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Progress in backreacted holographic QCD

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ENS, Paris

Strong Interactions in the LHC Era
Bad Honnef – 14 November 2014

1. Introduction and motivation
2. Basic properties of V-QCD
3. Results and applications
 - ▶ V-QCD at finite quark mass
 - ▶ The S-parameter
 - ▶ Four fermion deformations

1. Introduction

Motivation

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

- ▶ Often in holography: “quenched” or “probe” approximation, $N_f \ll N_c$, 't Hooft limit
- ▶ Here **Veneziano limit**: large N_f, N_c with $x = N_f/N_c$ fixed \Rightarrow backreaction

Backreaction \Rightarrow better modeling of (ordinary) QCD?

Important new features can be captured in the Veneziano limit, for example:

- ▶ Phase diagram of QCD (at zero temperature, baryon density, and quark mass), varying $x = N_f/N_c$
- ▶ QCD thermodynamics as a function of x

QCD phases as a function of N_f

QCD at fixed N_c (at zero T , μ , and quark mass)



Computations near the “transition” difficult

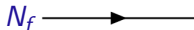
- ▶ Schwinger-Dyson approach, ...
- ▶ Lattice QCD

In the Veneziano limit (discrete) N_f replaced by (continuous) $x = N_f/N_c$

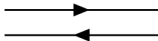
- ▶ True transition expected at some $x = x_c$
- ▶ Nature of the transition: BKT/Miransky (infinite order), or maybe first order (jumping)?

The QCD string in the Veneziano limit

Quarks:

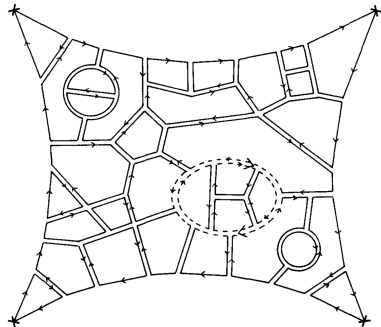


Gluons:

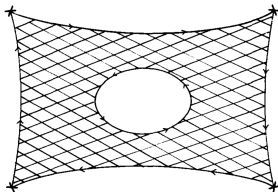


Leading diagrams in $1/N_c$:
gluonic with quark boundaries

[t Hooft]



Veneziano limit \Rightarrow boundaries not suppressed \Rightarrow open string loops!



$$= \mathcal{O}(N_f/N_c)$$

2. V-QCD

A step back: model for glue

“Improved holographic QCD” (IHQCD): well-tested string-inspired bottom-up model for pure Yang-Mills

[Gursoy, Kiritsis, Nitti arXiv:0707.1324, 0707.1349]

[Gubser, Nellore arXiv:0804.0434]

$$\mathcal{S}_g = M^3 N_c^2 \int d^5 x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

with the metric

$$ds^2 = e^{2A(r)} (dr^2 + \eta_{\mu\nu} x^\mu x^\nu)$$

- ▶ $A \leftrightarrow \log \Lambda$ energy scale
- ▶ $\lambda = e^\phi \leftrightarrow$ 't Hooft coupling $g^2 N_c$
- ▶ Modify V_g derived from string theory to match Yang-Mills β -function in the UV ($\lambda \rightarrow 0$)

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: model for glue inspired by string theory (dilaton gravity)

[Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions

[Klebanov, Maldacena; Bigazzi, Casero, Cotrone, Iatrakis, Kiritsis, Paredes]

Consider 1. + 2. in the Veneziano limit with **full backreaction**
 \Rightarrow V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Defining V-QCD

Degrees of freedom

- ▶ The tachyon $\tau \leftrightarrow \bar{q}q$, and the dilaton $\lambda \leftrightarrow \text{Tr}F^2$
- ▶ $\lambda = e^\phi$ is identified as the 't Hooft coupling $g^2 N_c$

$$\mathcal{S}_{\text{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] \\ - N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau)}$$

$$V_f(\lambda, \tau) = V_{f0}(\lambda) \exp(-a(\lambda)\tau^2); \quad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

Need to choose V_{f0} , a , and $\kappa \dots$

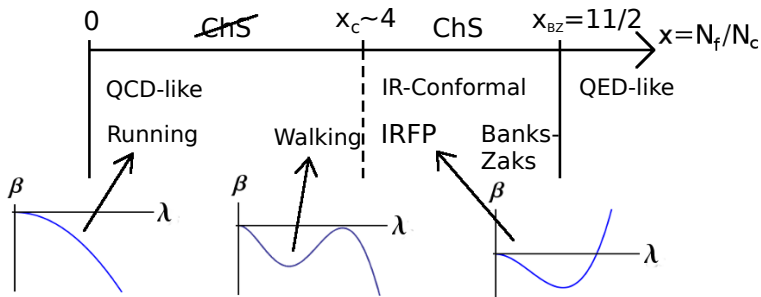
A simple strategy works (!):

- ▶ Match to perturbative QCD in the UV (asymptotic AdS₅)
- ▶ Logarithmically modified string theory predictions in the IR

Phase diagram of V-QCD

- ▶ Choose reasonable potentials
- ▶ Ansatz $\tau(r)$, $\lambda(r)$, $A(r)$ in equations of motion
- ▶ Construct numerically all vacua (various IR geometries)

QCD phase diagram reproduced:



- ▶ Matching to QCD perturbation theory \rightarrow Banks-Zaks
- ▶ Conformal transition (BKT) at $x = x_c \simeq 4$

(With tuned potentials, the phase diagram may change)

How does the phase structure arise?

Turning on a tiny tachyon in the conformal window

$$\tau(r) \sim m_q r^{\gamma_*+1} + \sigma r^{3-\gamma_*} \quad (IR, r \rightarrow \infty)$$

Breitenlohner-Freedman (BF) bound for γ_* at the IRFP

$$(\gamma_* + 1)(3 - \gamma_*) = \Delta_*(4 - \Delta_*) = -m_\tau^2 \ell_*^2 \leq 4$$

Violation of BF bound \Rightarrow **instability** \Rightarrow tachyon/chiral condensate

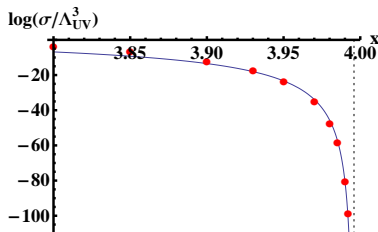
- ▶ \Rightarrow bound **saturated** at the conformal phase transition ($x = x_c$)
- ▶ $\gamma_* = 1$ at the transition
- ▶ BF bound violation leads to a BKT transition quite in general

Energy scales (at zero quark mass)

V-QCD reproduces the picture with Miransky scaling:

1. QCD regime: **single** energy scale Λ
2. Walking regime ($x_c - x \ll 1$): **two** scales related by **Miransky/BKT** scaling law

$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \exp\left(\frac{\kappa}{\sqrt{x_c - x}}\right)$$



3. Conformal window ($x_c \leq x < 11/2$): again one scale Λ , but slow RG flow

3. Applications

V-QCD literature

An ongoing program for studying V-QCD

Exploring the model at qualitative level (good match with QCD!):

- ▶ Phase diagram at finite T and μ

[Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516]

[Alho, MJ, Kajantie, Kiritsis, Rosen, Tuominen arXiv:1312.5199]

- ▶ Fluctuation analysis: meson spectra, S -parameter, quasi normal modes. . .

[Arean, Iatrakis, MJ, Kiritsis arXiv:1211.6125, 1309.2286]

[Iatrakis, Zahed arXiv:1410.8540]

- ▶ CP-odd terms: axial anomaly

[In progress with Arean, Iatrakis, Kiritsis]

- ▶ Phase diagram at finite quark mass

[MJ, arXiv:141n.xxxx]

This talk: selected results relevant for technicolor

Also just started: quantitative fit to QCD data

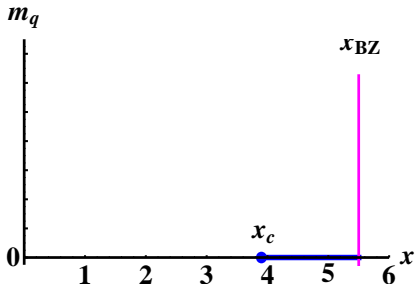
Turning on finite m_q

Quark mass defined through the tachyon boundary conditions in the UV:

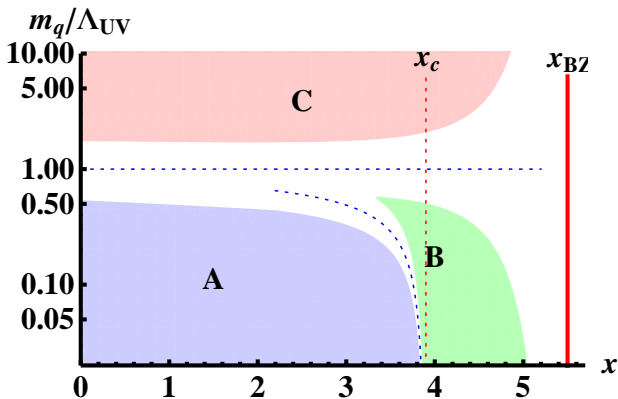
$$\tau(r) \simeq m_q (-\log r)^{-\gamma_0/\beta_0} r + \sigma (-\log r)^{\gamma_0/\beta_0} r^3$$

with $\sigma \sim \langle \bar{q}q \rangle$

- ▶ Finite (flavor independent) m_q implies nonzero tachyon and chiral symmetry breaking
- ▶ Conformal transition becomes a crossover
- ▶ Discontinuous change of IR geometry in the conformal window at $m_q = 0$

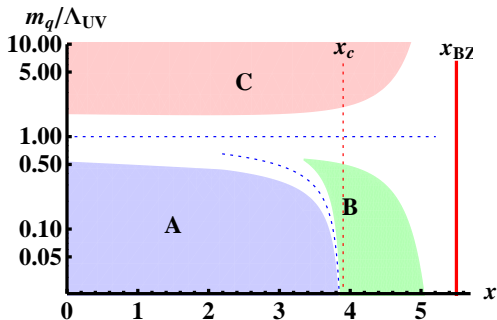


Analysis of the tachyon solution \Rightarrow separate different regimes:



Crossover between A and B: $m_q \sim \exp \left[-\frac{2K}{\sqrt{x_c - x}} \right] \sim \langle \bar{q}q \rangle$

► Regimes A and B “model independent”



A : m_q is a small perturbation

$$m_n(m_q) = m_{n0} + c_1 m_q + c_2 m_q^2 + \dots$$

B : “Hyperscaling” regime: amount of walking fixed by m_q

$$m_n \sim m_q^{1/(\gamma_*+1)}, \quad \langle \bar{q}q \rangle \sim m_q^{(3-\gamma_*)/(\gamma_*+1)}$$

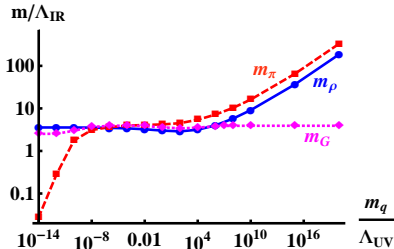
[Evans, Scott]

C : Large quark mass: model dependent,
in V-QCD large mass gap for mesons

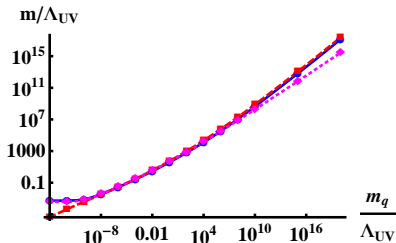
Example: masses for the walking case

$x_c - x \ll 1$, Masses in units

of IR (glueball) scale



of UV (RG flow) scale



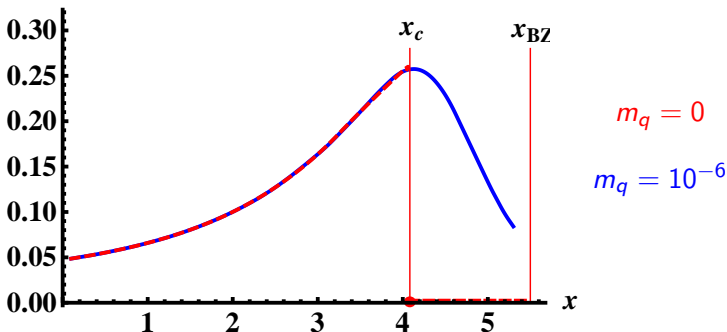
- ▶ $m_\pi/\Lambda_{\text{UV}}$ obeys same power laws in regimes A (GOR) and B
- ▶ Meson masses enhanced wrt glueballs at large m_q

S-parameter

After adding gauge fields dual to vector operators in the DBI action

$$S = 4\pi \frac{\partial}{\partial q^2} [q^2 \Pi_V(q^2) - q^2 \Pi_A(q^2)]_{q^2=0}$$

$S/(N_c N_f)$



- ▶ Discontinuity at $m_q = 0$ in the conformal window
- ▶ Qualitative agreement with field theory expectations

[Sannino]

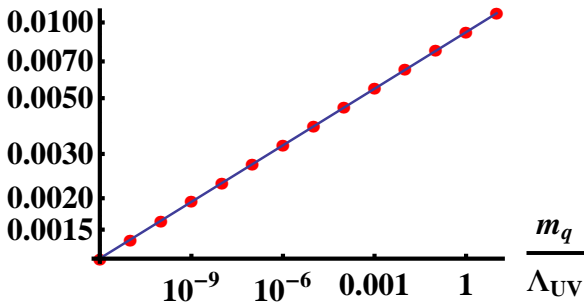
Scaling of the S-parameter

As $m_q \rightarrow 0$ in the conformal window,

$$S(m_q) \simeq S(0+) + c \left(\frac{m_q}{\Lambda_{UV}} \right)^{\frac{\Delta_{FF}-4}{\gamma_*+1}}$$

- ▶ Limiting value $S(0+) = \lim_{m_q \rightarrow 0+} S(m_q)$ is finite and positive (while $S(0) = 0$)
- ▶ Δ_{FF} is the dimension of $\text{tr}F^2$ at the fixed point

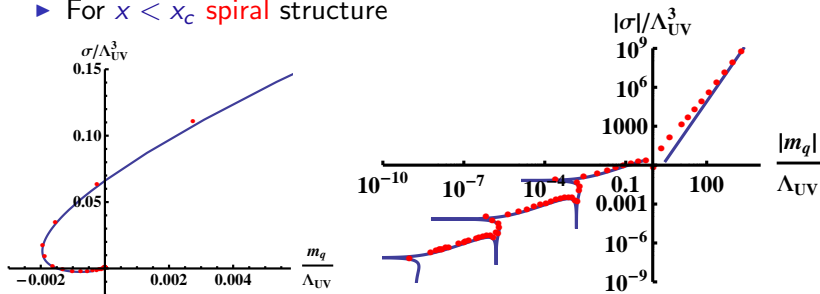
$$(S(m_q) - S(0+)) / N_c N_f$$



Chiral condensate

The dependence $\sigma(m_q)$ of the chiral condensate on the quark mass

- ▶ For $x < x_c$ spiral structure



- ▶ Dots: numerical data
- ▶ Continuous line: (semi)-analytic prediction

Allows to study the effect of double-trace deformations

Four-fermion operators

Witten's recipe: modified UV boundary conditions for the tachyon

For interaction term in field theory ($\mathcal{O} = \bar{q}q$)

$$W = m_q \int d^4x \mathcal{O}(x) + \frac{g}{2} \int d^4x \mathcal{O}(x)^2$$

Writing

$$\tau(r) \sim \alpha r (-\log r\Lambda)^\rho + \beta r^3 (-\log r\Lambda)^{-\rho}$$

require the boundary condition

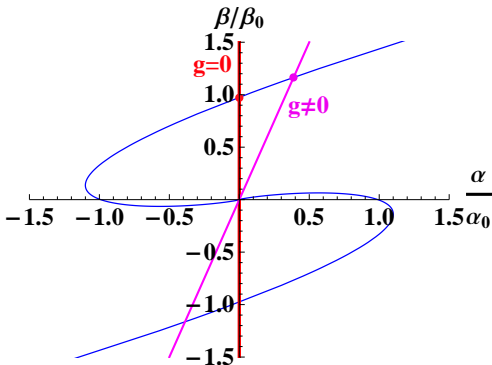
$$\alpha = \left. \frac{\delta W}{\delta \mathcal{O}(x)} \right|_{\mathcal{O}=\beta} = m_q + g\beta$$

Four-fermion operators at zero mass

Example: $x < x_c$
and $m_q = 0$

Efimov spiral:
all sols from holography

Straight lines:
boundary condition
 $\alpha = g\beta$



⇒ find all intersection points, check stability, ...

- ▶ Either an instability (typically when $g < 0$) or a smooth deformation of the $g = 0$ solution
- ▶ Location of conformal window unchanged

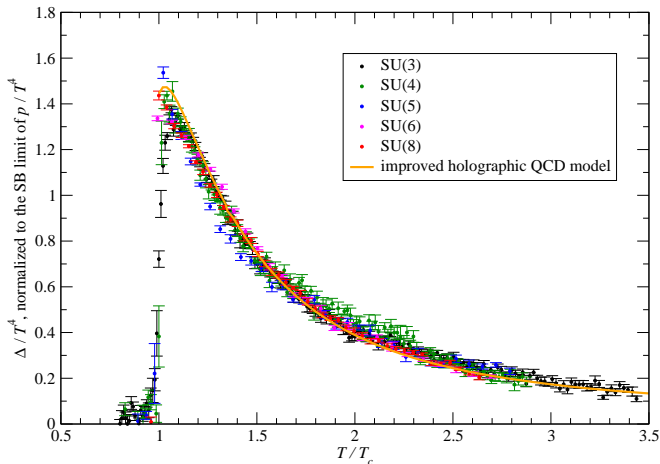
Conclusions

- ▶ V-QCD agrees with field theory results for QCD at qualitative level
- ▶ Next step: fitting the potentials of the model quantitatively to QCD data
- ▶ Toy model \rightarrow effective description of QCD and the conformal transition?

Extra slides

Example of fit to lattice data: interaction measure of Yang-Mills

Trace of the energy-momentum tensor



[Panero]

Second building block: Adding flavor

A recipe for adding quarks (in the fundamental of $SU(N_c)$ and in the probe approximation)

- ▶ Space-filling probe $D4 - \bar{D}4$ branes in 5D \rightarrow
 - ▶ Tachyon $T \leftrightarrow \bar{q}q$
 - ▶ Gauge fields $A_{L/R}^\mu \leftrightarrow \bar{q}\gamma^\mu(1 \pm \gamma_5)q$
- ▶ For the vacuum structure only the tachyon is relevant
- ▶ Sen-like tachyon DBI action with $V_T \sim \exp(-|T|^2)$
 - ▶ Confining IR asymptotics of the geometry triggers ChSB
 - ▶ Gell-Mann-Oakes-Renner relation
 - ▶ Linear Regge trajectories for mesons
 - ▶ A very good fit of the light meson masses

[Klebanov, Maldacena]

[Bigazzi, Casero, Cotrone, Iatrakis, Kiritsis, Paredes
hep-th/0505140, 0702155; arXiv:1003.2377, 1010.1364]

Vector correlators and S-parameter

1. Introduce bulk gauge fields dual to vector operators

$$A_{\mu}^{L/R} \leftrightarrow \bar{q} \gamma_{\mu} (1 \pm \gamma_5) q$$

2. Fluctuate full flavor action of V-QCD

$$S_f = -\frac{1}{2} M^3 N_c \text{Tr} \int d^4x dr \left(V_f(\lambda, T^{\dagger} T) \sqrt{-\det \mathbf{A}_L} + (L \rightarrow R) \right)$$
$$\mathbf{A}_{L/R MN} = g_{MN} + w(\lambda, T) F_{MN}^{(L/R)} +$$
$$+ \frac{\kappa(\lambda, T)}{2} \left[(D_M T)^{\dagger} (D_N T) + (D_N T)^{\dagger} (D_M T) \right]$$

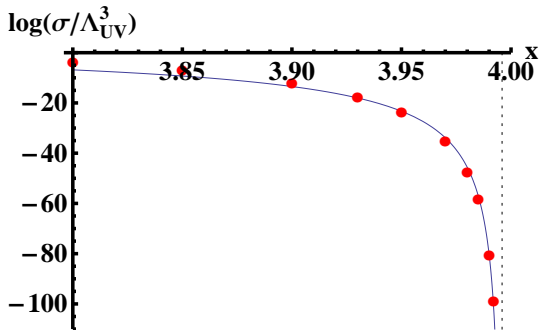
Here T and $A^{(L/R)}$ matrices in flavor space

3. Compute vector-vector correlators using standard recipes

$$-i \langle J_{\mu}^{a(V)} J_{\nu}^{b(V)} \rangle \propto \delta^{ab} (q^2 \eta_{\mu\nu} - q_{\mu} q_{\nu}) \Pi_V(q^2)$$

$$-i \langle J_{\mu}^{a(A)} J_{\nu}^{b(A)} \rangle \propto \delta^{ab} \left[(q^2 \eta_{\mu\nu} - q_{\mu} q_{\nu}) \Pi_A(q^2) + q_{\mu} q_{\nu} \Pi_L(q^2) \right]$$

Consequences of the BKT transition



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \rightarrow x_c$ from below
 - ▶ E.g., The chiral condensate $\langle \bar{q}q \rangle \propto \sigma$
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

Axial anomaly at large N_c

$U(1)_A$ anomalously broken in QCD

However: axial anomaly is suppressed at large N_c (in the 't Hooft limit)

- ▶ “Solved” in the Veneziano limit, where axial anomaly appears at LO
- ▶ η' meson (flavor-singlet pseudoscalar) is the corresponding “Goldstone mode”

[Witten, Veneziano]

$$m_{\eta'}^2 \simeq m_{\pi}^2 + \chi \frac{\chi}{\bar{f}_{\pi}^2}$$

- ▶ χ is the topological susceptibility (constant term in $F \wedge F$ correlator)
- ▶ \bar{f}_{π} is the pion decay constant with $N_{c,f}$ factors divided out
- ▶ Good agreement with experimental+lattice values for QCD

The CP-odd term in V-QCD

Bulk axion a

- ▶ dual to $\text{tr}F \wedge F$
- ▶ background value identified as θ/N_c , where θ is the theta angle of QCD

Tachyon Ansatz $T = \tau e^{i\xi} \mathbb{I}$

String motivated CP-odd term added in the action

$$S_a = -\frac{M^3 N_c^2}{2} \int d^5x \sqrt{-\det g} Z(\lambda) \\ \times [da - x(2V_a(\lambda, \tau)A - \xi dV_a(\lambda, \tau))]^2$$

[Casero, Kiritsis, Paredes]

Symmetry

$$A_\mu \rightarrow A_\mu + \partial_\mu \epsilon, \quad \xi \rightarrow \xi - 2\epsilon, \quad a \rightarrow a + 2x V_a \epsilon$$

reflects the axial anomaly in QCD (with $\epsilon = \epsilon(x_\mu)$)

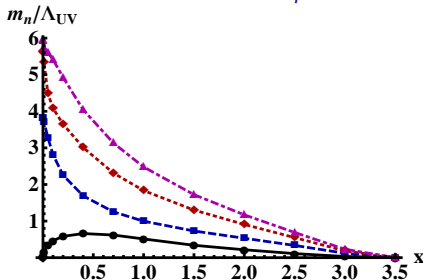
The mass of η' in V-QCD

Analytic derivation by perturbative analysis of the coupled flavor singlet (pseudoscalar meson+glueball) fluctuation equations \Rightarrow

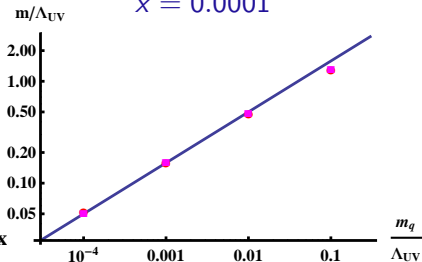
The Witten-Veneziano relation: η' becomes light as $x \rightarrow 0$

$$m_{\eta'}^2 \simeq m_{\pi}^2 + x \frac{\chi}{f_{\pi}^2}$$

PS masses at $m_q = 0$



π and η' masses at $x = 0.0001$



Finite T and μ – definitions

Add gauge field

$$\begin{aligned} \mathcal{S}_{V\text{-QCD}} = & N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] \\ & - N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \\ & \times \sqrt{-\det(g_{ab} + \kappa(\lambda) \partial_a \tau \partial_b \tau + w(\lambda) F_{ab})} \end{aligned}$$

$$F_{r0} = \partial_r \Phi \quad \Phi = \mu - nr^2 + \dots$$

A more general metric (A and f solved from EoMs)

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r) dt^2 + dx^2 \right)$$

Nontrivial blackening factor f : black hole solutions possible

Various solutions

Two classes of IR geometries:

1. Black hole solutions \rightarrow temperature and entropy through BH thermodynamics
 - ▶ $f'(r_h) = -4\pi T$; $s = 4\pi M^3 N_c^2 e^{3A(r_h)}$
2. Thermal gas solutions ($f \equiv 1$)
 - ▶ Any T and μ , zero s

Two types of tachyon behavior ($\tau \leftrightarrow \bar{q}q$, quark mass and condensate from UV boundary behavior):

1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

\Rightarrow **four** possible types of background solutions

Computation of pressure

Three phases turn out to be relevant (at small x)

- ▶ Tachyonic Thermal gas (chirally broken)
- ▶ Tachyonic BH (chirally broken)
- ▶ Tachyonless BH (chirally symmetric)

Nontrivial numerical analysis:

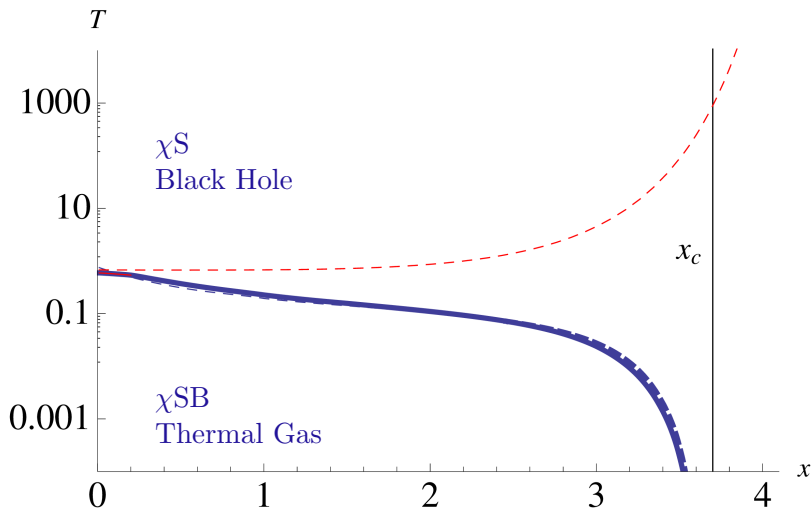
1. T , μ not input parameters, they need to be calculated first
2. Integrate numerically for each phase

$$dp = s dT + n d\mu$$

3. Phase with highest p dominates

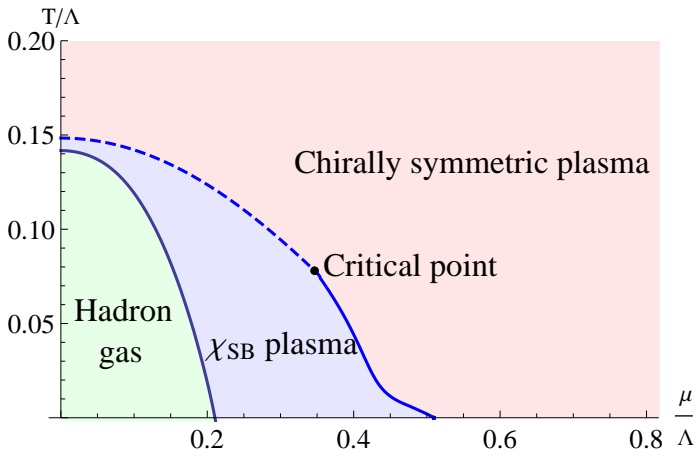
Phase diagram: example at zero μ

Phases on the (x, T) -plane – as expected from QCD



Phase diagram at finite μ (example at fixed x)

First attempt: $x = N_f/N_c = 1$, Veneziano limit, zero quark mass



- ▶ $AdS_2 \times \mathbb{R}^3$ IR geometry as $T \rightarrow 0$
- ▶ Finite entropy at zero temperature \Rightarrow instability?

Fluctuation analysis

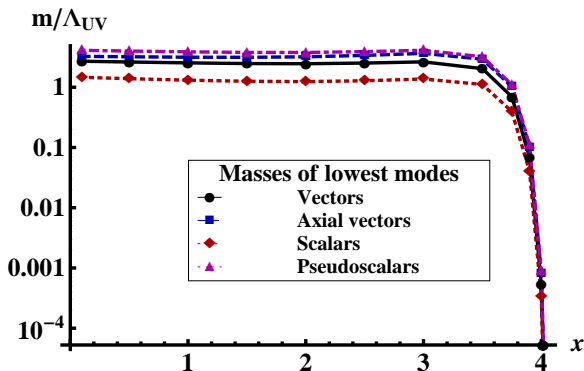
1. Meson spectra (at zero temperature and quark mass)
 - ▶ Implement (left and right handed) gauge fields in $\mathcal{S}_{V\text{-QCD}}$
 - ▶ Four towers: scalars, pseudoscalars, vectors, and axial vectors
 - ▶ Flavor singlet and nonsinglet ($SU(N_f)$) states

In the region relevant for “walking” technicolor ($x \rightarrow x_c$ from below):

- ▶ Possibly a light “dilaton” (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry.
Could the dilaton be the 125 GeV Higgs?

Meson masses

Flavor nonsinglet masses (Example: PotI)



► **Miransky** scaling:

$$m_n \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

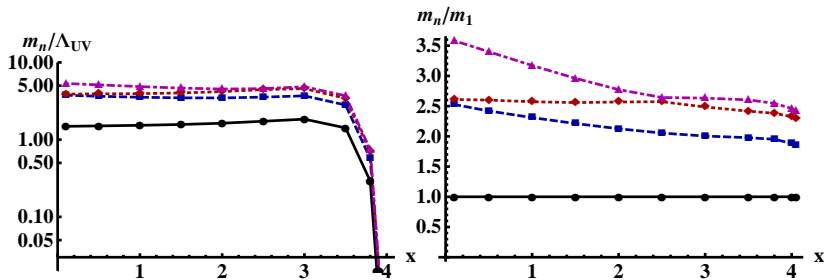
► Radial trajectories $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

Scalar singlet masses

Scalar singlet (0^{++}) spectrum (PotI):

In log scale

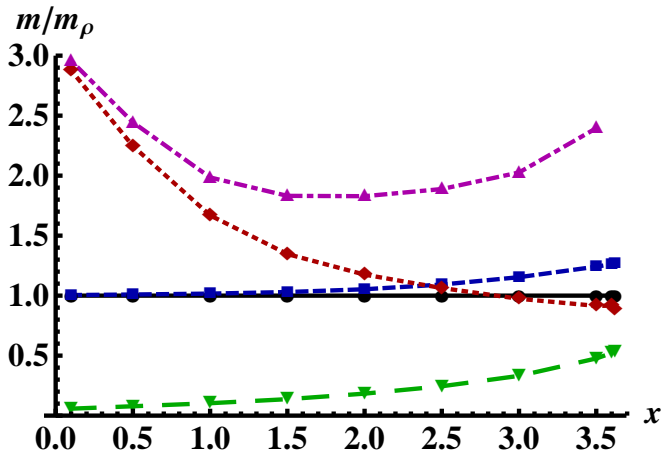
Normalized to the lowest state



► No light dilaton state as $x \rightarrow x_c$?

Meson mass ratios

Mass ratios (PotII): Lowest states normalized to ρ



All ratios tend to constants as $x \rightarrow x_c$: indeed **no dilaton**

S-parameter

$$S \sim \frac{d}{dq^2} q^2 [\Pi_V(q^2) - \Pi_A(q^2)]_{q^2=0}$$

where (at zero quark mass)

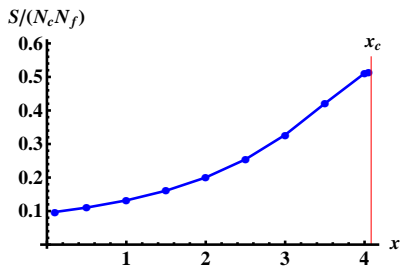
$$\Pi_{V/A}(q^2) (q^2 g^{\mu\nu} - q^\mu q^\nu) \delta^{ab} \propto \langle J_{V/A}^{\mu a} J_{V/A}^{\nu b} \rangle$$

in terms of the vector-vector and axial-axial correlators

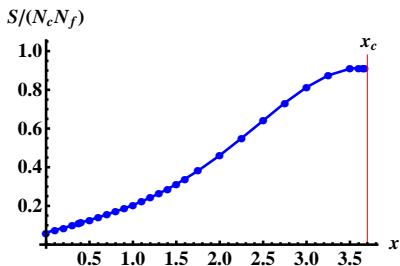
- ▶ The S-parameter might be reduced in the walking regime

Results:

PotI



PotII

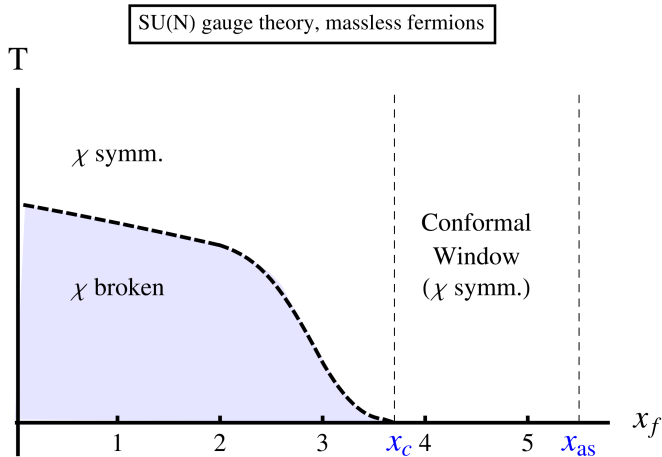


The S-parameter **increases** with x : **expected suppression absent**

Jumps discontinuously to zero at $x = x_c$

QCD at finite T (and x)

Expected phase structure at finite temperature (and x)



Potentials I

$$\begin{aligned}V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4} \frac{\lambda^2}{(1 + \lambda/(8\pi^2))^{2/3}} \sqrt{1 + \log(1 + \lambda/(8\pi^2))} \\V_f(\lambda, \tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33 - 2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2 \\a(\lambda) &= \frac{3}{22}(11 - x) \\\kappa(\lambda) &= \frac{1}{\left(1 + \frac{115 - 16x}{288\pi^2}\lambda\right)^{4/3}}\end{aligned}$$

In this case the tachyon diverges exponentially:

$$\tau(r) \sim \tau_0 \exp \left[\frac{81 \cdot 3^{5/6} (115 - 16x)^{4/3} (11 - x)}{812944 \cdot 2^{1/6}} \frac{r}{R} \right]$$

Potentials II

$$\begin{aligned}V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4} \frac{\lambda^2}{(1 + \lambda/(8\pi^2))^{2/3}} \sqrt{1 + \log(1 + \lambda/(8\pi^2))} \\V_f(\lambda, \tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33 - 2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2 \\a(\lambda) &= \frac{3}{22}(11 - x) \frac{1 + \frac{115 - 16x}{216\pi^2}\lambda + \lambda^2/(8\pi^2)^2}{(1 + \lambda/(8\pi^2))^{4/3}} \\\kappa(\lambda) &= \frac{1}{(1 + \lambda/(8\pi^2))^{4/3}}\end{aligned}$$

In this case the tachyon diverges as

$$\tau(r) \sim \frac{27 \cdot 2^{3/4} 3^{1/4}}{\sqrt{4619}} \sqrt{\frac{r - r_1}{R}}$$

Effective potential

For solutions with $\tau = \tau_* = \text{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, \tau_*) \right]$$

IHQCD with an **effective potential**

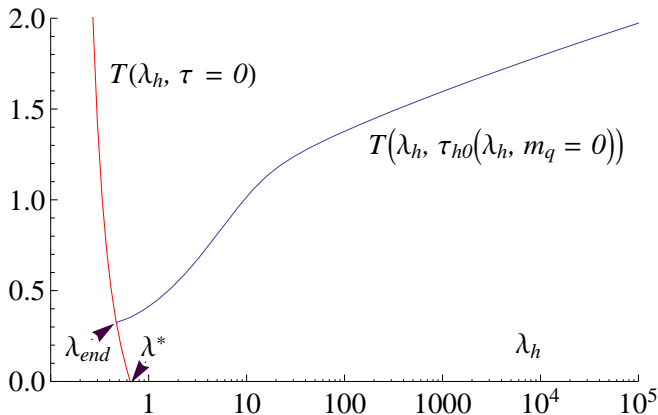
$$V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, \tau_*) = V_g(\lambda) - xV_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

- ▶ $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$;
fixed point with $V'_{\text{eff}}(\lambda_*) = 0$
- ▶ $\tau_* \rightarrow \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Black hole branches

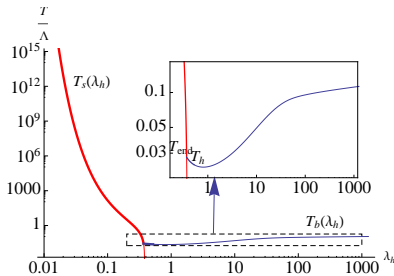
Example: PotII at $x = 3$, $W_0 = 12/11$



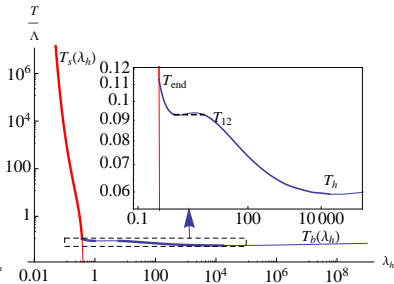
Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB



PotI at $x = 3.5$, $W_0 = 12/11$

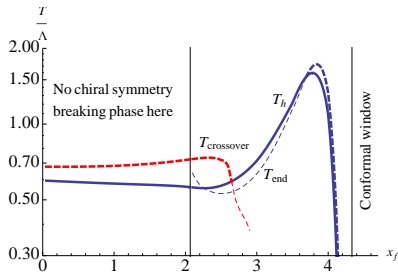


- ▶ Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- ▶ Right: Additional first order transition between BH phases with broken chiral symmetry

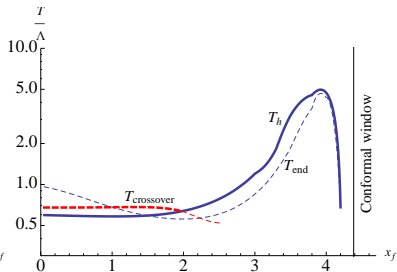
Also other cases ...

Phase diagrams on the (x, T) -plane

PotI* W_0 SB

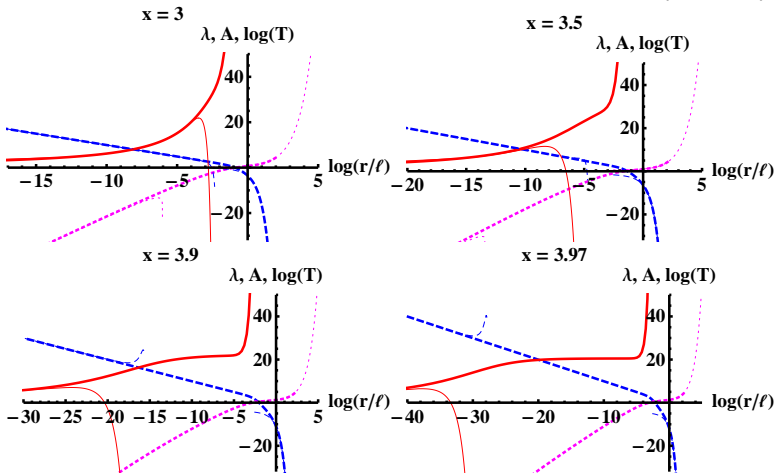


PotII* W_0 SB

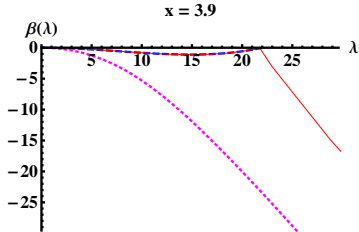
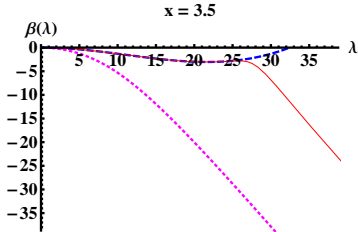
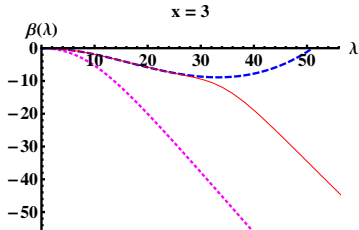
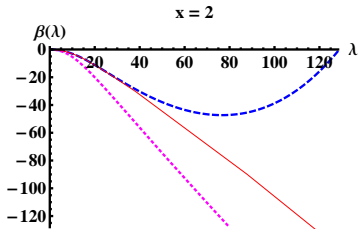


Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ , A , τ)



Beta functions **along the RG flow** (evaluated on the background),
 zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IHQCD

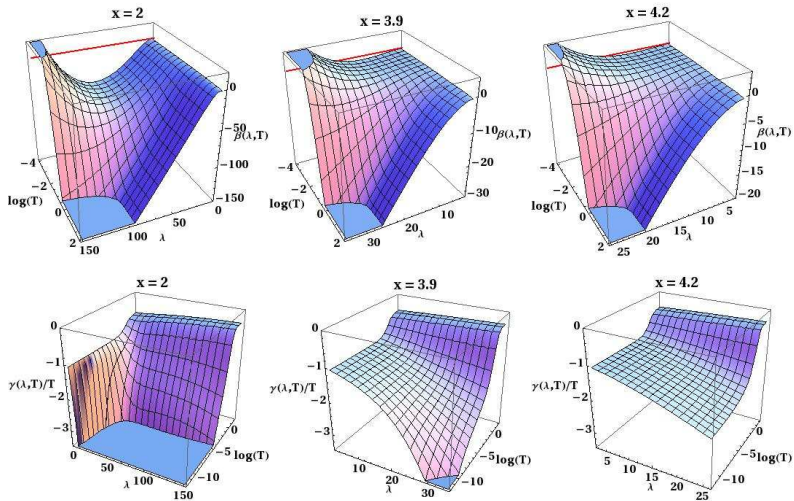
$$\beta(\lambda, \tau) \equiv \frac{d\lambda}{dA} ; \quad \gamma(\lambda, \tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\text{QCD}}}{d \log \mu} ; \quad \frac{dm}{d \log \mu}$$

The **full** equations of motion boil down to two first order partial non-linear differential equations for β and γ

“Good” solutions numerically (unique)



Miransky/BKT scaling

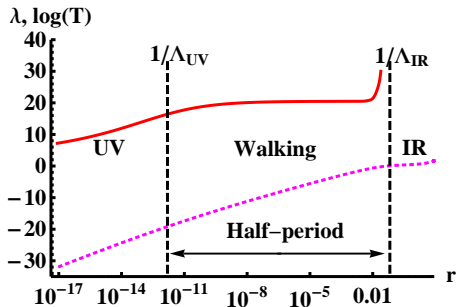
As $x \rightarrow x_c$ from below: walking, dominant solution

- ▶ BF-bound for the tachyon violated at the IRFP

- ▶ x_c fixed by the BF bound:

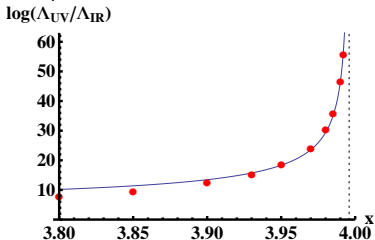
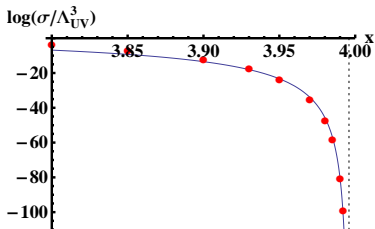
$$\Delta = 2 \text{ \& } \gamma_* = 1$$

at the edge of the conformal window



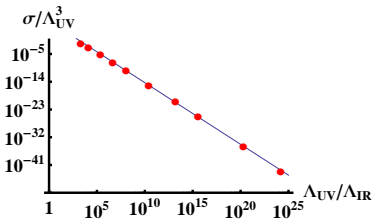
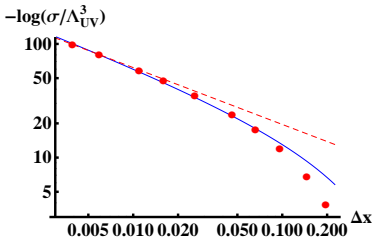
- ▶ $\tau(r) \sim r^2 \sin(\kappa \sqrt{x_c - x} \log r + \phi)$ in the walking region
- ▶ “0.5 oscillations” \Rightarrow Miransky/BKT scaling, amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa \sqrt{x_c - x}))$

As $x \rightarrow x_c$
with known κ



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$

$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$



γ_* in the conformal window

Comparison to other guesses

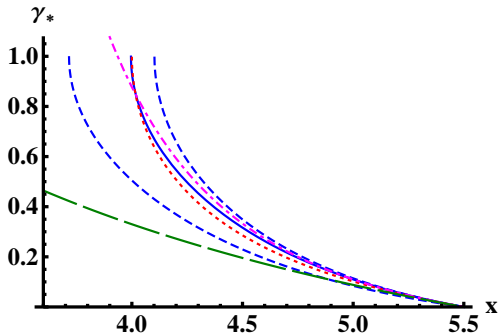
V-QCD (dashed: variation
due to W_0)

Dyson-Schwinger

2-loop PQCD

All-orders β

[Pica, Sannino arXiv:1011.3832]



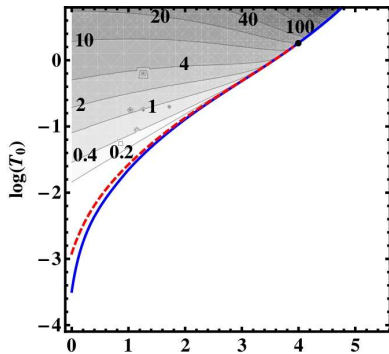
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

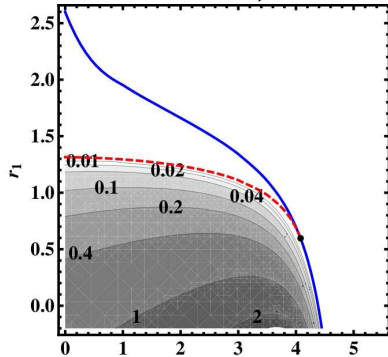
- ▶ YM or QCD with massless quarks: **no parameters**
- ▶ QCD with flavor-independent mass m : a **single** (dimensionless) parameter m/Λ_{QCD}
- ▶ In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- ▶ x has become continuous in the Veneziano limit

Map of all solutions

All “good” solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

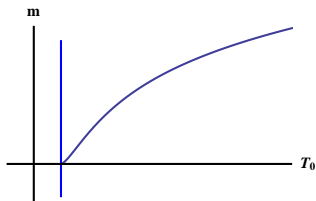


“Potentials I” $\leftrightarrow T_0$



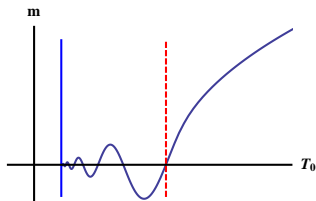
“Potentials II” $\leftrightarrow r_1$

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- ▶ For $m = 0$, unique solution with $\tau \equiv 0$
- ▶ For $m > 0$, unique “standard” solution with $\tau \neq 0$

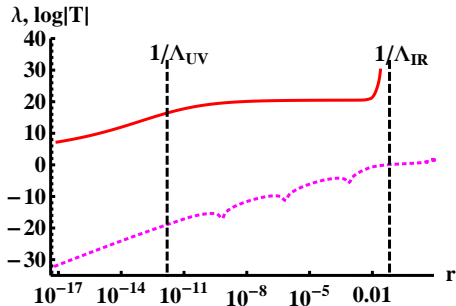


Low $0 < x < x_c$: **Efimov vacua**

- ▶ Unstable solution with $\tau \equiv 0$ and $m = 0$
- ▶ “Standard” stable solution, with $\tau \neq 0$, for all $m \geq 0$
- ▶ Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- ▶ Tachyon oscillates over the walking regime
- ▶ $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. “standard” solution



Effective potential: zero tachyon

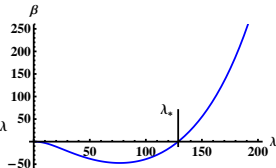
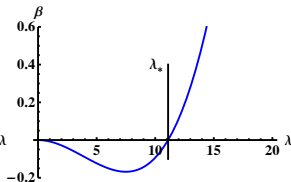
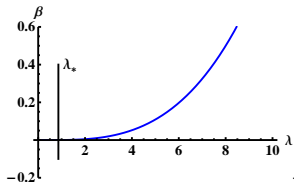
Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved
 ($\tau \leftrightarrow \bar{q}q$), $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

- ▶ V_{eff} defines a β -function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- ▶ Fixed point λ_* runs to ∞ either at finite $x(<x_c)$ or as $x \rightarrow 0$

Banks-Zaks
 $x \rightarrow 11/2$

Conformal Window
 $x > x_c$

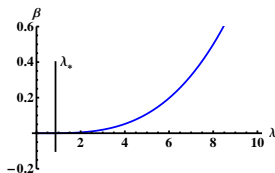
$x < x_c$??



Effective potential: what actually happens

Banks-Zaks

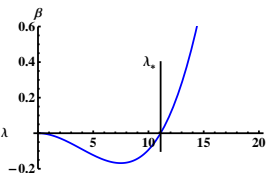
$$x \rightarrow 11/2$$



$$\tau \equiv 0$$

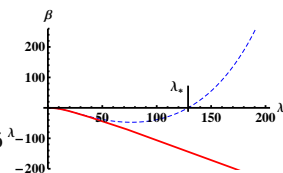
Conformal Window

$$x > x_c$$



$$\tau \equiv 0$$

$$x < x_c$$



$$\tau \neq 0$$

- ▶ For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- ▶ The tachyon **screens the fixed point**
- ▶ In the deep IR τ diverges, $V_{\text{eff}} \rightarrow V_g \Rightarrow$ dynamics is YM-like

Where is x_c ?

How is the edge of the conformal window stabilized?

Tachyon IR mass at $\lambda = \lambda_* \leftrightarrow$ quark mass dimension

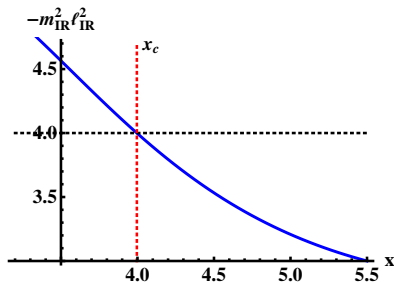
$$-m_{\text{IR}}^2 \ell_{\text{IR}}^2 = \Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\text{IR}} - 1$$

Breitenlohner-Freedman
(BF) bound (horizontal line)

$$-m_{\text{IR}}^2 \ell_{\text{IR}}^2 = 4 \Leftrightarrow \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details ... the question boils down to the linearized tachyon solution at the fixed point

- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) < 4$ ($x > x_c$):

$$\tau(r) \sim m_q r^{\Delta_{\text{IR}}} + \sigma r^{4 - \Delta_{\text{IR}}}$$

- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) > 4$ ($x < x_c$):

$$\tau(r) \sim Cr^2 \sin[(\text{Im}\Delta_{\text{IR}}) \log r + \phi]$$

Rough analogy:

Tachyon EoM \leftrightarrow Gap equation in Dyson-Schwinger approach

Similar observations have been made in other holographic frameworks

[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

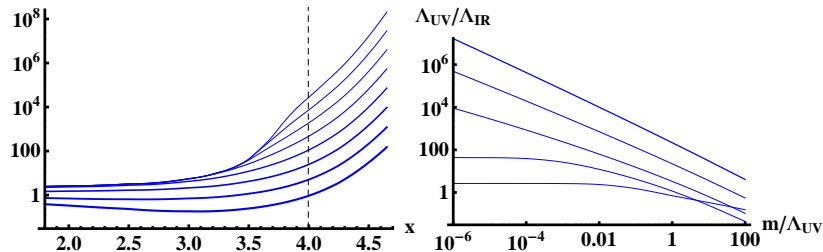
Mass dependence

For $m > 0$ the conformal transition disappears

The ratio of typical UV/IR scales $\Lambda_{UV}/\Lambda_{IR}$ varies in a natural way

$m/\Lambda_{UV} = 10^{-6}, 10^{-5}, \dots, 10$ $x = 2, 3.5, 3.9, 4.25, 4.5$

$\Lambda_{UV}/\Lambda_{IR}$



sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- ▶ $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c} .
- ▶ At $0 < x < 1$, the theory has a runaway ground state.
- ▶ At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- ▶ At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- ▶ At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- ▶ At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- ▶ At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) < 4$:
 $\tau(r) \sim m_q r^{4 - \Delta_{\text{IR}}} + \sigma r^{\Delta_{\text{IR}}}$
- ▶ For $\Delta_{\text{IR}}(4 - \Delta_{\text{IR}}) > 4$:
 $\tau(r) \sim Cr^2 \sin[(\text{Im}\Delta_{\text{IR}}) \log r + \phi]$
- ▶ Saturating the BF bound, the tachyon solutions will entangle
→ required to satisfy boundary conditions
- ▶ Nodes in the solution appear trough UV → massless solution

Saturating the BF bound (sketch)

Does the nontrivial (ChSB) massless tachyon solution exist?

Two possibilities:

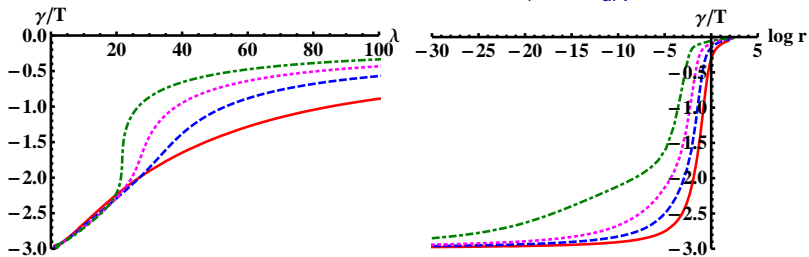
- ▶ $x > x_c$: BF bound satisfied at the fixed point \Rightarrow only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- ▶ $x < x_c$: BF bound violated at the fixed point \Rightarrow a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: **phase transition** at $x = x_c$

As $x \rightarrow x_c$ from below, need to approach the fixed point to satisfy the boundary conditions \Rightarrow nearly conformal, **“walking” dynamics**

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$



$x = 2, 3, 3.5, 3.9$