Breakdown of Perturbation Theory in Multi-Photon-Annihilation Processes

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(1) Introduction

Work out e^+e^--creation from an arbitrary number n of photons in leading order perturbation theory

Motivation:

- Optical wavelengths: $n \gtrsim 10^5$ photons required to annihilate in order to create e^+e^-
- Investigate: Can *perturbative* but *fully quantum* treatment recover and/or correct a *non-perturbative*, *background-field* treatment?

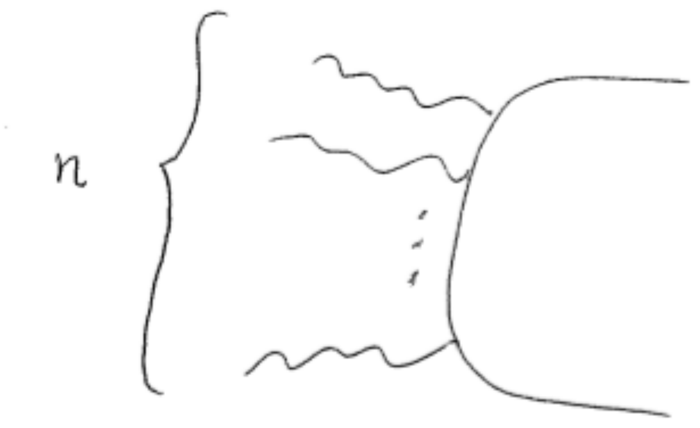

Figure: Leading order perturbation theory terms

(2) Calculations

- $n_1 + n_2 = n$ initial photons from *two* counter-propagating beams, approximated as *monochromatic*;
  Relevant to high-intensity-laser facilities such as *ELI*
- Treatment as *scattering* despite collinearity: justified from experiment (decoherence negligible over time-scale of propagation)
- Quantities computed: $\Gamma_{tree}(n_1, n_2 \to e^+e^-)$ in terms of parameters:
  n: number of photons annihilated, α: coupling, m: fermion mass, ω: photon energy, V: Volume.

(3) Loss of Perturbative Unitarity

Generic to all processes $n_1, n_2 \to e^+e^-$:

$$\Gamma_{tree} \sim n! \left(1 + O\left(\frac{1}{n}\right)\right) \tag{1}$$

- For n sufficiently large, perturbation theory breaks down for *all* parameter-values α, m, ω, V.
- Onset of unlimited growth:
  $$n_* \sim \frac{1}{\alpha} V \omega^3$$
- Constitutes 2nd known instance of lost tree-level unitarity in *weakly* coupled regime of SM besides Multi-Higgs/-W production[1, 2] $q\bar{q} \to nh + mV$.

(4) Relating to Experiment: Coherent State

For initial *coherent* superposition with symmetric spectrum

$$z(\boldsymbol{k}, s) = \sqrt{\langle N \rangle}\left(\delta^3(\boldsymbol{k} - \omega \boldsymbol{e}_z) + \delta^3(\boldsymbol{k} + \omega \boldsymbol{e}_z)\right)\delta_{s+}$$

and associated mean electric field

$$\langle \hat{\boldsymbol{E}} \rangle = \sqrt{\frac{\langle N \rangle \omega}{V}} \cos(\omega z)\left(\cos(\omega t), \sin(\omega t), 0\right), \tag{2}$$

the e^+e^--creation rate becomes weighted sum of individual rates:

$$\Gamma_z \equiv \sum_n^{\infty}\left(\sum_{n_1=1}^{n-1} \frac{\langle N \rangle^n}{n_1! n_2!} \Gamma_{tree}(n_1, n_2 \to e^+e^-)\right) \equiv \sum_n^{\infty} a_n \alpha^n \tag{3}$$

Factorial growth of (1) canceled → perturbativity depends on values of parameters $\frac{\omega}{m}$, $\frac{\langle N \rangle}{V}$ (cf. fig. below)

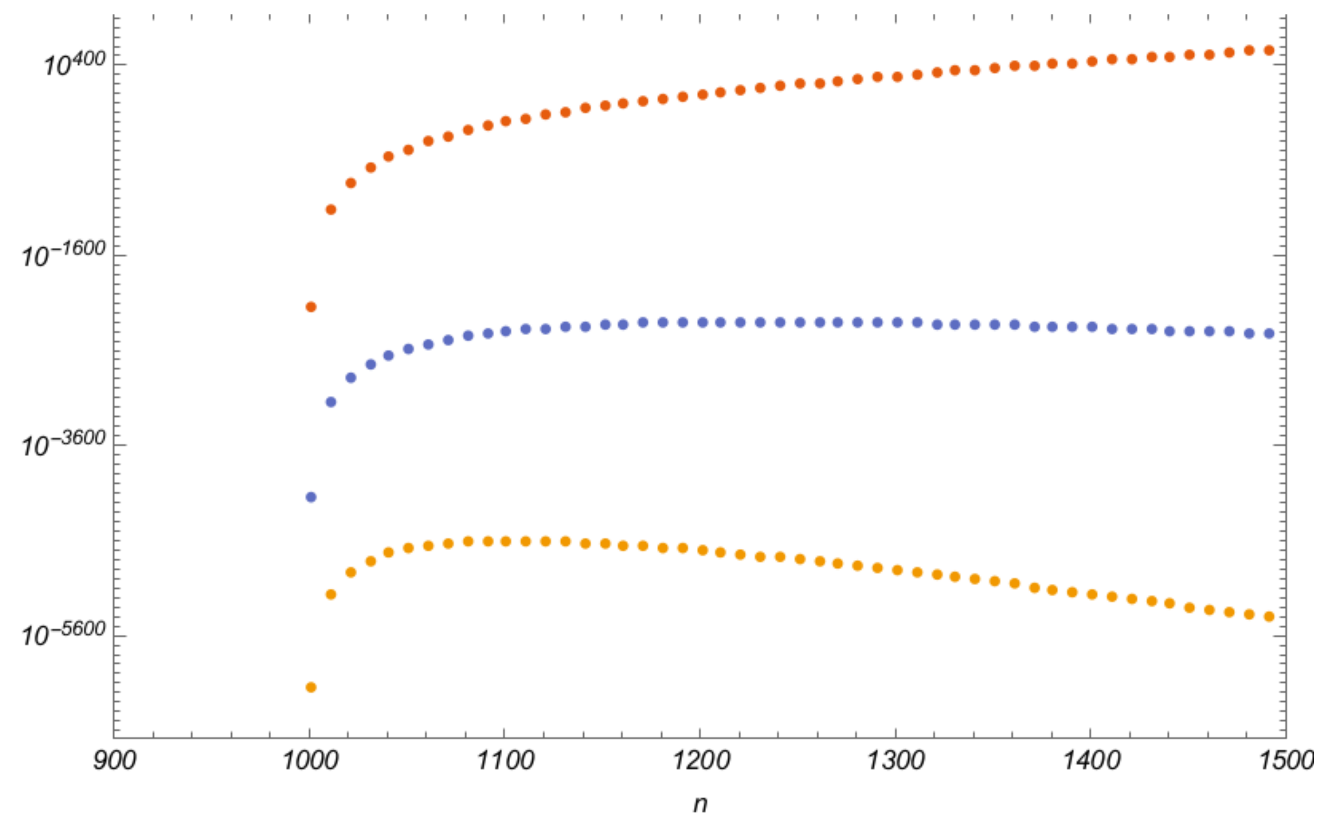

Figure: Series terms $a_n \alpha^n$ for three different parameter values of $\langle N \rangle / V$

(5) Connection to NP Calculations

Non-perturbative result[3] for rate Γ from background field $\boldsymbol{E} = E(\cos(\Omega t), \sin(\Omega t), 0)$ (which resembles mean field (2) in anti-nodes)

$$\Gamma(E, \Omega) \sim \alpha E^2 \begin{cases} \frac{1}{2\pi} e^{-\frac{\pi m^2}{gE}} & \text{for } m\Omega/gE \ll 1 \\ \frac{1}{8}\left(\frac{gE}{2m\Omega}\right)^{2m/\Omega} & \text{for } m\Omega/gE \gg 1, \end{cases} \tag{4}$$

interpolates between multi-photon- and non-perturbative regime.

→ This transition qualitatively captured by (3).

(6) Summary and Outlook

- **Perturbative unitarity loss** in *weakly* coupled regime of SM: Uncovered further incidence besides known case $q\bar{q} \to nh + mV$
- **Relation to background field calculations**
  - Transitory behaviour (4) of NP calculation qualitatively captured by coherent state
  - Calculation provides basis for looking into Quantum corrections from deviation of initial state from coherent one

Bibliography

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