Determination of $\Lambda_{\overline{\mathrm{MS}}}$ from the static quark-antiquark potential in momentum space

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Strong Interactions in the LHC Era
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Overview

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2. **Perturbation theory**
3. **Setup**
4. **The static quark-antiquark potential**
5. **Determination of $\Lambda_{\overline{MS}}$**

Motivation

- The parameter Λ defines the scale for dimensionful perturbative results.
- analogous to r_0 for lattice results
- Calculating dimensionful quantities requires a scale.
- In the $\overline{MS}$ renormalization scheme: $\Lambda_{\overline{MS}}$
- Determine a result by a lattice computation and a perturbative calculation $\rightsquigarrow$ solve for $\Lambda_{\overline{MS}}$
- suitable observable : quark-antiquark potential
  - simple computation on the lattice (no quark propagators)
  - precise analysis in perturbation theory

Why momentum space?

- Lattice theory is formulated in position space, but perturbation theory is formulated in momentum space.
- Existing work: Fourier transform perturbation theory before comparison to lattice results K. Jansen et al. [ETM Collaboration], JHEP 1201 (2012) 025 [arXiv:1110.6859 [hep-ph]].
- Problem in position space:

$$V_{pert}(r) = \int d^3p \mathrm{e}^{ipr} \tilde{V}_{pert}(p)$$

  is wrong for $p < \Lambda \quad (\approx 300 \text{ MeV})$
- Unavoidable systematic errors!
- result: $\boxed{\Lambda_{\overline{MS}} = 315(30)\mathrm{MeV}}$
- This work: get $V(p)$ directly and consider it only for large momenta $p \gg \Lambda$

Perturbation Theory

The static quark-antiquark potential:

$$\tilde{V}(p) = -\frac{16\pi}{3p^2}\alpha_s(\mu)\Big\{1 + \frac{\alpha_s(\mu)}{4\pi}P_1(L) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 P_2(L) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^3 [P_3(L) + a_{3\ln}\ln\alpha_s(\mu)] + ...\Big\}$$

with

$$L(\mu, p) = \ln\frac{\mu^2}{p^2} \equiv L \qquad P_i(L) : \text{polynomials in L} \qquad \text{scale parameter } \mu$$

requirements:

$$\mu \gg \Lambda_{\overline{MS}} \longrightarrow \alpha_s(\mu) \ll 1 \qquad \text{and} \qquad \mu \sim p$$

$\Lambda_{\overline{MS}}$ is related to α_s:

$$\ln\frac{\mu}{\Lambda} = \int \frac{\mathrm{d}\alpha_s}{\alpha_s}\frac{1}{\beta(\alpha_s)}\bigg|_{\alpha_s=\alpha_s(\mu=p)} + C, \qquad C = \frac{\beta_1}{2\beta_0^2}\ln(\frac{\beta_0}{4\pi})$$

with

$$\beta(\alpha_s): \quad \text{QCD } \beta \text{ function}$$

(I) straightforward integration

(II) series expansion of $\frac{1}{\beta(\alpha_s)}$ in α_s and subsequent integration

$\rightsquigarrow$ match perturbative potential $\tilde{V}(p)$ and lattice potential results by adjusting $\Lambda_{\overline{\mathrm{MS}}}$ appropriately

Lattice Computations

- use $n_f = 2$ gauge link configurations generated by the ETMC
- tree-level Symanzik improved gauge action
- Wilson twisted mass action
- $a = 0.042$ fm $\qquad \beta = 4.35 \qquad (L/a)^3 \times T/a = 32^3 \times 64$

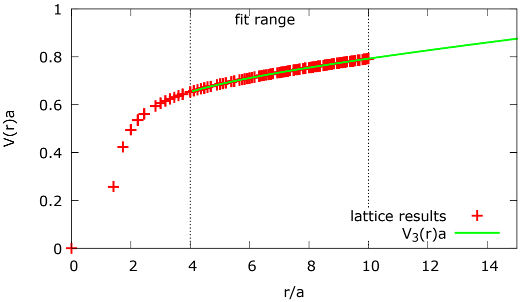
Potential in position space

1) Perform standard lattice computation of the static potential $V(\vec{r})$, i.e. extract $V(\vec{r})$ from the exponential decay of Wilson loop averages $\langle W(\vec{r},t)\rangle$ with respect to t:

- compute $V^{(\mathrm{effective})}(\vec{r},t) = \frac{1}{a}\ln\left(\frac{\langle W(\vec{r},t)\rangle}{\langle W(\vec{r},t+a)\rangle}\right)$
- perform χ^2-minimizing fit to $V^{(\mathrm{effective})}(\vec{r},t)$ in a suitable t-range

2) Model potential at large separations:

$$V_M(r) = A_0 + \sigma r + \sum_{m=1}^{M} \frac{A_m}{r^m}$$



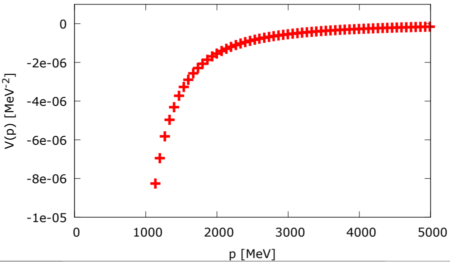
$\Longrightarrow$ keep in mind: choice of fit model is a new error source!

Transformation to momentum space

Discrete Fourier transform

$$\tilde{V}(\vec{k}) = \# \sum_{n_x, n_y, n_z = 0}^{N-1} V(n) e^{\frac{2\pi i \vec{k} \cdot \vec{n}}{N}}$$

- very sensitive to discontinuities in position space potential due to data modelling
- avoid lattice effects: use "cylinder cut"

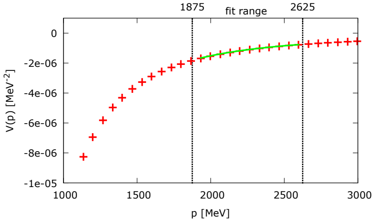


Variation of input parameters

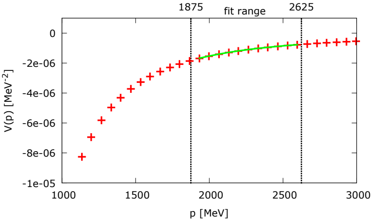
- data model in position space ($V_M(r)$)
- procedure (I) or (II) to determine $\Lambda_{\overline{\mathrm{MS}}}(\alpha_s(\mu))$
- fit range in momentum space

Results I

Exemplary fits: $M = 3$, procedure (I):



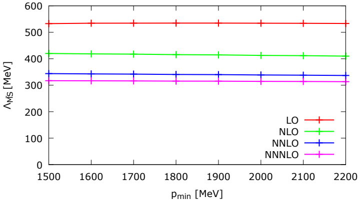
LO - $\Lambda_{\overline{\mathrm{MS}}} = 534$ MeV

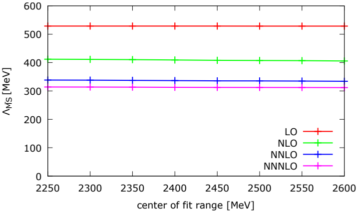


NNNLO - $\Lambda_{\overline{\mathrm{MS}}} = 315$ MeV

Results II

Results for different fit ranges in momentum space:





expression (I) $p_{\min}$ runs

expression (II) fixed fit interval

Errors

error source	this work	position space	comment
correlated syst. errors	$13\,\mathrm{MeV}$ $3\,\mathrm{MeV}$	$20-26\,\mathrm{MeV}$ $13-14\,\mathrm{MeV}$	NNLO and NNNLO NNNLO only
statistical errors	$\approx 2-4\,\mathrm{MeV}$	$\approx 2\,\mathrm{MeV}$	statistical error of lattice potential $V(r)$ propagated through to $\Lambda_{\overline{\mathrm{MS}}}$ via jackknife
lattice discr. errors	$\ll 8\,\mathrm{MeV}$	$\ll 6\,\mathrm{MeV}$	a rather conservative upper bound
lattice spacing errors	$\approx 13\,\mathrm{MeV}$		$\approx \Lambda_{\overline{\mathrm{MS}}} \times (\Delta a/a) \approx \Lambda_{\overline{\mathrm{MS}}} \times 0.04$

Final results for $\Lambda_{\overline{MS}}$

One finds (momentum space):

$\Lambda_{\overline{MS}} = 331(13)\mathrm{MeV}$ respectively $\Lambda_{\overline{\mathrm{MS}}} r_0 = 0.692(21)$
$\Lambda_{\overline{MS}} = 331(21)\mathrm{MeV}$ respectively $\Lambda_{\overline{\mathrm{MS}}} r_0 = 0.692(31)$

former result (position space):
$\Lambda_{\overline{MS}} = 331(20)\mathrm{MeV}$ respectively $\Lambda_{\overline{\mathrm{MS}}} r_0 = 0.692(31)$
$\Lambda_{\overline{MS}} = 315(30)\mathrm{MeV}$ respectively $\Lambda_{\overline{\mathrm{MS}}} r_0 = 0.658(55)$

without lattice discretization errors
with all errors

Conclusions

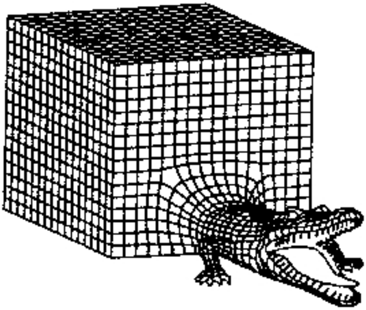
- Error on $\Lambda_{\overline{MS}}$ could be reduced!
- result very sensitive on choice of data model in position space

Future tasks

$\longrightarrow$ Continuous Fourier transform instead of discrete version $\surd$

$\longrightarrow$ precise lattice results at large distances needed

$\longrightarrow$ study the comparison of momentum space
and position space results in more detail



Thanks for your attention.