The role of the tetraquark at nonzero temperature

Achim Heinz

in collaboration with S. Strüber, F. Giacosa, D. H. Rischke

3rd February 2010

Outline

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   - What is a tetraquark?
2. Our model
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   - Our potential
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a possible solution to all these problems is to interpret the light scalars as tetraquark states

What is a tetraquark?

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combine two quarks to a coloured diquark and couple two diquarks two a colourneutral particle

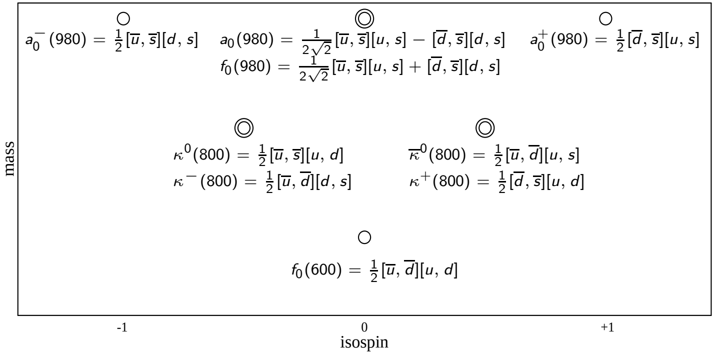
$$|qq\rangle = |\text{Space: } L = 0\rangle \, |\text{Spin: } (\uparrow\downarrow - \downarrow\uparrow)\rangle \, |\text{f: } (ud - du)\rangle \, |\text{c: } (RB - BR)\rangle$$

$$|qq\rangle = \rho_k = \sqrt{\tfrac{1}{2}}\epsilon_{ijk} q_i^t (C\gamma^5) q_j$$

$$\begin{array}{lll} u \Leftrightarrow [\bar{d}, \bar{s}] & d \Leftrightarrow [\bar{s}, \bar{u}] & s \Leftrightarrow [\bar{u}, \bar{d}] \\ \bar{u} \Leftrightarrow [d, s] & \bar{d} \Leftrightarrow [s, u] & \bar{s} \Leftrightarrow [u, d] \end{array}$$

out of this correspondency, you can build up a tetraquark nonet

What is a tetraquark?



- tertaquark picture generates many resonances
- $f_0(600)$ is an s-wave state, thus it can be lighter than 1 GeV
- mass degeneracy of $a_0(980)$ and $f_0(980)$ can be explained with constituent quarks

Our potential

the model we use is the $SU(2)_r \times SU(2)_l$ limit of the $SU(3)_r \times SU(3)_l$ case

F. Giacosa, **Phys.Rev.D75:054007 (2007)**

A. Heinz, S. Strüber, F. Giacosa, and D. H. Rischke, **Phys.Rev.D79:037502 (2009)**

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$$V = \frac{\lambda}{4}(\vec{\pi}^2 + \varphi^2 - F^2)^2 - \epsilon\varphi + \frac{1}{2}M_\chi^2\chi^2 - g\chi(\vec{\pi}^2 + \varphi^2)$$

tetraquark: $\chi = \rho^\dagger \rho$

diquark: $\rho = \sqrt{\frac{1}{2}\epsilon_{ij} q_i^t (C\gamma^5) q_j}$

for $SU(2)$ each diquark ρ is invariant under chiral transformation

Untwiddle the masses

$$V = \frac{\lambda}{4}(\vec{\pi}^2 + \varphi^2 - F^2)^2 - \epsilon\varphi + \frac{1}{2}M_\chi^2\chi^2 - g\chi(\vec{\pi}^2 + \varphi^2)$$

$$\varphi_0 = \frac{F}{\sqrt{1-(2g)/(\lambda M_\chi^2)}} + \frac{\epsilon}{2\lambda F^2} + \dots$$ quark condensate

$$\chi_0 = \frac{g}{M_\chi^2}\varphi_0^2$$ tetraquark condensate

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expanding the potential around the minimum:

$$V = \frac{1}{2}(\chi, \varphi)\begin{pmatrix} M_\chi^2 & -2g\varphi_0 \\ -2g\varphi_0 & M_\varphi^2 \end{pmatrix}\begin{pmatrix} \chi \\ \varphi \end{pmatrix} + \frac{1}{2}M_\pi^2\vec{\pi}^2 + \ldots$$

where $M_\varphi^2 = \varphi_0^2\left(3\lambda - \frac{2g^2}{M_\chi^2}\right) - \lambda F^2$ and $M_\pi^2 = \frac{\varepsilon}{\varphi_0}$

non diagonal mass matrix

$$-2g\varphi_0\varphi\chi$$

since the mass matrix is not diagonal we have to diagonalize the potential

Untwiddle the masses

H and S chosen to diagonalize the potential

$$\begin{pmatrix} H \\ S \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix} = B \begin{pmatrix} \chi \\ \varphi \end{pmatrix}$$

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$$V = \frac{1}{2}(\chi,\varphi)B^t B \begin{pmatrix} M_\chi^2 & -2g\varphi_0 \\ -2g\varphi_0 & M_\varphi^2 \end{pmatrix} B^t B \begin{pmatrix} \chi \\ \varphi \end{pmatrix} + \dots$$

$$= \frac{1}{2}(H,S) \begin{pmatrix} M_H^2 & 0 \\ 0 & M_S^2 \end{pmatrix} \begin{pmatrix} H \\ S \end{pmatrix} + \dots$$

$$\theta_0 = \frac{1}{2}\arctan\frac{4g\varphi_0}{M_\varphi^2 - M_\chi^2}, \quad -\frac{\pi}{4} < \theta_0 < \frac{\pi}{4}$$

$$M_H^2 = M_\chi^2 \cos^2\theta_0 + M_\varphi^2 \sin^2\theta_0 - 2g\varphi_0 \sin(2\theta_0)$$

$$M_S^2 = M_\varphi^2 \cos^2\theta_0 + M_\chi^2 \sin^2\theta_0 + 2g\varphi_0 \sin(2\theta_0)$$

$$\left(M_S^2 - M_H^2\right)^2 = \left(M_\varphi^2 - M_\chi^2\right)^2 + (4g\varphi_0)^2 \;\rightarrow\; \left|M_S^2 - M_H^2\right| \geq 4g\varphi_0$$

Nonzero T model

we employ the CJT-formalism in the Hartree-Fock approximation to calculate the T dependency of our masses, condensates and mixing angle

Nonzero T model

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masses, condensates and mixing angle become T dependent

$$M_H \rightarrow M_H(T) \quad M_S \rightarrow M_S(T) \quad M_\pi \rightarrow M_\pi(T)$$
$$\varphi_0 \rightarrow \varphi(T) \quad \chi_0 \rightarrow \chi(T) \quad \theta_0 \rightarrow \theta(T)$$

$$M_H(0) = M_H \quad M_S(0) = M_S \quad M_\pi(0) = M_\pi$$
$$\varphi(0) = \varphi_0 \quad \chi(0) = \chi_0 \quad \theta(0) = \theta_0$$

Range of the parameters

our potential

$$V = \frac{\lambda}{4}(\vec{\pi}^2 + \varphi^2 - F^2)^2 - \epsilon\varphi + \frac{1}{2}{M_\chi}^2\chi^2 - g\chi(\vec{\pi}^2 + \varphi^2)$$

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two known values

$M_\pi = 0.139$ GeV

$\varphi_0 = f_\pi = 0.0924$ GeV

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two known values

$M_\pi = 0.139$ GeV

$\varphi_0 = f_\pi = 0.0924$ GeV

three approximately known values

$M_H \approx 0.4$ GeV	$f_0(600) = 0.4$ GeV $-$ 1.2 GeV
$M_S \approx 1.2$ GeV	$f_0(1370) = 1.2$ GeV $-$ 1.5 GeV

g should be of the order of a few GeV

to manage these uncertainties we study variation of g, M_S and M_H

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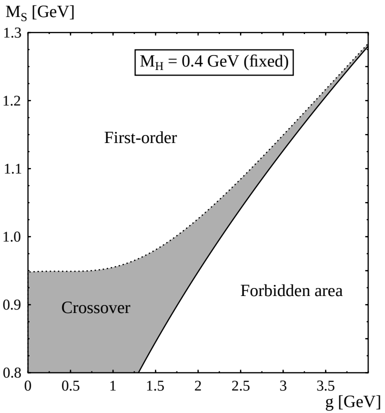
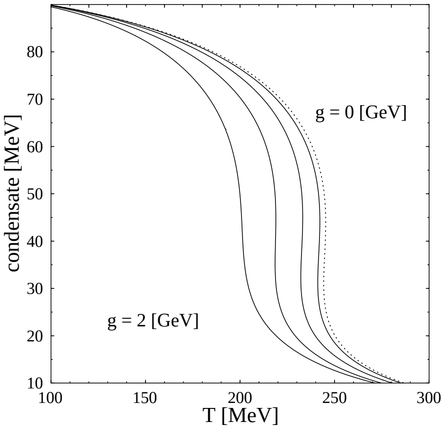
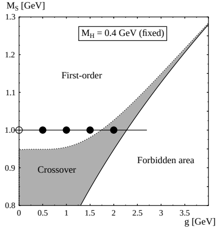
$M_H = 0.4 GeV$ fixed, g and M_S vary



- forbidden area arises from $\left|M_S^2 - M_H^2\right| \geq 4g\varphi_0$
- between first order and crossover region we find a second order phase transition
- $g \rightarrow 0$:
  χ and φ decouple
  $H \rightarrow \chi, \; S \rightarrow \varphi$
  $M_S > 0.948 GeV$:
  first order phase transition
  $M_S < 0.948 GeV$:
  crossover phase transition

T_c behaviour

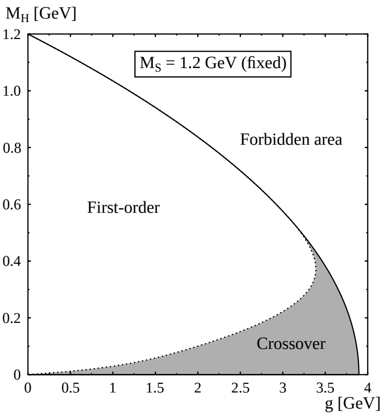




increasing of g:

- mixing increases
- T_c decreases
- first order softens
- crossover is obtained for large g

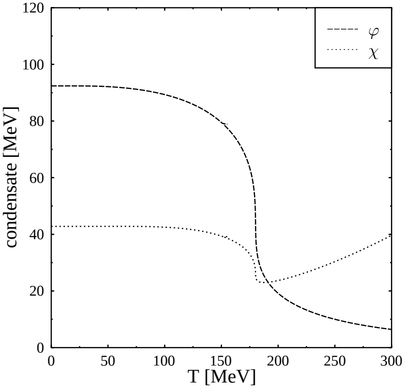
$M_S = 1.2 GeV$ fixed, g and M_H vary



- forbidden area arises from $\left|M_S^2 - M_H^2\right| \geq 4g\varphi_0$
- between first order and crossover region we find a second order phase transition
- $g \rightarrow 0$:
  χ and φ decouple
  $H \rightarrow \chi,\ \ S \rightarrow \varphi$
  we get first order phase transition
- to get a crossover for a large M_S we need a large gap between M_S and M_H

Condensate

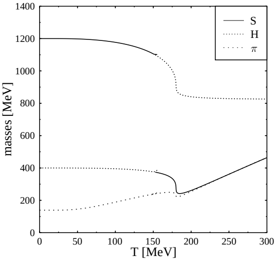
$M_H = 0.4$ GeV, $M_S = 1.2$ GeV and $g = 3.4$ GeV

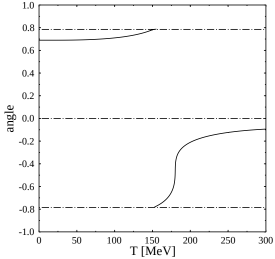


- crossover phase transition at $T_c \approx 170 MeV$
- $T < T_c$:
  $\chi(T)$ goes like $\frac{g}{M_\chi^2}\varphi(T)^2$
- $T > T_c$:
  $\chi(T)$ increases

Masses and mixing angle

$M_H = 0.4$ GeV, $M_S = 1.2$ GeV and $g = 3.4$ GeV





- predominantly $M_S(T)$ consists of quarkonium
  predominantly $M_H(T)$ consists of tetraquark
- mixing angle $\theta(T)$ increases till $T_s = 155 MeV$, then sign becomes negative
  T_s defined as $\theta(T_s) = \frac{\pi}{4}$
- at T_s both masses behave discontinously and the states interchange their roles
- for large T the mixing goes to zero and everything behaves like in the linear sigmar model

Conclusion and outlook

- a T dependent model including a tetraquark state
- order of phase transition changes with coupling g;
  if coupling g and mixing is large enough we also obtain a crossover phase transition for a mass of the chiral partner above 1 GeV
- mixing increases with T and at a temerature T_s a role interchange takes place

- include glueball states and vectormesons

Thank you
for your attention