SM EFT Contributions to Z Decay at One Loop

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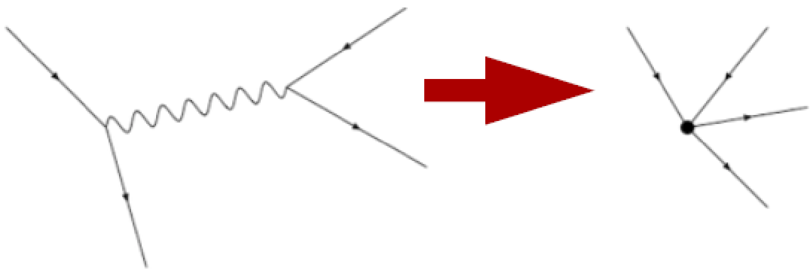
Introduction: EFT

- The canonical example of an EFT is Fermi's theory of weak decay
  - A real limit of the SM



- We still use this today!
- Captures physics in a particular energy regime
  - Able to be improved in precision systematically
- Ability to systematically improve theory predictions is the key virtue of EFTs

Tree-level Effects

- At tree level SM parameters are modified, e.g.

$$\delta M_Z^2 \equiv \frac{1}{2\sqrt{2}}\frac{\hat{m}_Z^2}{\hat{G}_F}C_{HD} + \frac{2^{1/4}\sqrt{\pi}\,\sqrt{\hat{\alpha}}\,\hat{m}_Z}{\hat{G}_F^{3/2}}C_{HWB},$$

$$\delta M_W^2 = -\hat{m}_W^2\left(\frac{\delta s_{\hat{\theta}}^2}{s_{\hat{\theta}}^2} + \frac{c_{\hat{\theta}}}{s_{\hat{\theta}}\sqrt{2}\hat{G}_F}C_{HWB} + \sqrt{2}\delta G_F\right),$$

$$\delta G_F = \frac{1}{\sqrt{2}\,\hat{G}_F}\left(\sqrt{2}\,C_{Hl}^{(3)} - \frac{C_{ll}}{\sqrt{2}}\right),$$

$$\delta s_\theta^2 = -\frac{s_{\hat{\theta}}\,c_{\hat{\theta}}}{2\sqrt{2}\,\hat{G}_F(1-2s_{\hat{\theta}}^2)}\left[s_{\hat{\theta}}\,c_{\hat{\theta}}\,(C_{HD} + 4\,C_{Hl}^{(3)} - 2\,C_{ll}) + 2\,C_{HWB}\right],$$

- Direct EFT effects also appear in processes at higher energies or with higher multiplicity

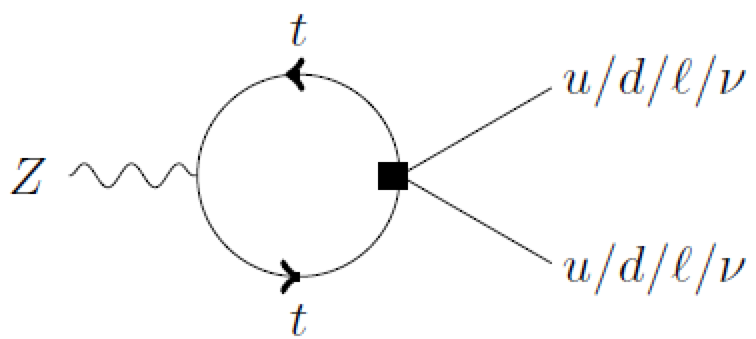
Why Loops?

- Electroweak observables have been measured with amazing precision
  - Theory calculations have to match this precision to get full value out of the data

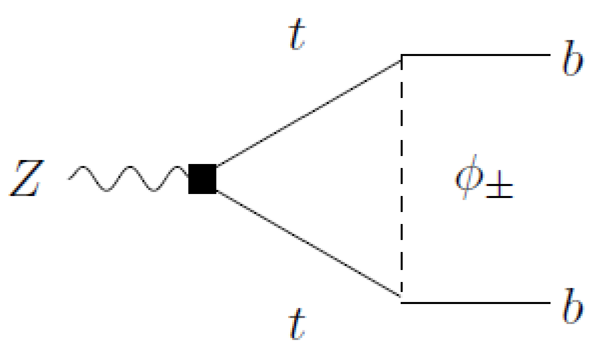
Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z$[GeV]	91.1875 ± 0.0021	[38]	-	-
$\hat{m}_W$[GeV]	80.385 ± 0.015	[39]	80.365 ± 0.004	[40]
σ_h^0 [nb]	41.540 ± 0.037	[38]	41.488 ± 0.006	[41]
Γ_Z[GeV]	2.4952 ± 0.0023	[38]	2.4942 ± 0.0005	[41]
R_ℓ^0	20.767 ± 0.025	[38]	20.751 ± 0.005	[41]
R_b^0	0.21629 ± 0.00066	[38]	0.21580 ± 0.00015	[41]
R_c^0	0.1721 ± 0.0030	[38]	0.17223 ± 0.00005	[41]
A_{FB}^ℓ	0.0171 ± 0.0010	[38]	0.01616 ± 0.00008	[42]
A_{FB}^c	0.0707 ± 0.0035	[38]	0.0735 ± 0.0002	[42]
A_{FB}^b	0.0992 ± 0.0016	[38]	0.1029 ± 0.0003	[42]

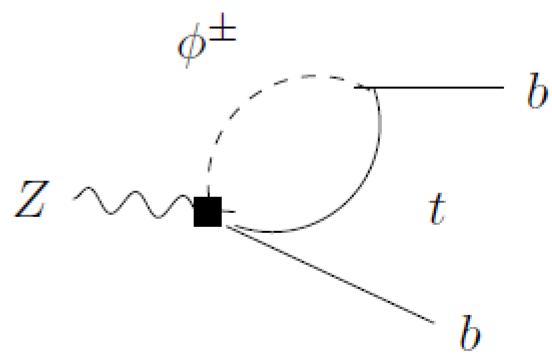
Contributing Operators

- 4-fermion operators:



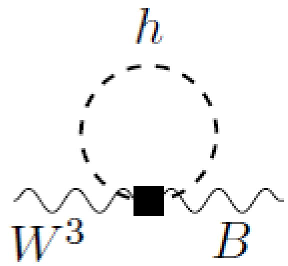
- Scalar-fermionic current operators:



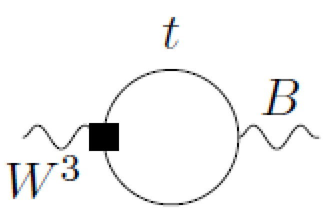


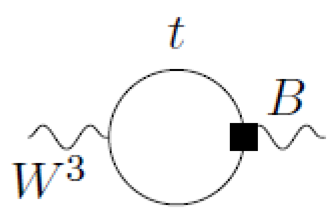
Contributing Operators

- Gauge-Higgs operators:



- Dipole operators:





Input Parameters

- Any calculation depends on the inputs used to set the theory parameters
- We use a canonical set of inputs for the SM
  - $\alpha_{EM}, G_F, M_Z, M_t, M_h$
- EFT gives corrections to the extraction of each
- We treat the Wilson coefficients in $\overline{MS}$ at the NP scale as EFT input parameters to be measured and/or constrained

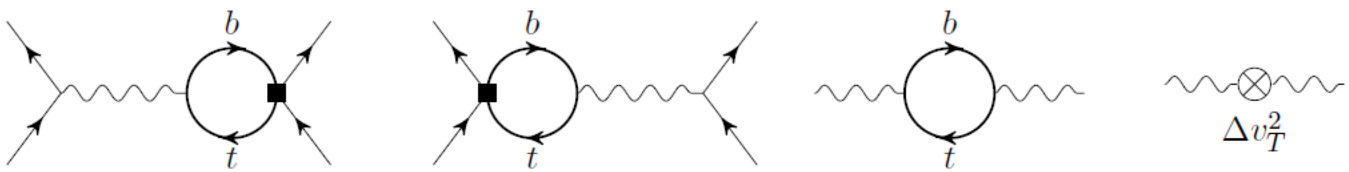
α_{EM} Corrections

- Matching contributions at low scales where α is measured are proportional to lepton masses
- Running cannot be neglected for α
- Two EFT effects here
  - Shift in Weinberg angle from HWB operator
  - Different running from HB, HW operators

$$
\begin{aligned}
\delta\alpha &= -\sqrt{2}\,\frac{4\pi\,\tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB}, \\
\Delta\alpha &= -\sqrt{2}\,\frac{4\pi\,\tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB} \left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F}\right) + \frac{\tilde{\alpha}}{\pi}\,\hat{m}_h^2 \left(C^{(r)}_{HB} + C^{(r)}_{HW}\right) \log\left[\frac{\hat{m}_Z^2}{p^2}\right], \\
&\simeq -\sqrt{2}\,\frac{4\pi\,\tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB} \left(\Delta V^2 + \frac{\Delta G_F}{\hat{G}_F}\right) + 0.03\,\hat{m}_h^2 \left(C^{(r)}_{HB} + C^{(r)}_{HW}\right).
\end{aligned}
$$

G_F Corrections

- G_F is extracted from the muon lifetime
  - Dominated by $\mu \to e\nu\nu$
- Corrected by 4-fermion ops and W mass shifts



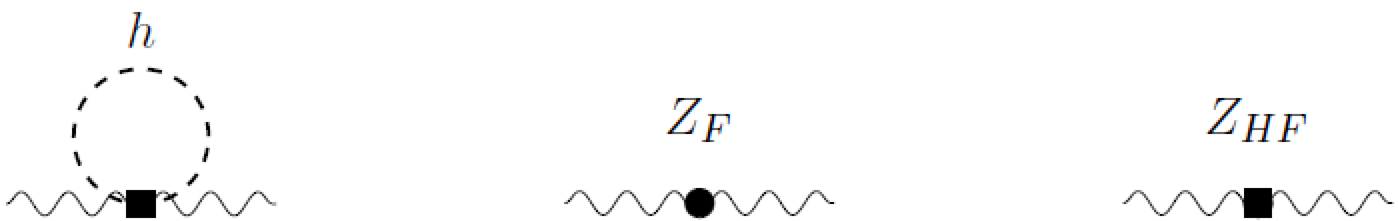
- Lagrangian parameter extracted as

$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2}\left(1 - \frac{\Delta V^2}{\bar{v}_T^2} - \frac{\Delta m_W^2}{\bar{m}_W^2}\right) - 4\,\hat{G}_F\,\delta G_F - \Delta\psi^4$$

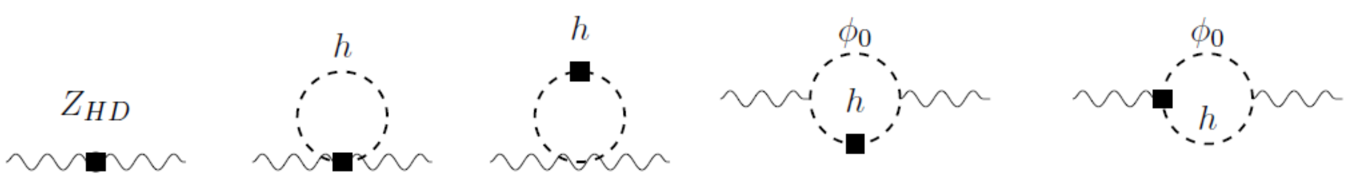
$$\Delta G_F = -\hat{G}_F\,\Delta V^2\,(1 - 2\sqrt{2}\,\delta G_F) - \frac{\Delta m_W^2}{\sqrt{2}\,\hat{m}_W^2} + \frac{\Delta\psi^4}{4\,\hat{G}_F} - \frac{\Delta m_W^2}{\sqrt{2}\,\hat{m}_W^2}\frac{\delta m_W^2}{\hat{m}_W^2}$$

M_Z Corrections

- Gauge-Higgs operators correct the Z mass through the graphs

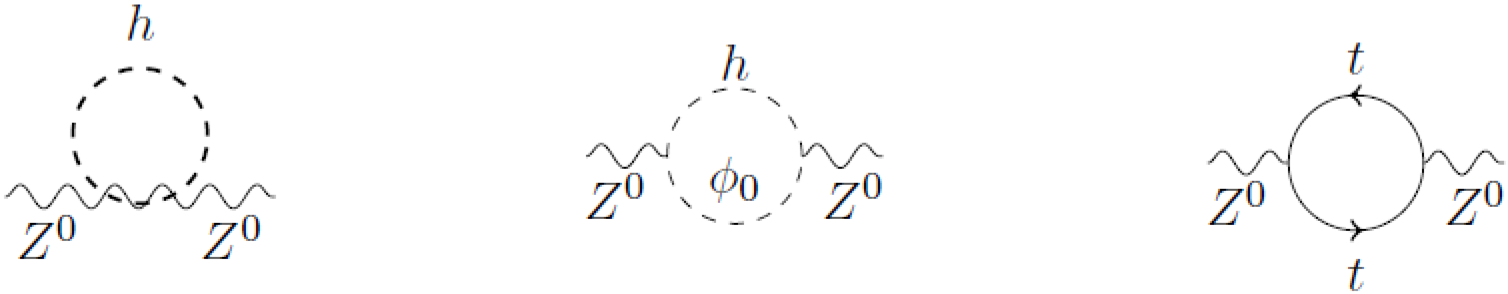


- The Higgs-derivative operator contributes too



Finite Field Normalizations

- Z boson R-factor arises from the graphs:



- B-quark R-factor:

$$= \frac{\hat{m}_t^2}{16\,\pi^2}\left(\sqrt{2}\hat{G}_F(1-2\delta G_F) + C^{\star}_{\substack{uH\\33}} - 2\,C^{(3)}_{Hq}\right)\left[-\frac{3}{4}-\frac{1}{2}\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right]$$

Sample Results

$$
\begin{aligned}
\Delta(g_L^d)_{rr} = {} & \Delta\bar{g}_Z (g_L^d)_{rr}^{SM} + \frac{N_c \hat{m}_t^2}{16\pi^2} \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \left[C^{(1)}_{\substack{qq\\33rr}} + C^{(1)}_{\substack{qq\\rr33}} - C^{(3)}_{\substack{qq\\33rr}} - C^{(3)}_{\substack{qq\\rr33}} - C^{(1)}_{\substack{qu\\rr33}} \right], \\
& - \frac{1}{2}\left(\frac{\Delta G_F}{\hat{G}_F} + \Delta V^2\right)\left(C^{(1)}_{\substack{Hq\\rr}} + C^{(3)}_{\substack{Hq\\rr}}\right) + \delta_{br} \frac{\hat{m}_t^2}{4\,\pi^2} \left[C^{(3)}_{\substack{qq\\3333}} \left(-1 + \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right)\right] - \frac{1}{3}\Delta s_\theta^2, \\
& - \delta_{br} \frac{\hat{m}_t^2}{16\,\pi^2} \left[\left(\frac{1}{4} - \frac{1}{2}\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right) C_{Hu} + C^{(1)}_{Hq}\right] - \delta_{br}\, \Delta R_b^L \left((g_L^d)_{rr}^{SM} + \delta(g_L^d)_{rr}\right), \\
& - \delta_{br} \frac{\hat{m}_t^2}{16\,\pi^2} C^{(3)}_{Hq} \left[\frac{1}{2} - \mathcal{Q}_b\, s_{\hat{\theta}}^2 + (3 - 2\,\mathcal{Q}_b\, s_{\hat{\theta}}^2) \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right], \\
& - \delta_{br} \frac{\hat{m}_t^2}{4\,\pi}\, \tilde{\alpha}\, (c_{\hat{\theta}}^2 - s_{\hat{\theta}}^2)\, C_{HWB}\, (\mathcal{Q}_u - 1) \left[\frac{3}{2} + \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right]\right],
\end{aligned}
$$

$$
\begin{aligned}
\Delta\Gamma_{Z\to Had} &= 2\,\Delta\Gamma_{Z\bar{u}u} + 2\,\Delta\Gamma_{Z\bar{d}d} + \Delta\Gamma_{Z\bar{b}b}, \\
&= \frac{\sqrt{2}\,\hat{G}_F \hat{m}_Z^3}{6\,\pi}\left[4\,(g_R^u + \delta g_R^u)\,\Delta g_R^u + 4\,(g_L^u + \delta g_L^u)\,\Delta g_L^u + 4\,(g_R^d + \delta g_R^d)\,\Delta g_R^d\right] \\
&+ \frac{\sqrt{2}\,\hat{G}_F \hat{m}_Z^3}{6\,\pi}\left[4\,(g_L^d + \delta g_L^d)\,\Delta g_L^d + 2\,(g_R^b + \delta g_R^b)\,\Delta g_R^b + 2\,(g_L^b + \delta g_L^b)\,\Delta g_L^b\right]
\end{aligned}
$$

Phenomenology

- Counting is all that's needed for the most important point
- NLO corrections have introduced dependence on (neglecting flavor indices):
  - 3 Higgs-gauge WCs
  - 2 Dipole WCs
  - 7 Higgs-fermion current WCs
  - 9 four-fermion WCs
- At this level of precision, we can measure only 5 Z pole observables (A_{FB} goes beyond NWA)

Phenomenology

- Recall that at tree level there were flat directions in Z pole observables
  - Lifted by TGC measurements
- With this increase in relevant parameters, all of EWPD not enough to constrain the EFT
- The lesson: loop corrections cannot be constrained by EWPD alone, thus EWPD bounds (at tree level) can never be more precise than a loop factor on WCs

Numerics

The δ correction to $\bar{\Gamma}_{Z\to\bar{d}\,d}$ (where $d=\{d,s,b\}$) is given by

$$\frac{\delta\bar{\Gamma}_{Z\to\bar{d}\,d}}{10^{-2}} = -0.939\,C_{Hd} - 1.58\,C_{HD} - 6.31\,C^{(3)}_{H\ell} + 5.10\left(C^{(1)}_{Hq}+C^{(3)}_{Hq}\right) - 0.510\,C_{HWB} + 3.15\,C_{\ell\ell}. \tag{7.21}$$

The $\delta\,\Delta$ correction to $\bar{\Gamma}_{Z\to\bar{d}\,d}$ (where $d=\{d,s\}$) has the contributions

$$\begin{aligned}\frac{\delta\Delta\bar{\Gamma}_{Z\to\bar{d}\,d}}{10^{-3}} = \Big[&(0.071\,\Delta\bar{v}_T+0.201)\,C_{Hd} - (0.115\,\Delta\bar{v}_T+0.144)\,C_{HD}, - (1.45\,\Delta\bar{v}_T+1.08)\,C^{(3)}_{H\ell}\\ &+(0.316\,\Delta\bar{v}_T-0.206)\left(C^{(1)}_{Hq}+C^{(3)}_{Hq}\right) - (0.024\,\Delta\bar{v}_T+0.064)\,C_{HWB} + 4.23\,\Delta\bar{v}_T,\\ &+(0.727\,\Delta\bar{v}_T+0.541)\,C_{\ell\ell} + 0.593\,C^{(3)}_{\ell q} + 0.072\,(C_{HB}+C_{HW})\Big],\end{aligned} \tag{7.22}$$

and the $\delta\,\Delta$ corrections to $\bar{\Gamma}_{Z\to\bar{d}\,d}$ (where $d=\{d,s\}$) also has the logarithmic terms

$$\begin{aligned}\frac{\delta\Delta\bar{\Gamma}_{Z\to\bar{d}\,d}}{10^{-3}} = \Big[&0.342\,C_{Hd} - 0.266\,C_{HD} - 0.995\,C^{(3)}_{H\ell} - 0.225\left(C^{(1)}_{Hq}+C^{(3)}_{Hq}\right) - 0.110\,C_{HWB},\\ &+1.09\,C_{\ell\ell} - 1.19\,C^{(3)}_{\ell q} + 0.176\left(C^{(1)}_{qd}-C^{(1)}_{ud}\right) + 1.92\left(C^{(3)}_{qq}-C^{(1)}_{qq}\right) + 0.958\,C^{(1)}_{qu},\\ &-0.091\,C_{uW} - 0.055\,C_{uB}\Big]\,\log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] + \Big[(2.43\times10^{-5}\,C_{HD} + 0.015\,C_{Hd},\\ &+0.103\,C^{(3)}_{Hl} - 0.083\left(C^{(1)}_{hq}+C^{(3)}_{hq}\right) - 0.005\,C_{HWB} - 0.052\,C_{\ell\ell}\Big]\,\log\left[\frac{\Lambda^2}{\hat{m}_h^2}\right]\end{aligned} \tag{7.23}$$

Numerics

The δ correction to $\bar{R}^b_\ell$ is given by

$$\frac{\delta R^0_b}{10^{-2}} = -0.192\, C_{Hd} + 0.039\, C_{HD} + 0.158\, C^{(3)}_{H\ell} + 2.13\, C^{(1)}_{Hq} - 0.055\, C^{(3)}_{Hq}, \\ -0.494\, C_{Hu} + 0.043\, C_{HWB} - 0.079\, C_{\ell\ell}. \tag{7.35}$$

Similarly, the $\delta\,\Delta$ correction to $\bar{R}^0_b$ has the contributions

$$\frac{\delta\Delta R^0_b}{10^{-3}} = \Big[(0.036\, \Delta\bar{v}_T + 0.083)\, C_{Hd} + (0.011\, \Delta\bar{v}_T + 0.013)\, C_{HD} + (0.084\, \Delta\bar{v}_T - 0.014)\, C^{(3)}_{H\ell}, \\ -(0.085\, \Delta\bar{v}_T + 0.152)\, C^{(1)}_{Hq} - (0.016\, \Delta\bar{v}_T + 0.019)\, C^{(3)}_{Hq} + (0.099\, \Delta\bar{v}_T + 0.208)\, C_{Hu}, \\ -(0.042\, \Delta\bar{v}_T - 0.007)\, C_{\ell\ell} + (0.013\, \Delta\bar{v}_T + 0.009)\, C_{HWB} - 0.015\, C^{(3)}_{\ell q}, \\ +0.597\, C^{(3)}_{qq} + 0.047\, C_{uH} - 0.006\,(C_{HB} + C_{HW}) - 0.106\, \Delta v\Big], \tag{7.36}$$

and the $\delta\,\Delta$ correction to R^0_b also has the logarithmic terms

$$\frac{\delta\Delta R^0_b}{10^{-3}} = \Big[0.129\, C_{Hd} + 0.025\, C_{HD} + 0.067\, C^{(3)}_{H\ell} - 0.559\, C^{(1)}_{Hq} + 0.383\, C^{(3)}_{Hq} + 0.240\, C_{Hu}, \\ +0.023\, C_{HWB} - 0.049\, C_{\ell\ell} + 0.030\, C^{(3)}_{\ell q} + 0.036\left(C^{(1)}_{qd} - C^{(1)}_{ud}\right) - 0.618\, C^{(3)}_{qq}, \\ -0.803\, C^{(1)}_{qq} + 0.494\, C^{(1)}_{qu} - 0.002\, C_{uB} + 0.032\, C_{uH} - 0.004\, C_{uW} - 0.186\, C_{uu}\Big] \log\left[\frac{\Lambda^2}{\hat{m}_t^2}\right] \\ + \Big[-8.94 \times 10^{-7}\, C_{HD} + \Big(0.313\, C_{Hd} - 3.49\, C^{(1)}_{Hq} + 0.090\, C^{(3)}_{Hq} - 0.258\, C^{(3)}_{H\ell}, \\ +0.808\, C_{Hu} + 0.129\, C_{\ell\ell} - 0.020\, C_{HWB}\Big) 10^{-2}\Big] \log\left[\frac{\Lambda^2}{\hat{m}_h^2}\right]. \tag{7.37}$$

Conclusions

- We have excellent data available, and must have enough respect for that to understand our new physics predictions at comparable precision
- In the case of LEP data, especially at the Z pole, this requires NLO accuracy
- In the most model-independent formulation of heavy new physics, the NLO predictions are under-constrained by low energy data
  - Setting shifts in EW observables to zero for the purposes of further searches does not give model-independent results
- A truly global analysis will be needed to properly constrain the EFT without UV assumptions
- Thank goodness we have the LHC with its forthcoming unprecedented data set to constrain new physics at higher energies!

Thank You!