Pinch Technique Gluon Propagator and Limits on a Dynamically Generated Gluon Mass

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Outline

- Pinch Technique
- Gluon mass and Pinch Technique SDE
- Regularity of the running coupling
- Pinch technique and CHSB
- Conclusion $m \simeq (0.4 - 0.7)\Lambda$

Motivation and Introduction

Pinch Technique J. Cornwall, PRD 1982

- PT- Technique to derive GAUGE INVARIANT and gauge fixing INDEPENDENT GFs in NaGT
- PT process and gauge fixing scheme independent running coupling are related directly with gauge boson propagators
- It is proved to all orders of QCD perturbation theory

"Literature" approach: QCD is NaGT, e.g. QCD classical Lagrangean is gauge invariant function of q^c and $A = A^a T^a$

$$L_c = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + g J_q . A + quarks$$

Quantization requires gauge fixing (to be able to write down gauge boson propagator), gauge fixed GFs do not satisfy WTI but STI and GFs receive ugly scheme dependence.

Popular example: covariant gauge fixing procedure, FP procedure introduces ghosts

AFter GFs we get free gluon propagator

$$G_{\mu\nu} = \frac{-t_{\mu\nu}}{q^2} - \xi \frac{q^\mu q^\nu}{(q^2)^2}$$

and PT one loop level gluon self-energy (fig!):

$$\Pi(q^2) = \frac{g^2 q^2}{16\pi^2}\left\{ (\frac{-13}{2} + \frac{\xi}{2}) N_c ln\frac{q^2}{\mu^2} + (97/36 + \frac{\xi}{2} + \frac{\xi^2}{4}) N_c \right\}$$

does not define any scheme invariant quantity (contrary to QED), although it is transverze $\Pi_{\mu\nu}(q) = t_{\mu\nu}\Pi(q^2)$, propagator is plagued by artificial ξ

$$G_{\mu\nu} = \frac{-t_{\mu\nu}}{q^2 - \Pi} - \xi\frac{q^\mu q^\nu}{(q^2)^2}$$

Pinch technique construction:

From S-matrix of scattered quarks and gluons.

Vertices and boxes involve "pinch parts" which are propagator like and are added to the propagator. Boxes involves vertex like contribution as well.

$$\Gamma_{ussual} = \Gamma_F + \Gamma_P$$

Γ_F are uniqely identified by WTI not by STI . Γ_P nontrivially contributes to the propagator givig us GI and GFI gluon propagator $\hat{d}$ and effective charge $\bar{g}$

$$\hat{d} = \frac{\bar{g}^2}{q^2}$$

1loop

$$\bar{g}^2 = \frac{g^2}{1 + bg^2 ln\frac{q^2}{\mu^2}} = \frac{1}{bln\frac{q^2}{\Lambda^2}}$$

$\Lambda \simeq 300 MeV$ defines validity of perturbation QCD.

$$b = \frac{11N_c - 2N_f}{48\pi^2}.$$

- Proved to all orders by Aquilar, Binosi, Papavassiliou.
- Extended to SM. To GUT (Brodsky).
- Equivalence with De Witt BFM quatization shown , PTGFs<=> $\xi = 1$ BFMGFs

PT SDE

Using WI defines PT SDE

Cornwall PRD 1982, Cornwall &Hou PRD 1986, Cornwall PRD 2009, Aguilar& Binosi& Papavassiliou 2002-2010

Solution has a mass (not massless pole)

Latest simple estimate based on study or RG invariant $g^2\hat{d}(q^2)$

$$g^2\hat{d}(q^2) = \bar{g}^2(q^2)\hat{H}(q^2)$$

where instead of $\hat{H}(q^2) = 1/q^2$

$$\hat{H}(q^2) = \frac{1}{q^2 - m^2(q) + i\varepsilon}$$

is considered in PT SDE Cornwall 2009:

$$\left[\bar{g}^2 \hat{d}(q^2)\right]^{-1} = q^2 bZ - \frac{ib}{\pi^2} \int d^{4k} \hat{H}(k) \hat{H}(k+q) \left[q^2 + \frac{m^2}{11}\right] + C \ ,$$

$$\left[\bar{g}^2 \hat{d}(q^2)\right]^{-1} = b \left[J(q)(q^2 + \frac{m^2}{11}) - J(m) \frac{12m^2}{11}\right] \ ,$$

renormalization with one loop ultraviolet asymptotic

$$J(q) = -\int_{4m^2}^{\infty} d\omega \frac{q^2}{\omega} \frac{\rho(\omega; m)}{q^2 - \omega + i\varepsilon} + 2 + 2\ln\left(m/\Lambda\right)$$

$$\rho(\omega; m) = \sqrt{1 - \frac{4m^2}{\omega}}$$

$$\begin{aligned} J(q) &= \rho \ln \left| \frac{1+\rho}{1-\rho} \right| + 2\ln(m/\Lambda) \\ &- i\pi\rho\theta(q^2 - 4m^2) \qquad \text{for } for 1 - 4m^2/q^2 > 0 \\ J(q) &= -i2\rho \text{ arctg} \left(\frac{i}{\rho} \right) + 2\ln(m/\Lambda)\,; \quad \text{for } 0 < q^2 < 4m^2 \end{aligned}$$

where $\rho = \sqrt{1 - \frac{4m^2}{q^2}}$.

Inverting the SDE (1)

$$b4\pi\alpha_{massive}(q^2) = b\bar{g}^2(q^2) = \frac{q^2 - m^2}{J(q)(q^2 + \frac{m^2}{11}) - J(m)\frac{12m^2}{11}}$$

The imaginary part of J for $q^2 > 4m^2$ is given by very simple phase space factor $pi\rho$, thus the imaginary and real the parts of the running coupling read

$$Im\, b\bar{g}^2 = (1-\gamma)\frac{\pi\rho(q^2)}{(ReI(q)-\gamma J(m))^2+(\pi\rho(q^2))^2}\,;$$

$$Re\, b\bar{g}^2 = (1-\gamma)\frac{ReJ(q)-\gamma J(m)}{(ReJ(q)-\gamma J(m))^2+(\pi\rho(q^2))^2}\,,$$

$\gamma = \frac{12m^2}{11q^2+m^2}$. For $q^2 < 4m^2$ the imaginary part vanishes.

Resulting property:

- increasing m α is decreasing
- α is singular at spacelike for $m < m_c^{II}$/ unacceptable Landau ghost
- α is singular for timelike q^2 for $m_c^{II} < m < m_c^{I}$ here it has two real poles , one simple and second non-simple
- up to threshold singularity $alpha$ is regular for $m > m_c^{I}$ but small to provide CHSB
- α does not satisfy KLR for any m

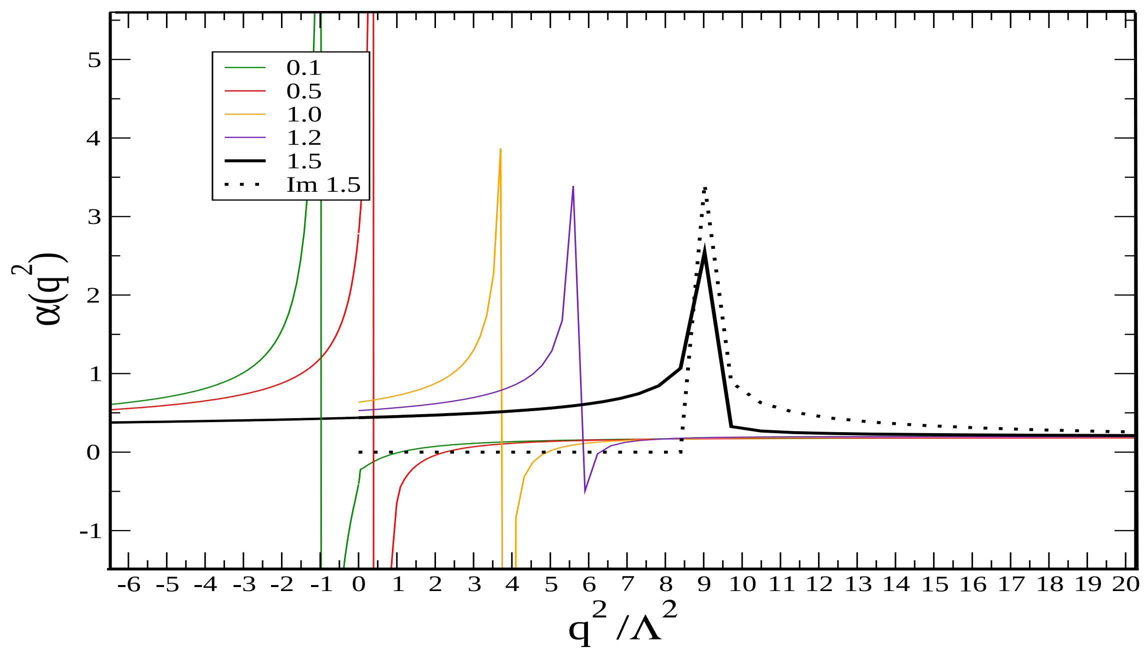

Figure 1: Pinch technique coupling α for a various ratio m/Λ. For better identifiation Im part only for regular solution with $m/\Lambda = 1.2$

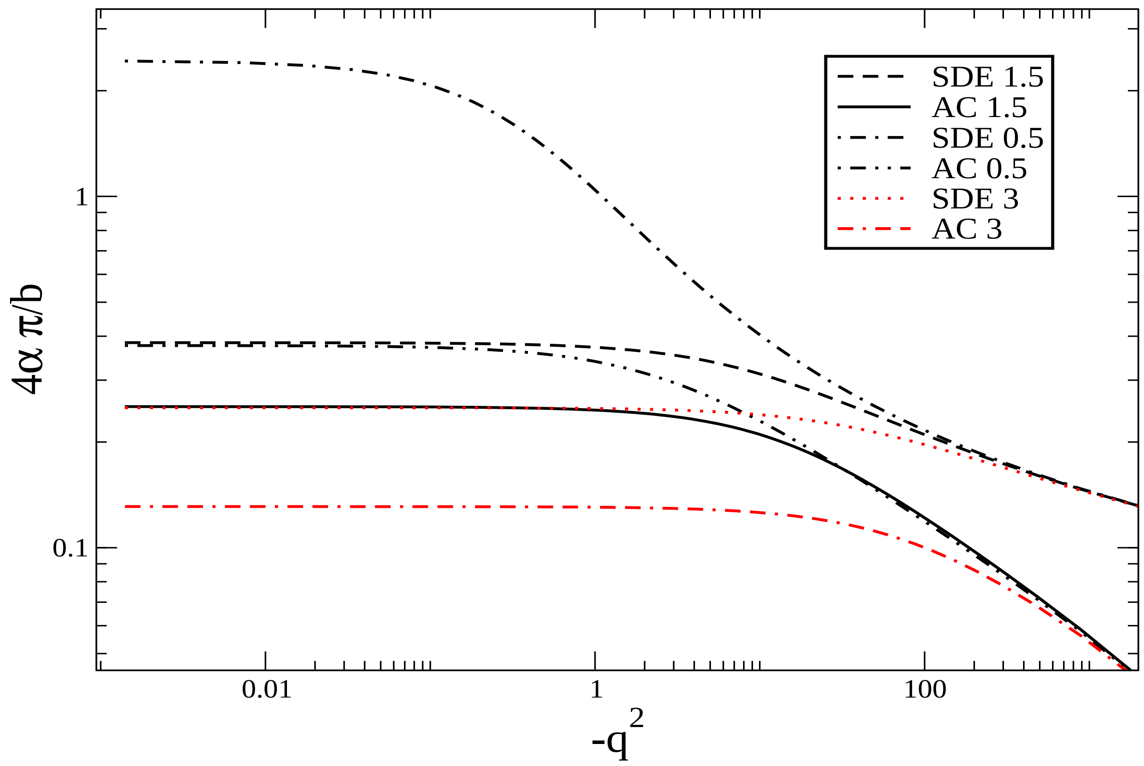

Figure 2: $b\bar{g}^2$ plotted for spacelike fourmomenta for various ratio of m. It is compared with analyticized coupling (AC) as described in the text.

KLR - Idea of AC Solovtsov, Shirkov

$$\alpha^{KLR}(q^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} dx \frac{\Im \alpha^{KLR}(x)}{x - q^2 - i\varepsilon}$$

Limts from CHSB

Untill now $m > 0.4$

In real QCD dynamical chiral symmetry breaking is phenomena responsible for the most of the u,d(s) quark mass generation aand simultaneously for the lightness of the pions(kaons). Various hadronic observables were calculated in the framework of Schwinger-Dyson equations during last two decades, it includes the meson spectra, decays and various form factors Roberts, Kurachi, Alkofer, Fischer.

- ladder quark SDE+ladder BSE. one gluon exchange is not enough to get CHSB and confinement

-more topologicaly complicated diagrams with gluons are needed to get nonrelativistic potential known from Wilson lines

Estimate of the effect on dynmical mass gebneration P. Bicudo,... arXiv:0912.1274, Schwinger-Dyson equations and the quark-antiquark static potential lesson: dynamical mass be already $M(o) \simeq \Lambda$ when ladder SDE with PT running coupling is used alone!

To get the correct description of hadrons with the PT running coupling we need $\alpha \simeq 2.0 \pm 0.5$ which gives $M(0) \simeq \Lambda$, th rest V_L gives further enhancement $M(0) \simeq few\Lambda$. and correct mass $m_\pi, m_\rho, ...$ and $f_{\pi,K}$ (assuming small scheme dependence, most of the hadronic observables is calculated in conventinal Landau or Coulomb gauge)

For dependence of $f_{\pi,K}, m_\pi$ on $(\alpha)\Lambda$ see also

$==>$ m must be closed to m_c^{II}, not to m_c^{I}.

Conclusion:

1-loop PT gluon SDE gives transverze, but massive gluon propagator with the effective gluon mass $m \simeq (0.4 - 0.7)\Lambda$. For m closed 0.4Λ –> Infrared Slavery still possible but for timelike q.

1)Ladder BSE reasonably approximate pseudoscalars, can we get approximate Regge trajectory with PT two pole gluon propagator?

2) If $\hat{H}$ has no real pole for real q^2 does it has the form of KLR with zero branch point (other branch points we excluded in a case of no real pole)

$$H^{KLR}(q^2) = \frac{1}{\pi} \int_0^\infty dx \frac{\Im H^{KLR}(x)}{x - q^2 - i\varepsilon}$$

3)More detailed estimate of upper boundary