Complex Masses in the S-Matrix

Complex Masses in the S-Matrix

George Rupp

CFIF, Instituto Superior Técnico, Lisbon

Collaborators: Eef van Beveren and Susana Coito

I. Why complex masses?

II. The model

III. Redefining the S-matrix

IV. Further issues to be addressed

I. Why complex masses?

⇒ Many hadrons have multiparticle strong decay modes. These can often be described as intermediate states with 1 or 2 resonances. Examples:

- $f_0(1370) \rightarrow \rho\rho\,,\ \sigma\sigma\,,\ \ldots \rightarrow 4\pi$ (see next slide)
- $K_2(1770) \rightarrow K_2^*(1430)\pi\,,\ Kf_2(1270)\,,\ \ldots \rightarrow K\pi\pi$
- $\phi(2170) \rightarrow \phi f_0(980) \rightarrow \phi\pi\pi\,,\ KKf_0(980) \rightarrow KK\pi\pi$

⇒ Processes of the type $2 \rightarrow 3$ and $2 \rightarrow 4$ are very difficult to deal with rigorously, as they require relativisitic multichannel Faddeev resp. Yakubovsky equations in the scattering region. Faddeev equations have been employed by Khemchandani, Martínez Torres and Oset for the generation of dynamical resonances, as e.g. the $\phi(2170)$ in ϕKK and $\phi\pi\pi$ (but not $KKf_0(980)$ and $KK\pi\pi$).

$f_0(1370)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	seen
Γ_2	4π	seen
Γ_3	$4\pi^0$	seen
Γ_4	$2\pi^+2\pi^-$	seen
Γ_5	$\pi^+\pi^-2\pi^0$	seen
Γ_6	$\rho\rho$	dominant
Γ_7	$2(\pi\pi)_{S\text{-wave}}$	seen
Γ_8	$\pi(1300)\pi$	seen
Γ_9	$a_1(1260)\pi$	seen
Γ_{10}	$\eta\eta$	seen
Γ_{11}	$K\overline{K}$	seen
Γ_{12}	$K\overline{K}n\pi$	not seen
Γ_{13}	6π	not seen
Γ_{14}	$\omega\omega$	not seen
Γ_{15}	$\gamma\gamma$	seen
Γ_{16}	e^+e^-	not seen

$f_0(1370)$ PARTIAL WIDTHS

$\Gamma(\gamma\gamma)$ — Γ_{15}

See $\gamma\gamma$ widths under $f_0(600)$ and MORGAN 90.

$\Gamma(e^+e^-)$ — Γ_{16}

VALUE (eV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
<20	90	VOROBYEV	88	ND	$e^+e^- \to \pi^0\pi^0$

$f_0(1370)$ BRANCHING RATIOS

$\Gamma(\pi\pi)/\Gamma_{total}$ — Γ_1/Γ

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.26±0.09	BUGG	96	RVUE	
<0.15	[26] AMSLER	94	CBAR	$\overline{p}p \to \pi^+\pi^-3\pi^0$
<0.06	GASPERO	93	DBC	$0.0\ \overline{p}n \to$ hadrons

[26] Using AMSLER 95B ($3\pi^0$).

$\Gamma(4\pi)/\Gamma_{total}$ — $\Gamma_2/\Gamma = (\Gamma_3+\Gamma_4+\Gamma_5)/\Gamma$

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
>0.72	GASPERO	93	DBC	$0.0\ \overline{p}n \to$ hadrons

⇒ Alternatives are:

- Use dispersive or purely phenomenological ansatzes, as e.g. done by Bugg, and Bugg, Sarantsev and Zou in describing the 4π channel when analysing the $f_0(1370)$.
- Include intermediate two-body states with 1 or 2 (broad) resonances as though they were channels of final states.

⇒ For the latter approach, there are two options:

1. Describe the final-state resonances like discretised mass distributions, as done by Albaladejo and Oller in their coupled-channel description of S-wave meson-meson scattering in the chiral unitary model.

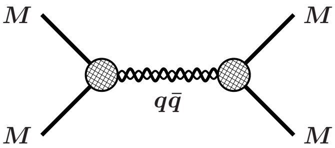
2. Represent the final-state resonances by complex masses corresponding to the resonance poles.

⇒ The first method leads to a proliferation of channels, besides the difficulty of mimicking a non-Breit-Wigner resonance like the σ. The second in principle destroys the two-body unitarity of the S-matrix, which must somehow be restored.

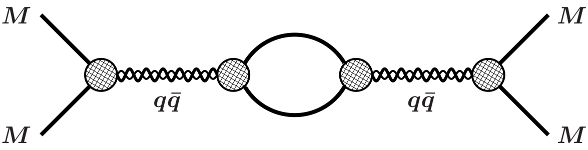
II. The model

⇒ Building blocks of Resonance Spectrum Expansion (RSE) are:

$V =$



$V\Omega V =$



- V is the effective two-meson potential;
- Ω is the two-meson loop function;
- the blobs are the $^3P_0$ vertex functions, modelled by a spherical δ shell in coordinate space, i.e., a spherical Bessel function in momentum space;
- the wiggly lines stand for s-channel exchanges of infinite towers of $q\bar{q}$ states, i.e., a kind of Regge propagators.

⇒ For N meson-meson channels and several $q\bar{q}$ channels:

$$V_{ij}^{(L_i,L_j)}(p_i, p'_j; E) = \lambda^2 r_0\, j^i_{L_i}(p_i r_0)\, j^j_{L_j}(p'_j r_0) \sum_{\alpha=1}^{N_{q\bar{q}}} \sum_{n=0}^{\infty} \frac{g_i^{(\alpha)}(n) g_j^{(\alpha)}(n)}{E - E_n^{(\alpha)}}$$

$$\equiv \mathcal{R}_{ij}(E)\, j^i_{L_i}(p_i r_0)\, j^j_{L_j}(p'_j r_0) .$$

⇒ The closed-form off-energy-shell T-matrix then reads

$$T_{ij}^{(L_i,L_j)}(p_i, p'_j; E) =$$

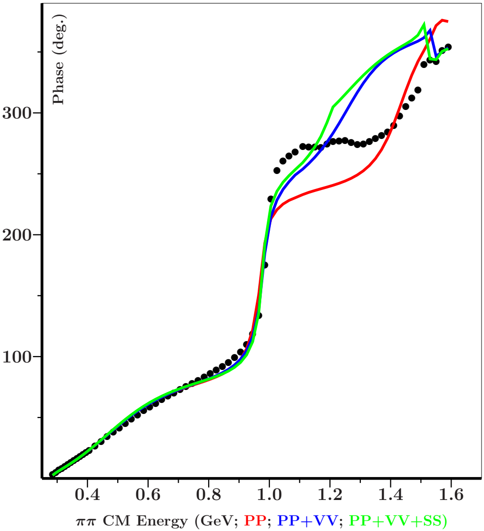
$$-2\lambda^2 r_0 \sqrt{\mu_i p_i \mu'_j p'_j}\, j^i_{L_i}(p_i r_0) \sum_{m=1}^{N} \mathcal{R}_{im}(E) \left\{[\mathbb{1} - \Omega\,\mathcal{R}]^{-1}\right\}_{mj} j^j_{L_j}(p'_j r_0) ,$$

$$\Omega = -2i\lambda^2 r_0\, \mathrm{diag}\left(j^n_{L_n}(k_n r_0) h^{(1)n}_{L_n}(k'_n r_0) \right) .$$

⇒ The corresponding unitary and symmetric S-matrix is given by

$$S_{ij}^{(L_i,L_j)}(k_i, k'_j; E) = \delta_{ij} + 2i T_{ij}^{(L_i,L_j)}(k_i, k'_j; E) .$$

⇒ Recall the results for the S-wave $\pi\pi$ phase shifts, with pseudoscalar-pseudoscalar, vector-vector, and scalar-scalar channels (the kinks are due to the sharp $\sigma\sigma$ and $\rho\rho$ thresholds:

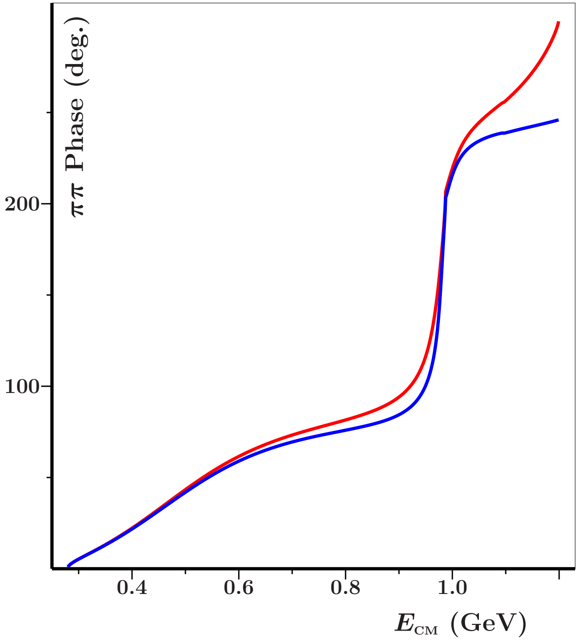


III. Redefining the S-matrix

⇒ If we take a complex mass in any of the meson-meson channels, the S-matrix ceases to be unitary, but stays symmetric. This requires a redefinition of the S-matrix.

- $S^\dagger S \equiv A$ is not unity anymore, but is always Hermitian, with *positive* real eigenvalues.
- So A can be diagonalised by a unitary matrix U: $A_d = US^\dagger SU^\dagger$.
- Define now $\tilde{S} \equiv SU^\dagger A_d^{-1/2} U$, where $A_d^{-1/2}$ is real.
- It is straightforward to show that $\tilde{S}^\dagger \tilde{S} = \tilde{S}\tilde{S}^\dagger = \mathbb{1}$.
- It is less straightforward to show that $\tilde{S}$ is also symmetric, but this has been checked numerically with a precision of better than one part in a trillion (10^{12}).

⇒ Just as an illustration, we show (next slide) preliminary results for the S-wave $\pi\pi$ phase shift with complex σ, ρ, and κ masses, compared to the original real case. No fit has been done yet.



Red curve: real masses; blue curve: complex masses.

IV. Further issues to be addressed

- Mathematical implications of the unitarisation procedure carried out here should be studied.
- Employed form factors for subthreshold suppression of closed channels should be reconsidered, due to the complex momenta for real energies in channels with complex masses.
- In the case of S-wave $\pi\pi$ scattering above 1.2 GeV, the $\pi(1300)\pi$ and $a_1(1260)\pi$ channels, listed in the PDG tables for the $f_0(1370)$ resonance, will have to be included as well.
- Model parameters are to be optimised in detailed fits to the available data sets.
- A comparison with the discretisation method of Albaladejo and Oller would also be interesting.

Do Videnia!