interacting UV fixed points : from 4D gauge theories to 4D quantum gravity

Daniel F Litim

University of Sussex

standard model

University of Sussex

local QFT for fundamental interactions

strong nuclear force
weak force
electromagnetic force

degrees of freedom

spin 0 (the Higgs has finally arrived)
spin 1/2 (quite a few)
spin 1

perturbatively renormalisable & predictive

standard model

University of Sussex

local QFT for fundamental interactions

strong nuclear force
weak force
electromagnetic force

challenges

Higgs, QED: maximum UV extension?
how does **quantum gravity** fit in?
...

interacting UV fixed points

UV fixed points

perturbation theory

theory with coupling α:

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = \quad - B\,\alpha^2$$

$\alpha_* \ll 1$

free fixed point

$$\alpha_* = 0$$

perturbation theory

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = \quad - B\,\alpha^2$$

$\alpha_* \ll 1$

free fixed point $\alpha_* = 0$

QED, Higgs $B < 0$ IR fixed point

predictive up to maximal UV extension

asymptotic freedom

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = \qquad - B\,\alpha^2$$

$\alpha_* \ll 1$

free fixed point $\qquad \alpha_* = 0$

QCD $\qquad B > 0 \qquad$ UV fixed point

perturbative renormalisability & asymptotic freedom
predictive up to highest energies

fundamental definition of QFT $\Leftrightarrow$ UV fixed point

Wilson '71

interacting fixed point

theory with coupling α**:** $t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

perturbative non-renormalisability: $A > 0$

interacting fixed point

theory with coupling α**:** $t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

interacting fixed point

theory with coupling α**:**

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$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

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if $A > 0\,,\ B > 0$**:** **IR** **UV**

interacting fixed point

theory with coupling α**:** $t = \ln \mu / \Lambda$

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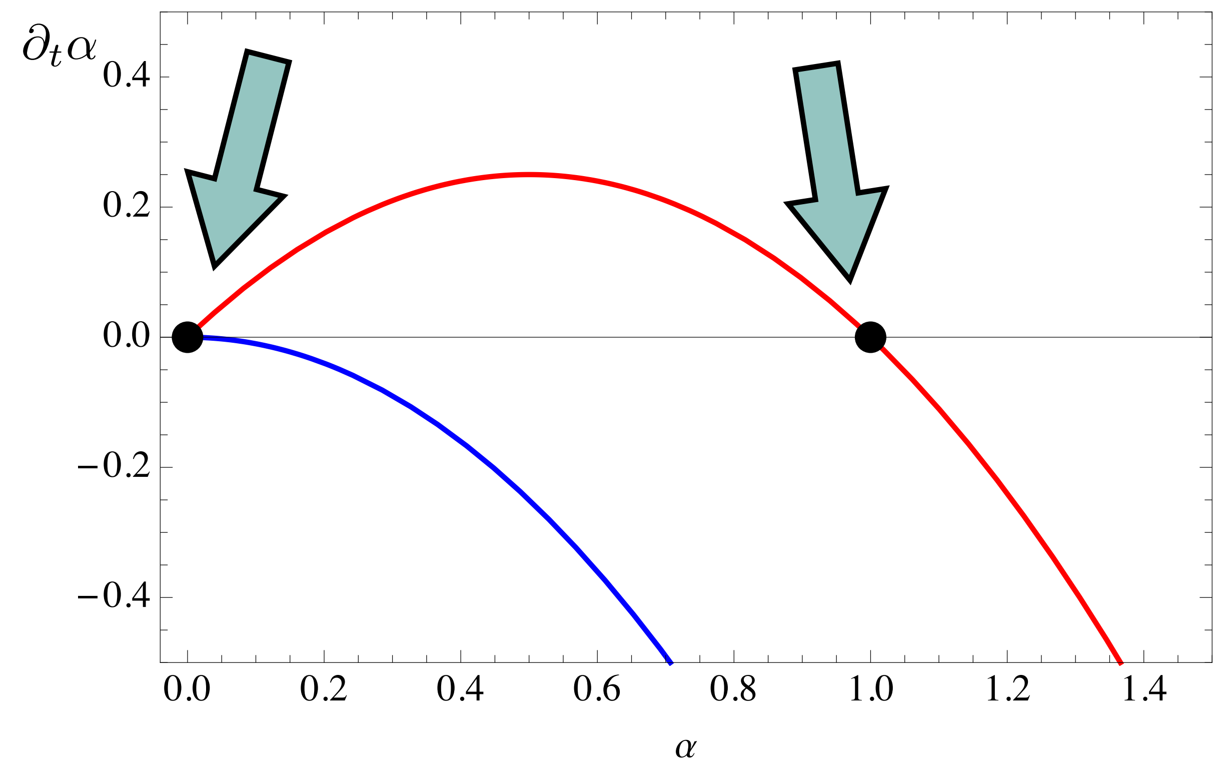


interacting fixed point

theory with coupling α:

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

epsilon expansion: $\epsilon = D - D_c$

large-N expansion: many fields

perturbation theory

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

gravitons	$D = 2 + \epsilon: \quad \alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Weinberg '79 Kawai et al '90
fermions	$D = 2 + \epsilon: \quad \alpha = g_{\rm GN}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4 + \epsilon: \quad \alpha = g^2_{\rm YM}(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2 + \epsilon: \quad \alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76

non-perturbative renormalisability

$$A = \epsilon \ll 1\,, \quad B = \mathcal{O}(1) > 0$$

UV fixed points in 4D gauge theories

with F Sannino
1406.2337

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

gauge theory with fermions

SU(NC) YM with NF fermions: $$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

large-NF,NC (Veneziano) limit:

ϵ **continuous**

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

Veneziano '79
Banks, Zaks '82

we consider $$0 < -B \equiv -B\,(\epsilon) \ll 1$$

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \dfrac{g^2\, N_c}{(4\pi)^2}$ $\quad t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2 + C \alpha_g^3 \qquad \alpha_* \ll 1$$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

interacting fixed points:

$B < 0 \;\; \& \;\; C < 0 :$ **UV fixed point**
no asymptotic freedom

$B > 0 \;\; \& \;\; C > 0 :$ **Caswell - Banks-Zaks**
IR fixed point

gauge theory with fermions

SU(NC) YM with NF fermions: $$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

consider the regime $0 < \epsilon \ll 1$

here: $$B = -\frac{4\epsilon}{3} < 0 \;\; \& \;\; C > 0$$

no physical fixed point

Caswell '74

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$$t = \ln \mu/\Lambda$$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$$\alpha_* \ll 1$$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

→ **scalar fields & Yukawa couplings required**

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

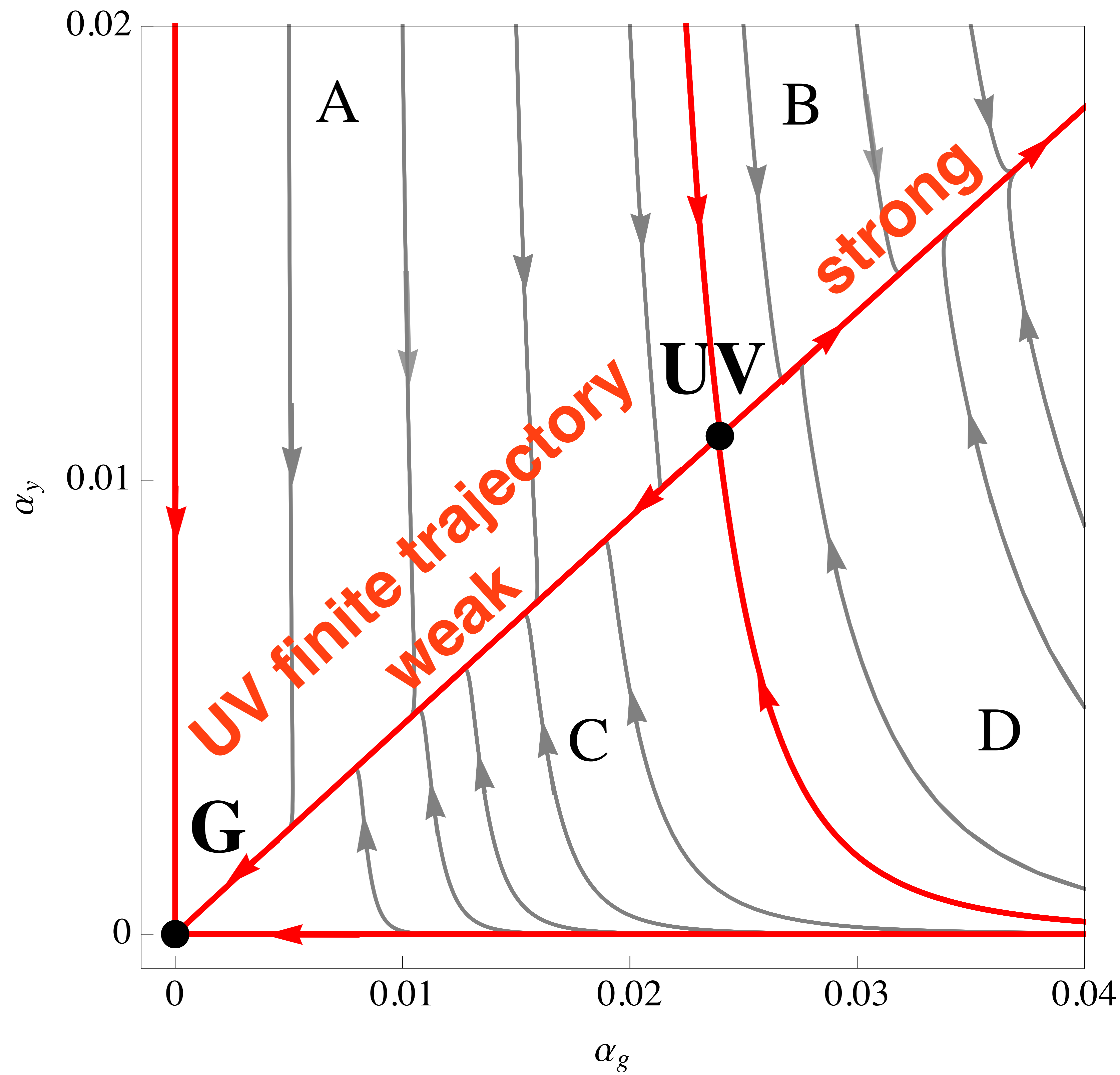
$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

→ **sensible interacting UV fixed point**

$$D\,F - C\,E \geq 0$$

results

phase diagram



exact UV FP
strict perturbative control

sample gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu} F_{\mu\nu}$$
$$L_F = {\rm Tr}\left(\overline{Q}\, i \not{D}\, Q\right)$$
$$L_Y = y\, {\rm Tr}\left(\overline{Q}\, H\, Q\right)$$
$$L_H = {\rm Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$$
$$L_U = -u\, {\rm Tr}\,(H^\dagger H)^2$$
$$L_V = -v\, ({\rm Tr}\, H^\dagger H)^2\,.$$

couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}$$
$$\alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}\,.$$

no asymptotic freedom

small parameter

$$0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

sample gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} \,.$$

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) \,.$$

sample gauge-Yukawa theory

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$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)\ .$$

UV fixed point

$$\begin{aligned} \alpha_g^* &= 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3)\,. \end{aligned}$$

vacuum stability

$$\alpha_h^* + \alpha_{v2}^* < 0 < \alpha_h^* + \alpha_{v1}^*$$

sample gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon \right) \alpha_g - 2 \left(\frac{11}{2} + \epsilon \right)^2 \alpha_y \right\}$$

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$$\beta_v = 12\alpha_h^2 + 4\alpha_v \left(\alpha_v + 4\alpha_h + \alpha_y \right) .$$

→ **sensible interacting UV fixed point**

UV fixed points in 4D quantum gravity

with K Falls, DL, K Nikolakopoulos & C Rahmede
1301.4191, 1410. 4815 & forthcoming

evidence for UV fixed point

overviews: DL **0810.3675** and **1102.4624**

gravitation

Einstein-Hilbert (Reuter '96, Souma '99, Reuter, Lauscher '01, DL '03)

higher dimensions, dimensional reduction (DL '03, Fischer, DL '05)

f(R), polynomials in R (Lauscher, Reuter, '02, Codello, Percacci, Rahmede '08, Machado, Saueressig '09 Benedetti,Caravelli'12, Dietz, Morris'12, Falls, DL, Nikolakopoulos, Rahmede '13)

local potential approximation (Benedetti,Caravelli'12, Dietz, Morris, '12, Demmel, Saueressig, Zanusso '12, Falls, DL, Nikolakopoulos, Rahmede '13, Benedetti '13, Benedetti, Guarnieri '13)

higher-derivative gravity (Codello, Percacci '05) (Benedetti, Saueressig, Machado '09, Niedermaier '09) (DL, Rahmede, in prep.)

conformally reduced gravity (Reuter,Weyer '09, Machado, Percacci '10, DL, Satz '12)

Holst action + Immirzi parameter (Daum, Reuter '10, Benedetti, Speciale '11)

signature effects (Manrique, Rechenberger, Saueressig '11)

gravitation + matter

matter (Percacci '05, Perini, Percacci '05, Narain, Percacci '09, Narain, Rahmede '09, Codello '11, Eichhorn et al '13)

Yang-Mills gravity

- **1-loop:** (Robinson,Wilzcek '05, Pietrokowski, '06,Toms '07, Ebert, Plefka, Rodigast '08)
- **beyond:** (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawlowski, 11, Harst, Reuter '11)

quantum gravity

running coupling $g(k) = G_N(k) k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N)\, g \qquad t = \ln k/\Lambda_c$$

quantum gravity

running coupling $$g(k) = G_N(k) k^{D-2}$$

$$\partial_t g = \left(D - 2 + \eta_N\right) g \qquad t = \ln k/\Lambda_c$$

$g_* \neq 0$ — **UV**

$g_* = 0$ — **IR**

fixed points

quantum gravity

running coupling $g(k) = G_N(k) k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N)\, g$$

$t = \ln k/\Lambda_c$

$g_* \neq 0$ $g_* = 0$

UV **IR**

fixed points

large **anomalous dimension** $\eta_N = \eta_N(g, \text{all other couplings})$

large **UV scaling exponents** $\vartheta \approx \mathcal{O}(1)$

strong **coupling effects** $g_* \approx \mathcal{O}(1)$

relevant **vs** **irrelevant**

invariants not known a priori

asymptotic freedom

$$g_* = 0$$

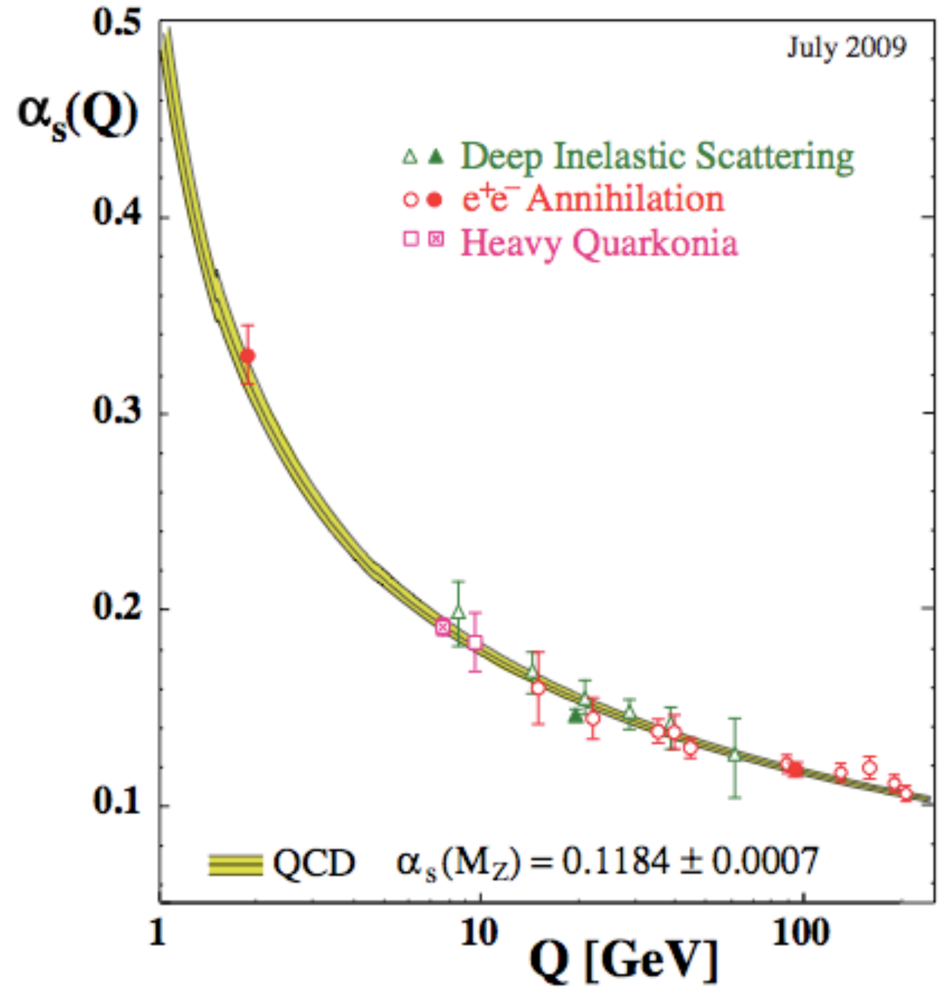
anomalous dimensions

$$\eta_A = 0$$

canonical power counting

$\{\vartheta_{\mathrm{G},n}\}$ are known

F^{256} irrelevant !



asymptotic freedom vs **asymptotic safety**

asymptotic freedom	asymptotic safety
$g_* = 0$	$g_* \neq 0$
anomalous dimensions	anomalous dimensions
$\eta_A = 0$	$\eta_N \neq 0$
canonical power counting	**non-canonical** power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known	$\{\vartheta_n\}$ are **not** known
F^{256} irrelevant !	R^{256} relevant / marginal / irrelevant **?**

pulling oneself over the fence...

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

... using a bootstrap

hypothesis operator ordering at UV FP follows canonical dimension

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

... using a bootstrap

hypothesis operator ordering at UV FP follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

→ **hypothesis** **supported** (refuted)

f(R) quantum gravity

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

invariants up to $D = 2(N-1)$

Step 2

RG flow
fixed point
scaling exponents

Step 3

enhance $N \to N+1$
& iterate

f(R) quantum gravity

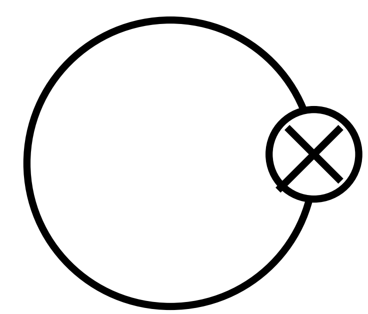
Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g}\, R^n$$

invariants up to $D = 2(N-1)$

functional renormalisation:

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] \quad = \frac{1}{2}$$



here:

M Reuter hep-th/9605030

DL hep-th/0103195
hep-th/0312114

Falls, DL, Nikolakopoulos, Rahmede **1301.4191.pdf** **1410.4815.pdf**

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909
P Machado, F Saueressig 0712.0445

f(R) quantum gravity

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

invariants up to $D = 2(N-1)$

Step 2

RG flow
fixed point
scaling exponents

Step 3

enhance $N \to N+1$
& iterate

iterate Step 1, 2 and 3

how often is enough?

well, it depends...

here:
34 consecutive orders

f(R) quantum gravity

Step 1

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

invariants up to $D = 2(N-1)$

Step 2

RG flow
fixed point
scaling exponents

Step 3

enhance $N \to N+1$
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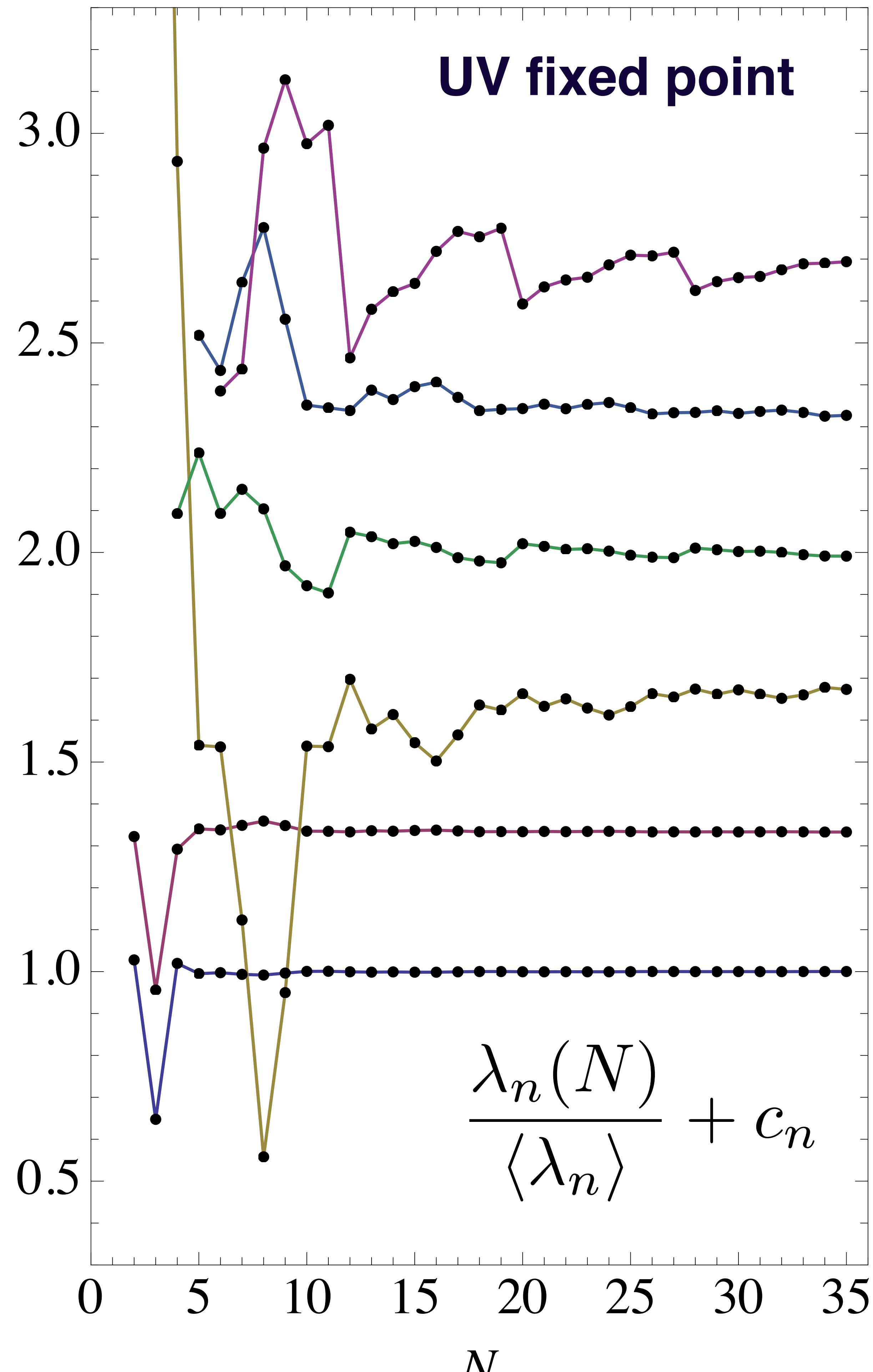
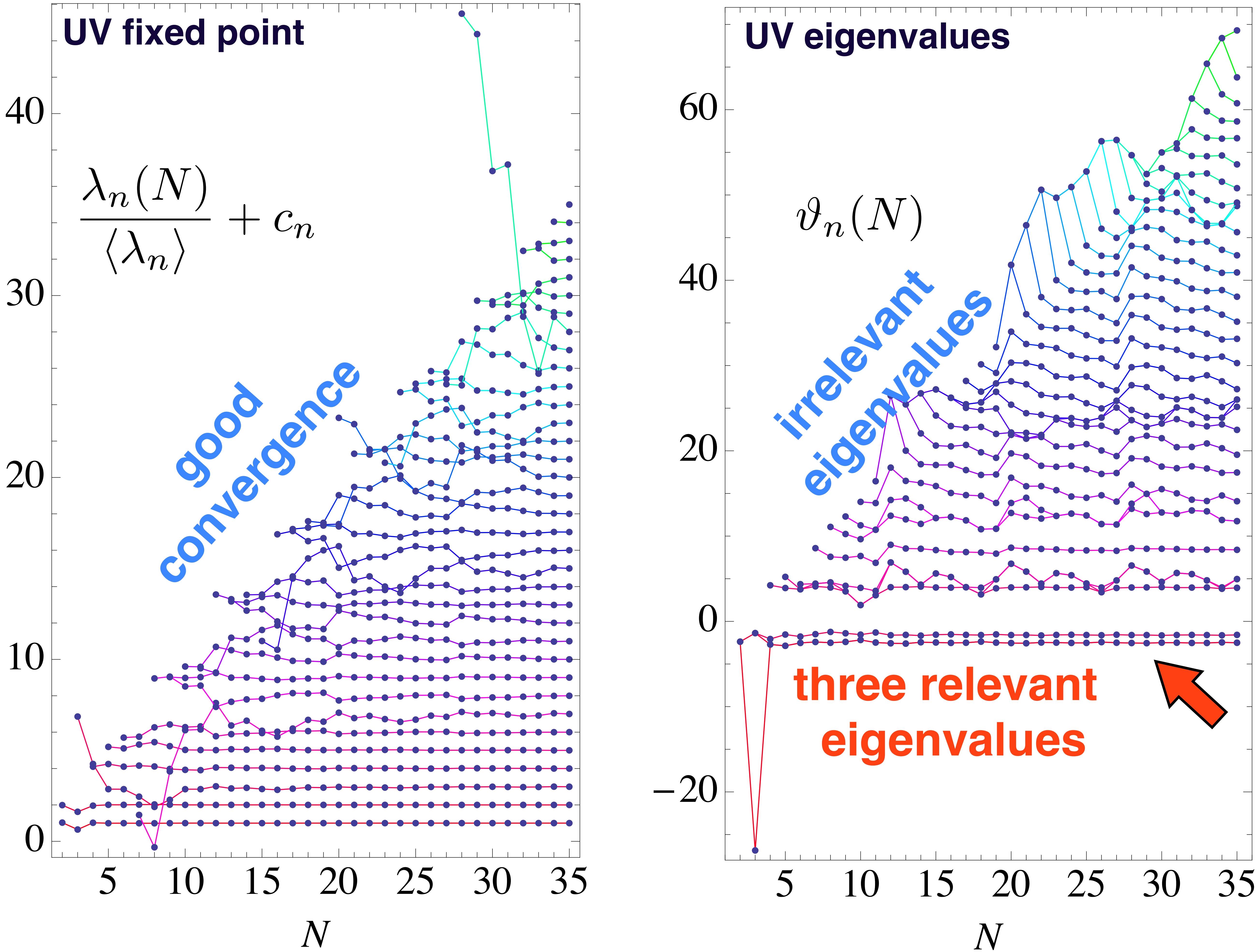
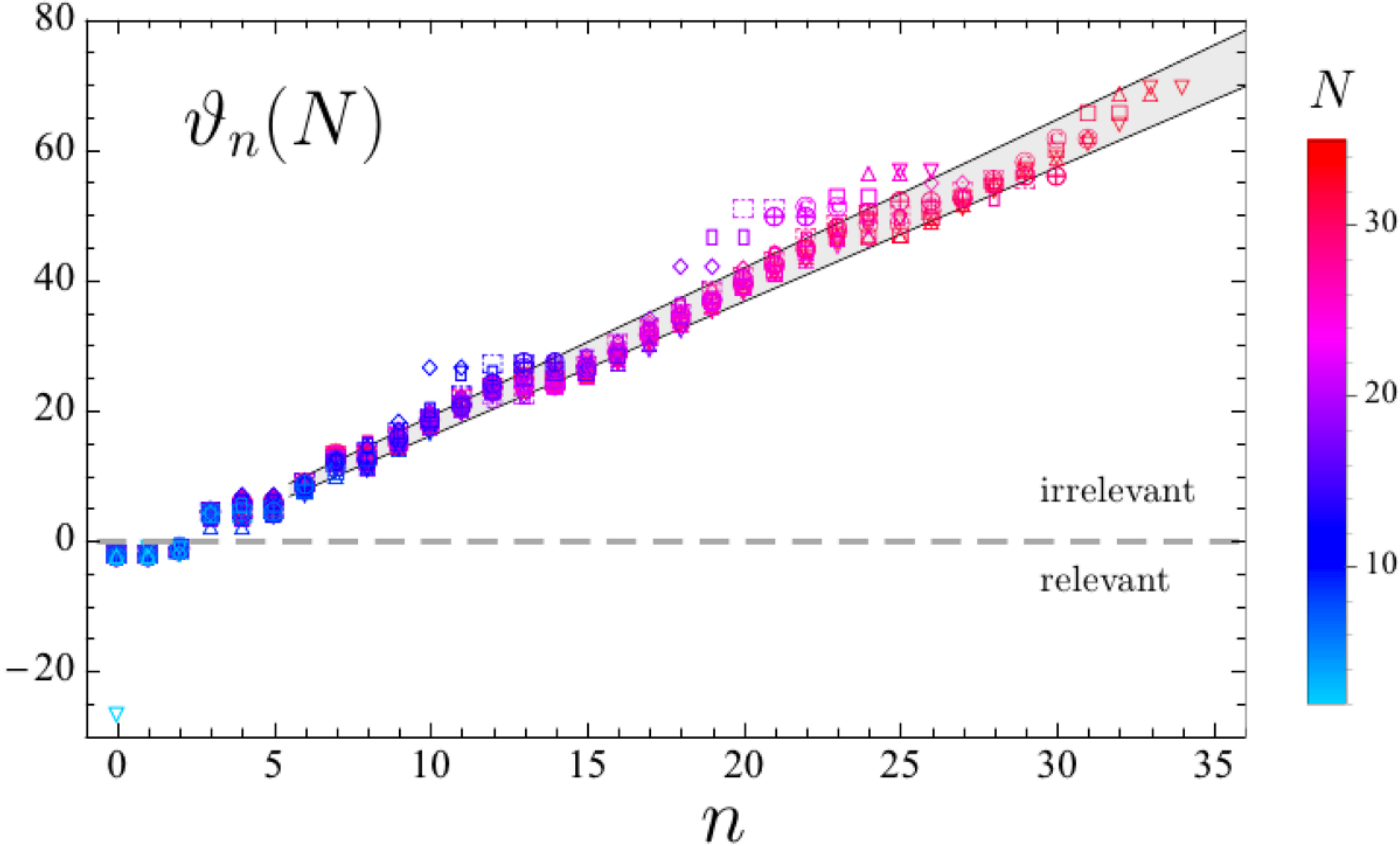
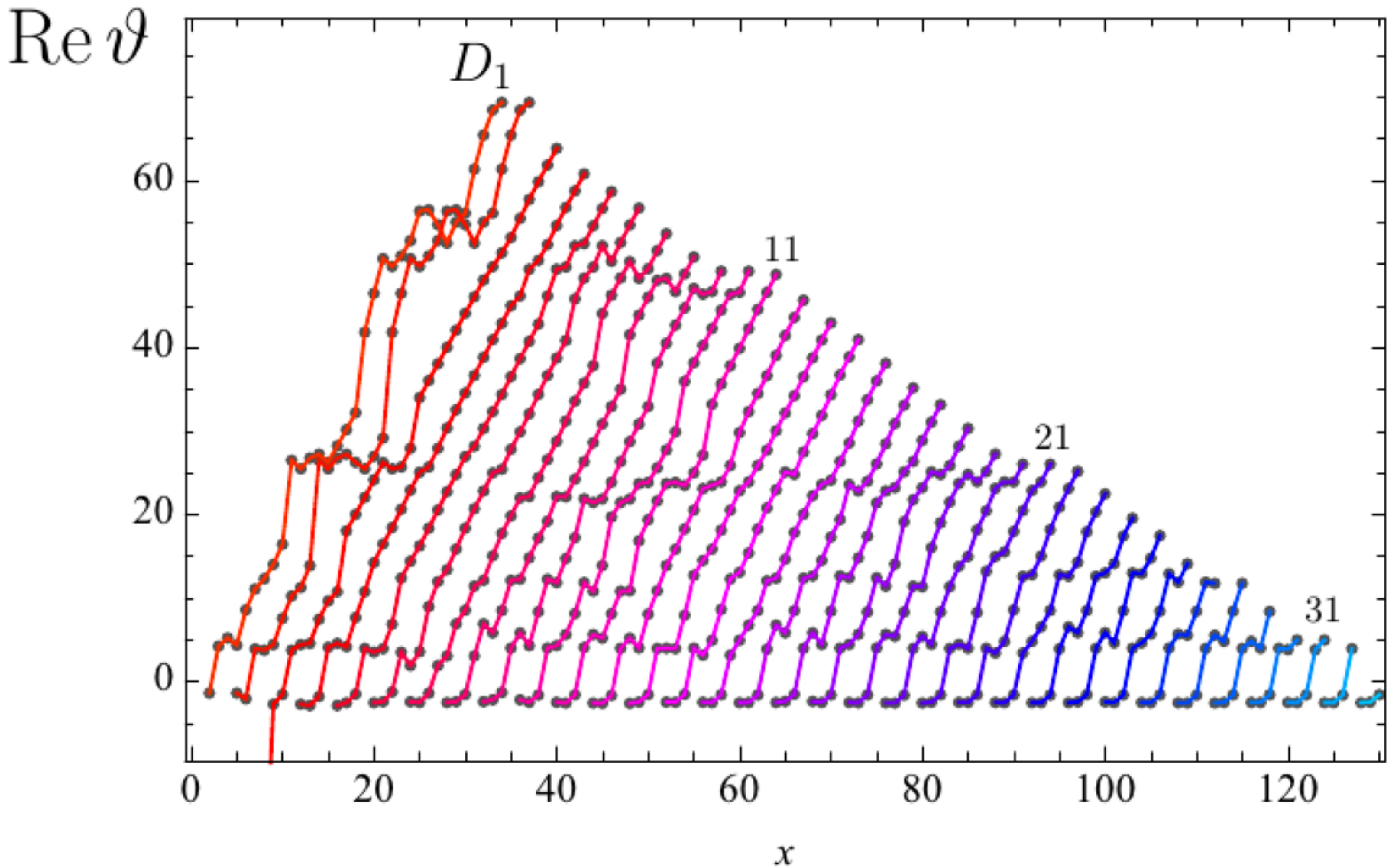



UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
N
UV eigenvalues
$\vartheta_n(N)$
irrelevant eigenvalues
three relevant eigenvalues
N

near-Gaussian



bootstrap test



f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g}\, [f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

$$\begin{aligned}\partial_t\Gamma[\bar{g},\bar{g}] =& \frac{1}{2}\mathrm{Tr}_{(2T)}\left[\frac{\partial_t\mathcal{R}_k^{h^T h^T}}{\Gamma^{(2)}_{h^T h^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\xi\xi}}{\Gamma^{(2)}_{\xi\xi}}\right] + \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma\sigma}}{\Gamma^{(2)}_{\sigma\sigma}}\right] + \frac{1}{2}\mathrm{Tr}_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{hh}}{\Gamma^{(2)}_{hh}}\right]\\ &+ \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma h}}{\Gamma^{(2)}_{\sigma h}}\right] - \mathrm{Tr}_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{C}^T C^T}}{\Gamma^{(2)}_{\bar{C}^T C^T}}\right] - \mathrm{Tr}_{(0)'}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma^{(2)}_{\bar{\eta}\eta}}\right] - \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma^{(2)}_{\bar{\lambda}\lambda}}\right]\\ &+ \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\omega\omega}}{\Gamma^{(2)}_{\omega\omega}}\right] - \mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma^{(2)}_{\bar{c}^T c^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\zeta^T\zeta^T}}{\Gamma^{(2)}_{\zeta^T\zeta^T}}\right] + \mathrm{Tr}'_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{s}s}}{\Gamma^{(2)}_{\bar{s}s}}\right]\end{aligned}$$

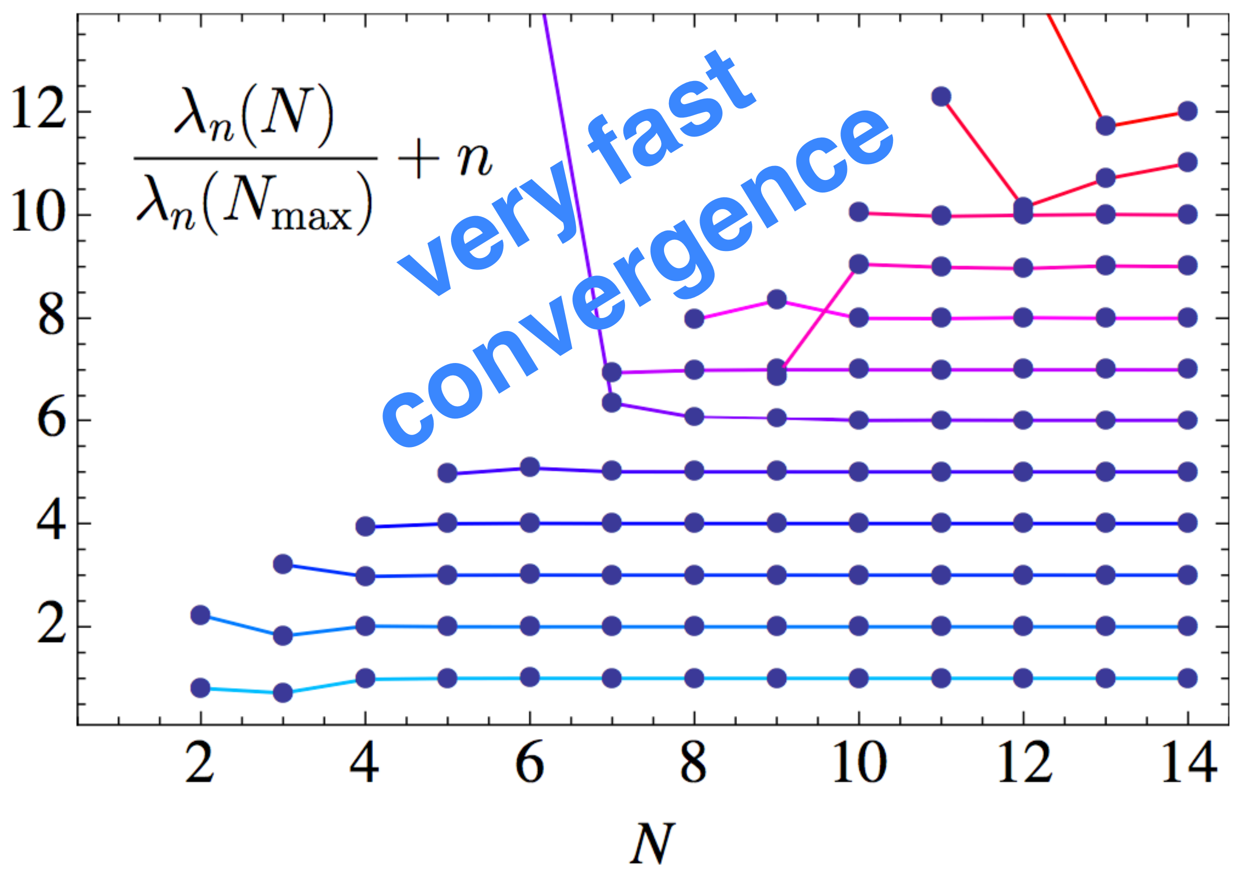
K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

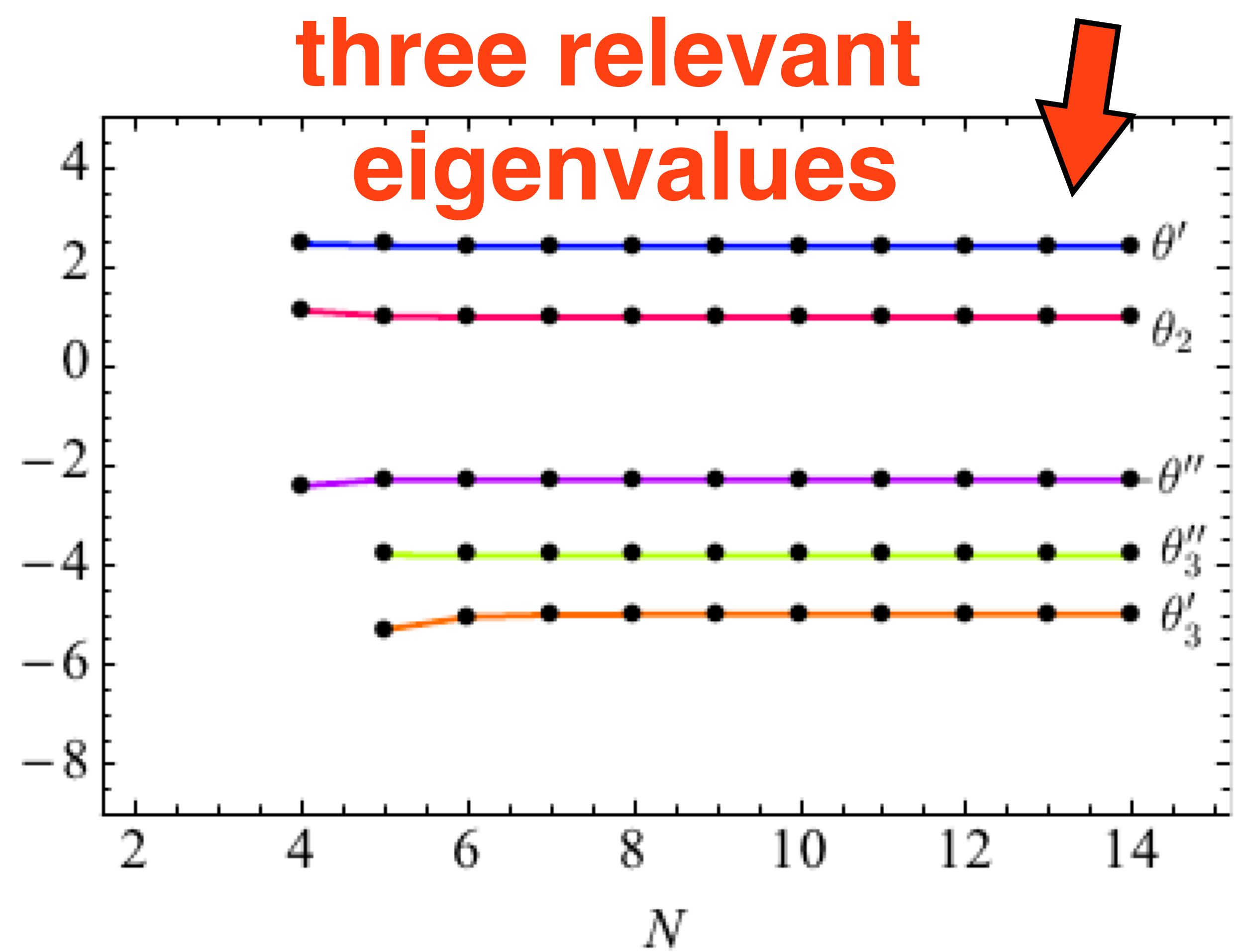
f(Ricci)

K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

$$\Gamma_k \propto \int d^d x \sqrt{g}\, [f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

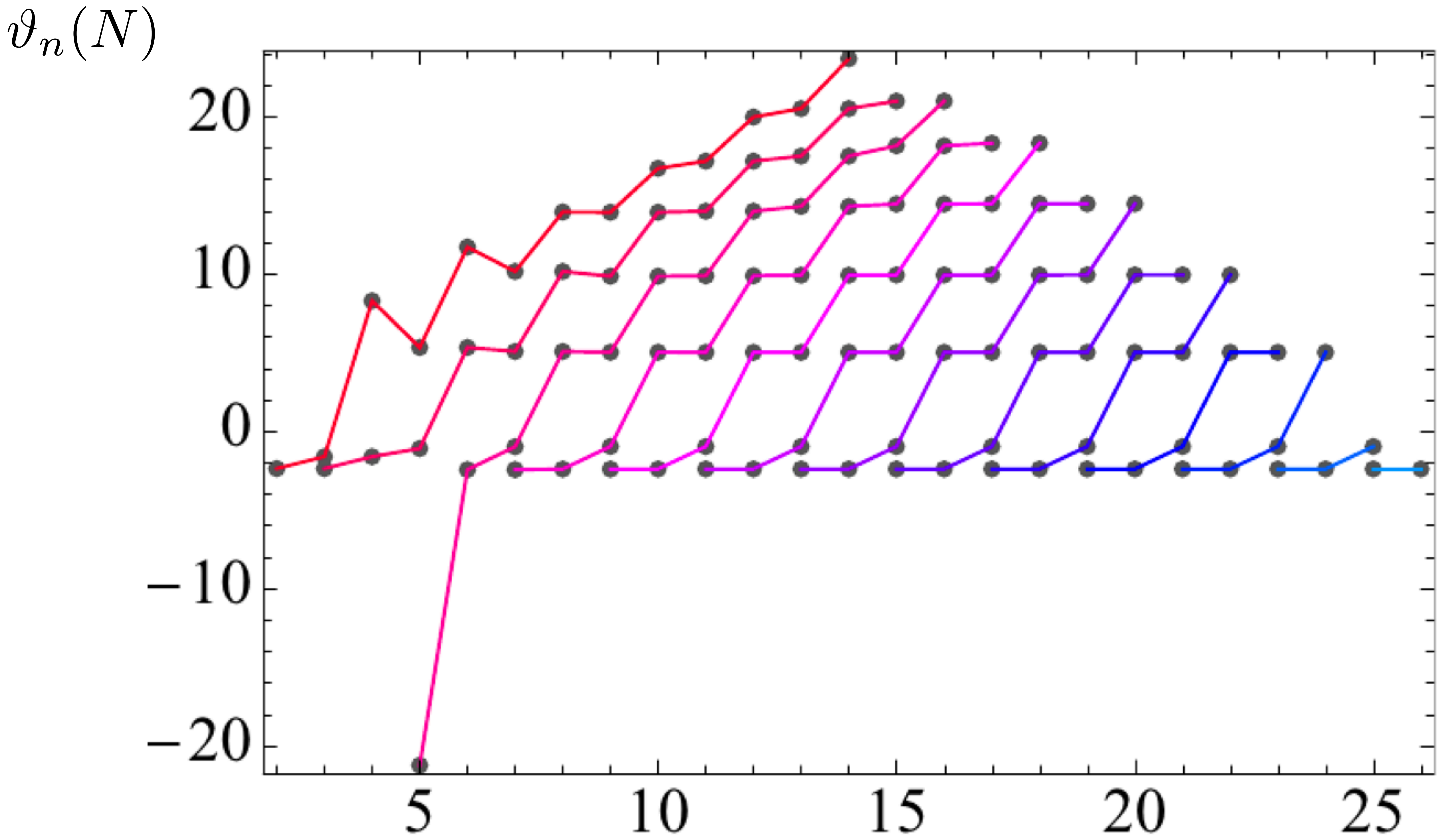
fixed point





bootstrap test

K Falls, DL, K Nikolakopoulos, C Rahmede (to appear, 2014)



conclusions

QFTs beyond asymptotic freedom

- 4D matter-gauge theories
  - perturbative proof of existence
  - all types of fields required
- 4D quantum gravity
  - systematic non-perturbative search strategies