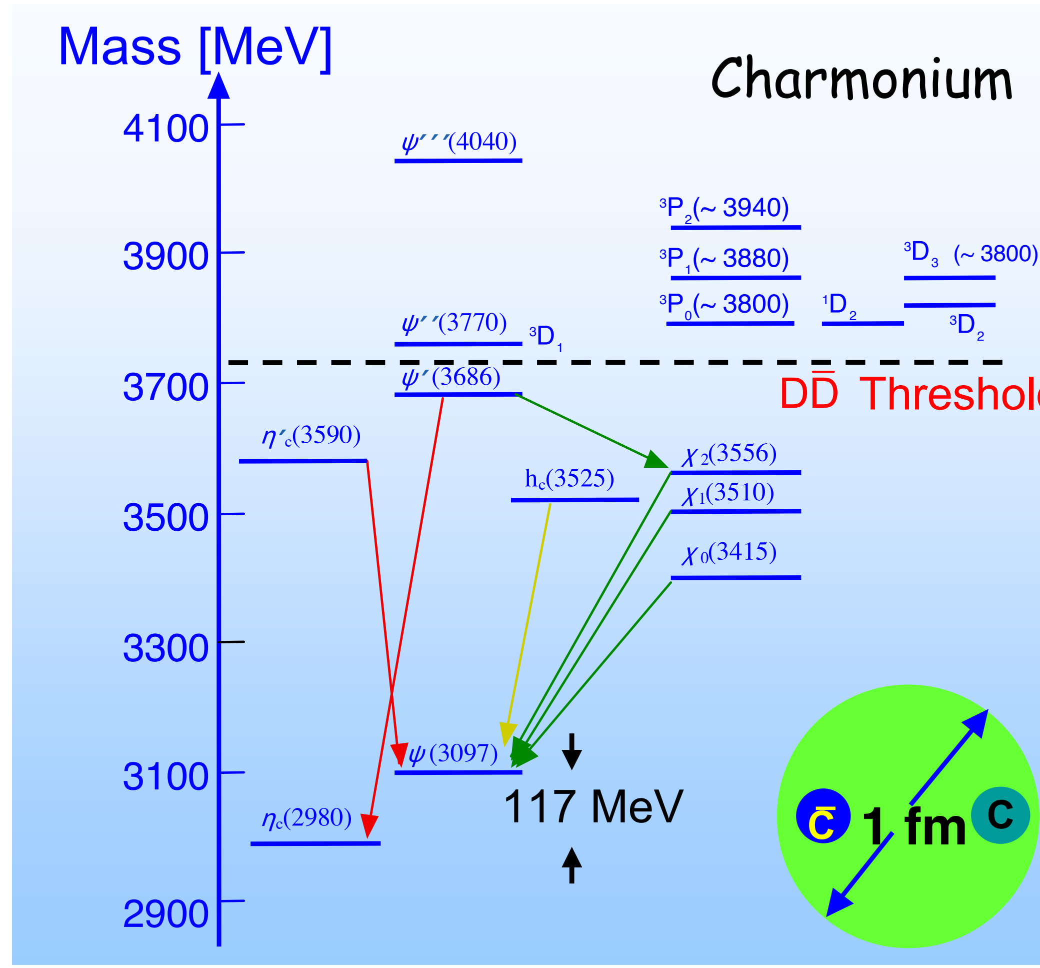
"The J/ψ is the Hydrogen atom of QCD"

QED



$$V(r) = -\frac{\alpha}{r}$$

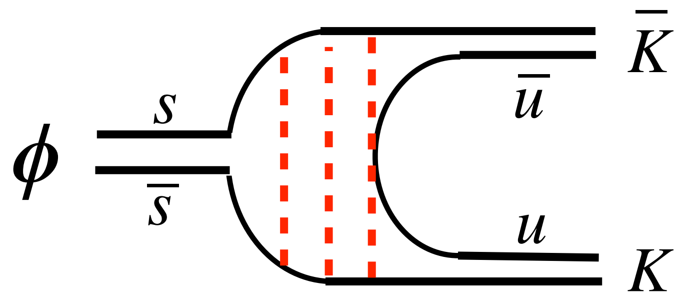
QCD



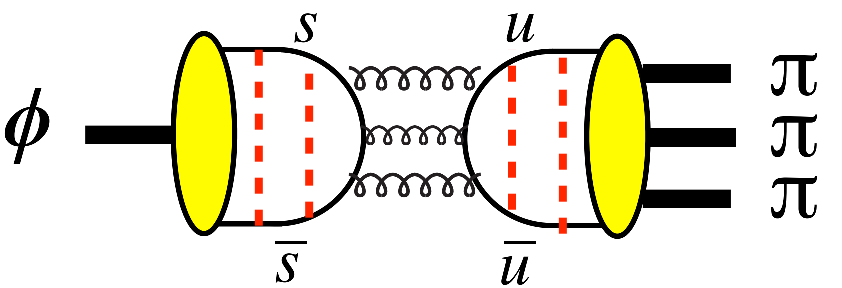
$$V(r) = c\,r - \frac{4}{3}\frac{\alpha_s}{r}$$

Okuba-Zweig-Iizuka Rule

Connected diagrams: Unsuppressed, string breaking due to linear potential

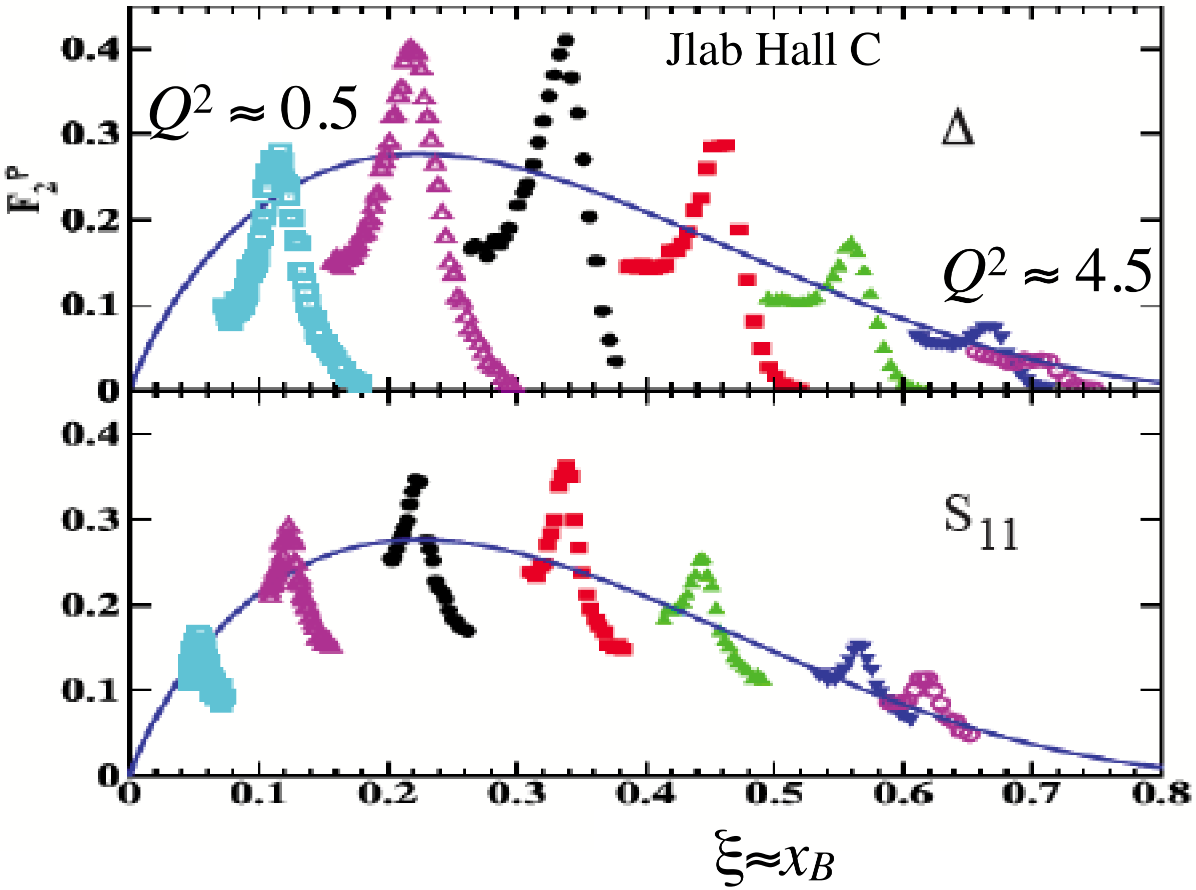
			ΔE	Br
$\phi(1020) \to K\bar{K}$			26 MeV	83.1 %

Disconnected diagrams: Suppressed due to perturbative gluons?

$\phi(1020) \not\to \pi\pi\pi$		610 MeV	15.3 %

Bloom-Gilman Duality

W. Melnitchouk et al, Phys. Rep. 406 (2005) 127

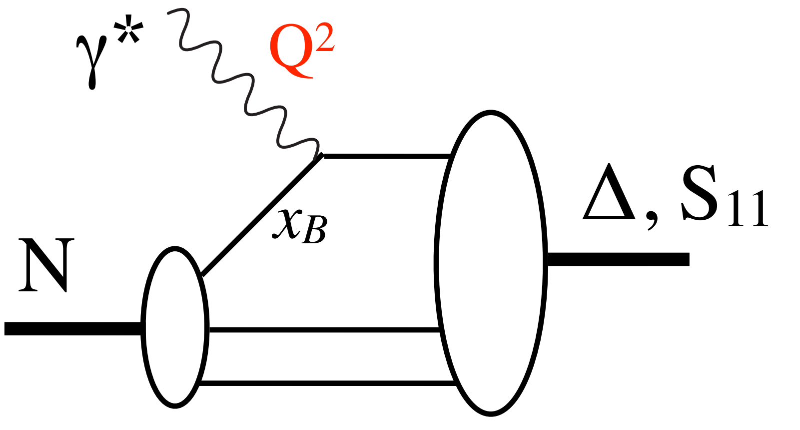


Resonance contributions

$ep \to eN^*$

build DIS scaling in

$ep \to eX$



$$m_{N^*}^2 = m_N^2 + Q^2\left(\frac{1}{x_B} - 1\right)$$

Scattering dynamics is built into hadron wave functions.

We must understand relativistic bound states in motion.

Can an analytic, 1st principles approach be excluded ?

Three apparently unassailable reasons:

I. Confinement does not arise at any finite order of α_s in PQCD

II. Chiral symmetry is preserved at all orders of α_s (for $m_q \to 0$)

III. Sea quarks and gluons abound in hadrons

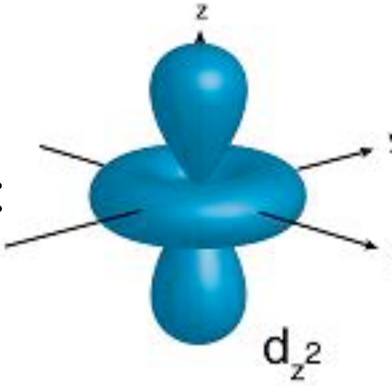
Nevertheless, the question is motivated by:

– the suggestive features of the data

– the potential significance of an analytic description

– the poorly understood structure of relativistic bound states

The unknown structure of Dirac states

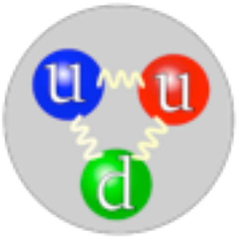


Schrödinger eq. specifies the electron wave function of atoms:

Relativistic Dirac wave functions have E < 0 components.
These reflect Fock states with extra pairs (cf. Klein paradox).

A more specific description seems to be lacking.
E.g.: what is the positron density distribution in the Dirac Hydrogen state?

Dirac states demonstrate that multi-particle states can have spectra which reflect only valence dof's.



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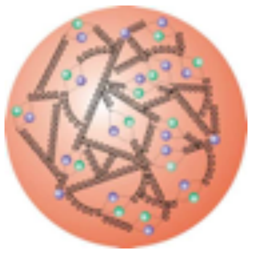
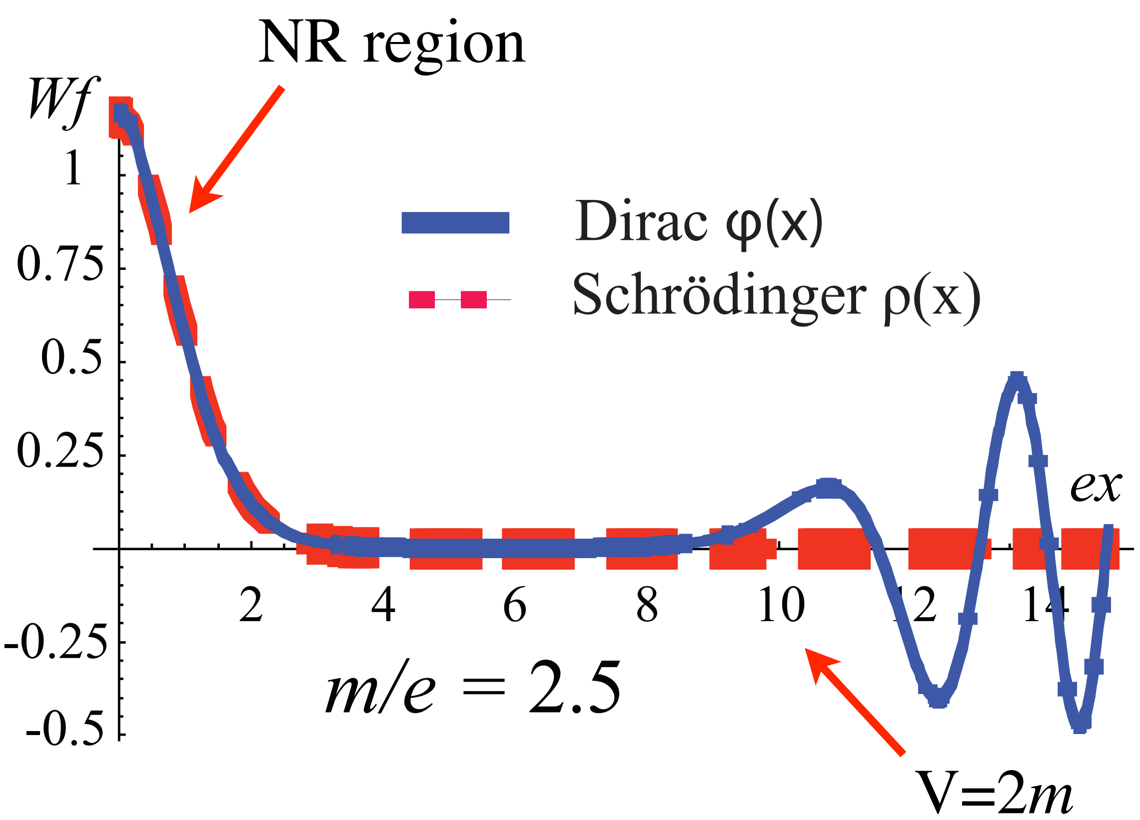


Analogy with Dirac states

Example: Dirac wf in D=1+1

Representing the Dirac matrices as 2x2 Pauli matrices, the Dirac eq. is:

$$\left[-i\sigma_1\partial_x + \tfrac{1}{2}e^2|x| + m\sigma_3\right]\begin{bmatrix}\varphi(x)\\ \chi(x)\end{bmatrix} = M\begin{bmatrix}\varphi(x)\\ \chi(x)\end{bmatrix}$$



For polynomial potentials the Dirac wave function is not normalizable, and the mass spectrum M is continuous.

Its normalizability for the $V(r) = -\alpha/r$ potential in D=3+1 is an exception.

Pair contributions are manifest for

$$V(x) = \tfrac{1}{2}e^2|x| \geq 2m$$

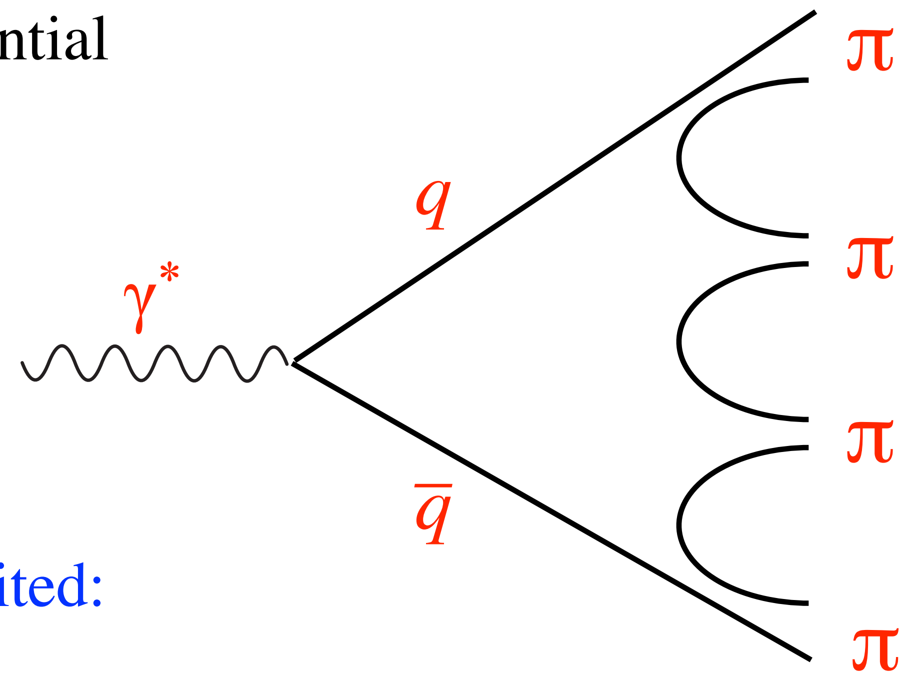
M. S. Plesset, Phys. Rev. **41** (1932) 278

Constant particle density for |x| $\to \infty$?!

$$\Psi(x \to \infty) \sim \exp(\pm i x^2/4) \quad \Rightarrow \quad \Psi^\dagger \Psi(x \to \infty) \sim const.$$

The virtual pairs created in the linear potential contribute to the Dirac wave function.

This reminds of
quark fragmentation in hadron physics:



The Dirac dynamics is instructive, but limited:

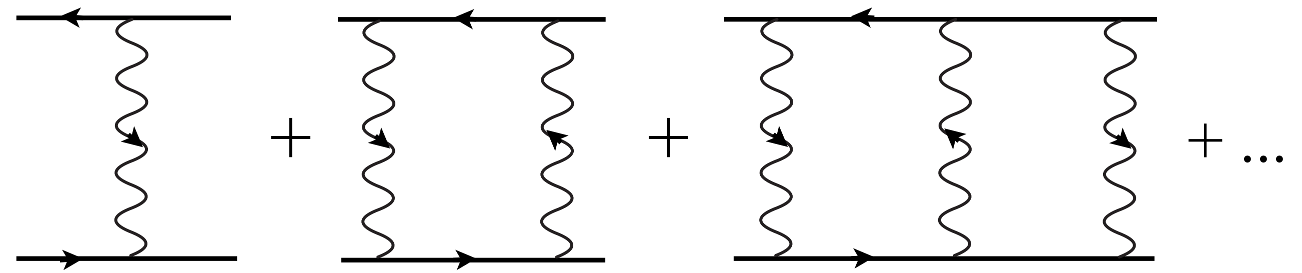
- String breaking is not included.
- The external potential violates translation invariance
  - There are no momentum eigenstates.

$\Rightarrow$ Consider $e^+ e^-$ states, bound by their mutual A^0 interaction.

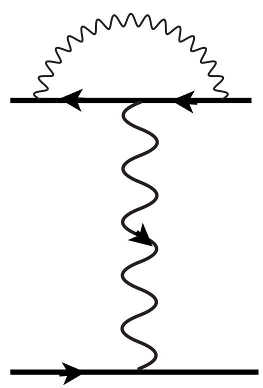
Recall QED bound states (atoms)

Perturbative description: Expansion in $\hbar$

Classical – α/r potential arises from the sum of ladder diagrams:



$\mathcal{O}(\hbar)$ corrections due to vertex corrections, etc:



Also for QCD hadrons an $\hbar$ expansion would have to start from an $\mathcal{O}(\hbar^0)$ Born term with a classical potential.

Summing the QCD series is hard – but we can ask:

Are there other classical solutions than – α/r ?

A homogeneous solution of Gauss' law in QED

For a state with e^- at x_1 and e^+ at x_2 $\quad \bar{\psi}(t,x_1)\psi(t,x_2)|0\rangle$

Gauss' law for A^0 reads (in QED) $-\boldsymbol{\nabla}^2 A^0(t,\boldsymbol{x}) = e\big[\delta^3(\boldsymbol{x}-\boldsymbol{x}_1) - \delta^3(\boldsymbol{x}-\boldsymbol{x}_2)\big]$

It has the homogeneous solution provided $\varkappa$ is independent of $\boldsymbol{x}$:

$$A^0(t,\boldsymbol{x}) = \kappa\, \boldsymbol{x}\cdot(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

This is usually excluded since $\lim_{|\boldsymbol{x}|\to\infty} A^0(\boldsymbol{x}) \neq 0$

and the squared field strength is independent of $\boldsymbol{x}$

$$\big[\boldsymbol{\nabla} A^0\big]^2 = \kappa^2\,(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2$$

These properties are similar to the Coulomb field in D=1+1.

The homogeneous solution leads to a linear potential in D=3+1.

A linear classical potential in D=3+1

$$A^0(t,\boldsymbol{x}) = \kappa\,\boldsymbol{x}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)$$

The $\boldsymbol{x}$-independent squared field strength $\left[\boldsymbol{\nabla} A^0\right]^2 = \kappa^2\,(\boldsymbol{x}_1-\boldsymbol{x}_2)^2$

implies an infinite contribution to the field energy $\propto \int d^3\boldsymbol{x}$

For this divergence not to depend on $\boldsymbol{x}_1$ or $\boldsymbol{x}_2$ we must have $\varkappa = \varkappa(\boldsymbol{x}_1, \boldsymbol{x}_2)$:

$$\kappa = \frac{\Lambda^2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

where Λ is an $O(e^0)$ universal constant.

The A^0 field then implies a linear potential energy to the pair,

$$V(\boldsymbol{x}_1,\boldsymbol{x}_2) \equiv \tfrac{1}{2}e\left[A^0(t,\boldsymbol{x}_1) - A^0(t,\boldsymbol{x}_2)\right] = \tfrac{1}{2}e\Lambda^2\,|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

Note: Translation invariance preserved only for neutral states (as in D=1+1)

Uniqueness: No other homogeneous A^0 solution gives translation invariance

The homogeneous solution of Gauss' law in QCD

An analogous argument gives a linear potential for color singlet mesons:

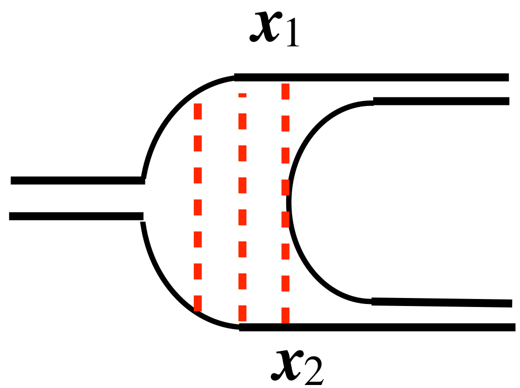
$$V_{\mathcal{M}}(\boldsymbol{x}_1-\boldsymbol{x}_2)=\tfrac{1}{2}\sqrt{C_F}\,g\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

For color singlet baryons the result is:

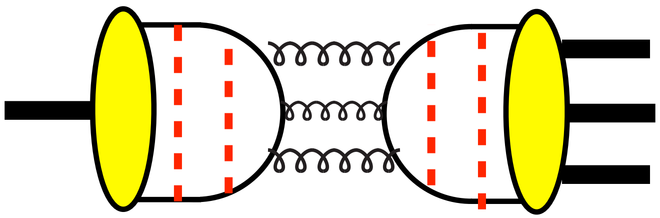
$$V_{\mathcal{B}}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)=\tfrac{1}{2\sqrt{2}}\sqrt{C_F}\,g\Lambda^2\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+(\boldsymbol{x}_2-\boldsymbol{x}_3)^2+(\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$$

Note: $V_{\mathcal{B}}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2)=V_{\mathcal{M}}(\boldsymbol{x}_1-\boldsymbol{x}_2)$

String breaking



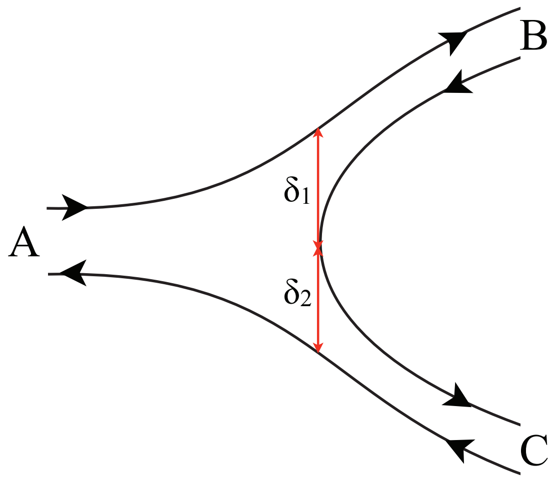
The $O(e^0)$ classical field in $\bar{\psi}(t,x_1)\psi(t,x_2)|0\rangle$ causes pair creation



There is no $O(e^0)$ field in the vacuum $|0\rangle$,
hence pair creation only at $O(e^n)$, $n \geq 2$

Pair production at $O(e^0)$ will be treated in a $1/N_c$ *expansion*.

At leading order in $1/N_c$, there is no string breaking.



Pair production can be evaluated from these solutions:
Resonance decays, hadron loop corrections...

Note: The exact $O(e^0)$ amplitudes must satisfy unitarity!

In the following I illustrate bound state results for U(1) gauge theory, at leading order in $1/N_c$ and (mostly) in D=1+1 dimensions.

$f\,\bar{f}$ bound states in D=1+1

A state with two fermions of energy E and momentum $P^1 = P$:

2x2 c-numbered wf.

$$|E,P\rangle = \int dx_1 dx_2\, \bar{\psi}(t,x_1)\exp\left[\tfrac{1}{2}iP(x_1+x_2)\right]\Phi(x_1-x_2)\psi(t,x_2)|0\rangle$$

field operators

With $\hat{P}^\mu\,|0\rangle = 0$ these are eigenstates of the translation generators:

$\hat{P}^1|E,P\rangle = P|E,P\rangle$ Bound state has momentum P (by construction)

$\hat{P}^0|E,P\rangle = E|E,P\rangle$ Bound state equation for $\Phi(x)$ from QED action:

$$i\partial_x\left\{\sigma_1,\Phi(x)\right\} + \left[-\tfrac{1}{2}P\sigma_1 + m\sigma_3, \Phi(x)\right] = \left[E - V(x)\right]\Phi(x)$$

where $V(x) = \frac{1}{2}e^2|x|$ and $\gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma^0\gamma^1 = \sigma_1$

Here the CM momentum P is a parameter, thus E and Φ depend on P .

Boost covariance

It is essential and non-trivial that the state is covariant under boosts:

$$|E + d\xi P, P + d\xi E\rangle = (1 - id\xi \hat{M}^{01})|E, P\rangle$$

M^{01} is the QED_2 boost generator

This holds only for a linear potential and ensures that $E(P) = \sqrt{P^2 + M^2}$

The P-dependence of the wave function Φ can be explicitly given:

$$\Phi^P(\sigma) = e^{\gamma_0\gamma_1\zeta/2}\Phi^{(P=0)}(\sigma)e^{-\gamma_0\gamma_1\zeta/2}$$

where $dx = -\dfrac{d\sigma}{E - V(x)}$ and $\tanh\zeta = -\dfrac{P}{E - V}$

The boost invariant length

The "kinetic 2-momentum" is $\Pi^\mu(x) \equiv (P - eA)^\mu = (E - V(x), P)$

For a linear potential the bound state equation can "miraculously" be expressed in terms of $\sigma = \Pi^2$ only (without frame dependent E, P), with

$$\Pi^2 \equiv \sigma \equiv (E - V)^2 - P^2 = M^2 - 2EV + V^2$$

The continuity condition imposed at $x = 0$, where $\sigma = E^2 - P^2$, ensures that the mass eigenvalues $M^2 = E^2 - P^2$ have the correct frame dependence.

Solutions of the bound state equation (cont)

To solve the bound state equation

$$i\partial_x \{\sigma_1, \Phi(x)\} + \left[-\tfrac{1}{2}P\sigma_1 + m\sigma_3, \Phi(x)\right] = \left[E - V(x)\right]\Phi(x)$$

we may expand the 2x2 wave function as $\Phi = \Phi_0+\sigma_1\Phi_1+\sigma_2\Phi_2+\sigma_3\Phi_3$.

We get two coupled equations, with no explicit E or P dependence:

$$-2i\partial_\sigma\Phi_1(\sigma) = \Phi_0(\sigma) \qquad\qquad -2i\partial_\sigma\Phi_0(\sigma) = \left[1 - \frac{4m^2}{\sigma}\right]\Phi_1(\sigma)$$

The general solution is

$$\Phi_1(\sigma) = \sigma\, e^{-i\sigma/2}\left[a\,{}_1F_1(1-im^2, 2, i\sigma) + b\,U(1-im^2, 2, i\sigma)\right]$$

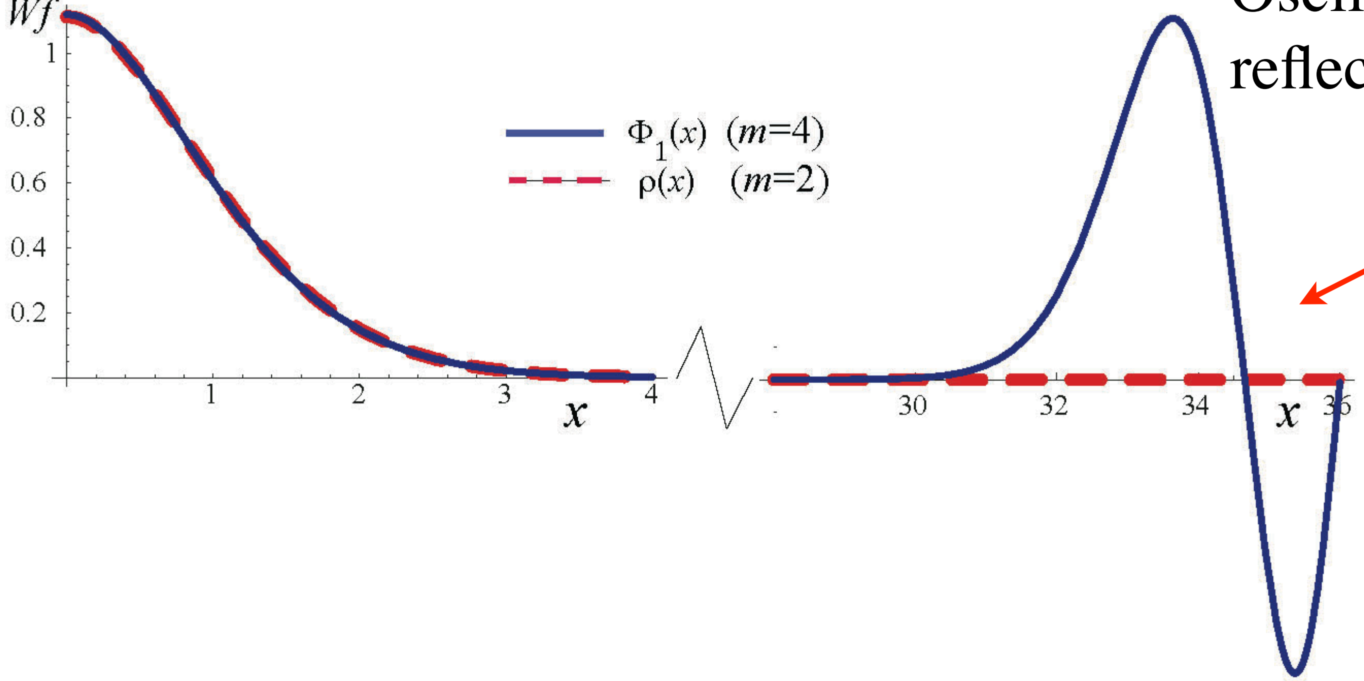
If $b \neq 0$ the wf Φ is singular at $\sigma = 0$. Requiring $b = 0$ the spectrum is discrete.

Note: This constraint only applies for $m \neq 0$.

Some numerical results

Nearly non-relativistic case: $m = 4.0e$

Schrödinger (Airy fn.) wf. $\varrho(x)$.



Oscillations for V(x) > 2m reflect virtual pair production

In the limit of small fermion mass m:

$$M_n^2 = \pi n + \mathcal{O}\left(m^2\right) \;\; ; \qquad n = 0, 1, 2, \ldots$$

Parity $= (-1)^{n+1}$ No parity doublets for $m \neq 0$

Infinite Momentum Frame (IMF)

The wf is frame invariant in terms of $\sigma = (E-V)^2 - P^2$. Since $V(x) = \frac{1}{2}|x|$:

$$x = 2\left(E \pm \sqrt{P^2 + \sigma}\right)$$

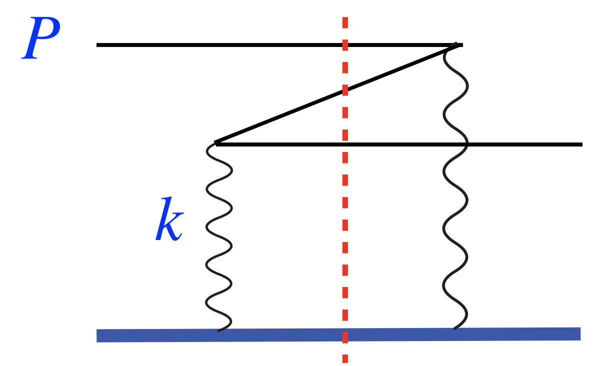
For $P \to \infty$ at fixed σ: $$x \simeq 2(E \pm P) \pm \frac{\sigma}{P} \simeq \begin{cases} 4P + \sigma/P \\ (M^2 - \sigma)/P \end{cases}$$

Lower solution: $x \propto 1/P$ Lorentz-contracted "valence" region.

Upper solution: $x \approx 4P \to \infty$ Pair production moves to infinite x.

Perturbatively: "Z-diagrams" get infinite energy ($k \to \infty$) in the $P \to \infty$ limit.

C.f.: $H|0\rangle = 0$ in LF quantization.



Explicitly: $$\Phi_{P\to\infty}(\sigma) = 2am\, P\gamma^+ e^{-i\sigma/2}\, {}_1F_1(1 - im^2, 2, i\sigma)$$

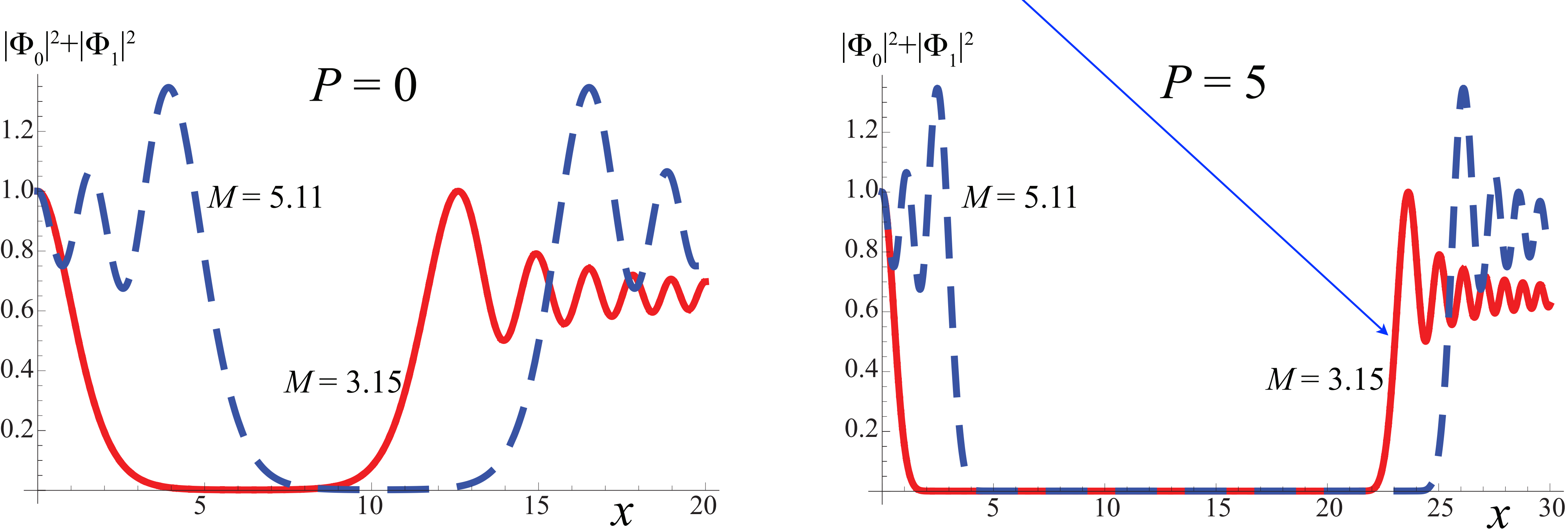
Frame (P) dependence of the solutions ($m_1 \neq m_2$)

Comparison of ground and excited state wave functions

for P=0 (CM frame) and for $P = 5e$.

m_1=1.0e m_2=1.5e

Moves away in IMF ($P \to \infty$ limit)



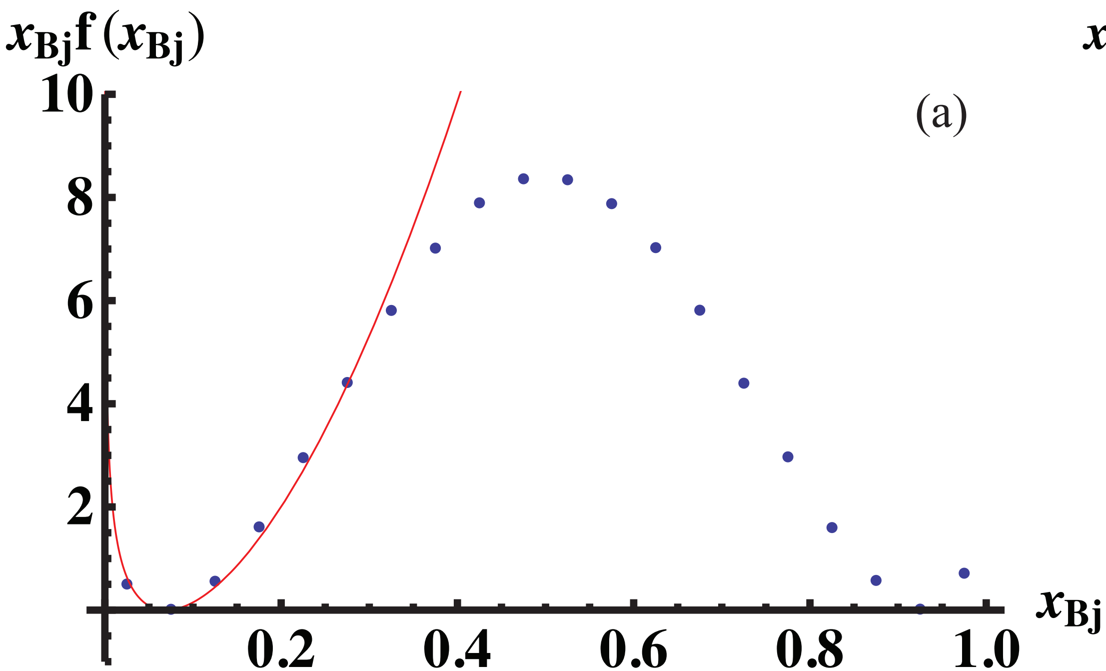
(a) (b)

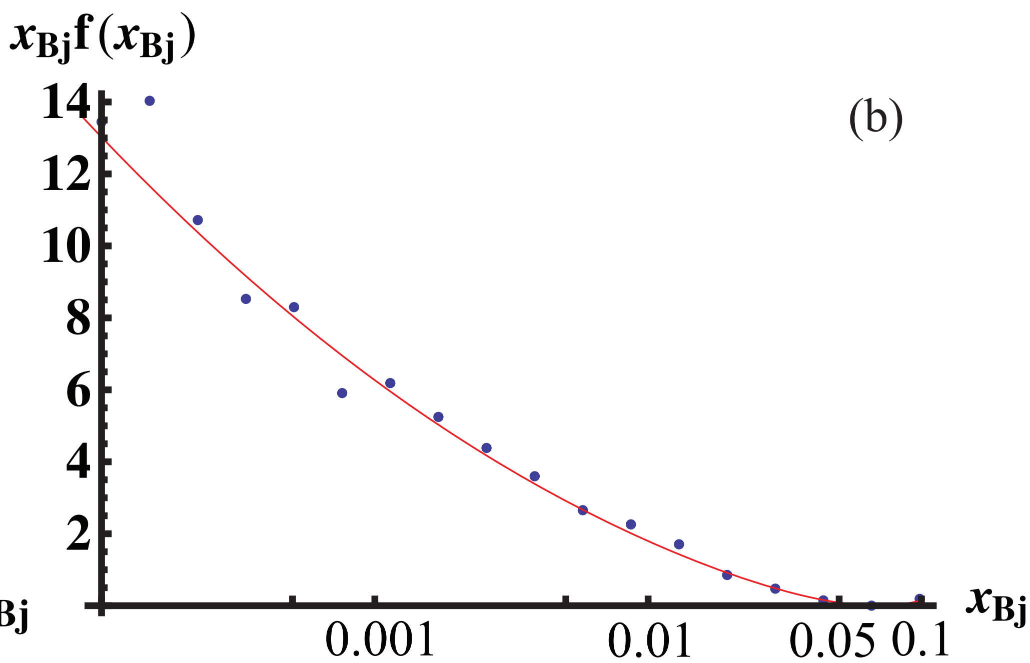
Note: In the IMF limit, only the normalizable, valence part of the wf remains.

Parton distributions have a sea component

The sea component is prominent at low m/e :

$$m/e = 0.1$$





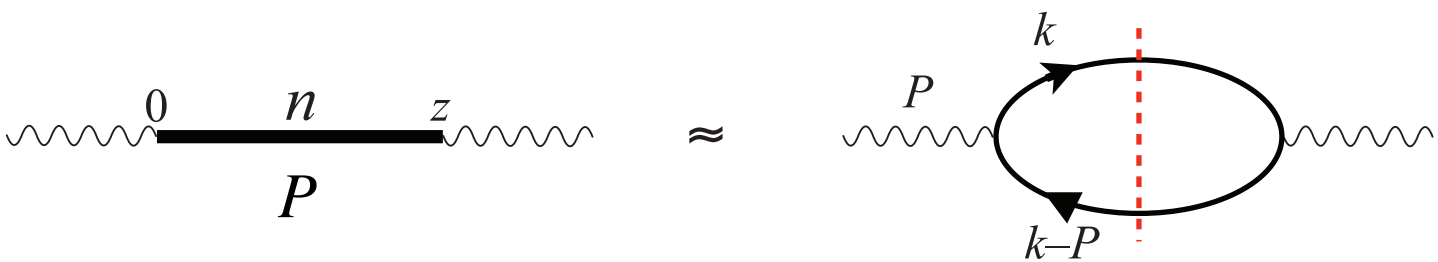
(log scale in x_{Bj})

The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

Note: Enhancement at low x is not due to Φ_A^{IMF}

Quark - Hadron duality

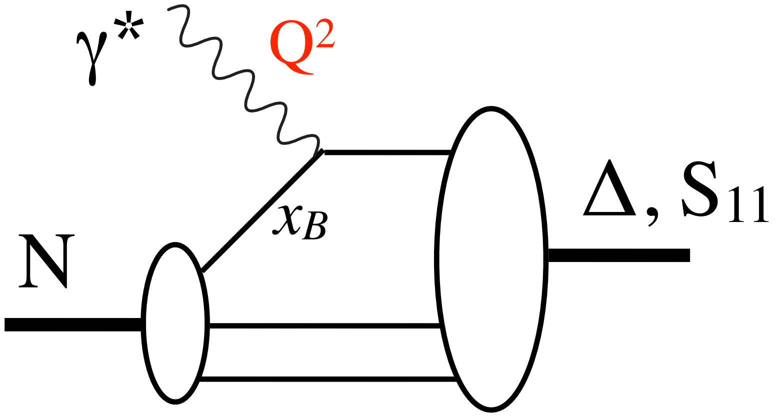
The wave functions of highly excited (large mass M) bound states are similar to free $f\bar{f}$ pairs (for $V(x) << M$). This determines their normalization:



$$\Rightarrow \quad |\Phi_0(x{=}0)|^2 = |\Phi_1(x{=}0)|^2 = \pi/2$$

The solutions are consistent with

Bloom-Gilman duality:



B-G Duality

Final remarks

- Hadron physics is fortunate: Theory (QCD) is known
  Much data on spectra, couplings, scattering
- Unprecedented features: Relativistic bound states, Confinement
- Simplicities in data: Hadron spectrum, Duality

- Above approach: Assume that regularities are not "accidental"
  Consider how/if they can be compatible with QCD

- Conclusions: "Non-perturbative" contribution may be limited to $\mathcal{O}\left({\alpha_s}^0\right)$
  Implies a different expansion point for perturbation theory.
  The approach is strongly constrained by the requirement of a perturbative expansion.

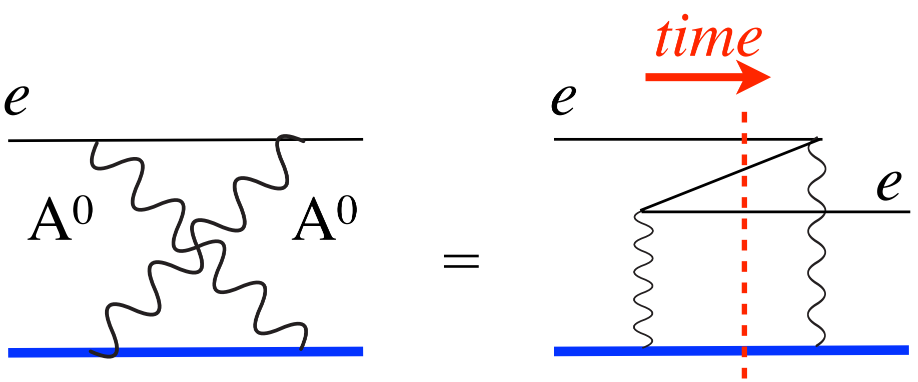
PH: arXiv:1409.4703

Back-up slides

Dirac wave function ψ(x) describes a multiparticle state

Consider a relativistic electron bound by a static (instantaneous) external A^0 potential.

The E < 0 components of the Dirac wave function represent pair production.



The $i\varepsilon$ prescription at the $p^0 < 0$ pole of the electron propagator is irrelevant for the bound state spectrum: We may use retarded boundary conditions.

$$S_R(p^0, \boldsymbol{p}) = i\,\frac{\not{p} + m_e}{(p^0 - E_p + i\varepsilon)(p^0 + E_p + i\varepsilon)}$$

!

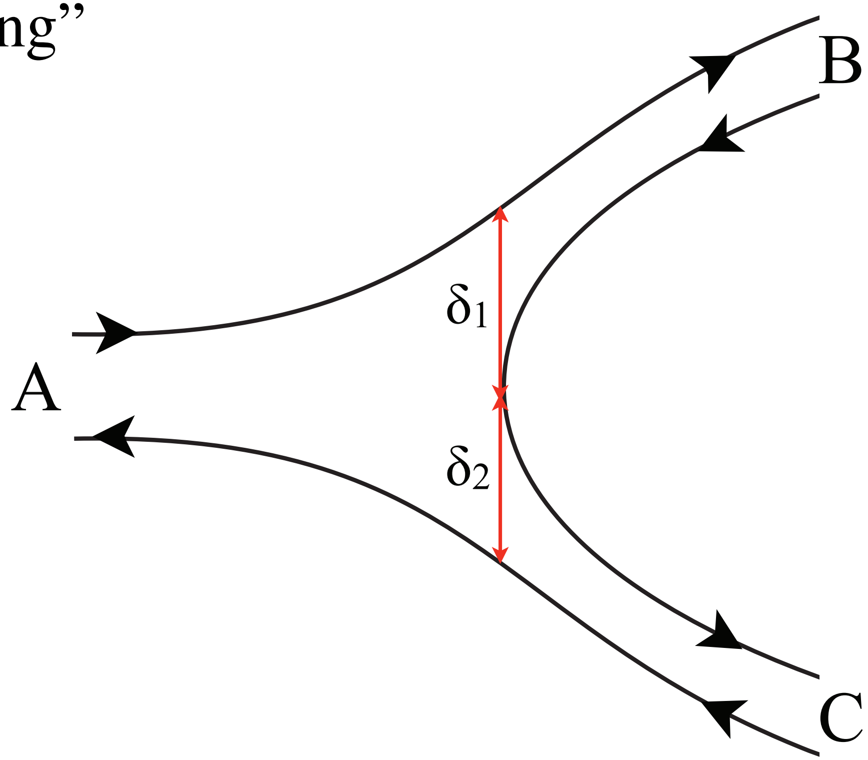
Also $p^0 < 0$ components move forward in time

$\Longrightarrow$

- The infinite number of pairs is described by a single electron wave function.
- $\psi^\dagger\psi(\boldsymbol{x})$ is an *inclusive* particle density.

String breaking: A → B+C

The linear potential induces "string breaking" at large separations of the quarks. The Poincaré invariant amplitude is given by the wave functions:



$$\langle B,C|A\rangle = -\frac{(2\pi)^3}{\sqrt{N_C}}\delta^3(\boldsymbol{P}_A-\boldsymbol{P}_B-\boldsymbol{P}_C)$$

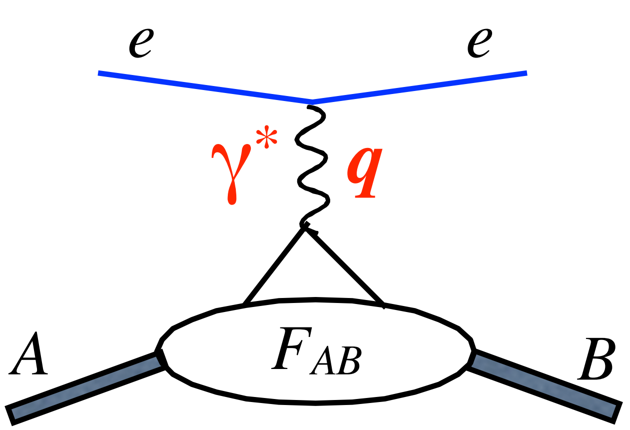
$$\times\int d\boldsymbol{\delta}_1 d\boldsymbol{\delta}_2\, e^{i\boldsymbol{\delta}_1\cdot\boldsymbol{P}_C/2-i\boldsymbol{\delta}_2\cdot\boldsymbol{P}_B/2}\mathrm{Tr}\left[\gamma^0\Phi_B^\dagger(\boldsymbol{\delta}_1)\Phi_A(\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)\Phi_C^\dagger(\boldsymbol{\delta}_2)\right]$$

When squared, this gives a hadron loop unitarity correction.

EM Form Factor

$$F^{\mu}_{AB}(z) = \langle B(P_B); t = +\infty | j^{\mu}(z)\, | A(P_A); t = -\infty \rangle$$

A, B: *in* & *out* states



EM current:

$$j^{\mu}(z) = \bar{\psi}(z)\gamma^{\mu}\psi(z) = e^{i\hat{P}\cdot z} j^{\mu}(0) e^{-i\hat{P}\cdot z}$$

Using anticommutators of fields:

$$F^{\mu}_{AB}(z) \; = \; e^{i(P_B - P_A)\cdot z}$$

$$\times \int_{-\infty}^{\infty} dx\, e^{i(P_B^1 - P_A^1)x/2} \Big\{ \mathrm{Tr}\left[\Phi_B^{\dagger}(x)\gamma^{\mu}\gamma^{0}\Phi_A(x)\right] - \eta_A\eta_B \mathrm{Tr}\left[\Phi_B(x)\gamma^{0}\gamma^{\mu}\Phi_A^{\dagger}(x)\right] \Big\}$$

Gauge invariance is valid: $\partial_{\mu} F^{\mu}_{AB}(z) = 0$ (also in D = 3+1)

The invariant form factor is frame independent (was checked numerically):

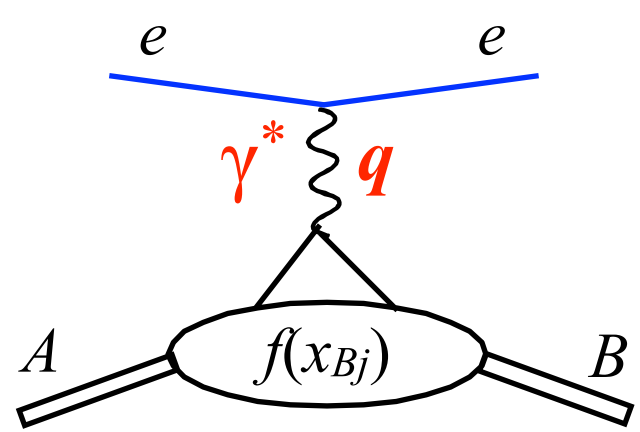
$$F_{AB}(Q^2) = -4i\frac{1-\eta_A\eta_B}{q^1}\int_0^{\infty} dx\, \sin\left(\frac{q^1 x}{2}\right)$$

$$\times \left[\Phi^*_{0B}(x)\Phi_{0A}(x) + \Phi^*_{1B}(x)\Phi_{1A}(x)\left(1 + \frac{4m^2}{\sigma_A\sigma_B}\tilde{\Pi}_A \cdot \Pi_B\right)\right]$$

Parton Distribution: γ*A → B

$$x_{Bj} = \frac{Q^2}{2p_A \cdot q} \quad \text{fixed}$$

$$M_B^2 = Q^2\left(\frac{1}{x_{Bj}} - 1\right) \to \infty$$



From analogy to D=3+1:

$$f(x_{Bj}) = \frac{1}{8\pi m^2}\frac{1}{x_{Bj}}|Q^2 F_{AB}(Q^2)|^2$$

For large M_B use asymptotic form of Φ_B .

Result scales in $v = xQ/2$ (Breit frame)

$$\sigma_A \simeq M_A^2 - \frac{|v|}{x_{Bj}} \equiv \tau_A$$

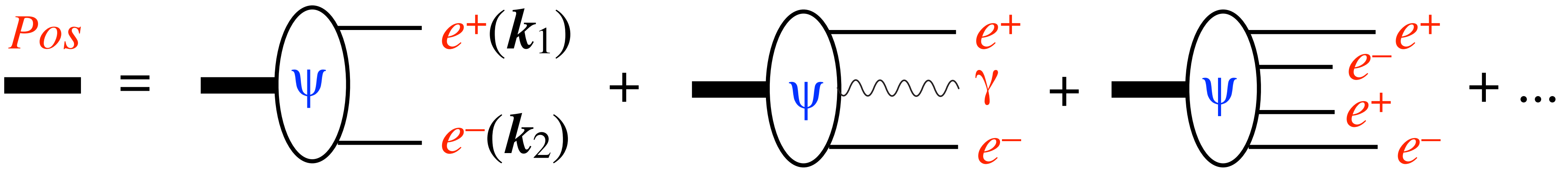
$$Q^2 F_{AB}(\eta_B = -) \simeq -4i\sqrt{2\pi}(1+\eta_A)$$

$$\times \int_0^\infty dv\, \sin v\left[\cos\left(\frac{v}{2x_{Bj}}\right) i\Phi_{0A}(\tau_A) - \sin\left(\frac{v}{2x_{Bj}}\right)\Phi_{1A}(\tau_A)\left(1 + \frac{2m^2}{x_{Bj}\tau_A}\right)\right]$$

An analytic/numerical evaluation shows a sea quark distribution at low x_{Bj}

The Positronium wave function

The Positronium state can be expanded in a complete set of Fock states:



$$|\boldsymbol{P}\rangle = \int \frac{d^3\boldsymbol{k}_1\, d^3\boldsymbol{k}_2}{(2\pi)^6 4E_1E_2}\psi^{\boldsymbol{P}}_{e^+e^-}(\boldsymbol{k}_1,\boldsymbol{k}_2)\delta^3(\boldsymbol{P}-\boldsymbol{k}_1-\boldsymbol{k}_2)\left|e^+(\boldsymbol{k}_1)e^-(\boldsymbol{k}_2)\right\rangle$$

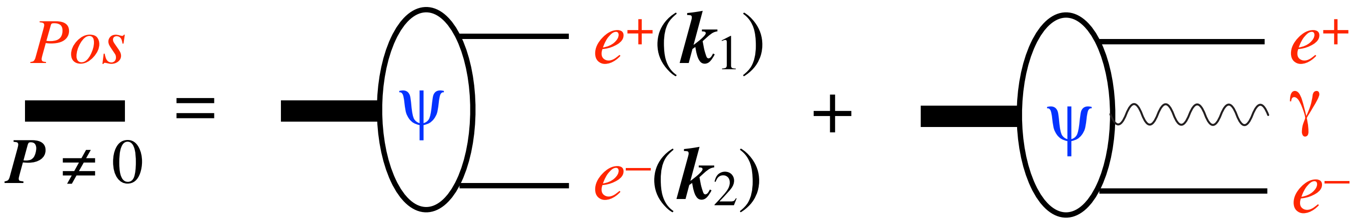
$$+\int [d\boldsymbol{k}_i]\psi^{\boldsymbol{P}}_{e^+e^-\gamma}(\boldsymbol{k}_i)\delta^3(\;)\left|e^+(\boldsymbol{k}_1)e^-(\boldsymbol{k}_2)\gamma(\boldsymbol{k}_3)\right\rangle \;+ \ldots$$

For $\boldsymbol{P} = 0$ and at lowest order in α, the $|\, e^+e^- \rangle$ Fock state dominates.

How does $\psi^{\boldsymbol{P}}_{e^+e^-}$ depend on $\boldsymbol{P}$? Do other Fock states contribute when $\boldsymbol{P} \neq 0$?

Transverse photons contribute for P ≠ 0

Transversely polarized photons contribute to $E(\boldsymbol{P} \neq 0$ at leading order in α)

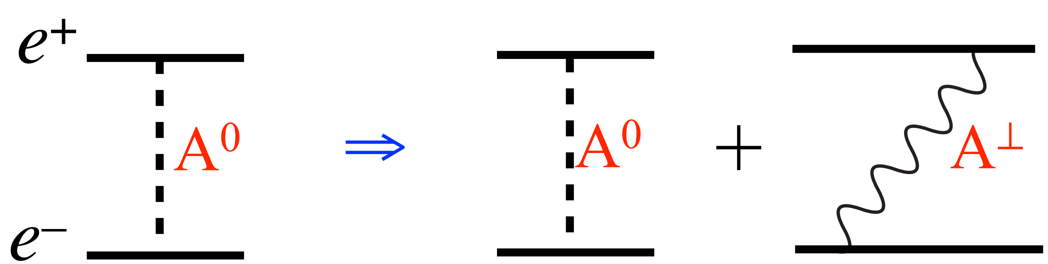


M. Järvinen, hep-ph/0411208

The $|e^+e^-\rangle$ Fock state wf contracts as in classical relativity.

The $|e^+e^-\gamma\rangle$ Fock state has a more complicated dependence on $\boldsymbol{P}$.

Cf. the single photon exchange amplitude:



$\boldsymbol{P} = 0$ $\qquad\qquad$ $\boldsymbol{P} \neq 0$

At higher orders of α more photons (and e^+e^- pairs) need to be included.

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The Dirac Electron in Simple Fields*

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(Received June 6, 1932)

The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in x, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in r, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

See also: E. C. Titchmarsh, Proc. London Math. Soc. (3) 11 (1961) 159 and 169; Quart. J. Math. Oxford (2), 12 (1961), 227.

A linear potential in D=3+1 QCD

Dokshitzer: Confinement in QCD is governed by classical fields (2013)

Zwanziger: No confinement without Coulomb confinement (2003)

Gribov: Coulomb interaction rearranges the vacuum for $\alpha > \alpha^{crit}$ (1997):

$$\alpha^{crit}(\text{QED}) = \pi\left(1-\sqrt{\frac{2}{3}}\right) \simeq 0.58 \qquad \gg \frac{1}{137}$$

$$\alpha_s^{crit}(\text{QCD}) = \frac{\pi}{C_F}\left(1-\sqrt{\frac{2}{3}}\right) \simeq 0.43 \qquad \gtrsim \alpha_s(m_\tau^2) \simeq 0.33$$

The Coulomb field is instantaneous, thus consistent with valence Fock states.

Gauss' law allows to express A^0 in terms of the propagating fields.