$$H = H_0 + H_1$$

$$H_0 = M_0 |S\rangle\langle S| + \sum_{k=\frac{2\pi n}{L}} \omega(k) |k\rangle\langle k|$$

$$H_1 = \sum_{k=\frac{2\pi n}{L}} g \frac{f(k)}{\sqrt{L}} \left(|k\rangle\langle S| + |S\rangle\langle k| \right)$$

Contin. limit $\sum_k \longmapsto L \int_{-\infty}^{\infty} \frac{dk}{2\pi}$, $|k\rangle \longmapsto \sqrt{\frac{2\pi}{L}}\, |k\rangle$.

$$a(t) = \langle S| e^{-iHt} |S\rangle = \frac{i}{2\pi} \int_{-\infty}^{\infty} dE \underbrace{\langle S| \frac{1}{E - H + i\varepsilon} |S\rangle}_{G_S(E) \text{ is the prop. of } S} e^{-iEt}$$

$$G_S(E) = \langle S| \frac{1}{E - H + i\varepsilon} |S\rangle = \frac{1}{E - M_0 + \underbrace{g^2 \Sigma(E)}_{\text{loop correction}} + i\varepsilon}$$

where (long calculation):

$$\Sigma(E) = - \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{f^2(k)}{E - \omega(k) + i\varepsilon}$$

$\Sigma(E)$ is, in general, a complex function.

Remark

$|k\rangle$ describes two particles: "the decay products" ...

$\frac{1}{2} = g \, \mathrm{Im}$

$$\Sigma(E) = \mathrm{Re}\,\Sigma(E) + i\,\mathrm{Im}\,\Sigma(E)$$

The 'new' (dressed) mass M is given by the eq:

$$M - M_0 + g^2\,\mathrm{Re}\,\Sigma(M) = 0$$

$$G_S^{BW}(E) = \frac{1}{E - M + g^2\,\mathrm{Im}\,\Sigma(M) + i\varepsilon}$$

$$a(t) \mapsto a^{BW}(t) = \frac{i}{2\pi}\int_{-\infty}^{\infty} dE\, G_S^{BW}\, e^{-iEt} = e^{-iMt}\, e^{-\Gamma/2\, t}$$

$$P(t) \mapsto P^{BW}(t) = e^{-\Gamma t}$$

$d_S(E)$ - recall

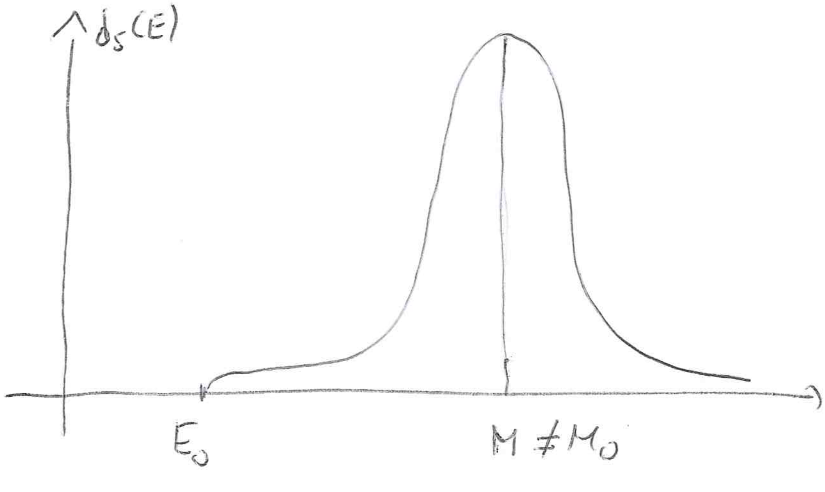
In general, one can write the propagator as the sum of many free propagators:

$$\frac{1}{E - M_0 + g^2 \Sigma(E) + i\varepsilon} = \underbrace{\int_0^\infty \frac{d_S(x)}{E - x + i\varepsilon} \, dx}$$

it is intuitive clear...

Sum of many free propagators $\frac{1}{E - x + i\varepsilon}$

$d_S(x)\,dx$ is the probability that the unstable particle $|S\rangle$ has a mass between x and $x + dx$.



$$d_S(E) = \frac{1}{\pi} \operatorname{Im}\left[\frac{1}{E - M_0 + g^2 \Sigma(E) + i\varepsilon}\right]$$

$g \mapsto 0 \quad d_S(E) = \delta(E - M_0)$; $\int_0^\infty d_S(E)\,dE = 1$!

The time evolution can be expressed as the Fourier transform of $d_S(E)$:

$$a(t) = \int_0^\infty d_S(x)\, e^{-ixt}\, dx.$$

Intuitive explanation:

$$|S\rangle = \int_0^\infty f(x)\, |x\rangle\, dx \quad \text{, where } |x\rangle \,/\, H|x\rangle = x|x\rangle \quad ; \; |f(x)|^2 = d_S(x)$$

$$a(t) = \langle S| e^{-iHt} |S\rangle = \langle S|x\rangle\langle x| e^{-iHt} |x'\rangle\langle x'|S\rangle =$$

$$= \int dx\, dx'\, f^*(x)\, f(x')\, \delta(x-x')\, e^{-ixt}$$

$$= \int dx\, d_S(x)\, e^{-ixt}$$

Ersoek

We have proven already that $P(t)$ is not an exponential. We can have a more formal argument in the following way.

$t=0 \quad |s>$

$$t_1>0 \quad e^{-iHt_1}|s> = \underbrace{a(t_1)}_{\text{surv. prob. amplitude}}|s> + \sum_K r_K(t_1)|K>$$

After $t = t_1+t_2$ we get

$$e^{-iH(t_1+t_2)}|s> = a(t_1+t_2)|s> + \sum_K r_K(t_1+t_2)|K> =$$

$$= e^{-iHt_2}\left(e^{-iHt_1}|s>\right) =$$

$$= e^{-iHt_2}\left(a(t_1)|s> + \sum_K r_K(t_1)|K>\right)$$

$$= a(t_1)\left(a(t_2)|s> + \sum_q r_q(t_2)|q>\right) + e^{-iHt_2}\sum_K r_K(t_1)|K>$$

$$= a(t_1)a(t_2)|s> + a(t_1)\sum_q r_q(t_2)|q> + \underbrace{e^{-iHt_2}\sum_K r_K(t_1)|K>}$$

The question is: what is this?

if we assume that:

$$\langle s| e^{-iHt_2} \sum_K r_K(t_1) |K\rangle = 0 ,$$

then

$$e^{-iHt_2} \sum_K r_K(t_1) |K\rangle = \sum_K r_K(t_1) e^{-iHt_2} |K\rangle =$$

$$= \sum_K r_K(t_1) \Big[\underbrace{b_K(t_2) |s\rangle}_{b_K(t_2)=0} + \sum_q S_{Kq}(t_2) |q\rangle \Big]$$

then: $e^{-iHt_2} \sum_K r_K(t_1) |K\rangle = \sum_K \# |K\rangle$

$$a(t_1+t_2) = a(t_1)\, a(t_2)$$

$$\Downarrow$$

$$a(t) = e^{\alpha t}$$

where $\alpha \in \mathbb{C} \rightarrow \alpha = i\left(M - i\frac{\Gamma}{2}\right)t$

$$e^{i(M - i\frac{\Gamma}{2})(t_1+t_2)} = e^{i(M - i\frac{\Gamma}{2})t_1}\, e^{i(M - i\frac{\Gamma}{2})t_2} ;$$

Then, if $|k\rangle$ cannot recombine to $|s\rangle$ $\mapsto$ we have an exp. decay.

But this is obviously not the case because:

- we have shown in a very general framework that the decay is not exp
- H_1 mixes $|k\rangle$ and $|s\rangle$ ... if I start from $|k\rangle$, the time evolution generates $|s\rangle$.

Then, $e^{-iHt_2}\left(\sum_k r_k(t_1)|k\rangle\right)$ $\quad \underline{\underline{\alpha |s\rangle}} + \text{rest}$

This recombination breaks the exp. decay... and gives rise to the non-exp. behavior. Without recombination no deviations from the exp. decay were possible.