

Strong Interactions in the LHC Era

Physikzentrum Bad Honnef

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Evidence for scale separation in many-flavor QCD

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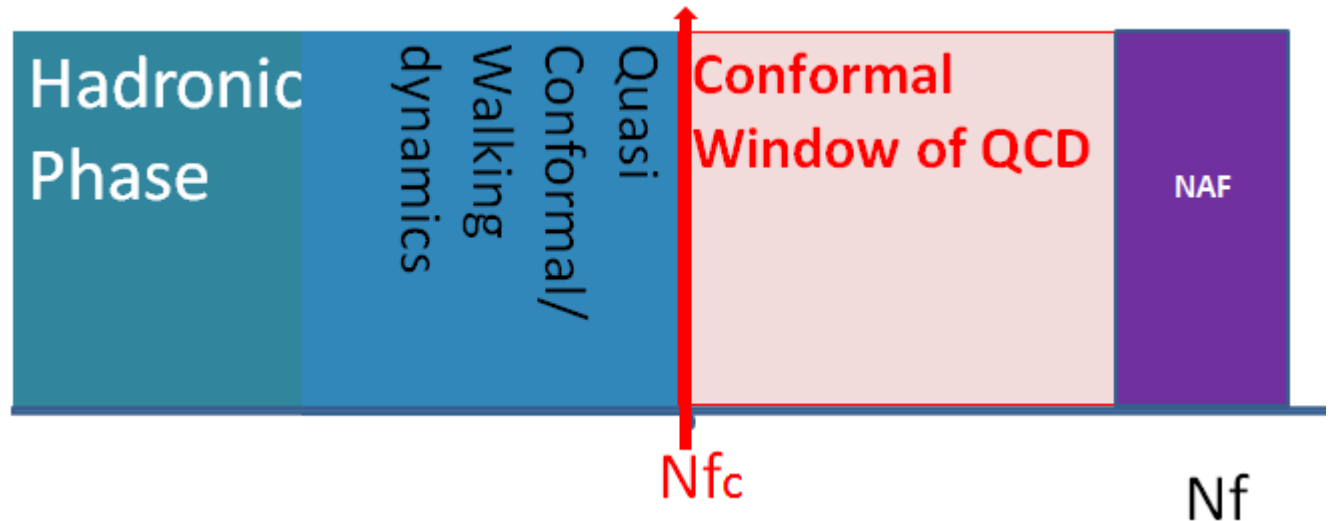
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We study $N_c=3$,
Fundamental fermions :

$N_f = 0$ --
 $N_f = 4$ --
 $N_f = 6$ --
 $N_f = 8$ --
 $N_f = 12$ --
 $N_f = 16$ --



At zero and non-zero temperature.

MpL, KM, TndS, EP:
arXiv:1410.0298
arXiv:1410.2036
arXiv:1411.1657

From UV to IR

$$\Lambda_{\text{IR}}/\Lambda_{\text{UV}} = \mathcal{O}(1).$$

Λ_{UV}

Λ_{IR}

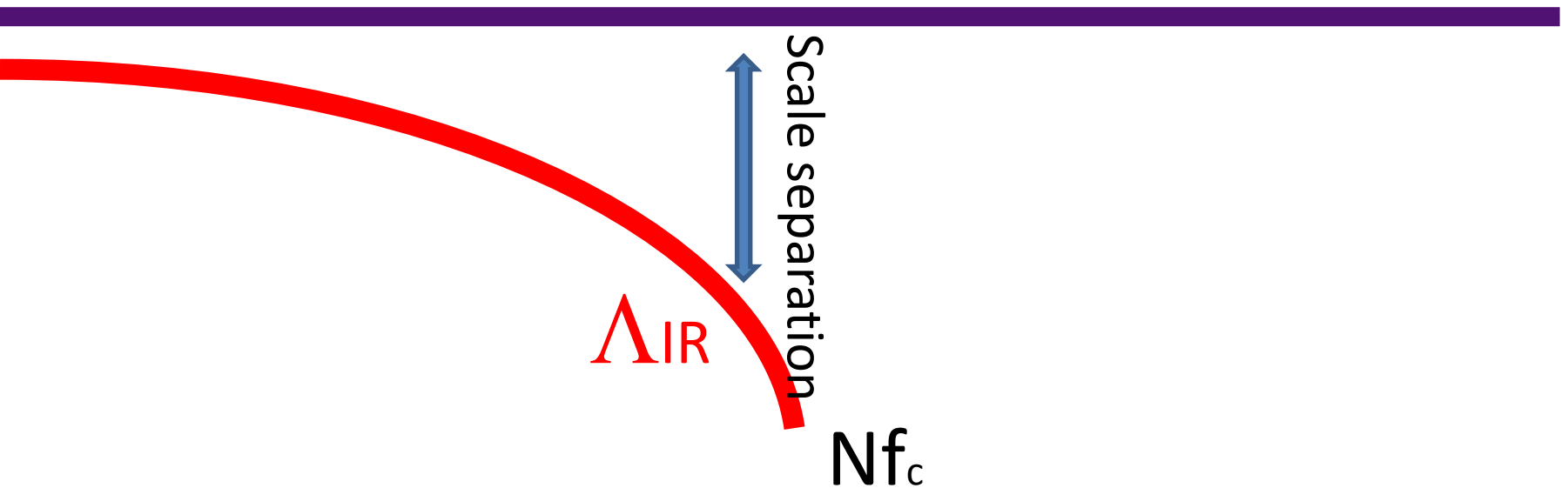
Nf_c

$$\frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

From UV to IR

$$\Lambda_{\text{IR}}/\Lambda_{\text{UV}} = \mathcal{O}(1).$$

Λ_{UV}



$$\frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

Physical scales & Lattice scales

✓ Lattice introduces two further technical scales a and L obscuring the UV and IR behaviour respectively

✓ Ratios of homogeneous quantities

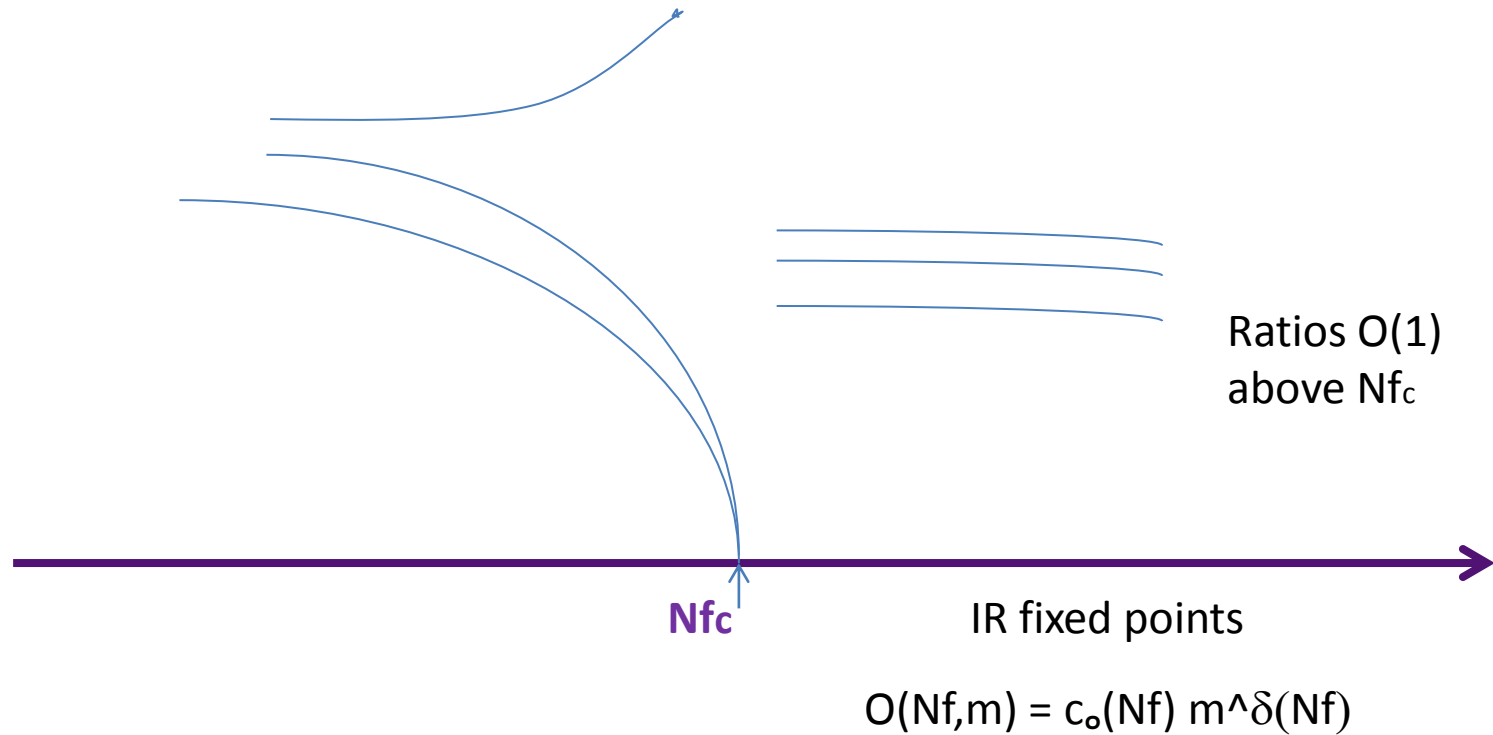
$$R = O1/O2$$

Miranski , Yamawaki
Braun, Gies
Kiritsis et al

useful: Help controlling a and L systematic effects
Display scale hierarchy with no need to fix the scale across different theories

✓ When $O2$ is an UV quantity – non critical at N_{fc} -- taking the ratio is de facto a scale fixing procedure for $O1$

Adimensional ratios below and above Nf_c



Plan

Conformal scaling and anomalous dimension
($N_f=12$)

Pre-conformal behaviour ($N_f=6,8$)

NF=12 CONFORMAL SCALING AND ANOMALOUS DIMENSION

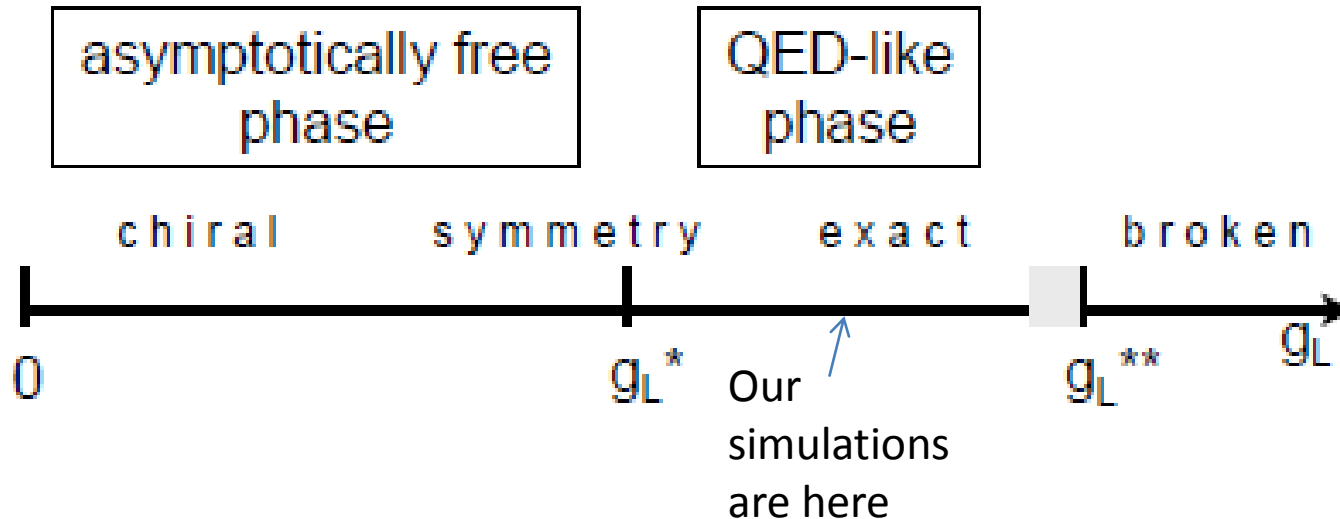
Lattice corrections to conformal scaling

1: Size $M_H = L^{-1} f_H(x), \quad x \equiv Lm^{1/y_m}$

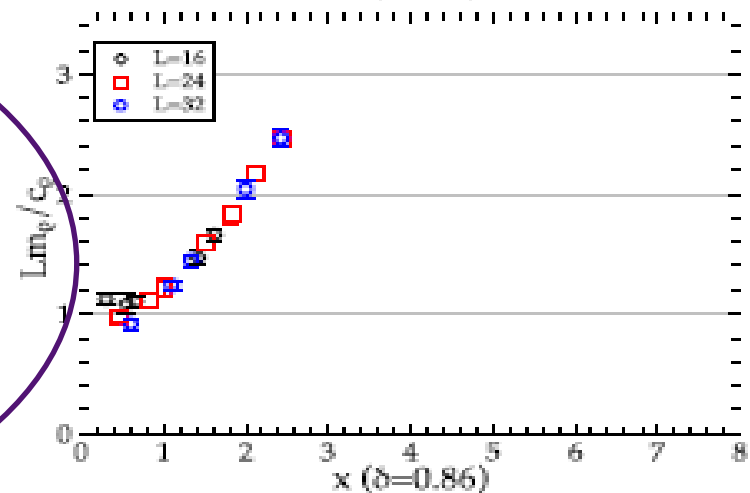
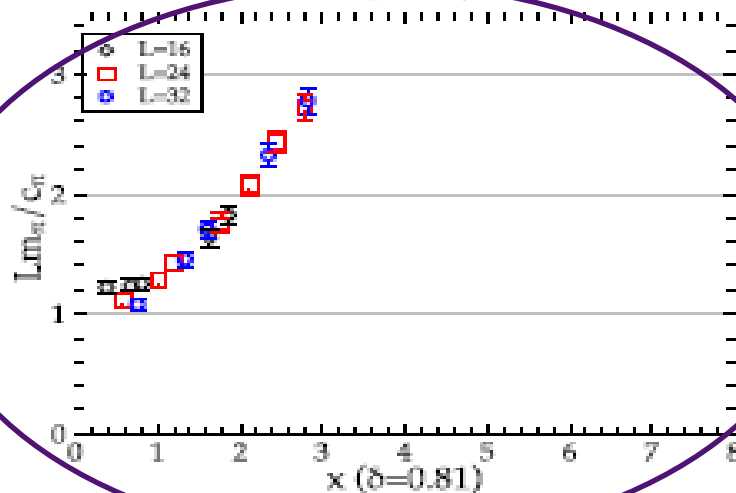
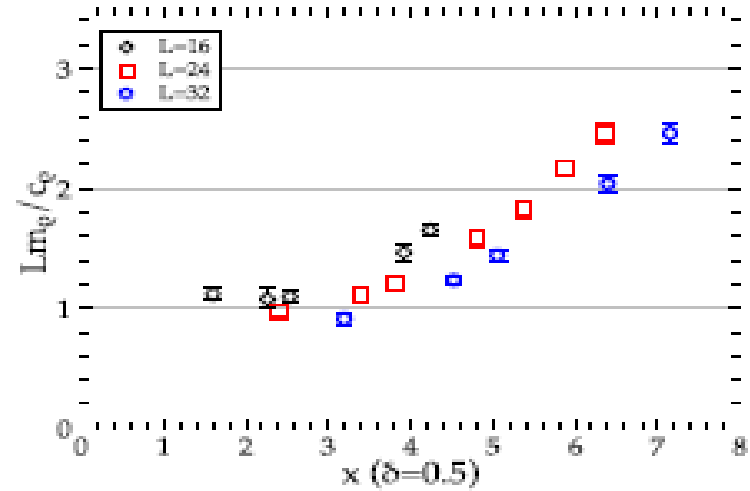
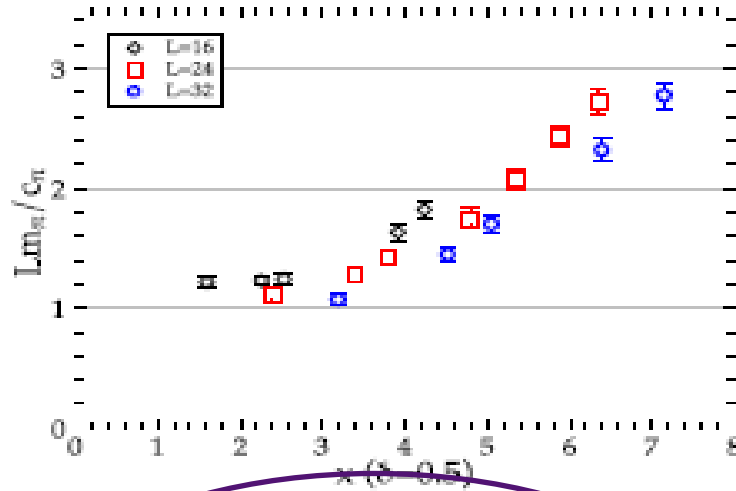
2: Coupling $M_H = L^{-1} f_H(x, g_0 m^\omega)$

Del Debbio, Zwicky;
Hasenfratz et al;
MpL, da Silva, Miura, Pallante

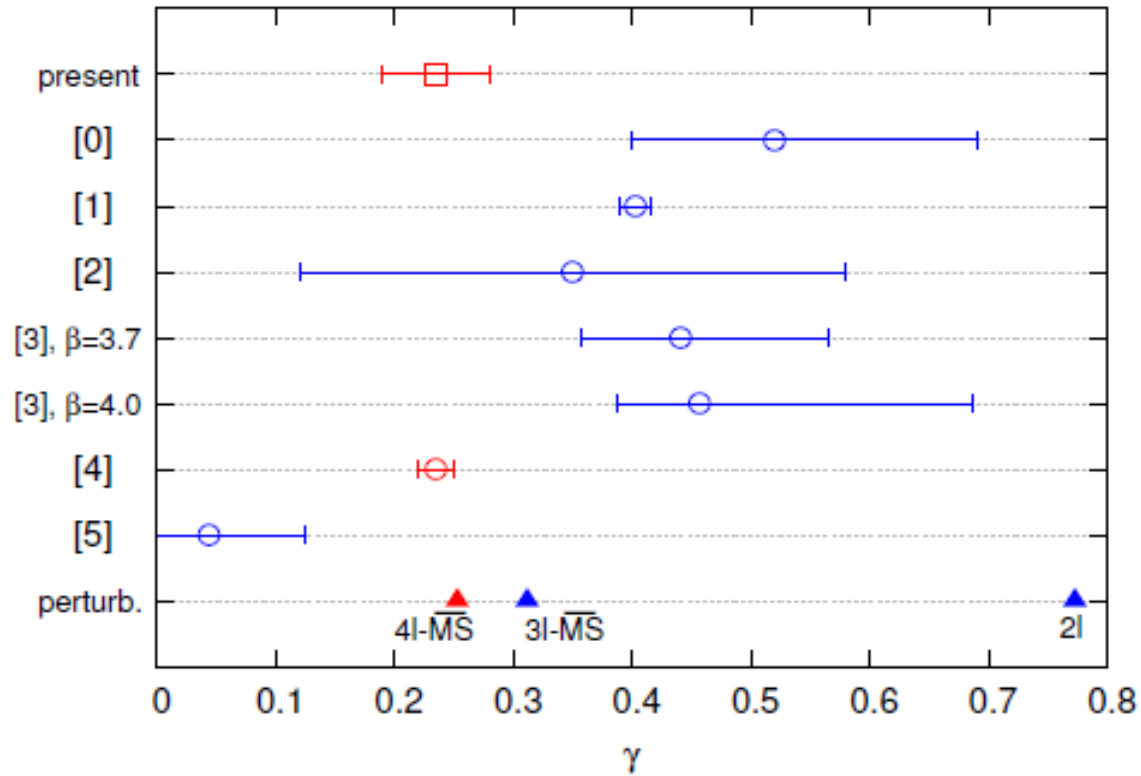
$$LM_H = F_H(x) \{1 + g_0 m^\omega G_H(x) + \mathcal{O}(g_0^2 m^{2\omega})\}$$

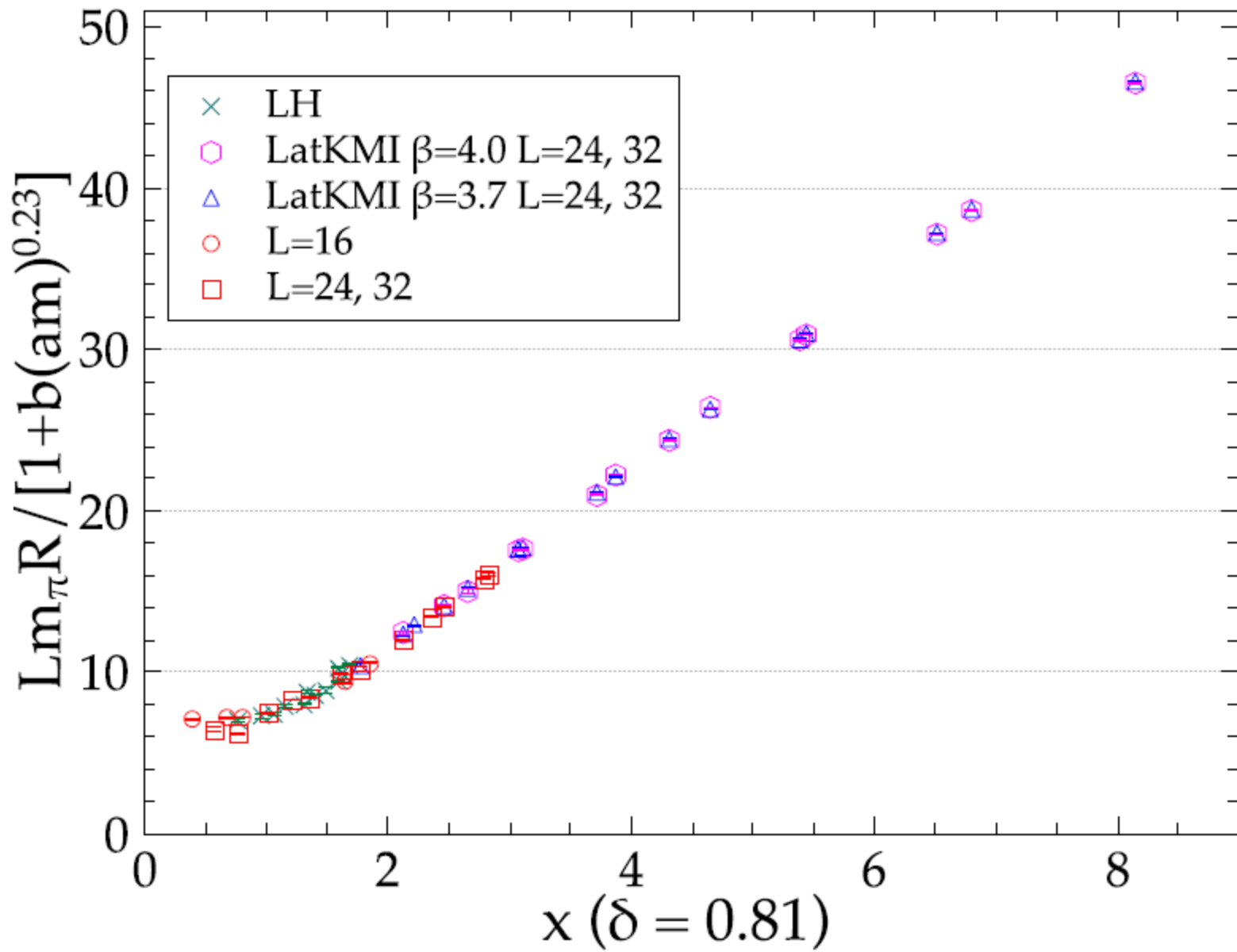


Anomalous dimension from the QED phase?

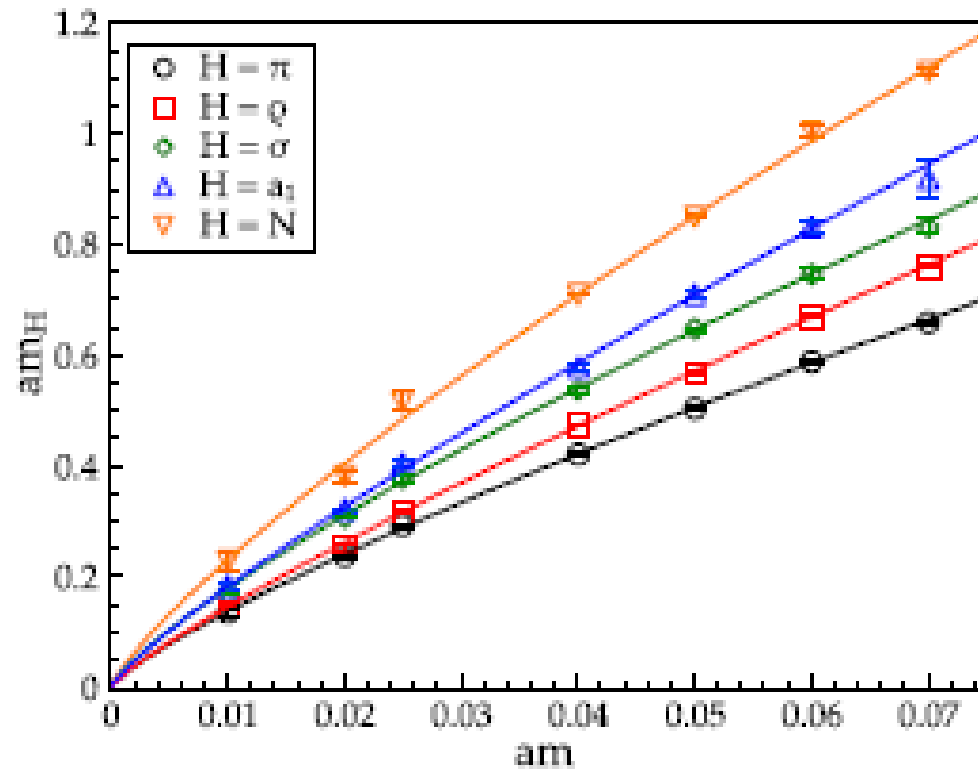


Summary of the results: accidental agreement??

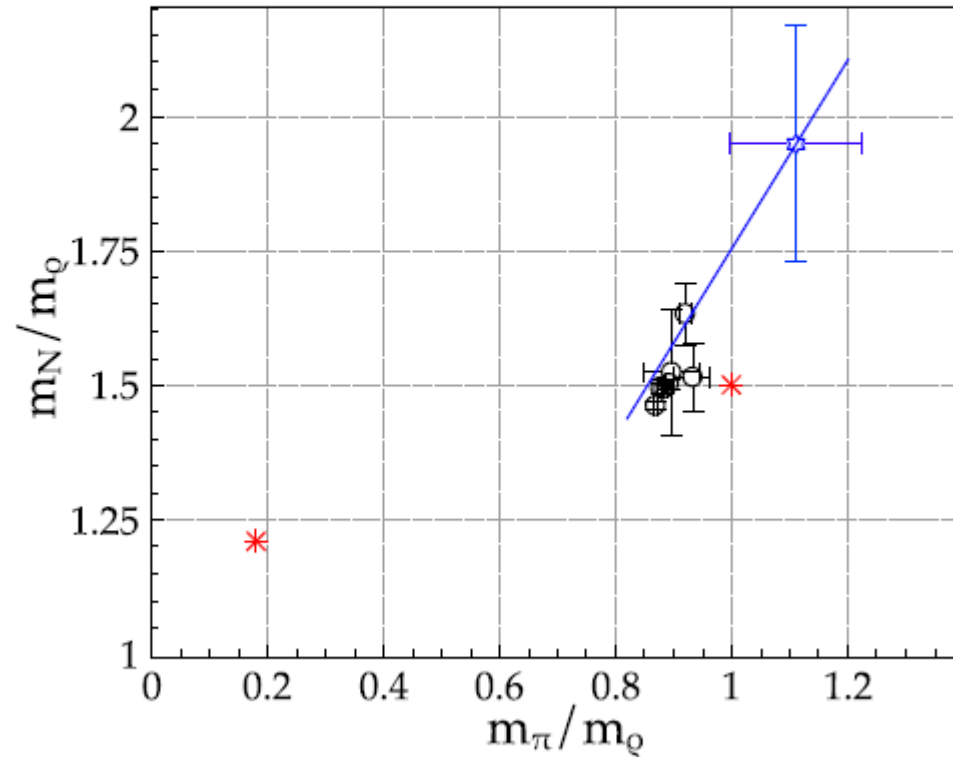




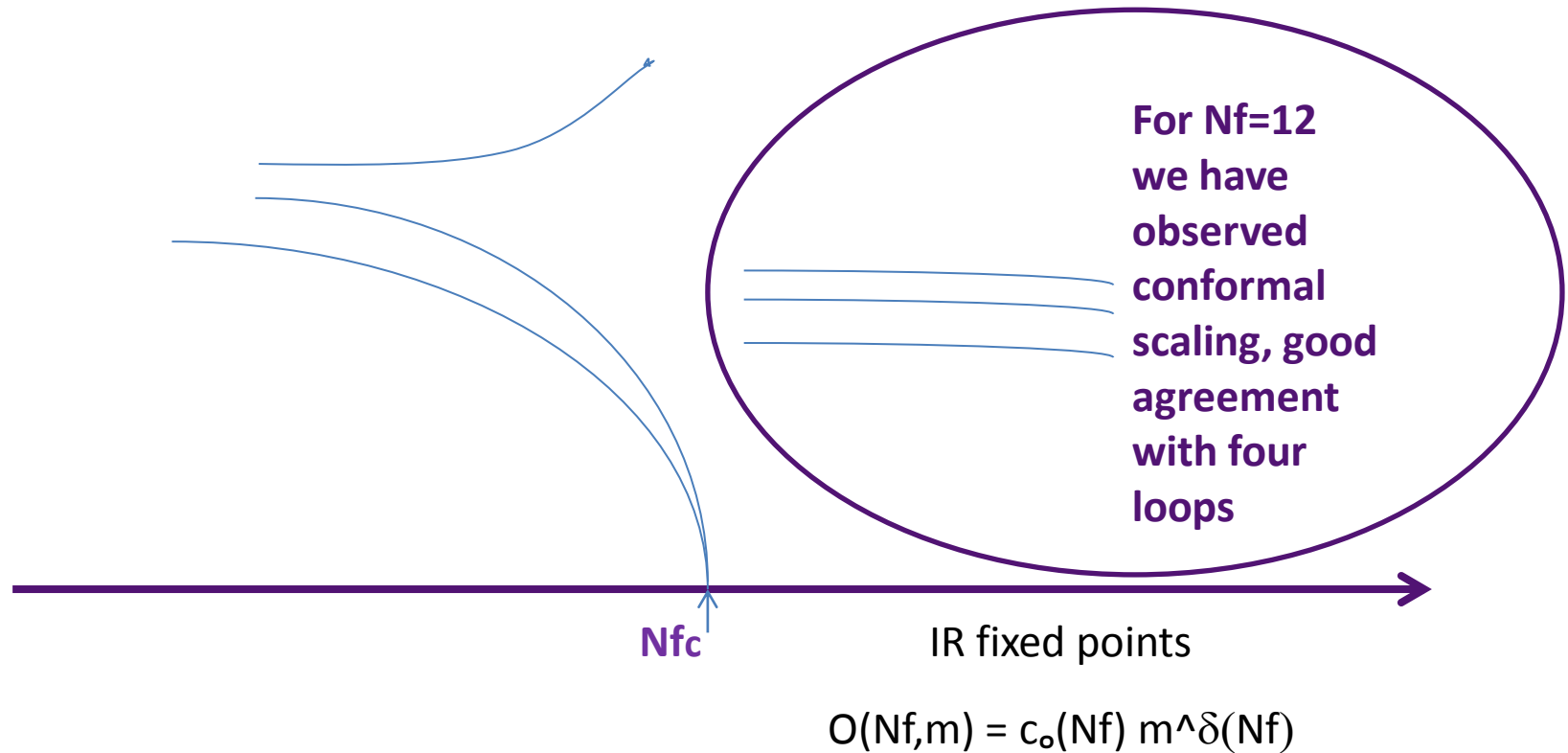
Hadron spectrum



Ratios in the conformal window at a glance: the Edinburgh plot



Adimensional ratios below and above $N_f c$



THE (PSEUDO) CRITICAL TEMPERATURE

Lattice setup

All simulations :

- Gauge Action: one loop Symanzyk improved
- Fermion Action: Tadpole improved AsqTad

$m = 0.02$: only one mass

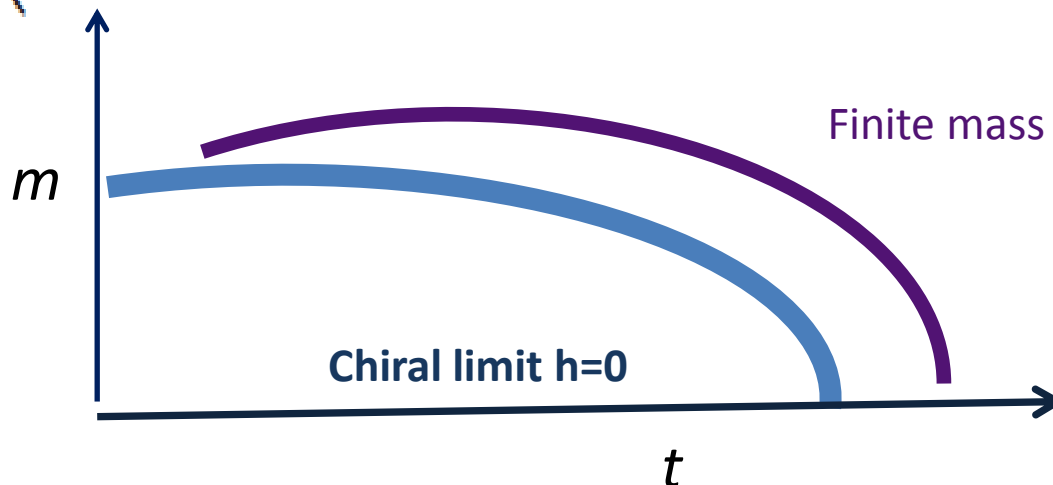
Scaling for essential singularities

Nogada, Hasegawa, Nemoto, PRL 2012

$$g(t, h, N^{-1}) = b^{-1} \hat{g}(e^{-(t/t_0)^{-x}} b, h b^{y_h}, N^{-1} b).$$

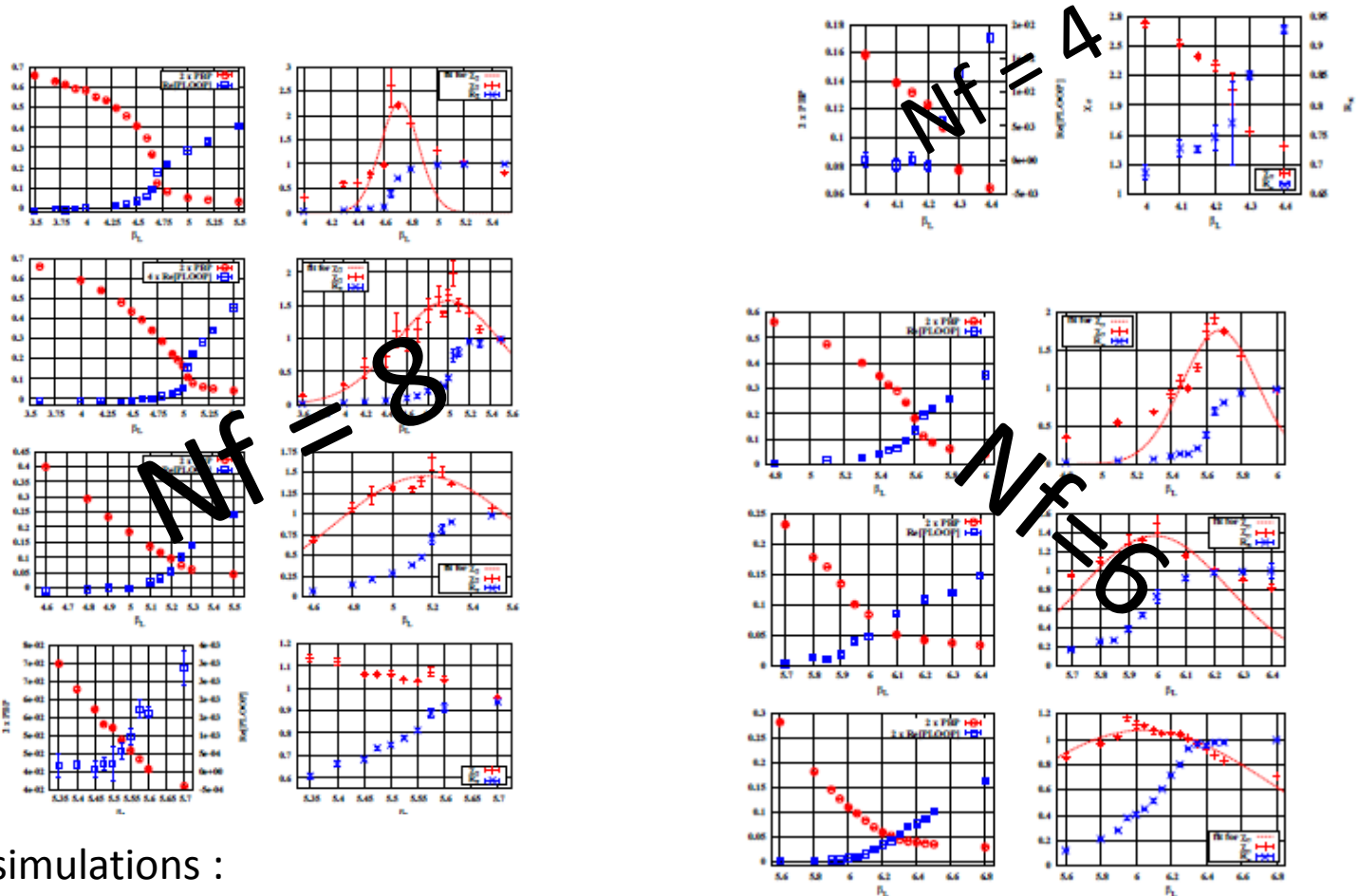
m \leftrightarrow Chiral Condensate
 h \leftrightarrow bare mass
 t \leftrightarrow Nfc – Nc

$$m \propto \begin{cases} e^{-(1-y_h)(t/t_0)^{-x}} & \text{for } h e^{y_h (t/t_0)^{-x}} \ll 1 \\ h^{y_h^{-1} - 1} & \text{for } h e^{y_h (t/t_0)^{-x}} \gg 1 \end{cases}$$

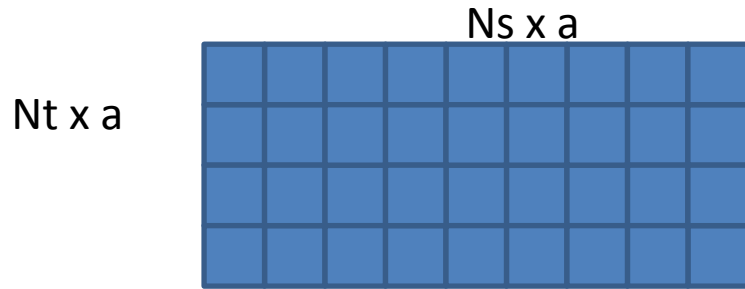


Within the scaling window data at finite mass contain information on the critical behaviour. They can be approximated as zero mass ones, but with a larger apparent critical point.

We studied the thermal transition for several N_f and several N_t



All simulations :
 Gauge Action one loop Sym.
 Tadpole improved AsqTad



From the Lattice..

$$T \equiv \frac{1}{a(\beta_L) \cdot N_t},$$

..to the continuum

Via old fashioned asymptotic scaling

$$\Lambda_L a(\beta_L) = \left(\frac{2N_c b_0}{\beta_L} \right)^{-b_1/(2b_0^2)} \exp \left[\frac{-\beta_L}{4N_c b_0} \right].$$

$$\frac{1}{N_t} = \boxed{\frac{T_c}{\Lambda_L}} \times \left(\Lambda_L a(\beta_L^c) \right).$$

Must be approx. constant for several Nt

The quest for continuum limit

$$\frac{T_c}{\Lambda_{L/E}} = \frac{R(g_{L/E})}{N_f} = (b_0 g_{L/E}^2)^{-b_1/(2b_0^2)} \exp\left[\frac{-1}{2b_0}\right],$$

Explore different prescriptions for T_c/Λ

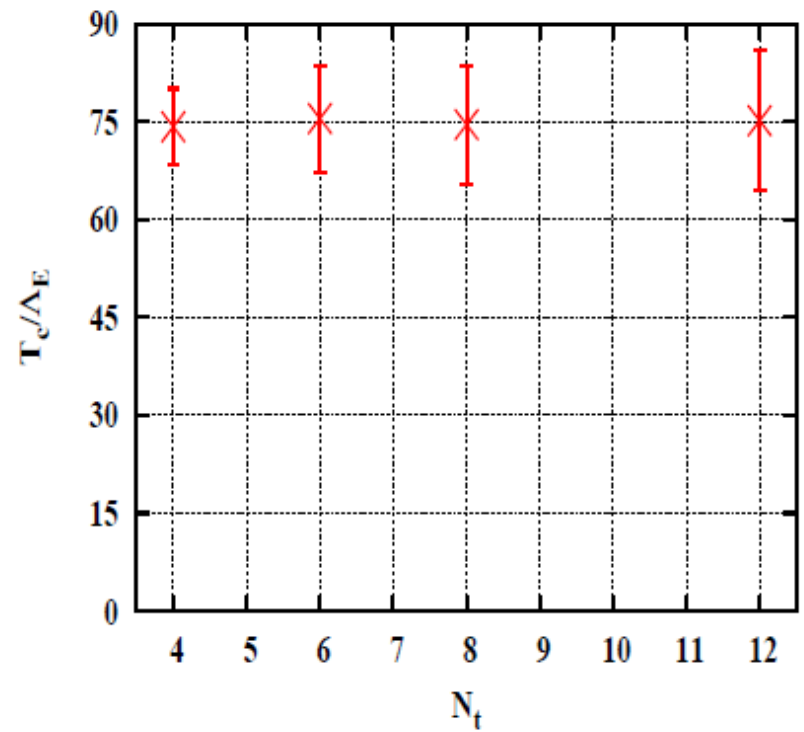
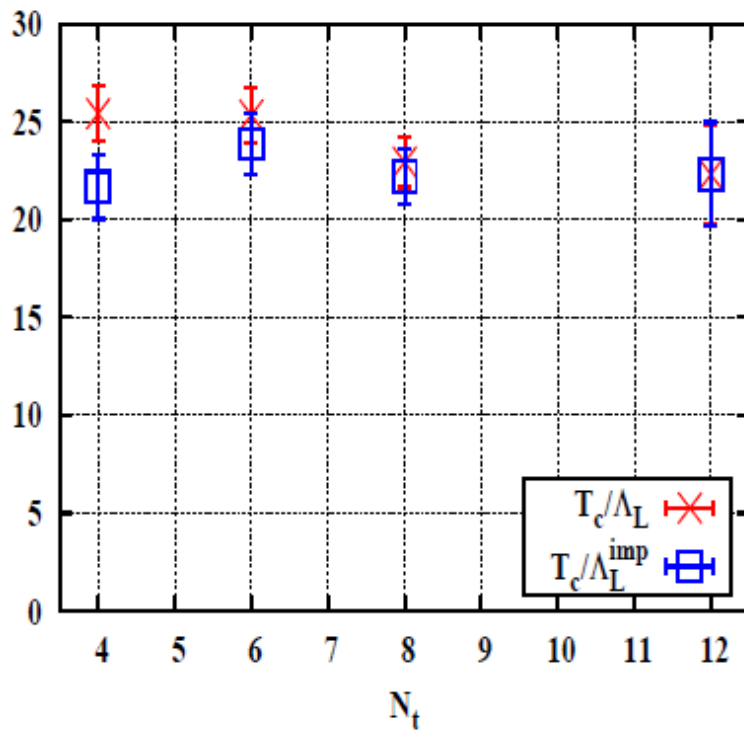
$$R(g_{L/E}) \equiv a(g_{L/E}) \Lambda_{L/E} = (b_0 g_{L/E}^2)^{-b_1/(2b_0^2)} \exp\left[\frac{-1}{2b_0 g_{L/E}}\right]$$

$$g_E = \sqrt{3(1 - \langle P \rangle(g_L))}$$

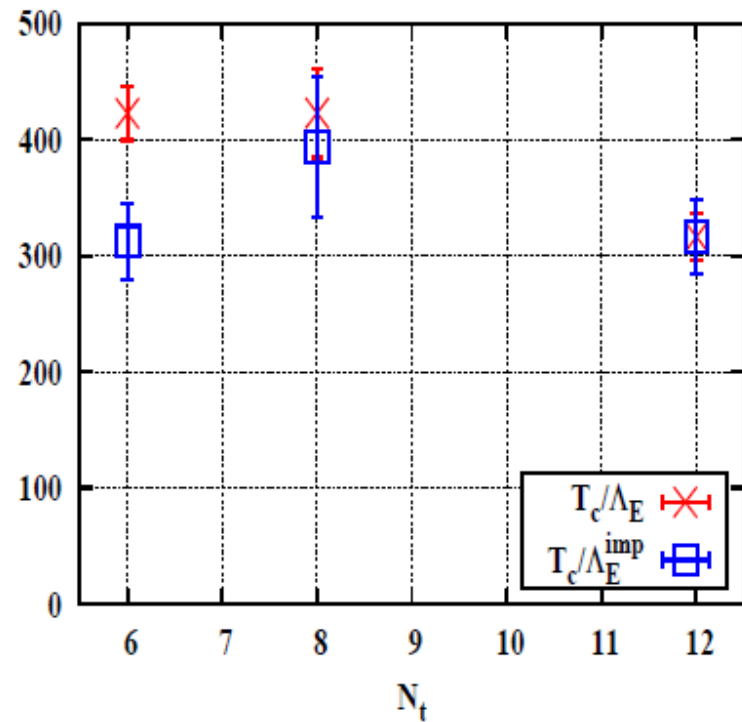
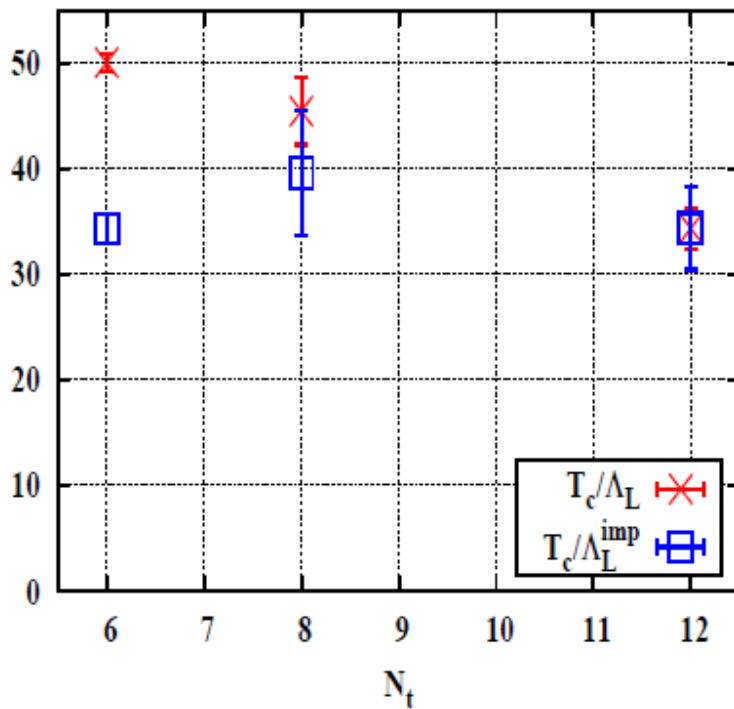
$$R^{\text{imp}}(\beta_{L/E}) = \Lambda_{L/E}^{\text{imp}} a(\beta_{L/E}) \equiv \frac{R(\beta_{L/E})}{1+h} \times \left[1 + h \frac{R^2(\beta_{L/E})}{R^2(\beta_0)} \right],$$

C. Allton, 2007

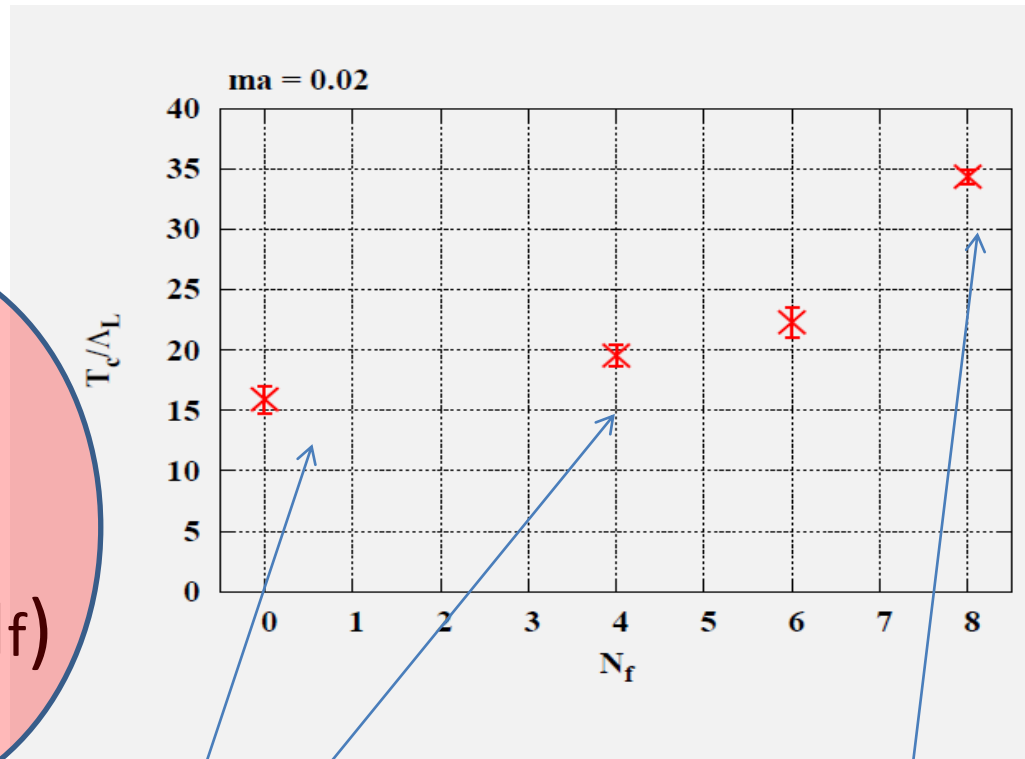
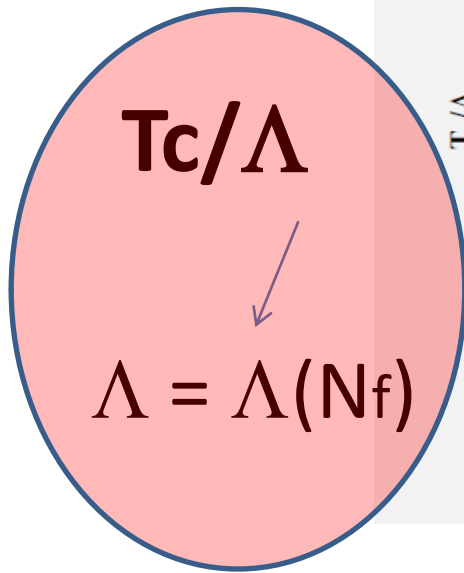
$N_f = 6$, asympt. scaling



$N_f = 8$, asympt scaling



T_c/Λ as a function of N_f

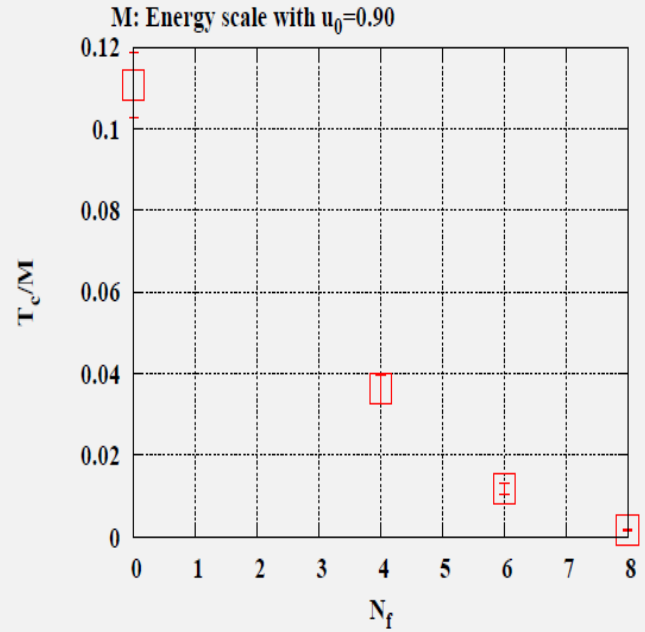
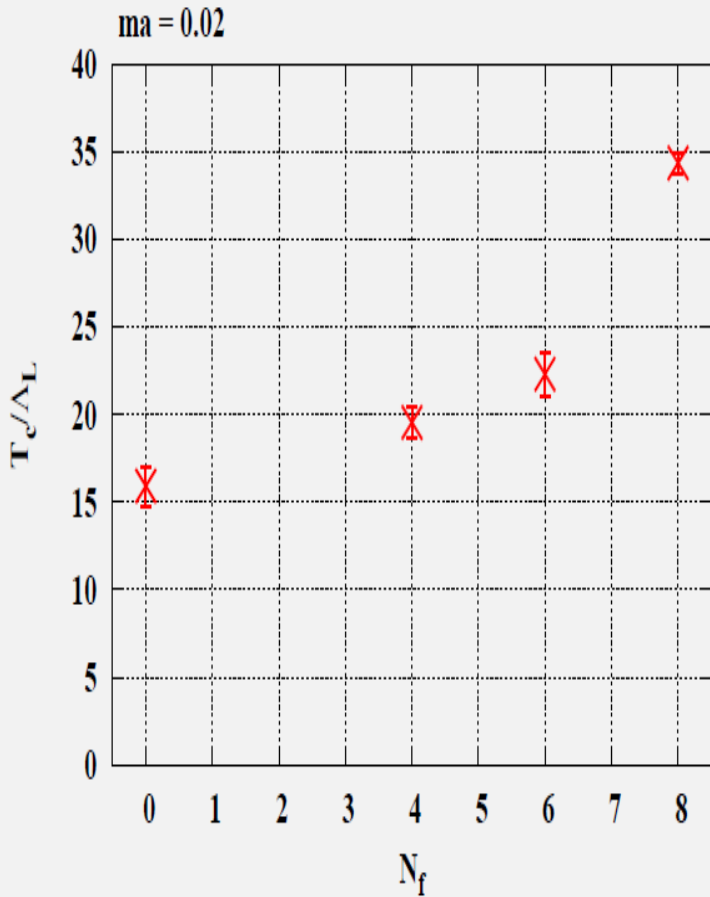


Conventional running

N_f

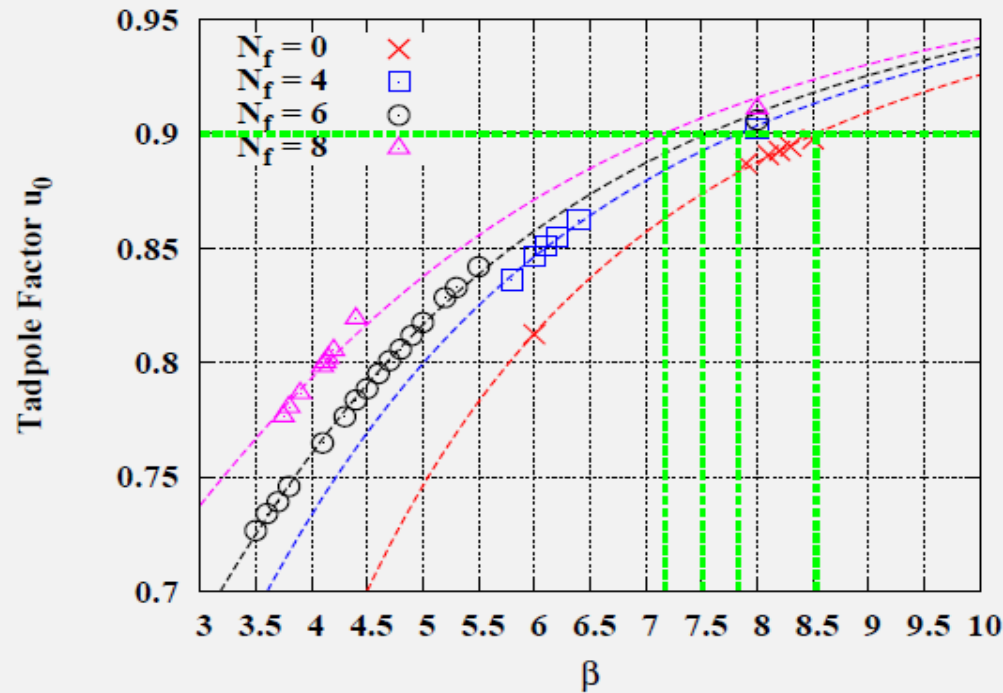
Scale separation ?

Solution: $\Lambda = \Lambda(N_f)$; use UV scale



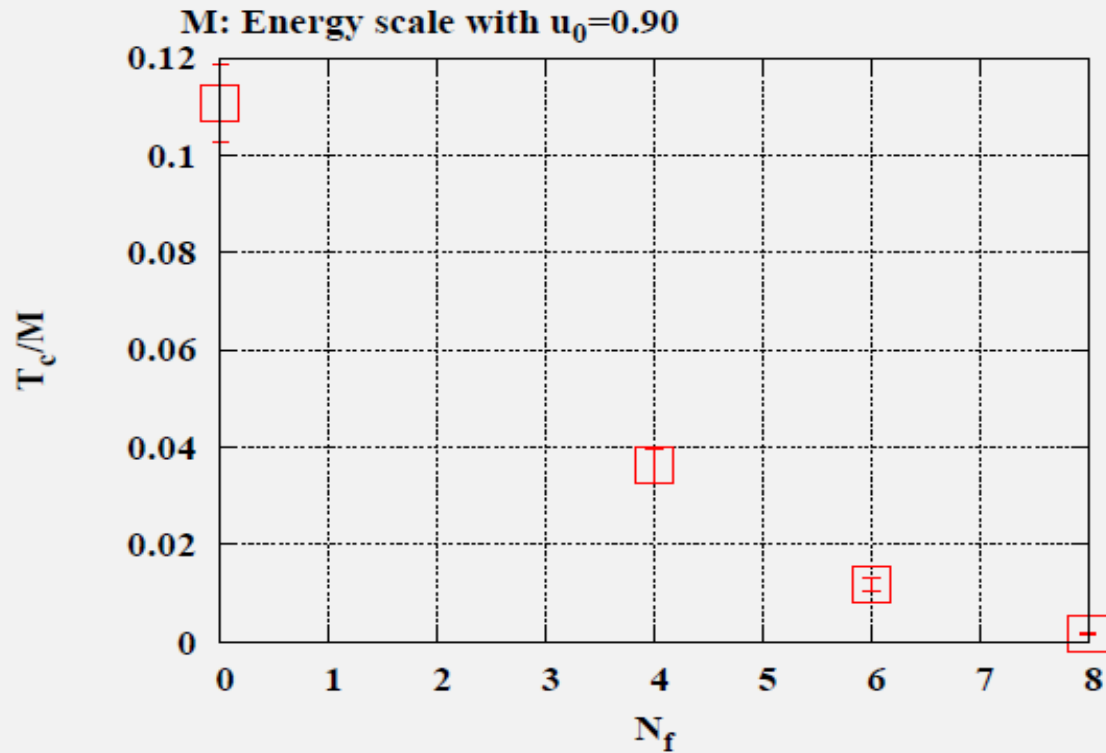
$$\frac{T_c}{M} = \frac{1}{N_t} \exp \left[\int_{g_{\text{ref}}}^{g_c} \frac{dg}{B(g)} \right].$$

Fixing an UV scale



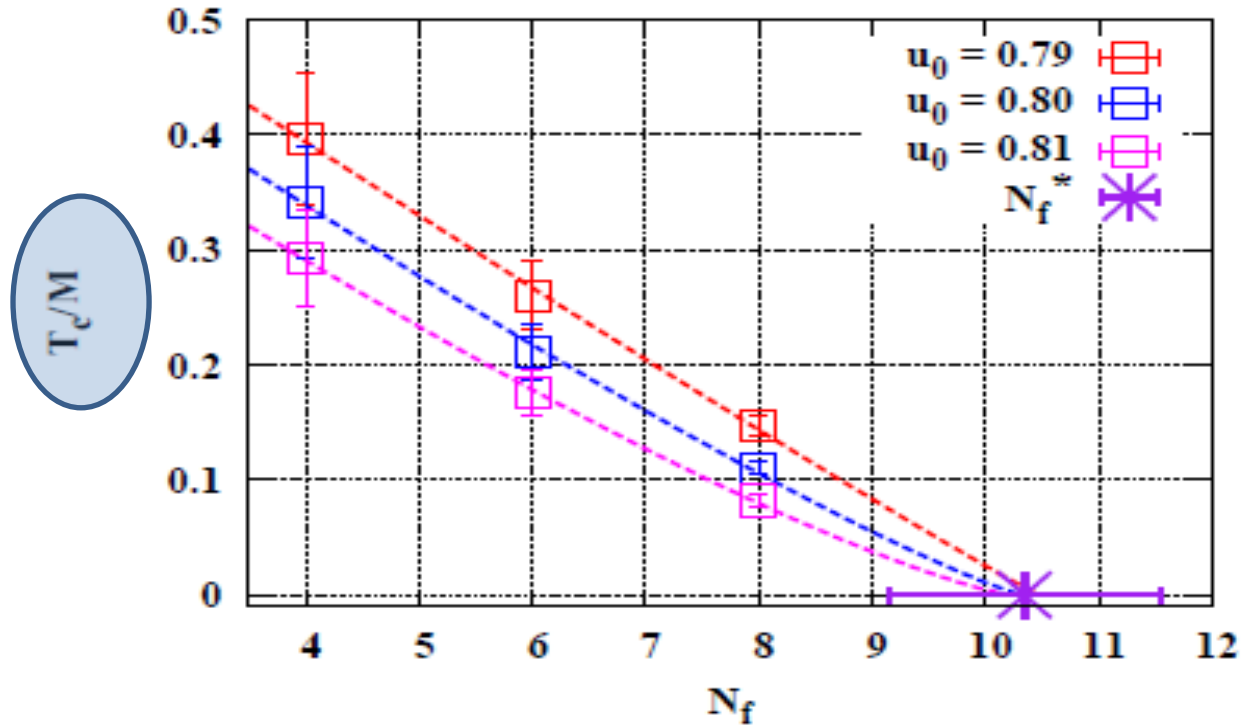
- We have measured the tadpole factor $u_0 = \langle \square \rangle^{1/4}$ at $T = 0$.
- We use the couplings obtained by the constant u_0 line to define a UV reference scale M .

T_c/M_{UV}



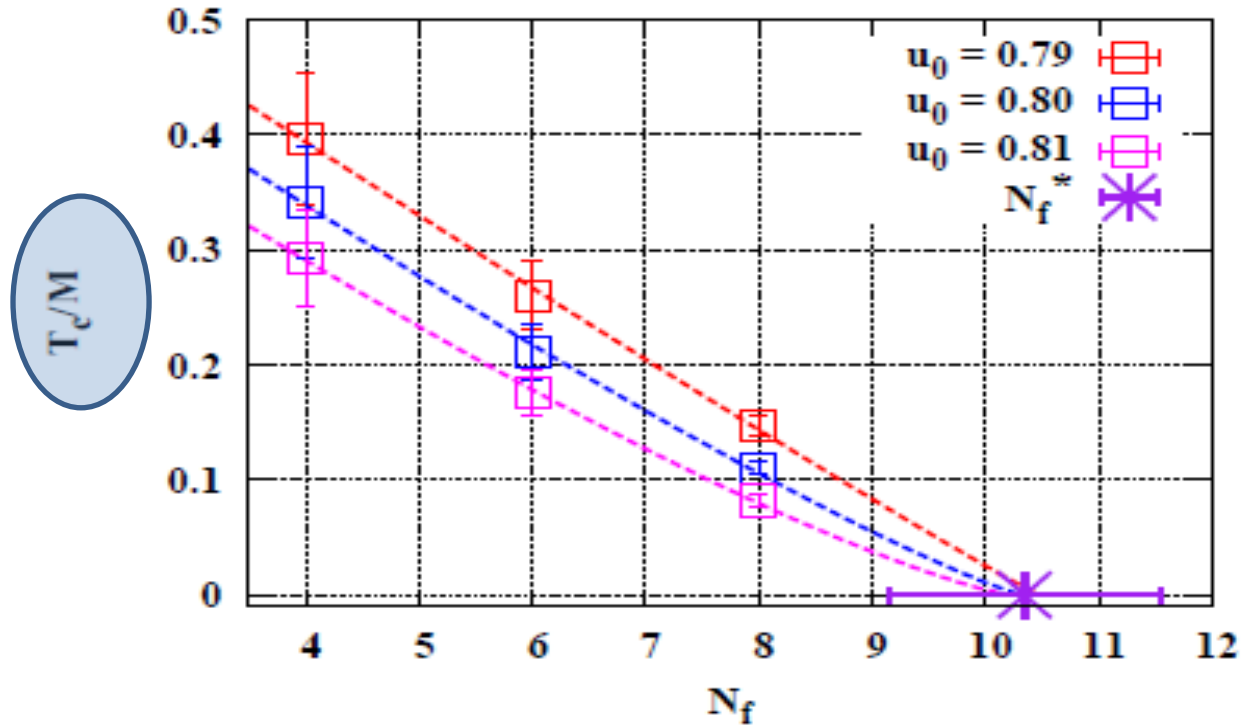
$$\frac{T_c}{M} = \frac{1}{N_t} \exp \left[\int_{g_{\text{ref}}}^{g_c} \frac{dg}{B(g)} \right] .$$

T_c/M extrapolates to zero for $N_f^* \sim 10.5$



T_c/M extrapolates to zero for $N_f^* \sim 10.5$

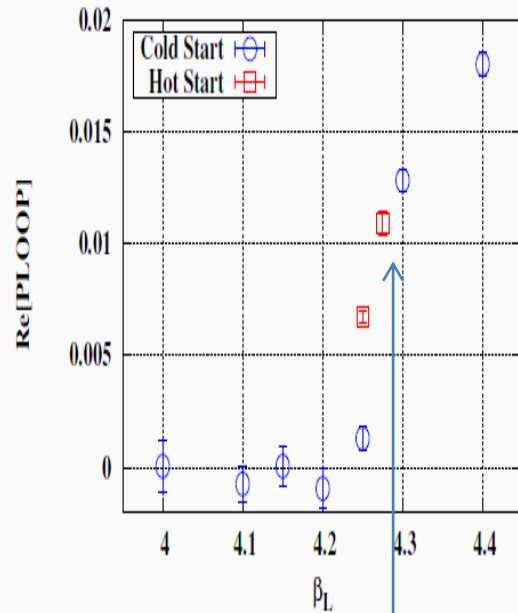
M fixed with the help of perturbation theory



THE STRING TENSION AND w_0

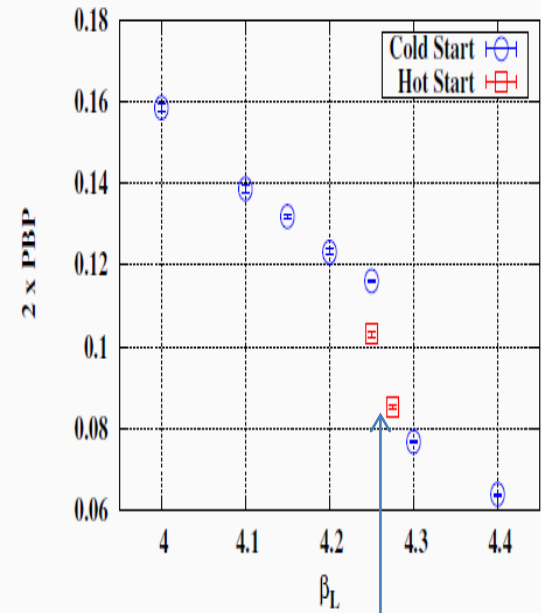
Lattice setup: β for $N_f=8$

Update for Miura-Lombardo Nucl. Phys. B ('13). c.f. Deuzeman et.al. Phys. Lett. B ('08).



$$\beta_L^c = 4.275 \pm 0.05$$

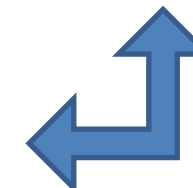
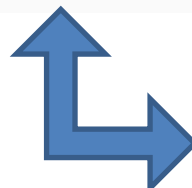
Update for Miura-Lombardo Nucl. Phys. B ('13). c.f. Deuzeman et.al. Phys. Lett. B ('08).



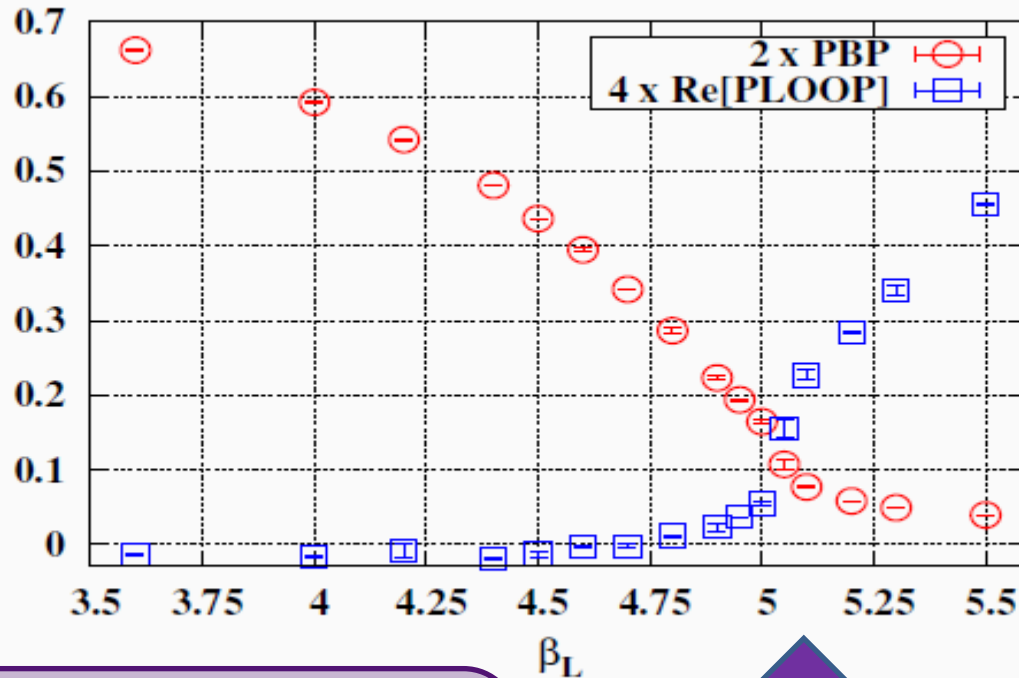
$$\beta_L^c = 4.275 \pm 0.05$$

Finite T
results $N_t=8$

Choice for the $T=0$
simulation



Lattice setup: β for $N_f=6$



Finite temperature, $N_t=6$ results

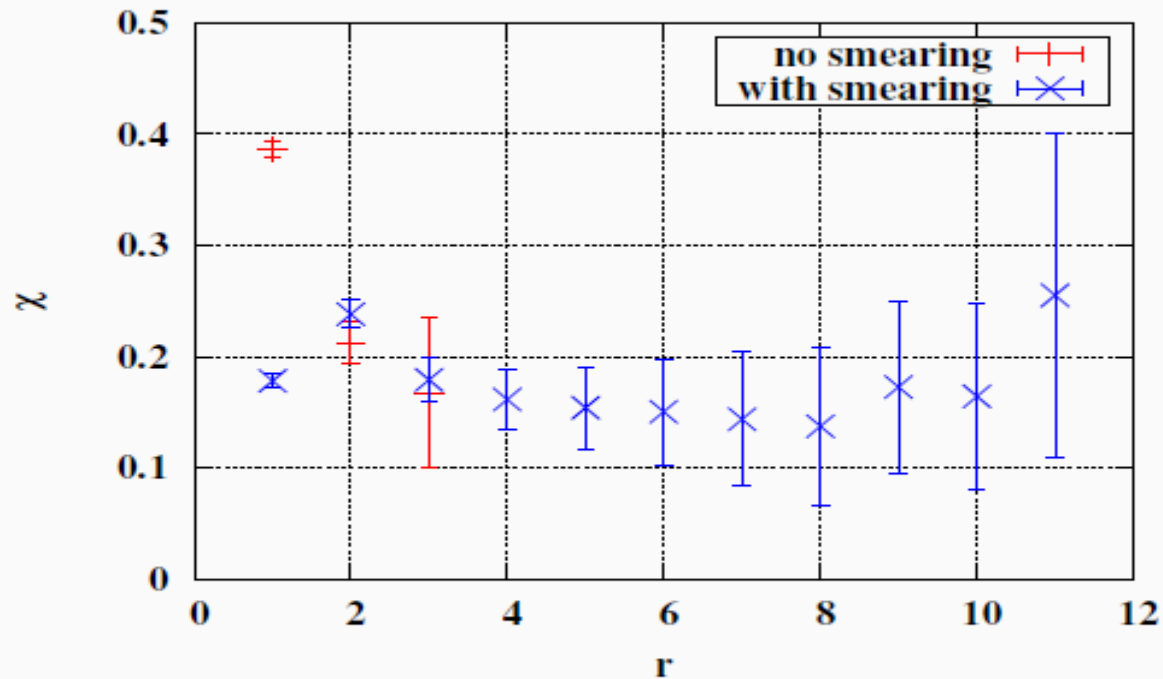
$\beta_c = 5.025$:
Choice for the $T=0$ simulations

And analogously for $N_t=8$

Nf=8: Creutz ratios

Measurements' code by M. Wagner and collaborators

Preliminary, $\beta = \beta_L^c = 4.275$, $ma = 0.02$, $32^3 \times 64$, $t = 3$

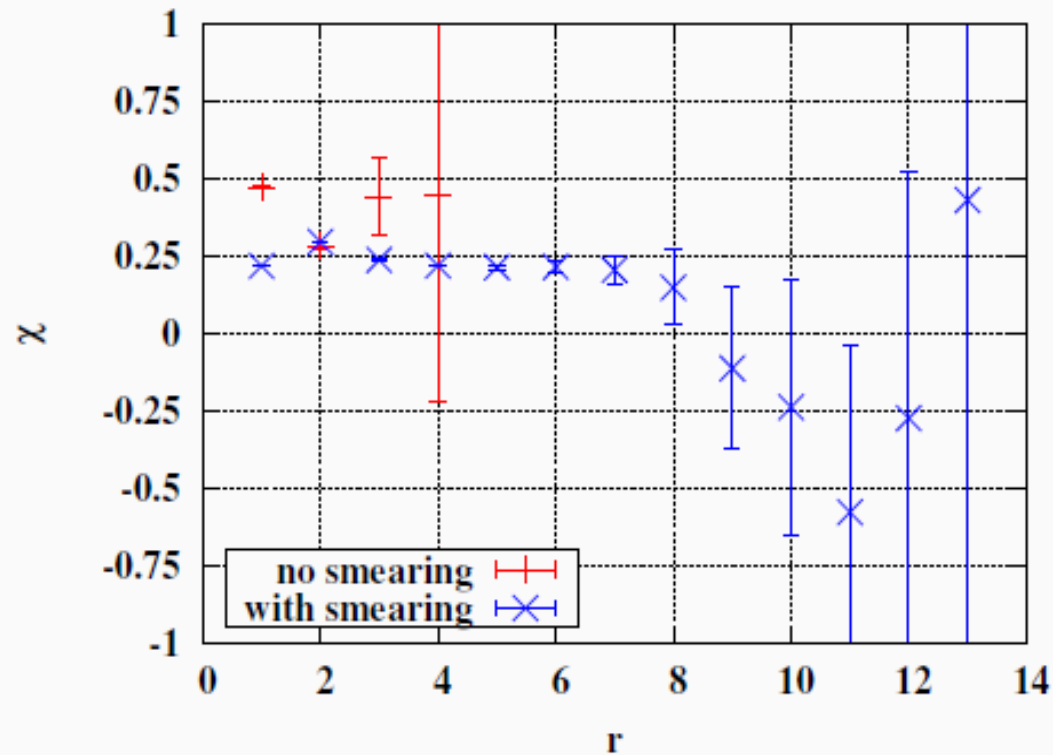


$$\chi_{r,t} = -\log \left[\frac{W_{r,t} W_{r+1,t+1}}{W_{r,t+1} W_{r+1,t}} \right] = \frac{\alpha}{\hat{r}(\hat{r} + 1)} + \hat{\sigma} .$$

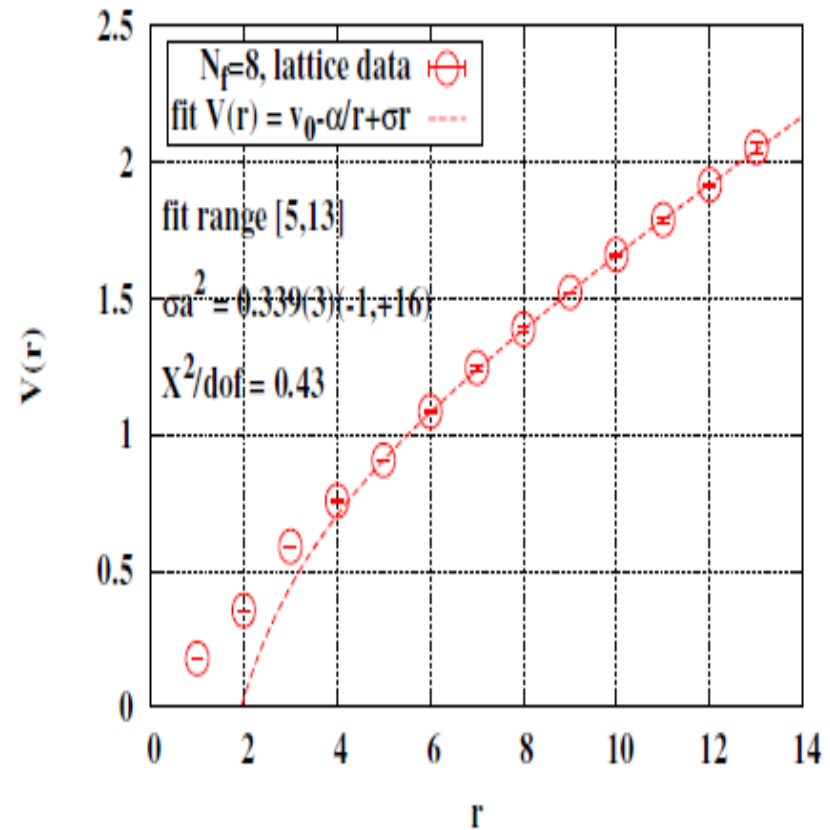
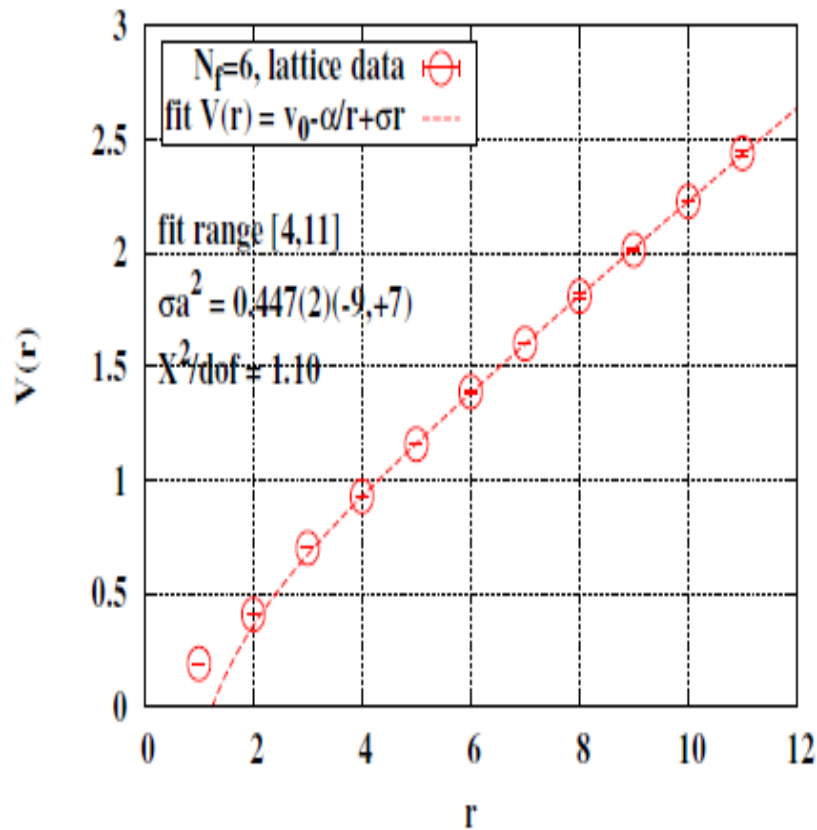
Nf=6: Creutz ratios

Measurements' code by M. Wagner and collaborators

Preliminary, $\beta = \beta_L^c = 5.025$, $ma = 0.02$, $32^3 \times 64$, $t = 3$

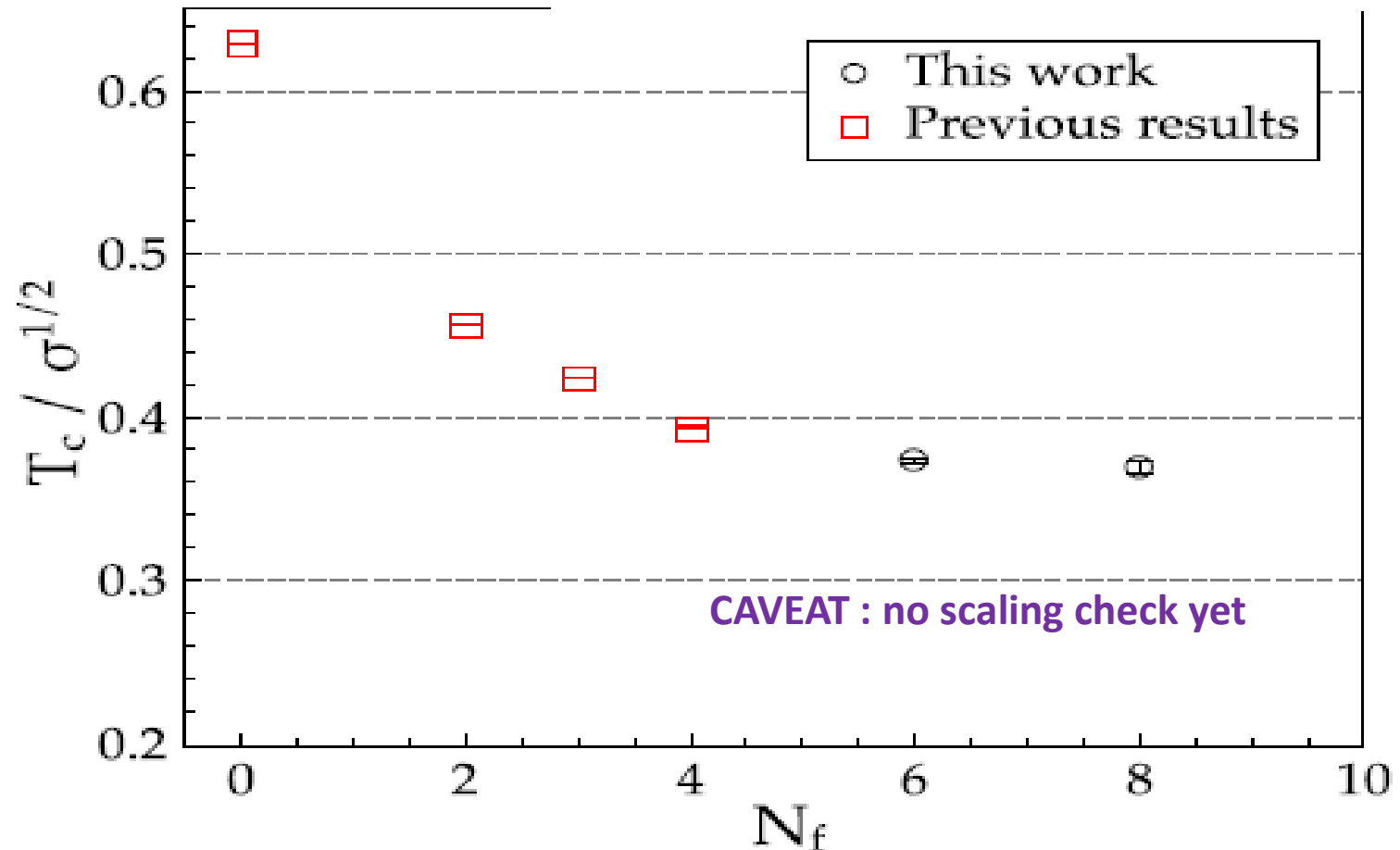


Heavy Quark Potential

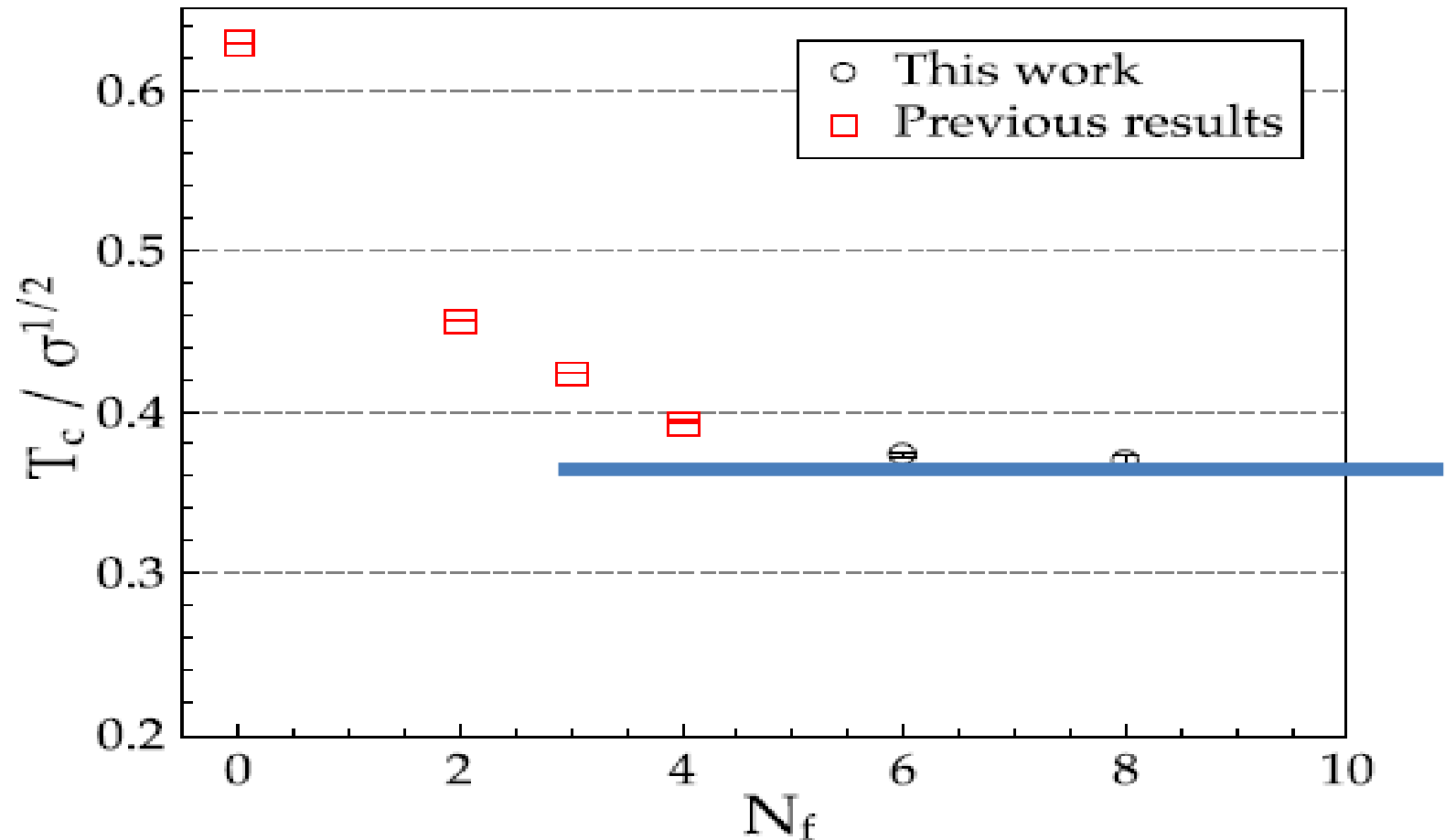


$T_c/\sqrt{\sigma}$

$$T_c/\sqrt{\sigma} = \begin{cases} 0.373(2)(+5, -6), & N_f = 6, \beta = 5.025, \\ 0.369(4)(+1, -5), & N_f = 8, \beta = 4.275. \end{cases}$$



Lim $N_f \rightarrow N_{fc}$ $T_c/v \sigma = \text{Const?}$



Wilson flow

$$\mathcal{E}(t) = t^2 \langle E(x, t) \rangle, \quad E(x, t) \equiv -\frac{1}{2} \text{tr} G_{\mu\nu}(x, t) G_{\mu\nu}(x, t)$$

$$w_0 : w_0^2 \mathcal{E}'(w_0^2) = 0.3,$$

- ✓ Computationally easy
- ✓ Naturally smooth
- ✓ Well behaved at short distance

W0 vs r0

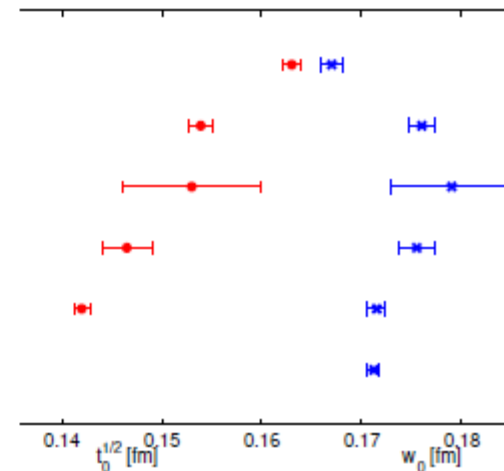
R. Sommer@Lat2013

Wilson, $N_f = 2$			tmQCD, $N_f = 2$			$N_f > 2$			
r_0 [fm]	from		r_0 [fm]	from	N_f	r_0 [fm]	r_1 [fm]	from	
0.503(10)	f_K	[14]	0.438(14)	f_K	[38]	2+1	0.466(4) ^a	0.313(2)	div. [39]
0.491(6) ^c	f_K	[9]				2+1		0.321(5)	Υ [1]
0.485(9) ^c	f_π	[9]	0.420(20)	f_π	[40]	2+1	0.470(4)	0.311(2)	f_π [41, 42]
0.501(15) ^b	m_p	[43]	0.465(16)	m_p	[44]	2+1	0.492(10) ^b		m_Ω [11]
0.471(17)	m_Ω	[13]				2+1	0.480(11)	0.323(9)	m_Ω [10]
						2+1+1		0.311(3)	f_π [45]

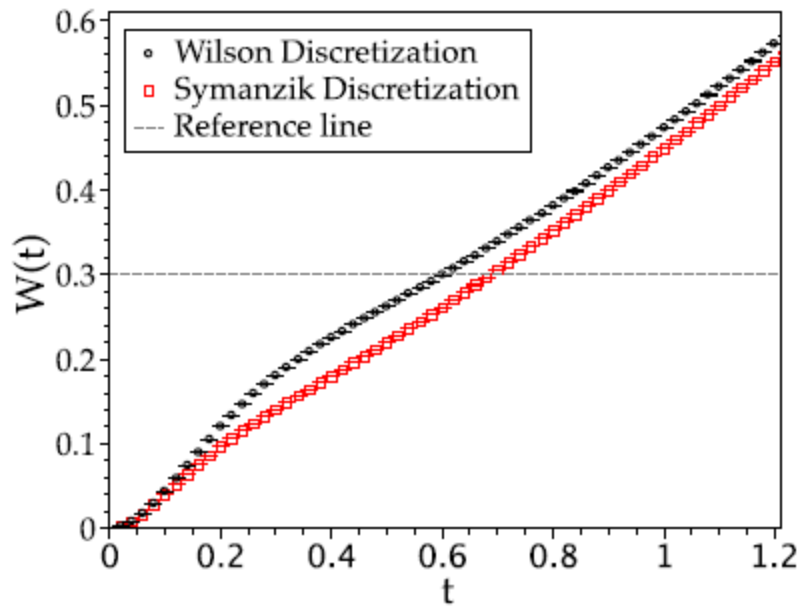
^a with r_0/r_1 and r_1/a from [46]

^c preliminary, at this conference

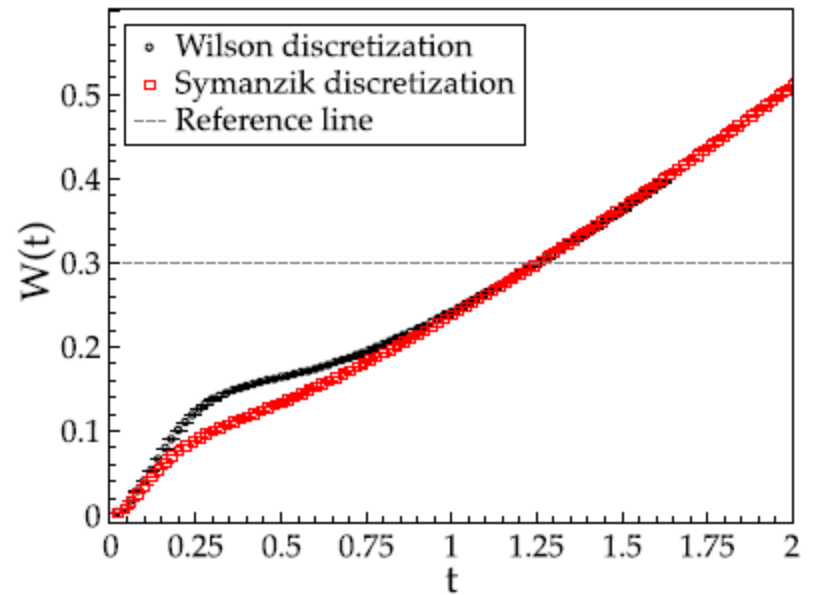
N_f	$\sqrt{t_0}$ [fm]	w_0 [fm]	from
0	0.1638(10)	0.1670(10)	$r_0 = 0.49$ fm [35, 30]
2	0.1539(12)	0.1760(13)	f_K [35, 9]
3	0.153(7)	0.179(6)	m_p [47]
3	0.1465(25)	0.1755(18)	m_Ω [33]
4	0.1420(8)	0.1715(9)	f_π [45]
4		0.1712(6)	f_π [34]



Scale from the flow, $N_f=6$

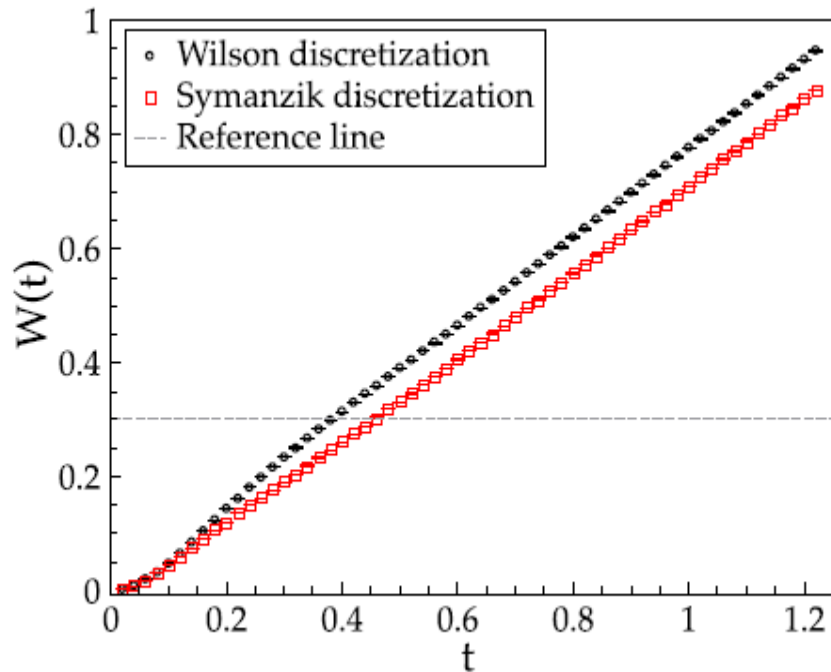


$$\beta = 5.025$$

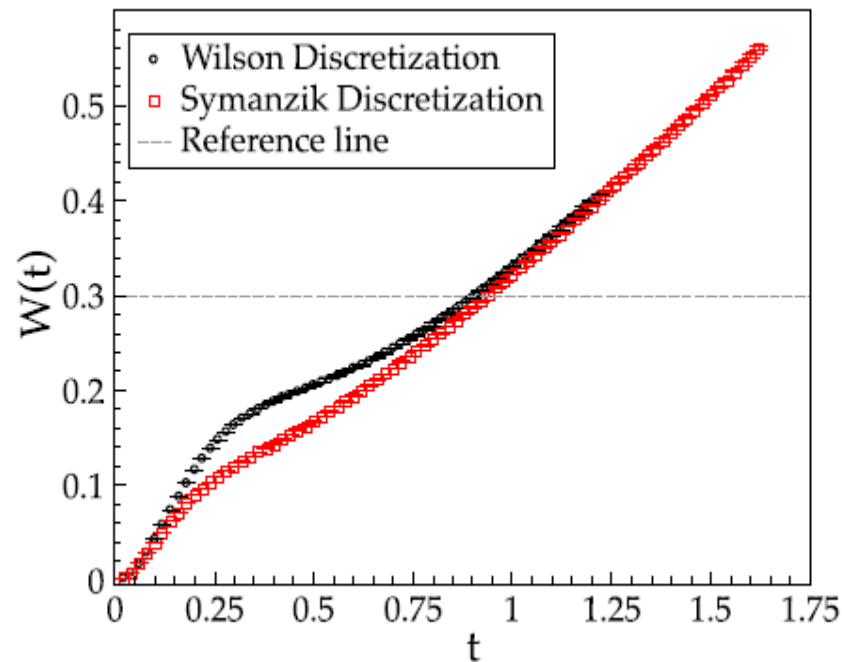


$$\beta = 5.200$$

Scale from the flow, $N_f=8$

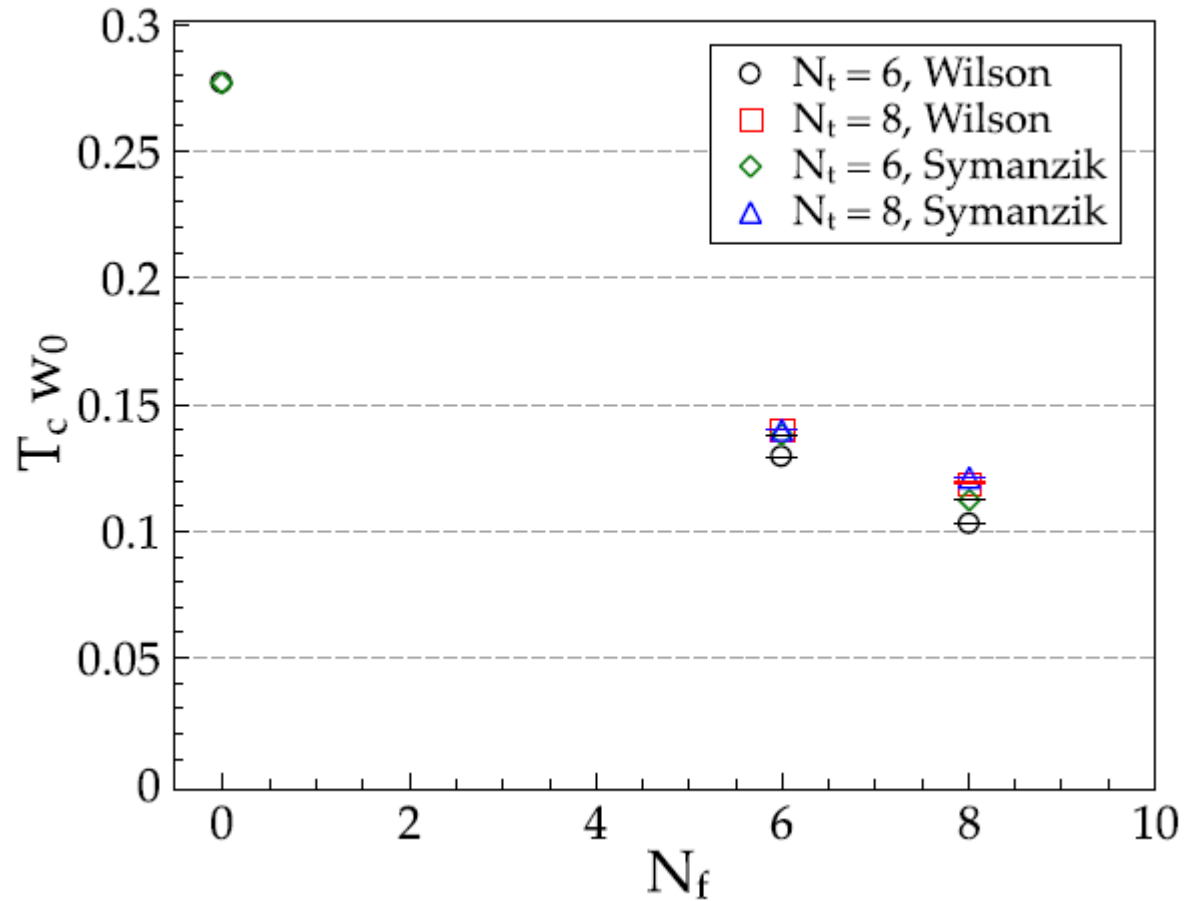


$\beta = 4.1125$



$\beta = 4.275$

Results for T_c on the $1/w_0$ scale



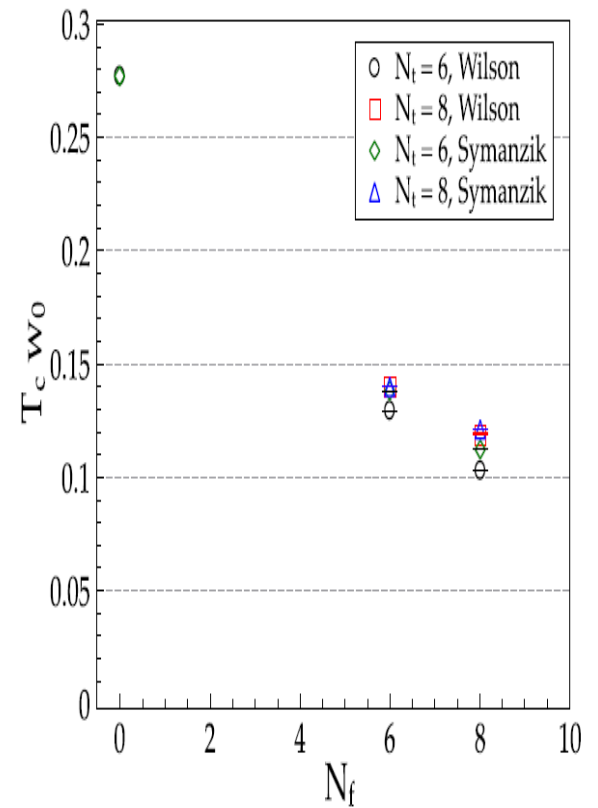
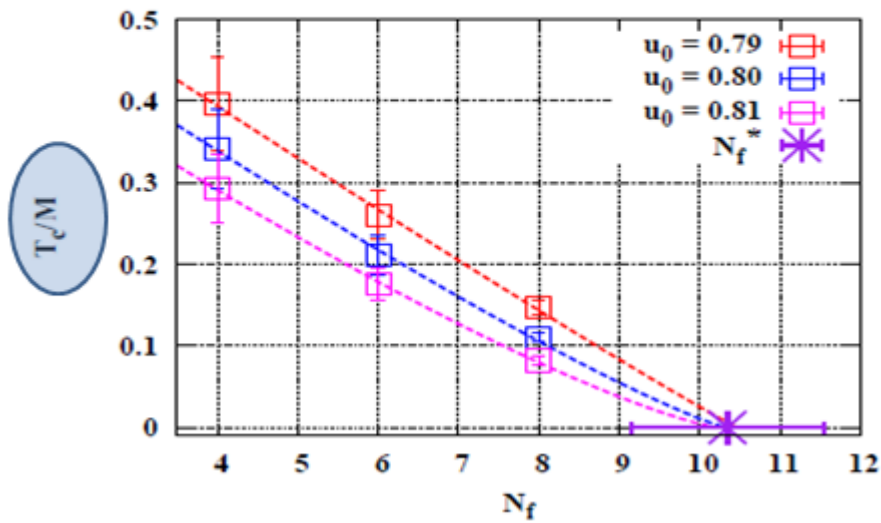
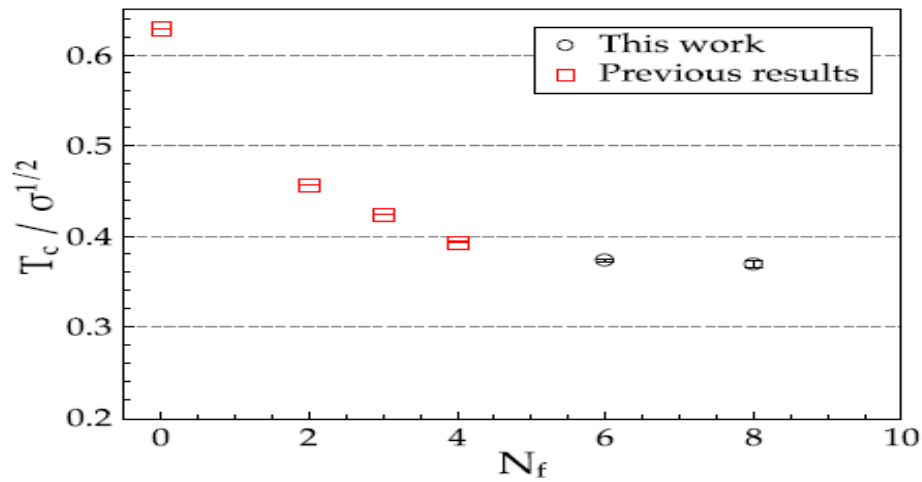
DIFFERENT SCALES

$N_f \rightarrow N_{fc}$:

$T_c/M = 0$

$T_c w_0 = 0$ (?)

$T_c / \sqrt{\sigma} \sim 0.3$



Different scales

M

Λ_{UV}

w_0

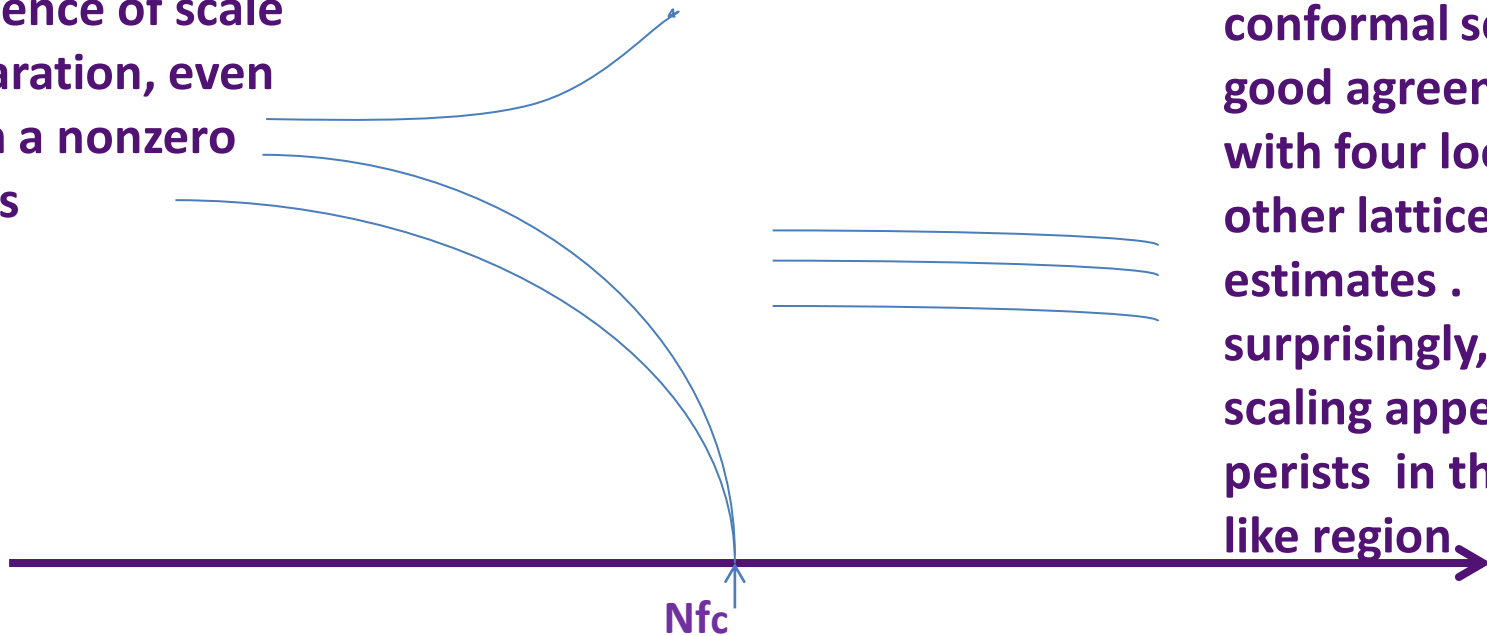
Λ_{IR}

$T_c, \nu\sigma$

Summary

For $N_f=8$ we have observed some evidence of scale separation, even with a nonzero mass

For $N_f=12$ we have measured conformal scaling, in good agreement with four loops and other lattice estimates. Perhaps surprisingly, the scaling appears to persist in the QED-like region



Summary

Indication of preconformality for $N_f=8$:

- Scale separation

- T_c measured on a UV scale approaches 0

T_c and the string tension have a similar sensitivity to the IRFP . Their ratio is weakly dependent on N_f

Anomalous dimension $\gamma = 0.235(46)$ for $N_f=12$:

Four loops 'almost exact'?

Next

- Better developed scaling for essential singularities maybe using models as a guidance
- Running coupling in the QCD phase from the potential and the flow
- Studies of mass dependence in the pre-conformal region
- Explicit check of conformal scaling and corrections with more masses and sizes in the conformal window