Holographic aspects of the worldline formalism

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Introduction

Strong interactions give rise to a wealth of phenomena, but frequently they overtax our computational abilities.
For example, for the strong nuclear force at sufficiently low energies color neutral objects are observed. However, the underlying theory – quantum chromodynamics (QCD) – refuses an analytic treatment of this infrared regime.
In the last decades holographic approaches have been applied to QCD and other strongly interacting systems and promised deeper analytic insight. Even for auspicious "bottom up" AdS/QCD models, however, a derivation from first principles is missing. In this context, worldline holography [1, 2, 3] maps sources of quantum fields in Mink_4 onto dynamical fields on AdS_5. Thus, it might actually be the missing link between QCD and QCD motivated AdS models for hadrons?

Worldline holography

An example

- massless fermion flavor ψ
- only vector source V
- coupling to gauge field G
- $\mathbb{D} = \partial - \mathrm{i}\underbrace{(G+V)}_{\mathbb{V}}$

$$Z = \langle \mathrm{e}^{w} \rangle = \int [\mathrm{d}G]\, \mathrm{e}^{w - \frac{\mathrm{i}}{4e^2}\int \mathrm{d}^4x\, G_{\mu\nu}^2},$$

where

$$w = \ln \operatorname{Det} \not{\mathbb{D}} = \ln \int [\mathrm{d}\psi][\mathrm{d}\bar\psi]\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \bar\psi \mathrm{i}\not{\mathbb{D}} \psi}.$$

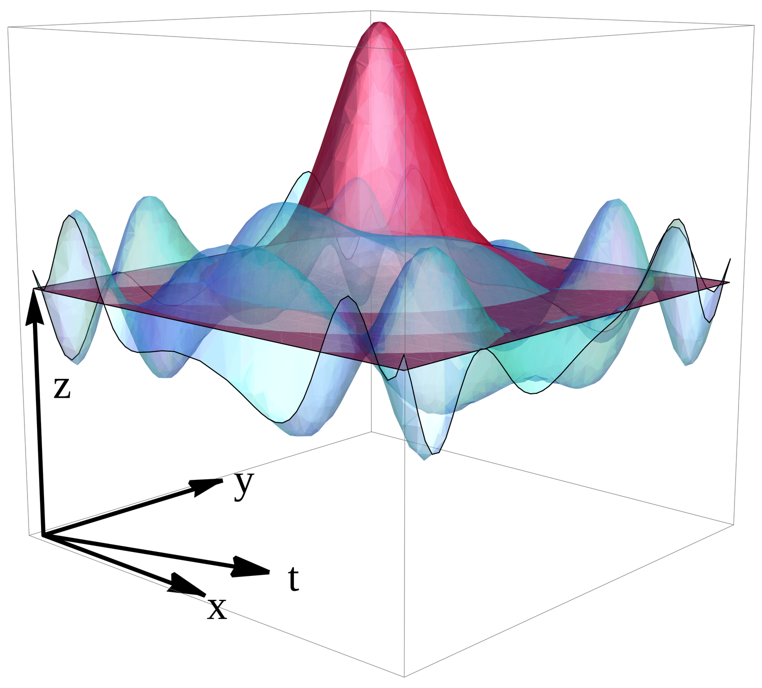

Figure 1: Functional-integral formalism: The background source distribution (blue) is probed by all configurations (one example in red) of the wave function (the quantum field) describing a single flavor.

Worldline formalism

The worldline formalism is a quantum point particle representation of the effective action w.

1. The integral representation of the logarithm introduces Schwinger's proper time T (the Dirac operator is squared to obtain a positive spectrum),
$$w = \frac{1}{2}\operatorname{Tr}\ln \not{\mathbb{D}}^2 = -\int_{\varepsilon>0}^{\infty} \frac{\mathrm{d}T}{2T} \operatorname{Tr} \mathrm{e}^{-T\not{\mathbb{D}}^2} + \text{normalization}.$$
2. The evaluation of the functional trace results in a path integral over periodic paths in Mink_4 for a point particle in a background potential U,
$$w = \int \mathrm{d}^4x \int_{\varepsilon>0}^{\infty} \frac{\mathrm{d}T}{2T^3}\, \mathfrak{L},$$
$$\mathfrak{L} = -\frac{\mathcal{N}}{(4\pi)^2}\int_p [\mathrm{d}y]\, \operatorname{tr} \hat{P}\, \mathrm{e}^{-\int_0^T \mathrm{d}\tau \left\{\frac{\dot{y}^2}{4} + U[\mathbb{V}(y+x),\dot{y}]\right\}}.$$

$\not{\mathbb{D}}^2 = \mathbb{D}^2 - \frac{\mathrm{i}}{2}\sigma^{\mu\nu}[\mathbb{D}_\mu, \mathbb{D}_\nu]_-$ is translated into the worldline action.

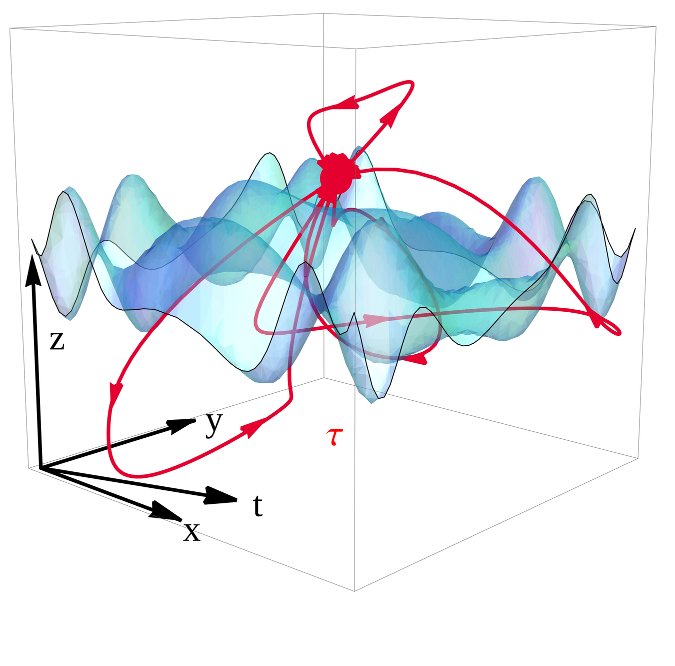

Figure 2: Worldline formalism: The background source distribution (blue) is probed by all possible closed particle trajectories (several examples in red).

The AdS_5 structure

The effective action for the sources transpires to be actually formulated over AdS_5, because [3, 4]

1. all volume elements turn out to be AdS_5 volume elements,
$$\mathrm{d}s^2_{\text{AdS}_5} \overset{g}{=} \frac{\mathrm{d}T^2}{4T^2} + \frac{\mathrm{d}x\cdot\mathrm{d}x}{T} \qquad \longrightarrow \qquad \sqrt{g} = \frac{1}{2T^3}.$$
2. all contractions of Mink_4 spacetime indices are contractions with AdS_5 metrics.

Concretely, integrating out the gluon fields and organizing Z in an expansion w.r.t. powers of gradients and source fields V, we find the four-dimensional rudiments of a classical AdS_5 action for $V(x)$,

$$Z = \iint_{\varepsilon}^{\infty} \mathrm{d}^5x\sqrt{g} \sum_{n_\partial, n_W} \#_{n_\partial,n_V} (g^{\circ\circ})^{\frac{n_\partial+n_V}{2}} (\partial_\circ)^{n_\partial}[V_\circ(x)]^{n_V},$$

where '$\circ$' indicates contractions in four dimensions.

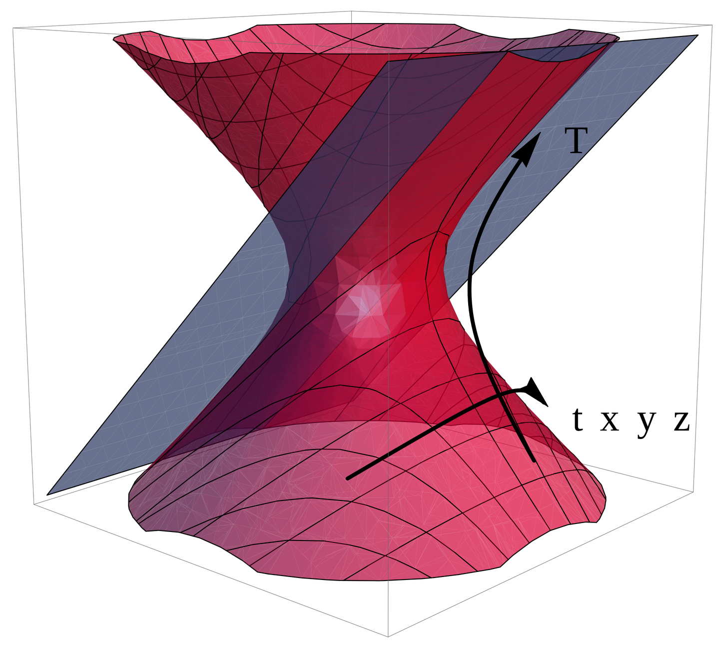

Figure 3: Three-dimensional embedding of AdS_2 space. $T \in (0,\infty)$ is Schwinger's proper time and sets a length (inverse energy $[T] = \text{energy}^{-2}$) scale. $T \to 0$ is the AdS-boundary. The coordinate singularity is regulated by $T > \varepsilon > 0$.

Fifth-dimensional components

Asking for the independence of Z from the value of the unphysical UV regulator ε implies [4, 5]

- a Wilson-Polchinski renormalisation condition,
$$0 \overset{!}{=} \varepsilon\partial_\varepsilon \ln Z_\varepsilon.$$
- an appropriate scaling of the sources $\to$ scale invariance of Z.

This is realized by,

- raising Z to a manifestly conformally invariant action for sources on AdS_5,
$$\mathcal{Z} = \iint_{\varepsilon}^{\infty} \mathrm{d}^5x\sqrt{g} \sum_{n_\partial, n_V} \#_{n_\partial,n_W} (g^{\bullet\bullet})^{\frac{n_\partial+n_V}{2}} (\nabla_\bullet)^{n_\partial}[\mathcal{V}_\bullet(x,T)]^{n_V},$$
where '$\bullet$' indicates complete five-dimensional AdS_5 contractions.
- maintaining local flavor invariance in five dimensions such that we are allowed to eliminate previously inexistent components through the (axial) gauge condition $\mathcal{V}_T = 0$.
- imposing a boundary condition for saddle-point solution
$$\mathcal{V}_\mu(x, T=\varepsilon) = V_\mu(x).$$

Towards low-energy QCD

Including the dominant currents for low energy QCD (axial vector A, vector V, pseudoscalar P, and scalar S) and neglecting gluon contributions [4, 5, 6],

$$w = \operatorname{Tr}\ln(\not{\partial} - \mathrm{i}\Gamma), \qquad \Gamma = \not{V} + \gamma^5\not{A} + S + \mathrm{i}\gamma^5 P,$$

worldline holography yields up to fourth order in fields as well as gradients and expressed in left-right-handed basis

$$V \pm A = L/R \quad \longrightarrow \quad \mathcal{L}/\mathcal{R}(x,T),$$
$$\sqrt{T}(S+\mathrm{i}P) = \sqrt{T}\Phi \quad \longrightarrow \quad \varPhi(x,T),$$

the five-dimensional free AdS_5 action

$$\mathcal{Z} \supset -\frac{4}{(4\pi)^2}\iint_{\varepsilon}^{\infty} \mathrm{d}^5x\,\sqrt{g}\,\operatorname{tr}_{f,c}\Big[-\frac{1}{2}g^{MN}(\mathcal{D}_M\varPhi)^\dagger(\mathcal{D}_N\varPhi) - \#|\varPhi|^2 + \frac{1}{12}g^{MK}g^{NJ}(\mathcal{L}_{MN}\mathcal{L}_{KJ} + \mathcal{R}_{MN}\mathcal{R}_{KJ})\Big],$$

where $\mathcal{D}\varPhi \equiv \partial\varPhi - \mathrm{i}\mathcal{L}\varPhi + \mathrm{i}\varPhi\mathcal{R}$ is the flavor-covariant derivative, and $\#$ is some real number, which is not fixed by maintaining local flavor invariance, but by maintaining the scaling dimension of the mass operator.

Worldline holography reproduces the structure of holographic models of hadrons [7, 8]. This extends to particle masses and warping due to confining interactions.

Summary

The worldline formalism together with the requirement for regulator independence (renormalization condition) links,

QFT in Mink_4	$\longleftrightarrow$	field theory over AdS_5
source fields in Mink_4	$\longleftrightarrow$	dynamical fields in AdS_5
proper-time UV regulator	$\longleftrightarrow$	UV brane regularization
Schwinger's proper time	$\longleftrightarrow$	fifth (bulk) dimension of AdS_5

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The authors would like to thank D. H. Rischke for his support.