

Nonlinear k_{\perp} -factorization and the unintegrated gluon distribution of a nucleus

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Outline

Color dipole cross section \longrightarrow unintegrated gluon distribution

Unintegrated gluon distribution of a nucleus: boundary condition

Nonlinear small- x evolution of the nuclear glue

Small- x evolution of quasielastic diffraction

How photon-jet correlations can probe the nuclear unintegrated glue

Summary and outlook

References

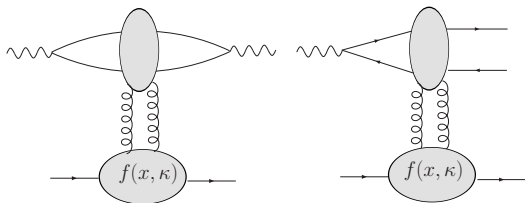
-  N.N. Nikolaev & W.S.
Phys. Rev. D74, 074021 (2006).
-  N.N. Nikolaev, W.S. & B.G. Zakharov
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-  N.N. Nikolaev, W.S., B.G. Zakharov & V.R. Zoller
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color dipole cross section \leftrightarrow unintegrated glue

- at high energies, when $\Lambda_{QCD} \ll p_{\perp} \ll \sqrt{s}$, we should take parton transverse momenta explicitly into account \rightarrow **unintegrated parton distributions**.
- equivalence of **color dipole-cross section** and **unintegrated gluon distribution** (Nikolaev & Zakharov '94):

$$\int \frac{d^2\mathbf{r}}{(2\pi)^2} \sigma(x, \mathbf{r}) \exp(-i\mathbf{p}\mathbf{r}) = \sigma_0(x) \delta^{(2)}(\mathbf{p}) - f(x, \mathbf{p})$$
$$f(x, \mathbf{p}) = \frac{4\pi\alpha_S}{N_c} \frac{1}{\mathbf{p}^4} \frac{\partial G(x, \mathbf{p}^2)}{\partial \log \mathbf{p}^2}; \quad \sigma_0(x) = \int d^2\mathbf{p} f(x, \mathbf{p}).$$

- nb: diffractive amplitude $\propto \int d^2\mathbf{r} \exp(-i\mathbf{q}\mathbf{r}) \sigma(x, \mathbf{r}) \psi_{q\bar{q}}(z, \mathbf{r})$,
 \mathbf{q} = transverse momentum of q or \bar{q} diffractive jet.



small- x evolution: adding $q\bar{q}(ng)$ Fock-states:

Nikolaev & Zakharov '94, Mueller '94

- as we increase energy Fock states $q\bar{q}g, q\bar{q}gg, \dots q\bar{q}(ng)$ with strongly ordered light-cone momenta $z_n \ll \dots \ll z_2 \ll z_1 \ll 1$ will be coherent over the target.
- their effect can be resummed and absorbed into the x -dependent dipole cross section:

$$\frac{\partial \sigma(x, \mathbf{r})}{\partial \log(1/x)} = \int d^2 \boldsymbol{\rho} K(\boldsymbol{\rho}, \boldsymbol{\rho} + \mathbf{r}) \left[\sigma_{q\bar{q}g}(x, \boldsymbol{\rho}, \mathbf{r}) - \sigma(x, \mathbf{r}) \right]$$
$$\sigma_{q\bar{q}g}(x, \boldsymbol{\rho}, \mathbf{r}) = \frac{N_c^2}{N_c^2 - 1} [\sigma(x, \boldsymbol{\rho}) + \sigma(x, \boldsymbol{\rho} + \mathbf{r})] - \frac{1}{N_c^2 - 1} \sigma(x, \mathbf{r})$$

$$K(\boldsymbol{\rho}, \boldsymbol{\rho} + \mathbf{r}) \propto \left| \psi(\boldsymbol{\rho}) - \psi(\boldsymbol{\rho} + \mathbf{r}) \right|^2, \quad \psi(\boldsymbol{\rho}) = \frac{\sqrt{C_{F\alpha_S}}}{\pi} \frac{\boldsymbol{\rho}}{\rho^2} F(\mu_G \boldsymbol{\rho})$$

- $\mu_G^2 \sim 0.5 \text{ GeV}^2$, 'gluon mass' - a smooth cutoff for long wavelength gluons, which respects 'gauge cancellations'.

...in momentum space it is BFKL:

- the equivalence of dipole and momentum space approaches extends to the small- x evolution:

$$\begin{aligned}\frac{\partial f(x, \mathbf{p})}{\partial \log(1/x)} &= 2 \int d^2 \kappa K(\mathbf{p}, \mathbf{p} + \kappa) f(x, \kappa) - f(x, \mathbf{p}) \int d^2 \kappa K(\kappa, \kappa + \mathbf{p}) \\ &= \mathcal{K}_{\text{BFKL}} \otimes f(x, \mathbf{p})\end{aligned}$$

- the kernel:

$$K(\mathbf{p}_1, \mathbf{p}_2) = K_0 \cdot \left| \frac{\mathbf{p}_1}{\mathbf{p}_1^2 + \mu_G^2} - \frac{\mathbf{p}_2}{\mathbf{p}_2^2 + \mu_G^2} \right|^2, \quad K_0 = \frac{C_A \alpha_S}{2\pi^2}$$

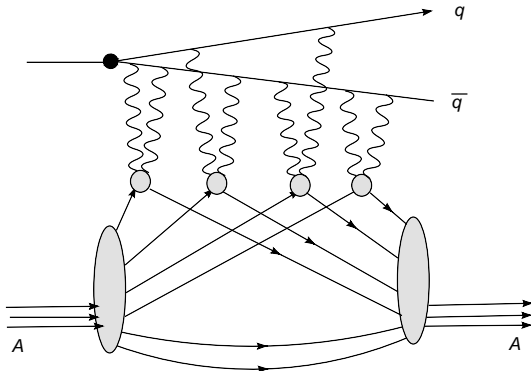
- nonperturbative parameters: μ_G , freezing of α_S .

unintegrated glue of a nucleus:

- amplitude for scattering of a $q\bar{q}$ dipole \mathbf{r} off a nucleus at fixed impact parameter $\mathbf{b} \leftrightarrow$ the **nuclear unintegrated glue**:

$$\int \frac{d^2\mathbf{r}}{(2\pi)^2} \Gamma(\mathbf{b}, \mathbf{x}, \mathbf{r}) \exp(-i\mathbf{p}\mathbf{r}) = (1 - w_0(\mathbf{b}, \mathbf{x})) \delta^{(2)}(\mathbf{p}) - \phi(\mathbf{b}, \mathbf{x}, \mathbf{p})$$

- a physical observable: hard diffractive dijets in $\pi A \rightarrow \text{Jet}_1 \text{Jet}_2 A$ acquire \mathbf{p} from gluons \rightarrow diffractive amplitude directly proportional to $\phi(\mathbf{b}, \mathbf{x}, \mathbf{p})$.



Nuclear unintegrated glue at $x \sim x_A$

- at not too small $x \sim x_A = (R_A m_p)^{-1} \sim 0.01$ only the $\bar{q}q$ state is coherent over the nucleus, and $\Gamma(\mathbf{b}, x, \mathbf{r})$ can be constructed from Glauber-Gribov theory:

$$\Gamma(\mathbf{b}, x_A, \mathbf{r}) = 1 - \exp[-\sigma(x_A, \mathbf{r}) T_A(\mathbf{b})/2].$$

- nuclear coherent glue per unit area in impact parameter space:

$$\phi(\mathbf{b}, x_A, \mathbf{p}) = \sum w_j(\mathbf{b}, x_A) f^{(j)}(x_A, \mathbf{p})$$

- collective glue of j overlapping nucleons :

$$f^{(j)}(x_A, \mathbf{p}) = \int \left[\prod_{i=1}^j d^2 \kappa_i f(x_A, \kappa_i) \right] \delta^{(2)}(\mathbf{p} - \sum \kappa_i)$$

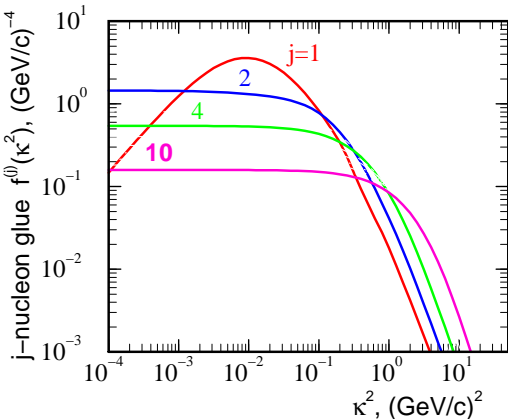
- probab. to find j overlapping nucleons

$$w_j(\mathbf{b}, x_A) = \frac{\nu_A^j(\mathbf{b}, x_A)}{j!} \exp[-\nu_A(\mathbf{b}, x_A)], \quad \nu_A(\mathbf{b}, x_A) = \frac{1}{2} \alpha_S(q^2) \sigma_0(x_A) T_A(\mathbf{b}),$$

- impact parameter $\mathbf{b} \rightarrow$ effective opacity ν_A , $q^2 =$ the relevant hard scale.

Nuclear unintegrated glue: salient features

collective glue $f^{(j)}(x_A, \kappa)$



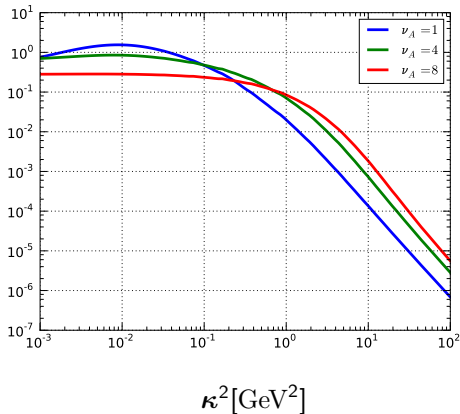
- nuclear coherent glue per unit area in impact parameter space:

$$\phi(\nu_A, x_A, \kappa) = \sum w_j(\nu_A) f^{(j)}(x_A, \kappa)$$

- typical scale: the saturation scale $Q_A^2 \sim 0.8 \div 1.5 \text{ GeV}^2$ for realistic glue and heavy nuclei.
- large- κ^2 Cronin-type antishadowing enhancement
- furnishes linear k_{\perp} -factorization of inclusive deep inelastic, forward single jets in DIS, and diffractive dijets.
- straightforward unitarity cut interpretation.

Nuclear unintegrated glue: salient features, $x_A = 0.01$

collective nuclear glue $\phi(\nu_A, x_A, \kappa)$



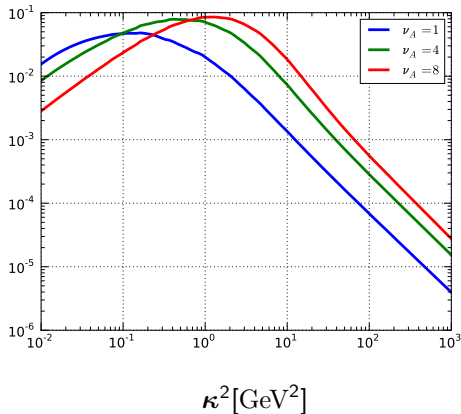
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saturation scale grows with ν_A , $x_A = 0.01$

$$\kappa^2 \phi(\nu_A, x_A, \kappa) \propto \partial G_A(x_A, \kappa^2) / \partial \kappa^2$$



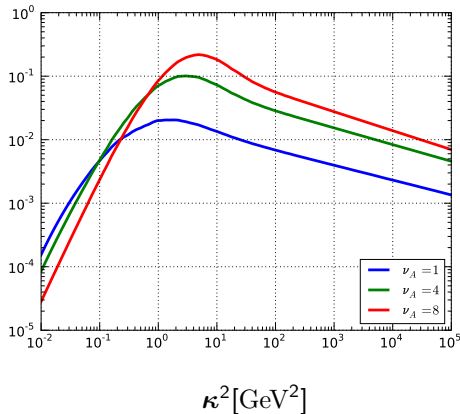
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- straightforward unitarity cut interpretation.

...behaviour at large κ^2 , $x_A = 0.01$

$$\kappa^4 \phi(\nu_A, x_A, \kappa) \propto \partial G_A(x_A, \kappa^2) / \partial \log \kappa^2$$



- nuclear coherent glue per unit area in impact parameter space:

$$\phi(\nu_A, x_A, \kappa) = \sum w_j(\nu_A) f^{(j)}(x_A, \kappa)$$

- typical scale: the saturation scale $Q_A^2 \sim 0.8 \div 1.5$ GeV² for realistic glue and heavy nuclei.
- large- κ^2 Cronin-type antishadowing enhancement
- furnishes linear k_{\perp} -factorization of inclusive deep inelastic, forward single jets in DIS, and diffractive dijets.
- straightforward unitarity cut interpretation.

Nuclear unintegrated glue: small- x evolution

- again, add the $q\bar{q}g$ Fock-state:
- small- x evolution [Balitsky–Kovchegov '96–'98](#):

$$\Gamma_{q\bar{q},A}(\nu_A, x_A, \mathbf{r}) \rightarrow \Gamma_{q\bar{q},A}(\nu_A, x_A, \mathbf{r}) + \log(x_A/x) \delta\Gamma_{q\bar{q},A}(\nu_A, \mathbf{r})$$
$$\delta\Gamma_{q\bar{q},A}(\nu_A, \mathbf{r}) \propto \int d^2\rho K(\rho, \rho + \mathbf{r}) \left(\Gamma_{q\bar{q}g,A}(\nu_A, \rho, \mathbf{r}) - \Gamma_{q\bar{q},A}(\nu_A, \rho) \Gamma_{q\bar{q},A}(\nu_A, \rho + \mathbf{r}) \right)$$

$$\Gamma_{q\bar{q}g,A}(\nu_A, \rho, \mathbf{r}) = \Gamma_{q\bar{q},A}(\nu_A, \rho) + \Gamma_{q\bar{q},A}(\nu_A, \rho + \mathbf{r}) - \Gamma_{q\bar{q},A}(\nu_A, \rho) \Gamma_{q\bar{q},A}(\nu_A, \rho + \mathbf{r})$$

- evolution of **unintegrated glue**:

$$\frac{\partial\phi(\nu_A, x, \mathbf{p})}{\partial \log(1/x)} = \mathcal{K}_{BFKL} \otimes \phi(\nu_A, x, \mathbf{p}) + \mathcal{Q}[\phi](\nu_A, x, \mathbf{p})$$

$$\mathcal{Q}[\phi](\nu_A, x, \mathbf{p}) = \int d^2\mathbf{q} d^2\boldsymbol{\kappa} \phi(\nu_A, x, \mathbf{q}) \left\{ \left[K(\mathbf{p} + \boldsymbol{\kappa}, \mathbf{p} + \mathbf{q}) - K(\mathbf{p}, \boldsymbol{\kappa} + \mathbf{p}) - K(\mathbf{p}, \mathbf{q} + \mathbf{p}) \right] \phi(\nu_A, x, \boldsymbol{\kappa}) \right. \\ \left. - \phi(\nu_A, x, \mathbf{p}) \left[K(\boldsymbol{\kappa}, \boldsymbol{\kappa} + \mathbf{q} + \mathbf{p}) - K(\boldsymbol{\kappa}, \boldsymbol{\kappa} + \mathbf{p}) \right] \right\}$$

properties of the nonlinear term:

- first piece of the nonlinear term looks like a diffractive cut of a triple-Pomeron vertex **Nikolaev & WS '05**:

$$\begin{aligned} \int d^2\mathbf{q} d^2\boldsymbol{\kappa} \phi(\nu_A, x, \mathbf{q}) \left[K(\mathbf{p} + \boldsymbol{\kappa}, \mathbf{p} + \mathbf{q}) - K(\mathbf{p}, \boldsymbol{\kappa} + \mathbf{p}) - K(\mathbf{p}, \mathbf{q} + \mathbf{p}) \right] \phi(\nu_A, x, \boldsymbol{\kappa}) \\ = -2K_0 \left| \int d^2\boldsymbol{\kappa} \phi(\nu_A, x, \boldsymbol{\kappa}) \left[\frac{\mathbf{p}}{\mathbf{p}^2 + \mu_G^2} - \frac{\mathbf{p} + \boldsymbol{\kappa}}{(\mathbf{p} + \boldsymbol{\kappa})^2 + \mu_G^2} \right] \right|^2 \end{aligned}$$

- at large \mathbf{p}^2 the nonlinear term is a **pure higher twist**, it is dominated by the **'anticollinear'** region $\boldsymbol{\kappa}^2 > \mathbf{p}^2$. It cannot be written as a square of the integrated gluon distribution.

$$\begin{aligned} \mathcal{Q}[\phi](\nu_A, x, \mathbf{p}) &\approx -\frac{2K_0}{\mathbf{p}^2} \left| \int_{\mathbf{p}^2} \frac{d^2\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2} \phi(\nu_A, x, \boldsymbol{\kappa}^2) \right|^2 \\ &\quad - 2K_0 \phi(\nu_A, x, \mathbf{p}^2) \int_{\mathbf{p}^2} \frac{d^2\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2} \int_{\boldsymbol{\kappa}^2} d^2\mathbf{q} \phi(\nu_A, x, \mathbf{q}^2) \end{aligned}$$

- for the lowest 'conformal spin' component, it involves only 1-dim integrations, which greatly helps numerics.

Quasielastic diffractive production $\gamma^* A \rightarrow VA^*$

- diffractive production with breakup of the target nucleus, no particle production in the nuclear hemisphere:

$$\begin{aligned} & \left. \frac{d\sigma(\gamma_i^* A \rightarrow V_f A^*)}{d\mathbf{\Delta}^2} \right|_{\mathbf{\Delta}^2=0} = \\ & \sum_{A^* \neq A} \left| \langle A^* \otimes V | \int d^2\mathbf{b}_+ d^2\mathbf{b}_- (1 - \hat{S}_A(\mathbf{b}_+, \mathbf{b}_-)) | A \otimes \gamma^* \rangle \right|^2 \\ & = \int d^2\mathbf{b} T_A(\mathbf{b}) \left| \langle V | \sigma(\mathbf{r}) \exp[-\frac{1}{2}\sigma(\mathbf{r}) T_A(\mathbf{b})] | \gamma^* \rangle \right|^2 + \dots \end{aligned} \quad (2)$$

- a simple generalization of quasielastic pA scattering ([Glauber & Matthiae, Czyż et al. 1970](#))
- incoherent sum over \mathbf{b} , effective diffraction operator:
 $\Omega(\mathbf{r}, \mathbf{b}) = \sigma(\mathbf{r}) \exp[-\frac{1}{2}\sigma(\mathbf{r}) T_A(\mathbf{b})]$
- valid at $x \sim x_A$

Quasielastic diffractive production $\gamma^* A \rightarrow VA^*$

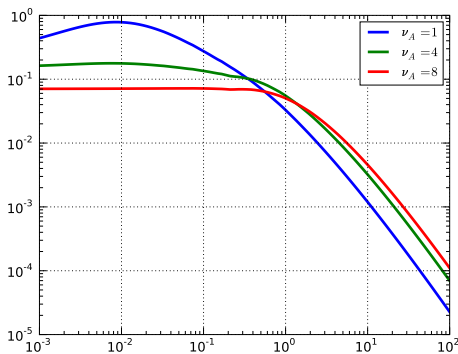
- for $x \ll x_A$ we must add the $q\bar{q}g$ -Fock-state of the photon/vector meson, and absorb the effect of the gluon into the effective x -dependent diffraction operator $\Omega(x, \mathbf{r}, \mathbf{b})$.
- coupled equations for $\Omega(x, \mathbf{r}, \mathbf{b})$ and the $q\bar{q}$ - S -matrix, $S_{q\bar{q}}(x, \mathbf{r}) = 1 - \Gamma_{q\bar{q}}(x, \mathbf{r})$:

$$\frac{\partial S_{q\bar{q}}(x, \mathbf{r})}{\partial \log(1/x)} = \int d^2\rho K(\rho, \rho + \mathbf{r}) \left(S_{q\bar{q}}(x, \rho) S_{q\bar{q}}(x, \rho + \mathbf{r}) - S_{q\bar{q}}(x, \mathbf{r}) \right)$$
$$\frac{\partial \Omega(x, \mathbf{r})}{\partial \log(1/x)} = \int d^2\rho K(\rho, \rho + \mathbf{r}) \left(S_{q\bar{q}}(x, \rho) \Omega(x, \rho + \mathbf{r}) + S_{q\bar{q}}(x, \rho + \mathbf{r}) \Omega(x, \rho) - \Omega(x, \mathbf{r}) \right)$$

- The boundary condition is $\Omega(x_A, \mathbf{r}) = \sigma(x_A, \mathbf{r}) S_{q\bar{q}}(x_A, \mathbf{r})$.
- Notice, that after small- x evolution $\Omega(x, \mathbf{r}) \neq \sigma(x, \mathbf{r}) S_{q\bar{q}}(x, \mathbf{r})$
- a similar set of coupled equations holds for the rapidity-gap evolution of the central diffractive process $pA \rightarrow p + \text{Higgs} + A$, where it sums up all multipomeron diagrams.

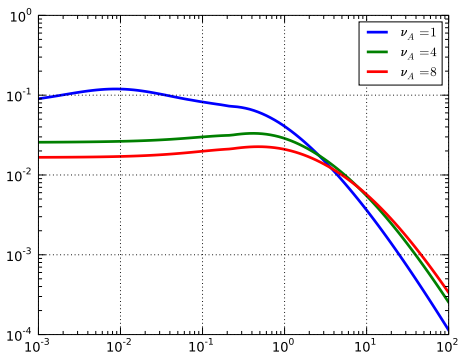
The evolved nuclear glue $\phi(\nu_A, x, \mathbf{p})$:

$$\phi(\nu_A, x, \mathbf{p}), x = 10^{-4}$$



$\mathbf{p}^2[\text{GeV}^2]$

$$\phi(\nu_A, x, \mathbf{p}), x = 10^{-6}$$

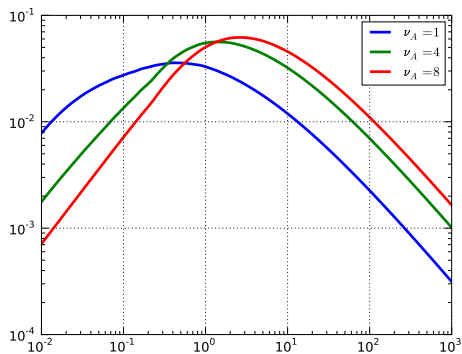


$\mathbf{p}^2[\text{GeV}^2]$

- $\mu_G^2 = 0.5\text{GeV}^2$, running α_S with 'freezing'.

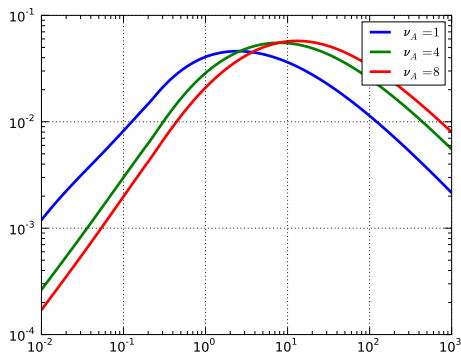
Evolution of the saturation scale:

$$\mathbf{p}^2 \phi(\nu_A, x, \mathbf{p}), \quad x = 10^{-4}$$



$$\mathbf{p}^2 [\text{GeV}^2]$$

$$\mathbf{p}^2 \phi(\nu_A, x, \mathbf{p}), \quad x = 10^{-6}$$

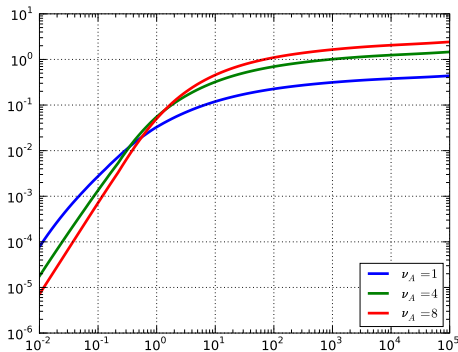


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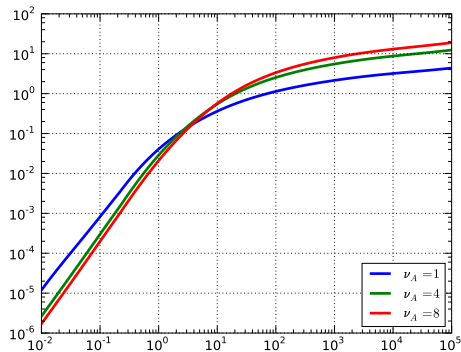
Large p^2 -behaviour of the unintegrated glue:

$$p^4 \phi(\nu_A, x, p), x = 10^{-4}$$



$$p^2 [\text{GeV}^2]$$

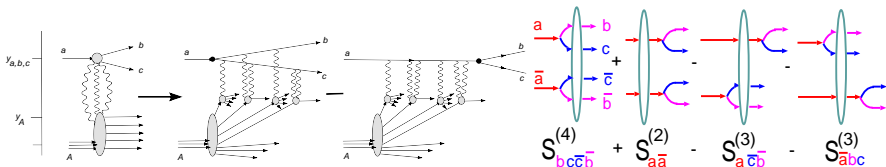
$$p^4 \phi(\nu_A, x, p), x = 10^{-6}$$



$$p^2 [\text{GeV}^2]$$

- the effective large- p^2 'anomalous dimension', $\gamma(x) > 2$ in $(\mu^2/p^2)^{\gamma(x)}$ is due to the linear BFKL piece.

Production as excitation of beam partons $a \rightarrow bc$



- To calculate dijet correlations, we need the two parton density matrix.
- In general this involves S -matrices of up to 4-parton states, and a coupled channel problem in the space of color representations. In momentum space, the pertinent observables are **nonlinear functionals of the unintegrated nuclear glue**.
- great simplification in $q \rightarrow q\gamma$: γ does not interact, hence **only 2-parton($q\bar{q}$) S -matrices are involved**. Furthermore, the problem becomes an abelian one.
- $q \rightarrow q\gamma$ satisfies a linear k_{\perp} -factorization theorem.

Linear k_{\perp} -factorization of $q \rightarrow q\gamma$

- on the free nucleon:

$$\frac{2(2\pi)^2 d\sigma_N(q \rightarrow q\gamma)}{dzd^2\mathbf{p}d^2\mathbf{\Delta}} = f(x, \mathbf{\Delta}) \left| \psi(z, \mathbf{p}) - \psi(z, \mathbf{p} - z\mathbf{\Delta}) \right|^2$$

with

$$\left| \psi(z, \mathbf{p}_1) - \psi(z, \mathbf{p}_2) \right|^2 = P_{\gamma q}(z) \left| \frac{\mathbf{p}_1}{\mathbf{p}_1^2 + \varepsilon^2} - \frac{\mathbf{p}_2}{\mathbf{p}_2^2 + \varepsilon^2} \right|^2$$

- $z, \mathbf{p} \rightarrow$ photon momentum, $\varepsilon^2 = zm_q^2$, $P_{\gamma q}(z) =$ splitting function
- $\mathbf{\Delta} = \mathbf{p} + \mathbf{p}_q =$ decorrelation momentum
- notice the collinear pole at $\mathbf{p} = z\mathbf{\Delta}$, from final state photon emission of the scattered quark.
- exact over the phase space of the γq -pair.
- decorrelation momentum distribution maps out the unintegrated glue

Linear k_{\perp} -factorization of $q \rightarrow q\gamma$

- on the nucleus:

$$\frac{(2\pi)^2 d\sigma_A(q \rightarrow q\gamma)}{dzd^2\mathbf{p}d^2\mathbf{\Delta}d^2\mathbf{b}} = \phi(\nu_A, x, \mathbf{\Delta}) \left| \psi(z, \mathbf{p}) - \psi(z, \mathbf{p} - z\mathbf{\Delta}) \right|^2 \\ + w_0(\nu_A) \delta^{(2)}(\mathbf{\Delta}) \left| \psi(z, \mathbf{p}) - \psi(z, \mathbf{p} - z\mathbf{\Delta}) \right|^2$$

- a potential factorization violating diffractive contribution vanishes
- the (parton-level) 'nuclear modification factor' **doesn't depend** on \mathbf{p} :

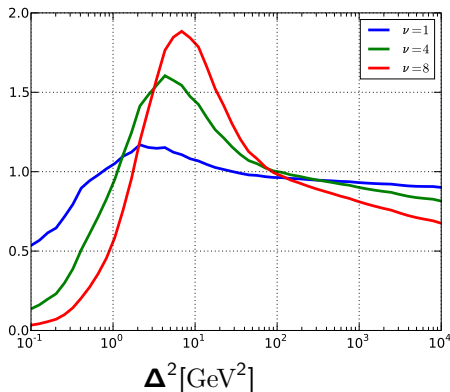
$$R_{pA}(\nu_A, \mathbf{p}, \mathbf{\Delta}) = \frac{d\sigma_A}{T_A(\mathbf{b})d\sigma_N} = \frac{\phi(\nu_A, x, \mathbf{\Delta})}{\nu_A f(x, \mathbf{\Delta})}$$

- similarly the central-to-peripheral ratio:

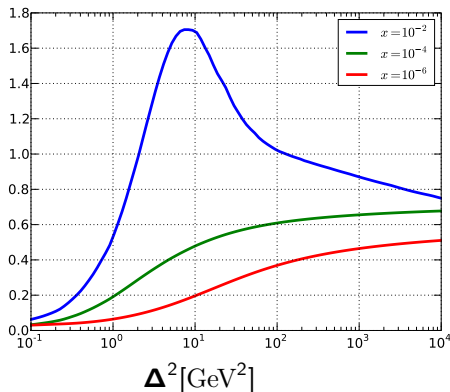
$$R_{CP}(\nu_>, \nu_<, \mathbf{p}, \mathbf{\Delta}) = \frac{\nu_< \phi(\nu_>, x, \mathbf{\Delta})}{\nu_> \phi(\nu_<, x, \mathbf{\Delta})}$$

R_{pA} (left panel), R_{CP} (right panel)

$$\frac{\phi(\nu_A, x, \Delta)}{\nu_A f(x, \Delta)}, x = 0.01$$



$$\frac{\nu_{<} \phi(\nu_{>}, x, \Delta)}{\nu_{>} \phi(\nu_{<}, x, \Delta)}, \nu_{>} = 8, \nu_{<} = 1$$



- left: a Cronin-type enhancement around the saturation scale
- ...which is quenched by small- x evolution (right panel)

Summary

- we presented numerical solutions of an impact parameter (or opacity-) dependent nonlinear evolution equation
- The diffraction operator for quasielastic diffractive processes obeys a novel evolution equation, coupled with the BK equation.
- γ -jet correlations in the proton fragmentation region can map out the unintegrated gluon distribution \rightarrow **experimental determination of the saturation scale**
- Outlook
 - $\phi(\nu, x, \mathbf{p})$ as a universal 'saturation glue', even for the free nucleon:

$$f_N(x, \mathbf{p}) = 2 \int d^2 \mathbf{b} \phi(\nu_N(\mathbf{b}), x, \mathbf{p}), \quad \nu_N(\mathbf{b}) = \frac{1}{2} \alpha_S(q^2) \sigma_0 t_N(\mathbf{b}).$$

- some fine tuning of the input $f(x_0, \mathbf{p}), \mu_G^2, \dots$, nucleon profile $t_N(\mathbf{b})$ may be required