Naturalness and Hierarchy

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Why New physics?

Motivations:

* Elegance, simplicity, predictivity;
* Trying to explain existing phenomena (Dark matter, Dark energy, unification with quantum gravity ...)
* Naturalness

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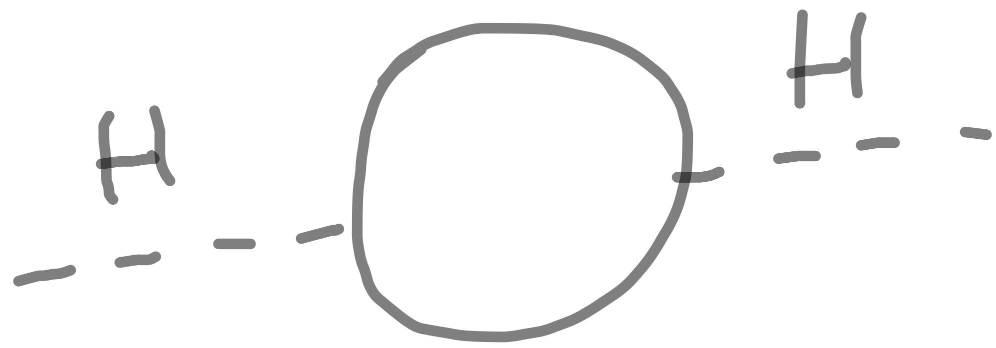
Naturalness problems

① UV-sensitivity

e.g. Higgs mass



② Vacuum super-selection

e.g. vacuum θ-angle in QCD

$$\theta F \tilde{F}$$

The hierarchy problem has a meaning because of gravity:

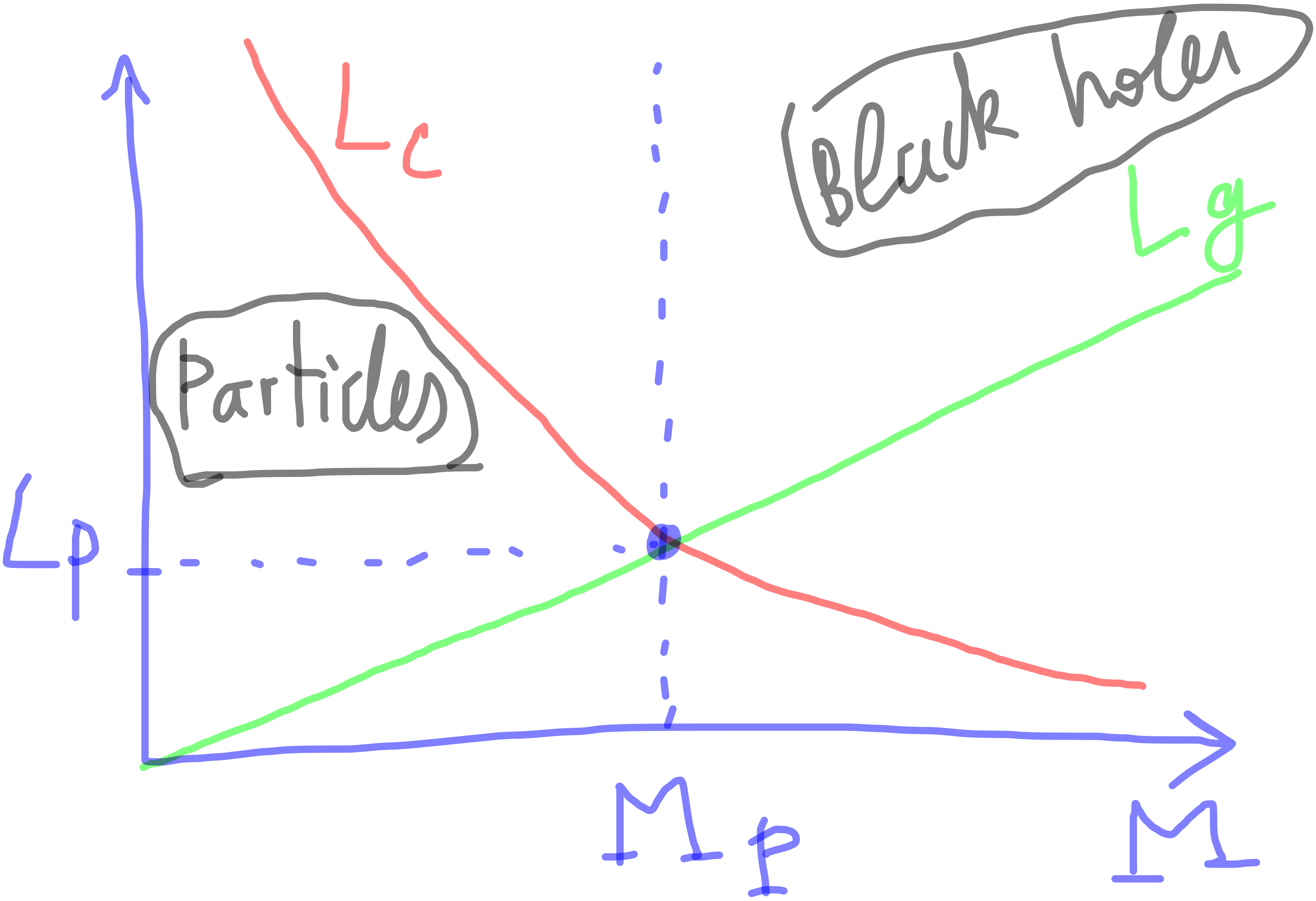
$$M_P \equiv \frac{\hbar}{L_P}\,, \quad L_P^2 \equiv \hbar G_N$$

$M_P \sim 10^{19}\,\mathrm{GeV}$, $L_P \sim 10^{-33}\,\mathrm{cm}$

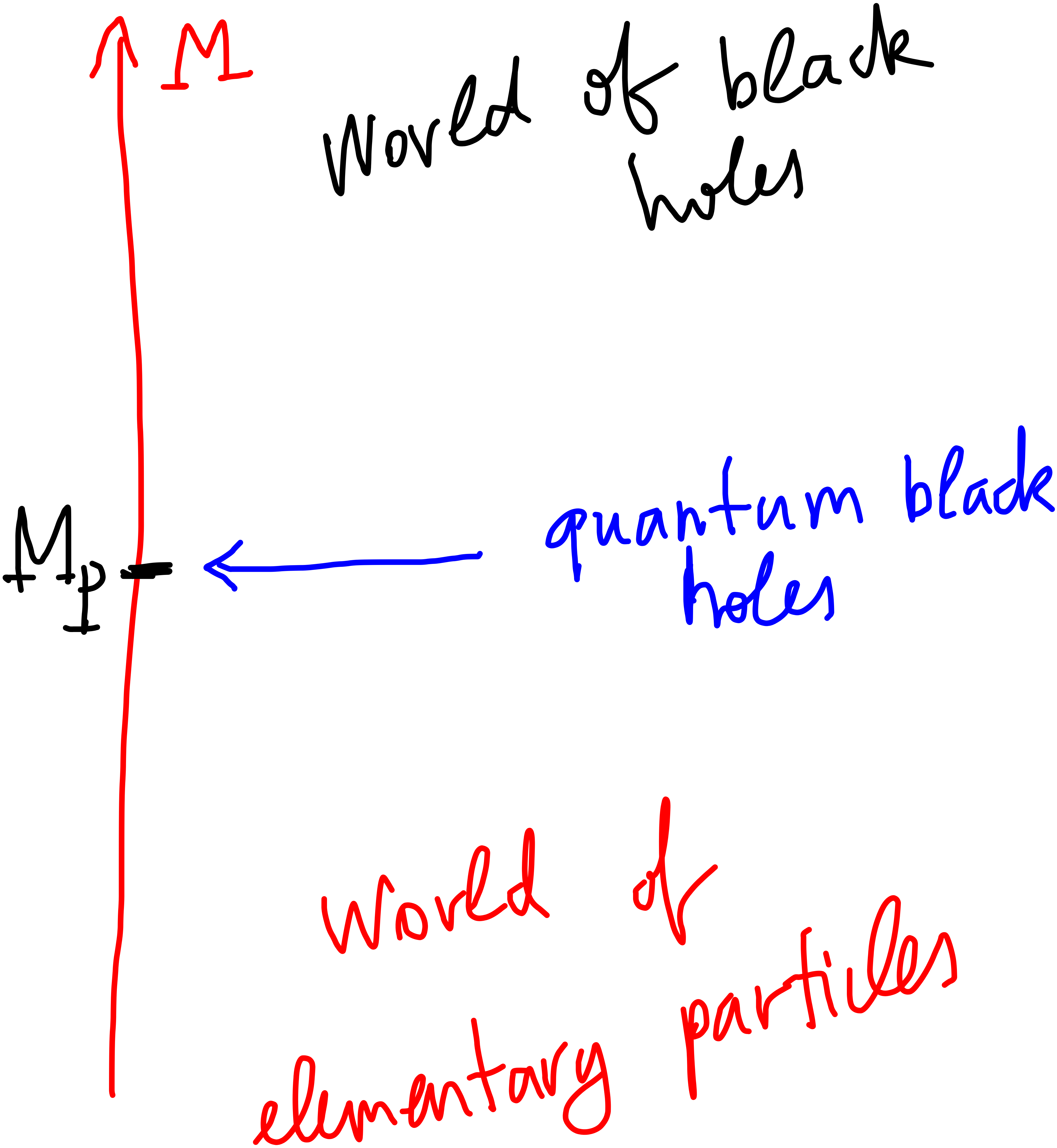
Particles heavier than M_P do not exist: they are black holes!

A particle of mass M has two length scales:

$$L_c \equiv \frac{\hbar}{M} \qquad L_g \equiv \frac{M}{M_p^2}\hbar$$



$L_c = L_g = L_p$ for $M = M_p$


M
World of black holes
M_P
quantum black holes
World of elementary particles

Hierarchy problem:

Why $m_H \ll M_P$?

or

Why Higgs is not a quantum black hole?

Can the physics that solves the Hierarchy Problem be arbitrarily weakly coupled?

$\Downarrow$

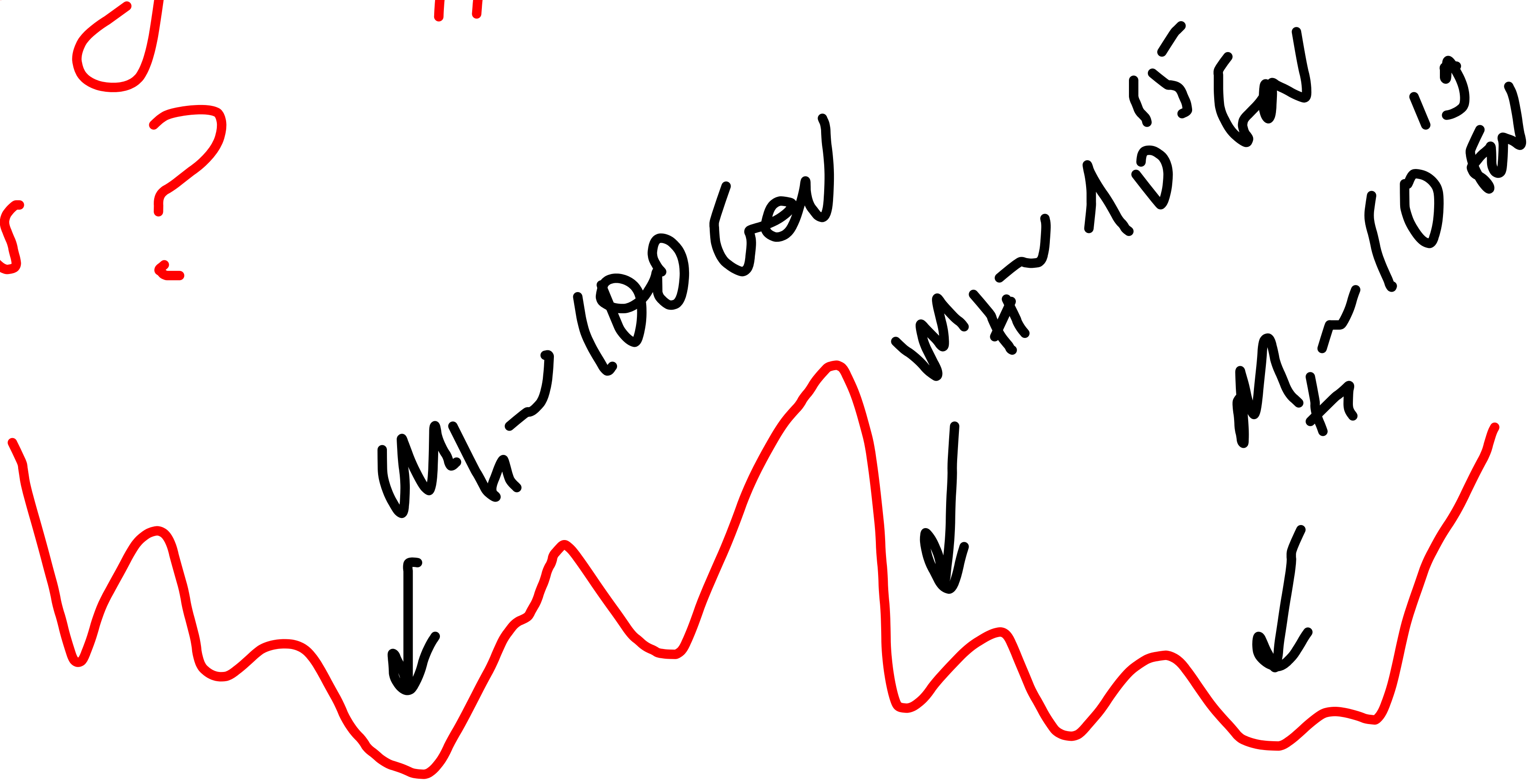
Hierarchy Problem = Vacuum selection problem

Can the hierarchy problem be promoted into a problem of vacuum selection?

Instead of picking up one among many theories, we pick up one vacuum among many vacua of the same theory.

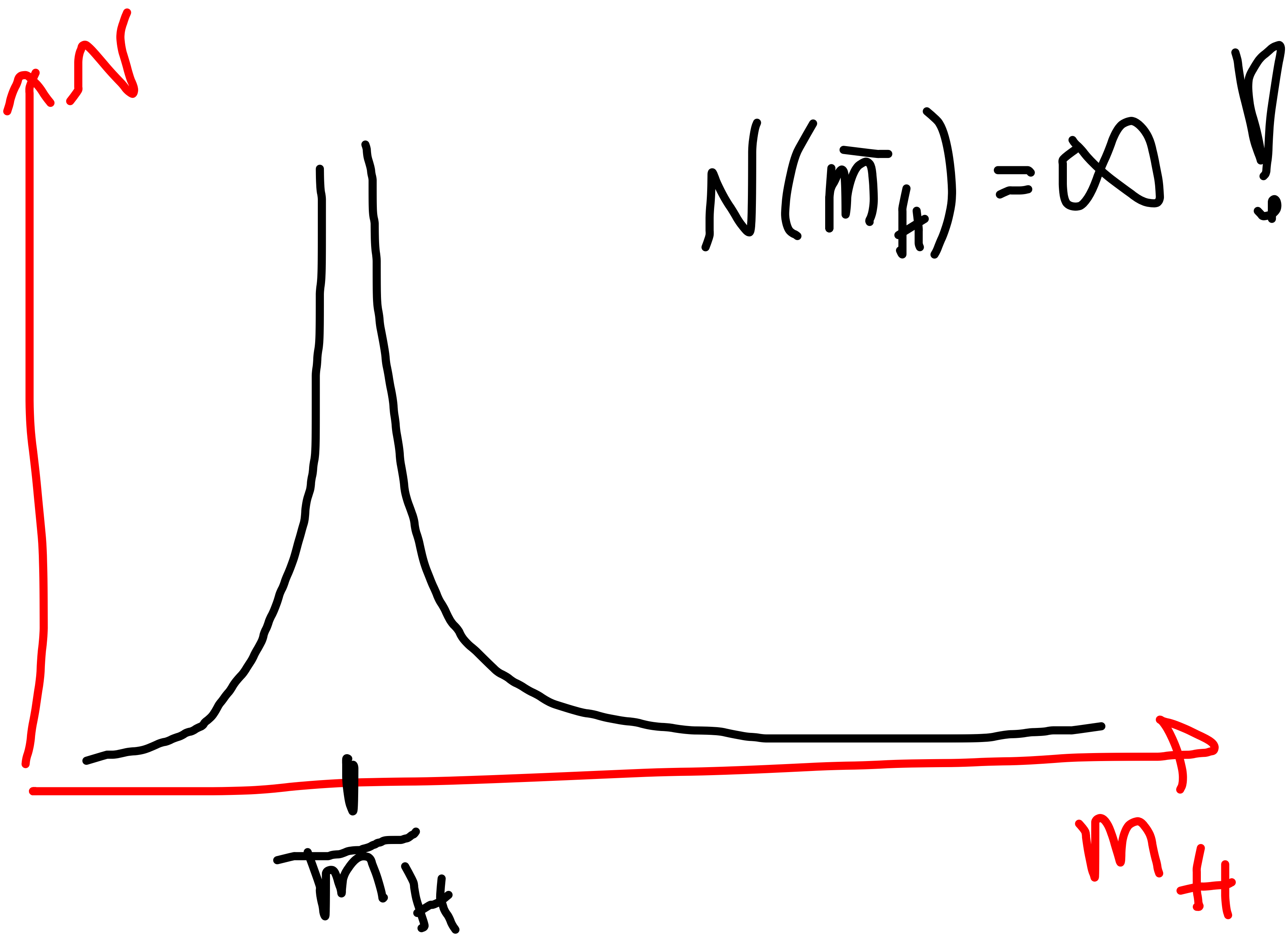
Of course, this is not any better unless you have a mechanism for selection:

Why m_H is what it is?



Vacuum attractor mechanism:

Higgs mass (and VEV) controls the number density of vacua G.D., A.Vilenkin '03



No New Below-Planck-Scale-
-Solution to the Hierarchy
Problem.

G.D., A.Vilenkin, PRD70(2004)
63501, hep-th/0304043;

G.D., PRD 74.025018,
hep-th/0410286

Attractors on Landscape

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with A. Vilenkin hep-th/0304043
G. D. hep-th/0410286

Outline:

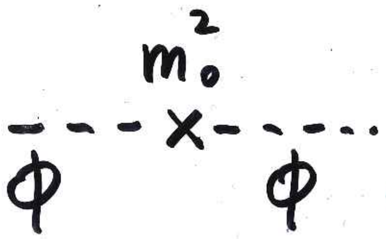
* Motivation
* Attractor phenomenon
* UV-stability
* Models & preditions:

  1. Z_{2N} - family symmetry
  2. The Higgs VEV is set by Λ_{QCD}

* Implications for cosmological constant.

The usual view is that naturalness requires some new UV-regurating physics at ~TeV scale, for solving the hierarchy problem.

Quadratic UV-sensitivity of the Higgs mass



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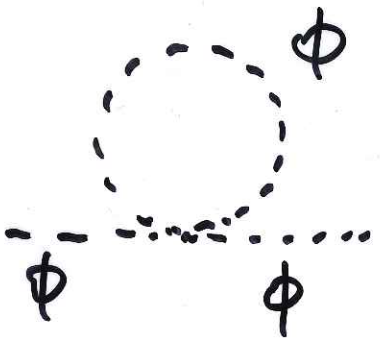
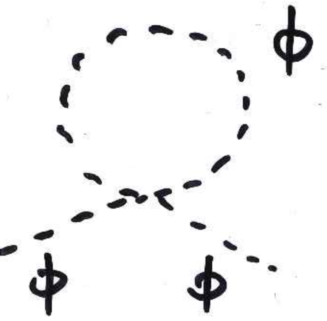


$$= m_0^2 + \frac{\lambda^2}{16\pi^2}\Lambda^2$$



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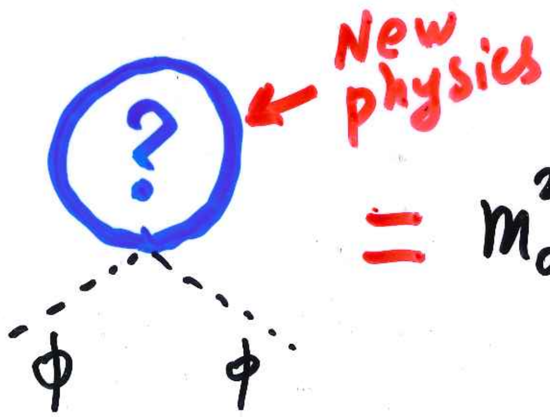


$$= m_0^2\left(1 + \frac{\lambda^2}{16\pi^2}\ln\Lambda\right.$$

Is this true?

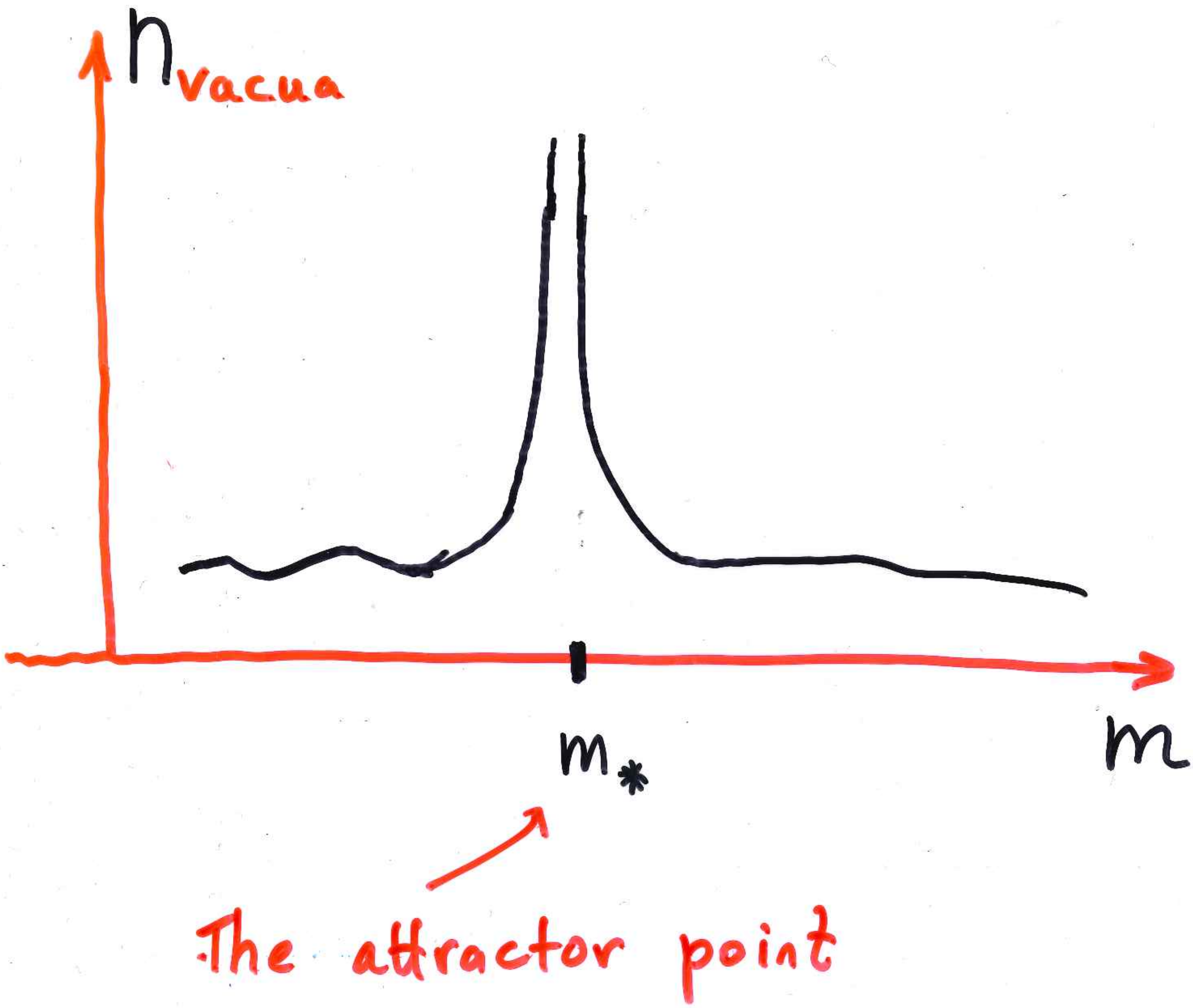
We shall present a framework in which no UV-regulating new physics is required around the TeV scale. Yet, the hierarchy problem is solved.

New ideas about naturalness:

The Attractors

These are the places where

$$N_{vacua} \longrightarrow \infty$$

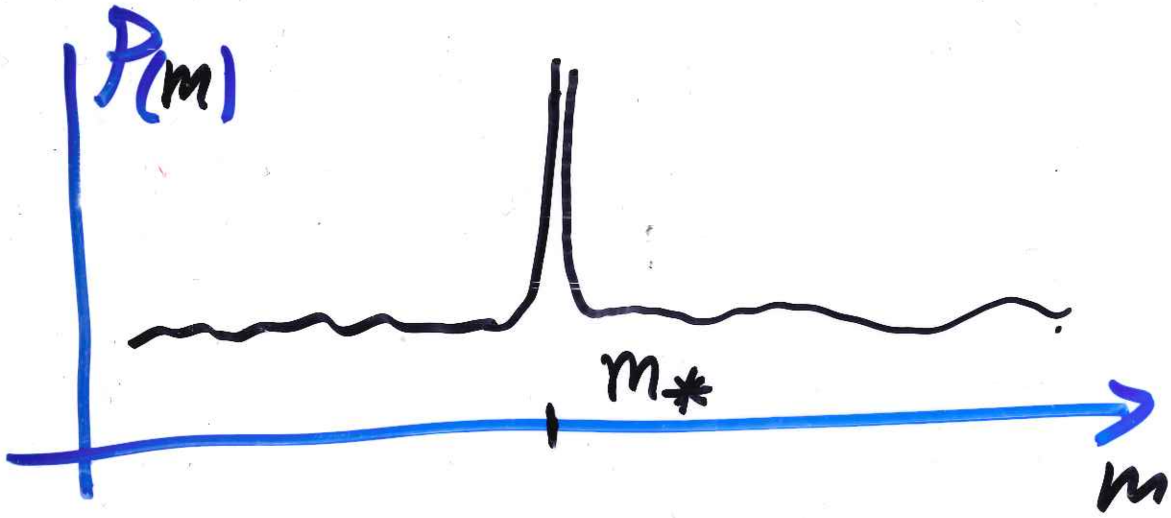


$$n^{(m)}_{vacua} \sim \frac{1}{(m - m_*)^N}$$

The question I ask in this talk:

What if $P(m)$ blows up for some $m = m_*$?

Can the picture be like this?



$$P(m) \sim \frac{1}{(m - m_*)^N}$$

In my talk I will heavily rely on the existence of

3-forms and 2-branes (membranes)

(Domain Walls)

The gauge freedom:

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + d_{[\alpha}\Omega_{\beta\gamma]}$$

n 4D eliminates all the propagating degrees of freedom.

Therefore, the field strength

$$F_{\alpha\beta\gamma\nu} \equiv d_{[\alpha}C_{\beta\gamma\nu]} = \text{Constant}$$

in any given vacuum!

The action:

$$\int_{3+1} F^2$$

Equation of motion

$$\partial^\mu F_{\mu\nu\alpha\beta} = 0$$

is solved by

$$F_{\mu\nu\alpha\beta} = F_0 \, \epsilon_{\mu\nu\alpha\beta}$$

F_0 = arbitrary constant.

In the absence of other interactions, 3-form just adds an arbitrary integration constant to Lagrangian.

Coupling to the Higgs field ϕ:

$$\mathcal{L} = |\partial_\mu \phi|^2 - \frac{F^2}{2\cdot 4!} + |\phi|^2\left(m^2 + \frac{F^2}{2\cdot 4!\, M^2}\right) - \frac{\lambda}{2}|\phi|^4 + \ldots$$

Equation

$$\partial^\mu\left[\left(1 - \frac{|\phi|^2}{M^2}\right) F_{\mu\alpha\beta\gamma}\right] = 0$$

is solved by:

$$F_{\mu\alpha\beta\gamma} = \frac{F_0}{\left(1 - \frac{|\phi|^2}{M^2}\right)} \epsilon_{\mu\alpha\beta\gamma}$$

Effective equation for the Higgs VEV (for $|\phi|^2/M^2 \ll 1$):

$$\left[-m^2 + \frac{F_0^2}{2M^2}\right]\phi + \lambda|\phi|^2\phi = 0$$

The Higgs VEV

$$\langle |\phi|^2 \rangle = \frac{-1}{\lambda}\left[-m^2 + \frac{F_0^2}{2M^2}\right]$$

depends on integration constant F_0!

Theory has a continuum of vacuum states. In particular there is a vacuum with

$$\langle \phi \rangle = 0, \qquad F_0 = 2m^2 M^2$$

But, there is a superselection rule:

Every F_0-vacuum is a <u>good vacuum</u>.

What have we achieved?

The hierarchy problem, from the problem of UV-stability, got promoted into the super-selection problem:

F_0-vacua are similar to Θ-vacua in QCD:

any choice of F_0 is good.

$\langle \phi \rangle = 0$ vacuum is as good as the vacuum with

$\langle \phi \rangle \sim M_{Pl}$

In order to solve the hierarchy problem, we need to:

* Promote F_0 into a dynamical quantity;

* Find the symmetry reason that will ensure that the vacuum with $\langle \phi \rangle = 0$ is the preferred one.

This is accomplished by coupling to 2-branes (Domain Walls)

F can change if there are 2-branes:

$$S = Q \int_{2+1} C + \int_{3+1} F^2$$

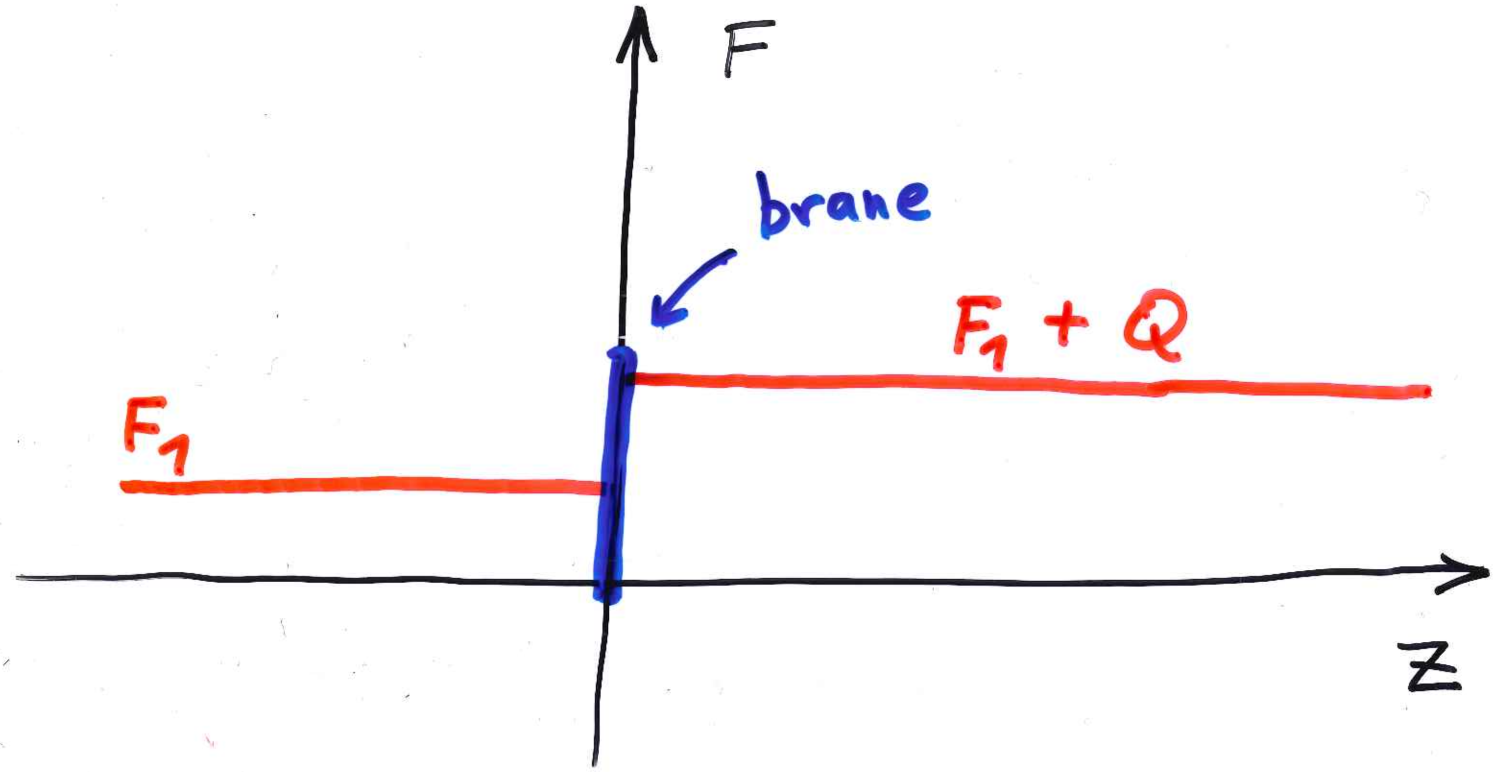
2-brane charge

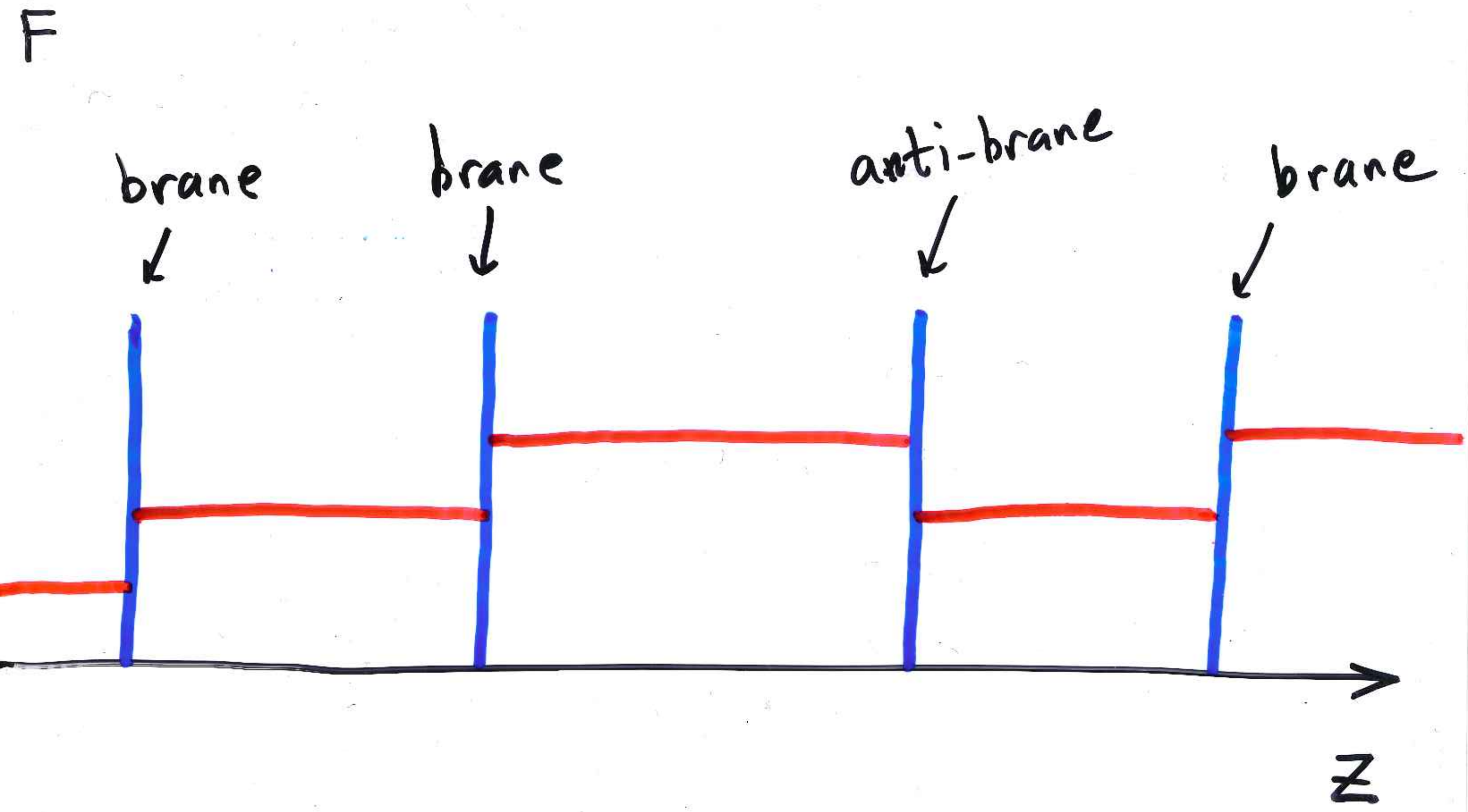
$$\partial^\mu F_{\mu\alpha\beta\gamma} = -Q\,\delta(z)\,\epsilon_{\mu\alpha\beta\gamma}$$

$$F_{\mu\alpha\beta\gamma} = (nQ + \text{constant})\,\epsilon_{\mu\alpha\beta\gamma}$$

brane separates two vacua in either of which F is constant, and the two values differ by

$$\Delta F = |Q|$$


F
brane
$F_1 + Q$
F_1
Z


F
brane
brane
anti-brane
brane
Z

Because of branes the superselection rule got broken, and transition between the different $\langle\phi\rangle$ - vacua is now possible.

Integrating out F, we get the following effective vacuum equation for ϕ - VEV (for small $\frac{\phi}{M} \ll 1$)

$$\left[-m^2 + \frac{(nQ)^2}{2M^2}\right]\phi + \lambda|\phi|^2\phi = 0$$

The key point:

We must promote Q in function of ϕ, such that

$$Q(\phi) \longrightarrow 0 \quad \text{for} \quad \phi \to 0$$

To guarantee this we need a Z_{2N} - symmetry:

$$\phi \to e^{i\frac{\pi}{N}} \phi$$

which acts as "brane charge conjugation"

brane $\longleftrightarrow$ anti-brane

$$Q = \phi^N + h.c.$$

$$Q \xleftrightarrow{Z_{2N}} -Q$$

Technicality:

$$Q \neq \text{const.} \longrightarrow \partial_\alpha J^{\alpha\beta\gamma} \neq 0$$

So the gauge-invariant coupling is

$$J_{(T)}^{\alpha\beta\gamma} C_{\alpha\beta\gamma}$$

transverce part

A compensating Goldstone-Stückelberg field

$$B_{\beta\gamma} \to B_{\beta\gamma} + \Omega_{\beta\gamma}$$

$$C_{\alpha\beta\gamma} \longrightarrow C_{\alpha\beta\gamma} + d_{[\alpha}\Omega_{\beta\gamma]}$$

Gauge-invariant couplings:

$$\left[C_{\alpha\beta\gamma} - d_{[\alpha}B_{\beta\gamma]}\right] J^{\alpha\beta\gamma} +$$

$$+ L^{\beta\gamma}\,\partial^{\alpha}\left[C_{\alpha\beta\gamma} - d_{[\alpha}B_{\beta\gamma]}\right]$$

$$\Big\Downarrow$$

$$J_{T}^{\alpha\beta\gamma}\, C_{\alpha\beta\gamma}$$

A simple model:

$$\mathcal{L} = |\partial_\mu \Phi|^2 - \frac{F^2}{48} - V(\Phi, F) - C_{\alpha\beta\gamma} J_T^{\alpha\beta\gamma}$$

where

$$J^{\alpha\beta\gamma}_{(x)} = \int d^3Y^{\alpha\beta\gamma} Q(Y)\, \delta^4(x - Y)$$

$$Q = \Phi^N + \text{h.c.}$$

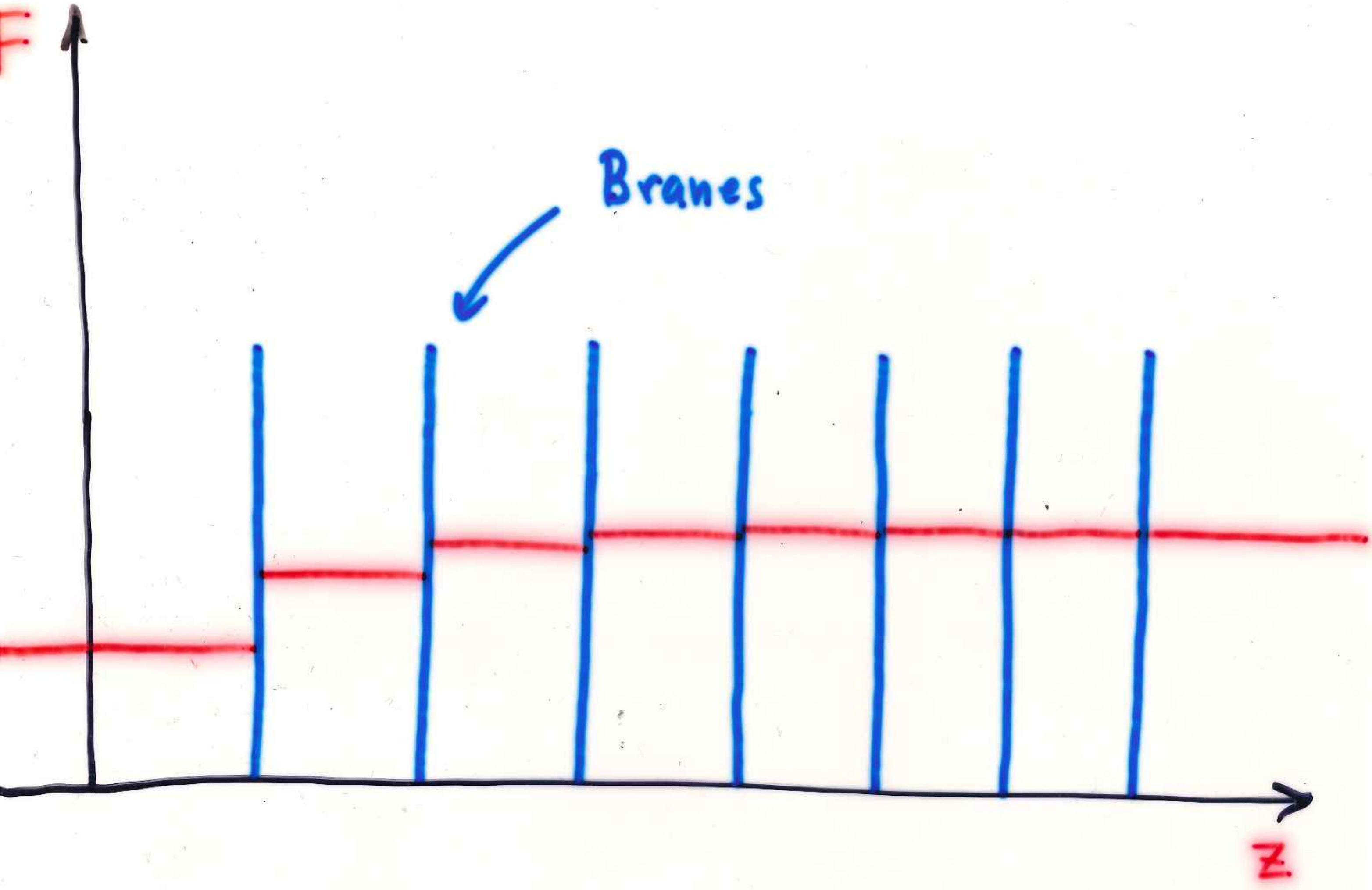
$$V(\phi, F) = \left(-m^2 - \frac{F^2}{48}\right)|\phi|^2 + \frac{\lambda}{2}|\phi|^4$$

Integrating out F from

$$\partial^{\mu}\left[(1-|\phi|^2)\,F_{\mu\alpha\beta\gamma}\right] = \epsilon_{\alpha\beta\gamma\nu}\,\Theta^{\nu z}\left[\Phi^N\,\delta(z)\right]$$

we get the following effective vacuum equation for ϕ (for small $\phi \ll 1$)

$$-\left[m^2 - \frac{n^2}{6}\,(\Phi^N + h.c)^2\right]\phi + \lambda|\phi|^2\phi = 0$$



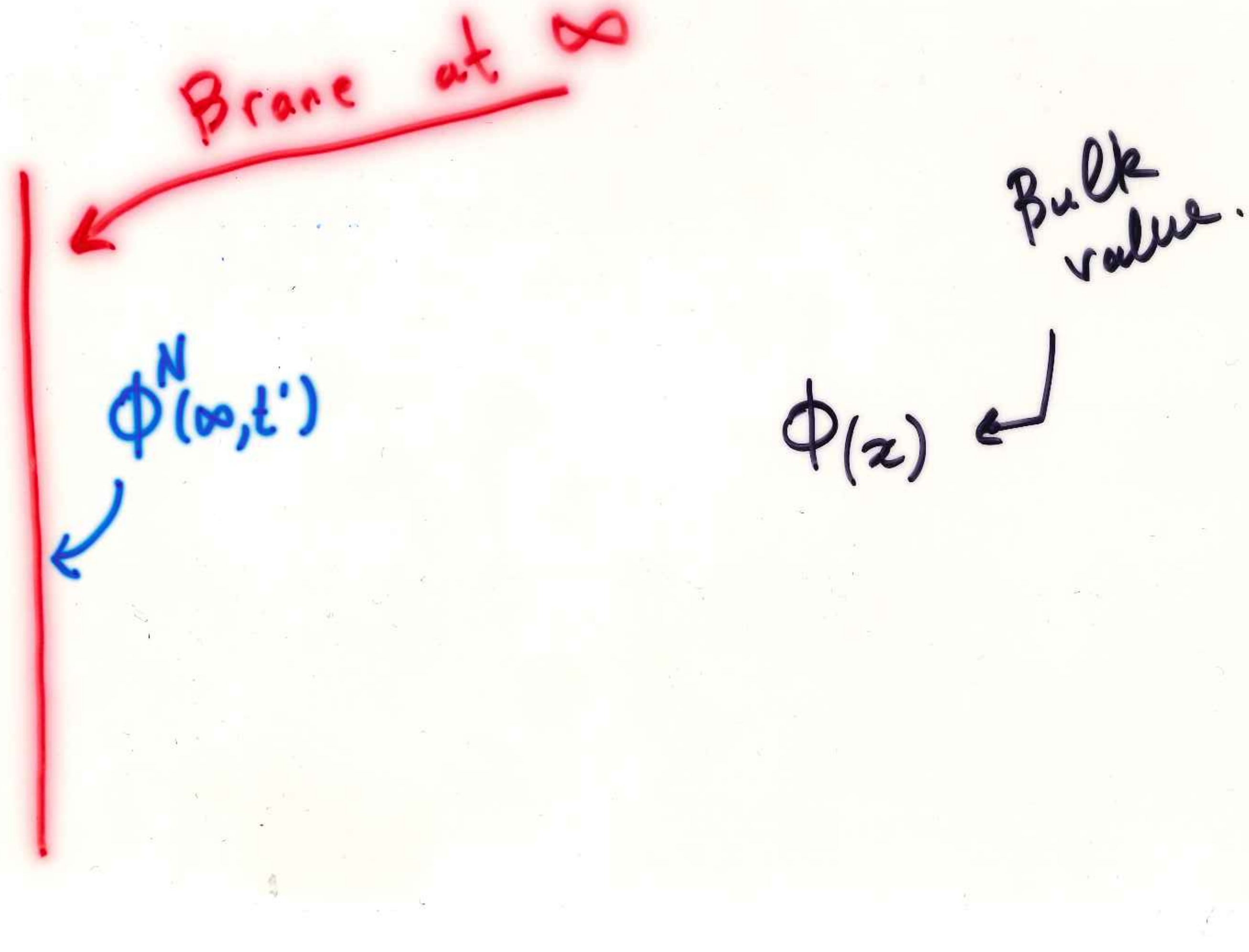
$$\Delta F \longrightarrow 0$$

Effective equation for ϕ

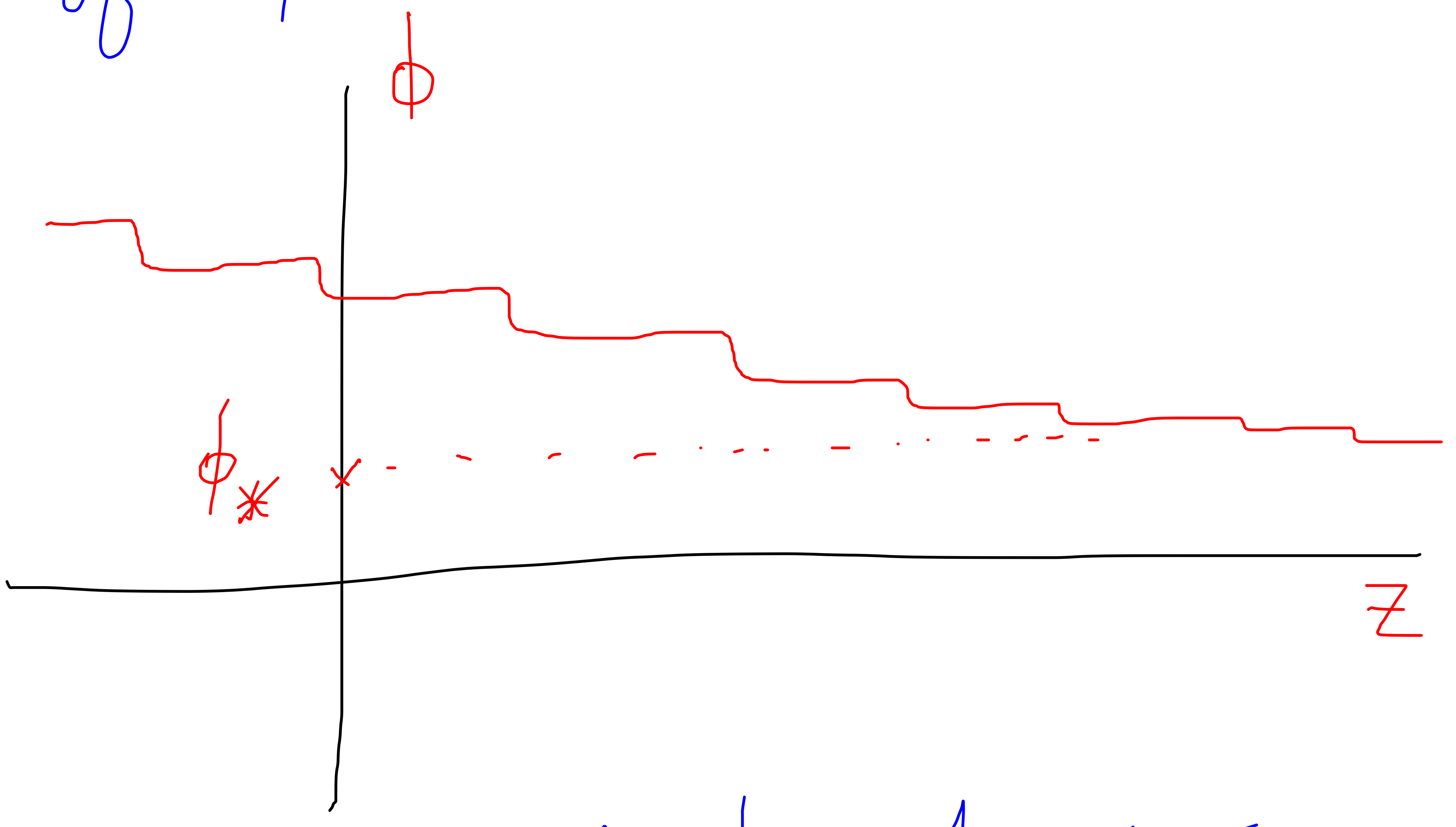
$$\Box\phi_{(x)} - \left[m^2 - \left\{n \int d^2x\, G(x-x')\, \phi^N_{(\infty,t')}\, \delta(z'-\infty)\right\}^2\right]\phi_{(x)}$$

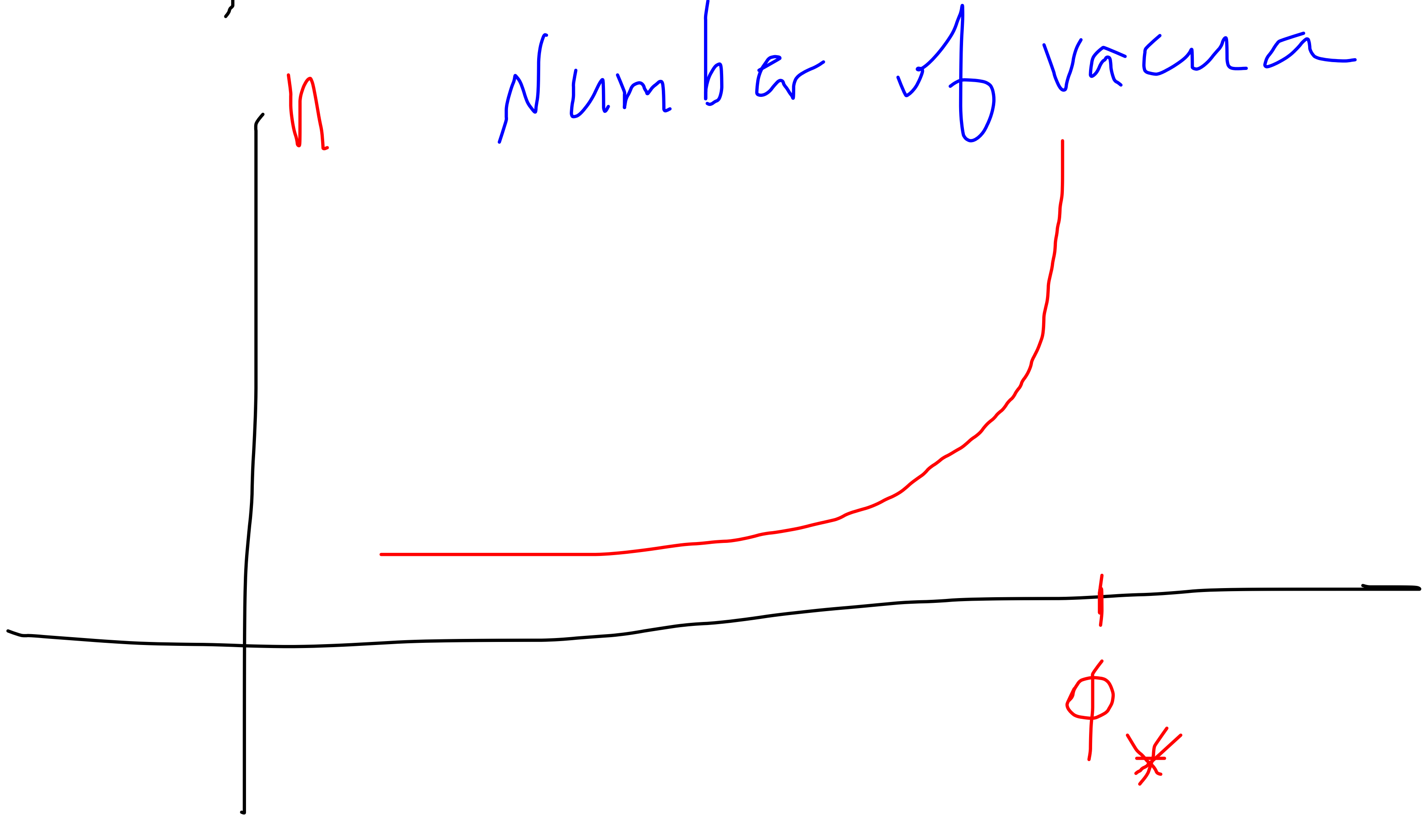
$$+ \phi^3_{(x)} = 0$$

$$G \rightarrow \frac{\partial_z}{\Box}$$



Real-space profile of ϕ





Realistic Models & Predictions

(*) Need for the second doublet

$Q(\phi)$ is $SU(2)\otimes U(1)$-invariant, and

$$Q \xrightarrow{Z_{2N}} -Q$$

This is impossible with a single doublet ϕ.

But Standard Model has a second doublet: Quark condensate

$$\langle \bar{q}_L q_R \rangle \neq 0 \, !$$

So,
$$Q = \left(\phi \bar{q}_L q_R\right)^N + h.c.$$

$$\phi \bar{q}_L q_R \xrightarrow{Z_{2N}} e^{i\frac{\pi}{N}} \phi \bar{q}_L q_R$$

(*) Because Z_{2N} acts as a family symmetry, the charge becomes:

$$Q = \left\{ (\phi \bar{q}_L q_R)^N - (\bar{q}_L q_R \bar{q}_L q_R)^K \right\}$$

The attractor point is at:

$$\langle \phi \rangle \equiv M_P \left(\frac{\Lambda_{QCD}}{M_P} \right)^{6\frac{K}{N} - 3}$$

Prediction:

The weak scale is set by Λ_{QCD}:

(*) $0 < \langle \phi \rangle \ll M_P$

(*) Zeros in Youkawa couplings

A complete model

$$\mathcal{L} = |D_\mu \phi|^2 - F^2 + |\phi|^2(m^2 + F^2) - |\phi|^4$$
$$- C_{\alpha\beta\gamma} J_T^{\alpha\beta\gamma} - M_b |\phi|^2$$

where

$$Q = \left\{ (\phi q_L u_R)^N - (\bar{q}_L u_R q'_L d'_R)^K \right\} + \text{h.c.}$$

Then attractor is at

$$\langle \phi \rangle \sim (\Lambda_{QCD})^{3\frac{K}{N} - \frac{3}{2}}$$

$$\frac{K}{N} \simeq \frac{4}{5}$$

General prediction:
zeros in Yukawa matrix.

$$m_u = 0$$

would be the simplest, but it is (probably?) ruled out.

The VEV H_* for which $Q(H_*) = 0$ is attractor:

The number of vacua with $H \to H_*$ diverges.

Since $\langle \bar{q}_L q_R \rangle = \Lambda_{QCD}^3$

So $$H_* = M_P \left(\frac{\Lambda_{QCD}}{M_P} \right)^{4\frac{k}{h} - 2}$$

For $\frac{k}{n} \sim \frac{5}{7}$, $H_* \sim 100\,\text{GeV}$.

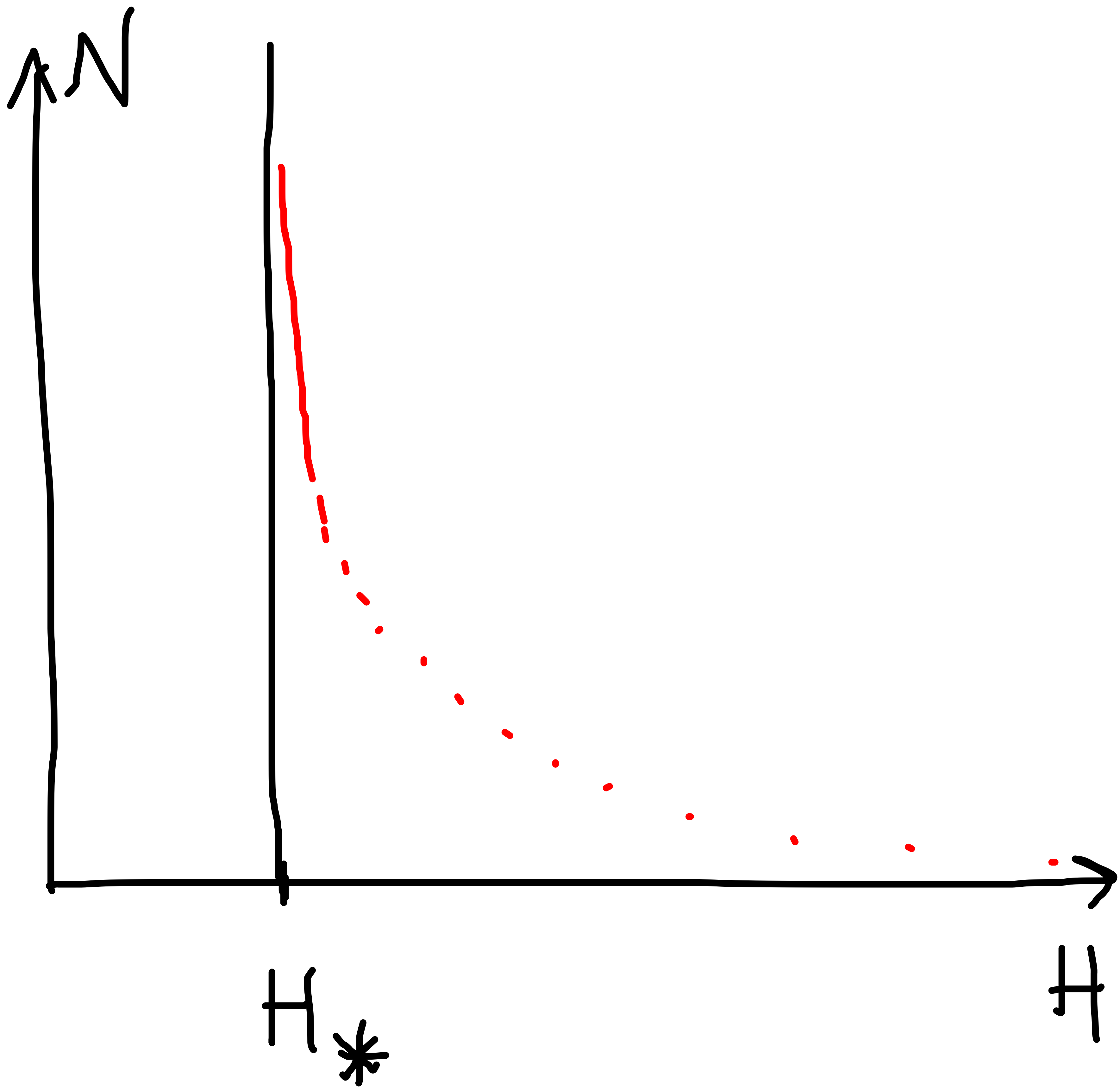
From vacuum H one step towards H_* is

$$\delta H \sim (H_* - H)\,\varepsilon$$

$$\varepsilon \sim 10^{-484}$$

Number of vacua near H_*

$$N \sim 10^{484} \ln(H - H_*)$$


N
H
H*

We have promoted the hierarchy problem into a problem of <u>vacuum super-selection</u>.

No new physics is required around the weak scale.

However there is <u>no free lunch</u>.

The Z_{2n} symmetry must be a family symmetry, because

$$H\bar{q}_L q_R \rightarrow e^{i\frac{\pi}{n}} H\bar{q}_L q_R$$

in order to allow the Yukawa couplings

Secondly, there is a propagating $\frac{1}{M_p}$-coupled massless axion in this theory.

This is a Stückelberg field $B_{\mu\nu}$, which ensures gauge invariance of the coupling with brane current.

$$C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma} + \partial_{[\alpha}\Omega_{\beta\gamma]}$$

$$\left(C_{\alpha\beta\gamma} - \partial_{[\alpha}B_{\beta\gamma]}\right) J^{\alpha\beta\gamma}$$

$$B_{\mu\nu} \to B_{\mu\nu} + \Omega_{\mu\nu}$$

Constraint

$$\chi^{\mu\nu}\partial^{\alpha}\left(C_{\mu\nu\alpha} - \partial_{\mu}B_{\nu\alpha}\right)$$

$$C_{\alpha\beta\gamma} J_T^{\alpha\beta\gamma}$$

$B_{\mu\nu}$ propagates massless waves

$$\Box\left(\left(1-\frac{H^2}{M_p^2}\right)F\right)=\partial_z\left(Q(H)\,\delta(z)\right)$$

$$F(z,t)=\frac{1}{\left(1-\frac{H^2}{M_p^2}\right)}\Big[\theta(z)\,Q(t-z)$$
$$-\,Q(t+z)\,\theta(-z)+f_0\Big]$$

We can extend this mechanism for solving doublet-triplet splitting problem in GUTs.

SU(5) Higgs, Σ, 5_H

$$\bar{5}\left[\Sigma^2 + \mu\Sigma + M^2 - \frac{F^2}{M_p^2}\right]5$$

$$\Sigma = \mathrm{diag}\left[2,2,2,-3,-3\right] M_G$$

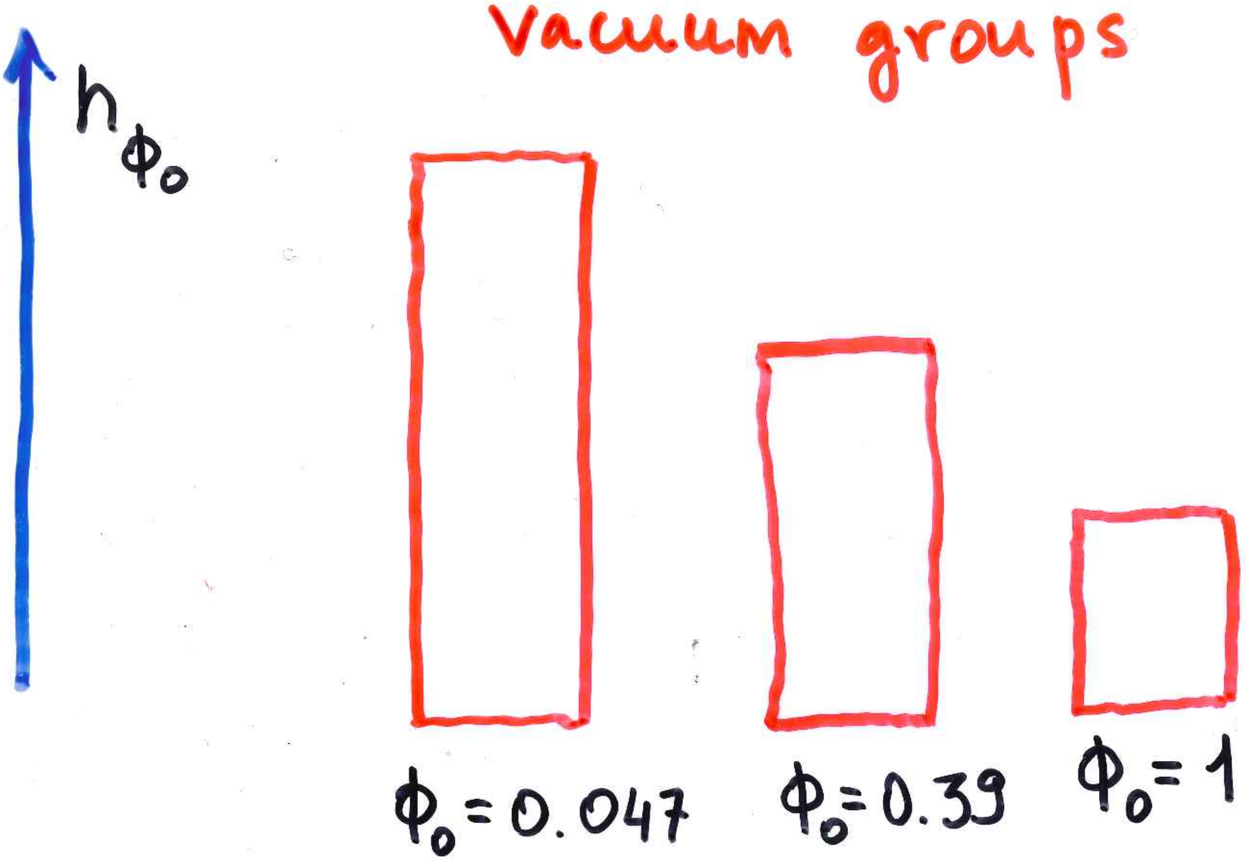
Outlook

* Attractors may be the (only) way in which ∞-vacua theories could make sharp low energy predictions.

* Althou there is no UV-regulating physics, the attractor models have testable predictions

Vacuum Counting

vacuum groups

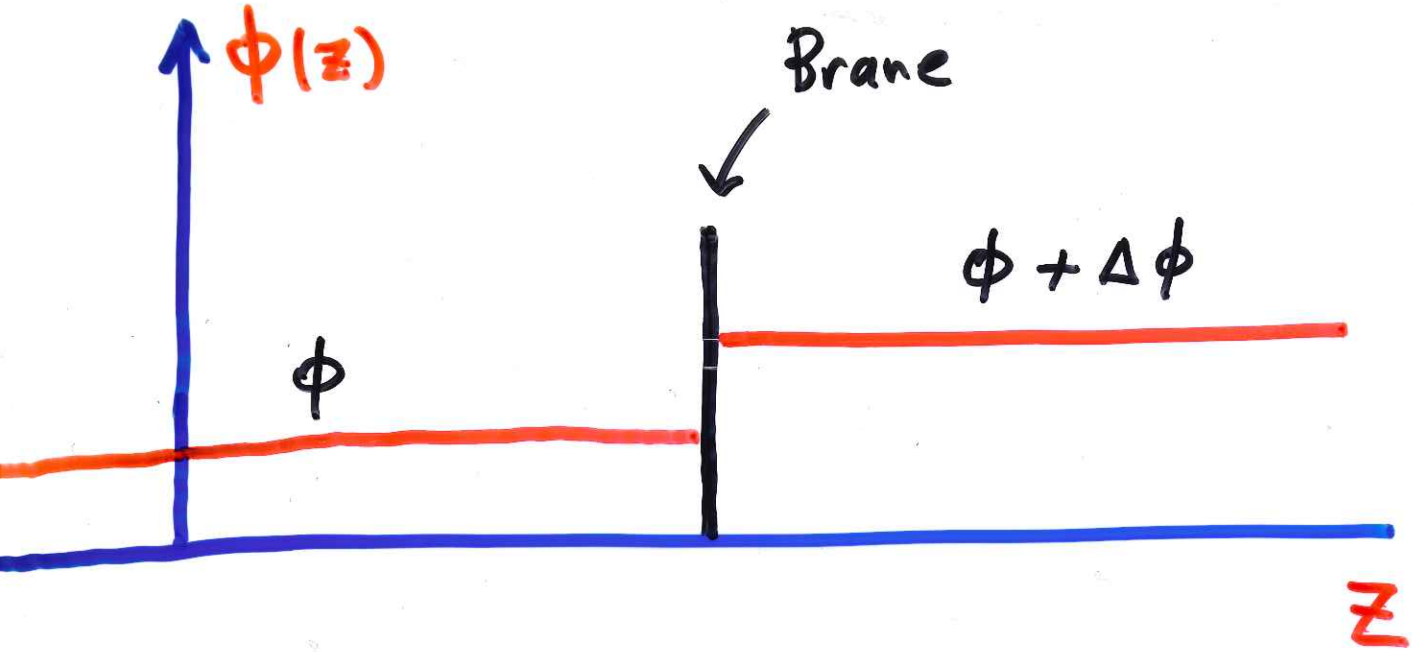


ϕ_0-th group $\ni$ All vacua $\phi = \phi_0 1.XXX...$

Because in each step $\frac{\Delta\phi}{\phi} \sim Q$

$\Downarrow$

$$n_{\phi_0} \sim \frac{1}{Q(\phi_0)} \sim \frac{1}{\phi_0^{N-2}}$$



$$\frac{\Delta\phi}{\phi} \sim Q \;\longrightarrow\; n_{\phi_0} \sim \frac{1}{Q(\phi_0)} \sim \frac{1}{\phi_0^{N-2}}$$

Number of vacua with $\phi > \phi_0$

$$n_{(>\phi_0)} \sim \int_{\phi_0}^{1} n_{\phi} \frac{d\phi}{\phi^2} \sim \begin{cases} \ln\phi & \text{for } N=1 \\ \dfrac{1}{\phi^{N-1}} & \text{for } N>1 \end{cases}$$