Decays involving fermions

As we have discussed, the SM is "full" of fermions.

We have:

$$\underbrace{e^-, \mu^-, \tau^-}_{\text{copies of } e^-}, \nu_e, \nu_\mu, \nu_\tau$$

$$u, d, s, c, b, t$$

proton = uud ....
neutron = udd ..

The basic eq. for fermion is the Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi \longrightarrow (i\partial\!\!\!/ - m)\psi = 0.$$

$$\partial\!\!\!/ = \gamma^\mu \partial_\mu$$

$\gamma^\mu \equiv$ Dirac Matrices

$$\boxed{\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}} \qquad \mu, \nu = 0, 1, 2, 3 \qquad ; \boxed{\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$(\gamma^5)^2 = 1$$

$$\gamma^{5\dagger} = \gamma^5$$

The solution of the Dirac eq. involves plane waves:

$$\psi(t,\vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}=\frac{2\pi\vec{n}}{L}} \sum_{r=1,2} \sqrt{\frac{m}{E}} \left(b_{\vec{p}}^{(r)} U^{(r)}(\vec{p}) e^{-ipx} + d_{\vec{p}}^{(r)\dagger} V^{(r)}(\vec{p}) e^{ipx} \right)$$

$$\left\{ b_{\vec{p}}^{(r)}, b_{\vec{p}'}^{(r')\dagger} \right\} = \left\{ d_{\vec{p}}^{(r)}, d_{\vec{p}'}^{(r)\dagger} \right\} = \delta_{\vec{p}\vec{p}'} \delta_{rr'}.$$

(All the other anticommutators vanish...)

The spinors are given by:

$$U^{(r)}(\vec{p}) = \frac{\not{p}+m}{\sqrt{2m(E+m)}} U^{(r)}(0) \overset{\text{Dirac}}{=} \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \chi^r \\ \frac{\vec{\sigma}\vec{p}}{2m} \chi^r \end{pmatrix}$$

$\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$V^{(r)}(\vec{p}) = \frac{-\not{p}+m}{\sqrt{2m(E+m)}} V^{(r)}(0) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \frac{\vec{\sigma}\vec{p}}{2m} \chi^r \\ \chi^r \end{pmatrix}$$

Projectors:

$$\Lambda_+(\vec{p}) = \sum_{r=1,2} U^{(r)}(\vec{p})\,\bar{U}^{(r)}(\vec{p}) = \frac{\not{p}+m}{2m}$$

If we explicitly write the elements:

$$\left(\Lambda_+^{(\vec{p})}\right)_{ab} = \sum_{r=1,2} U_a^{(r)}\,\bar{U}_b^{(r)}(\vec{p}) = \left(\frac{\not{p}+m}{2m}\right)_{ab}$$

$$\Lambda_-(\vec{p}) = -\sum_{r=1,2} V^{(r)}(\vec{p})\,\bar{V}^{(r)}(\vec{p}) = \frac{-\not{p}+m}{2m}$$

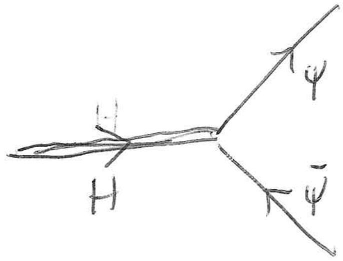
If we explicitly write the elements

$$\left(\Lambda_-^{(\vec{p})}\right)_{ab} = -\sum_{r=1,2} V_a^{(r)}(\vec{p})\,\bar{V}_b^{(r)}(\vec{p}) = \left(\frac{-\not{p}+m}{2m}\right)_{ab}$$

Decay: $H \mapsto \bar{\psi}\psi$

Let us study the decay of a scalar field into two fermions. This is the typical framework to calculate the decay of the Higgs.

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{1}{2}\left[(\partial_\mu H)^2 - M_H^2 H^2\right] + \underline{\underline{g}}\, H \bar{\psi}\psi$$



g = coupling constant

it can be determined!

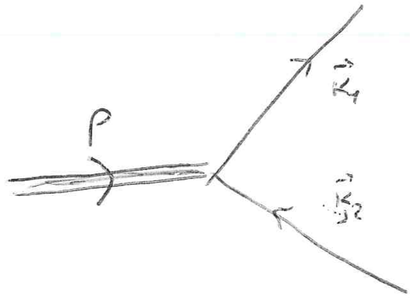
See later...

$$\Gamma_{H \mapsto \bar{\psi}\psi} = \underbrace{\frac{|\vec{k}|^2}{8\pi M_H^2}}_{\text{usual factor}} \underbrace{\left(\sqrt{2m}\right)^{n_f = 2}}_{\text{change of norm. due to } n_f \text{ fermions}} \cdot \overline{|-iM|^2}$$

usual factor

change of norm. due to n_f fermions which are part of the process (here a fermion and an antifermion in the final states)

average decay amplitude



$\sim$ fermion : $\bar{u}^a(\vec{k}_1)$

$\rightsquigarrow$ antifermion $v^b(\vec{k}_2)$

$$-iM^{ab} = i g \, \bar{u}^a(\vec{k}_1) v^b(\vec{k}_2)$$

$$\sum_{a,b} |-iM^{ab}|^2 = g^2 \sum_{a,b} \left(\bar{u}^a(\vec{k}_1) v^b(\vec{k}_2)\right)^\dagger \left(\bar{u}^a(\vec{k}_1) v^b(\vec{k}_2)\right) =$$

$$= g^2 \left(v^{b\dagger}(\vec{k}_2) \gamma^{0\dagger} u^a(\vec{k}_1)\right) \left(\bar{u}^a(\vec{k}_1) v^b(\vec{k}_2)\right)$$

$$= g^2 \left(\bar{v}^b(\vec{k}_2) u^a(\vec{k}_1)\right) \left(\bar{u}^a(\vec{k}_1) v^b(\vec{k}_2)\right)$$

$$= g^2 \left(\bar{v}^b_\alpha(\vec{k}_2) u^a_\alpha(\vec{k}_1)\right) \left(\bar{u}^a_\beta(\vec{k}_1) v^b_\beta(\vec{k}_2)\right)$$

$$= g^2 \left(\frac{\not{k}_1 + m}{2m}\right)_{\alpha\beta} \left(\frac{-\not{k}_2 - m}{2m}\right)_{\beta\alpha}$$

$$= \frac{g^2}{4m^2} \operatorname{Tr}\left[(\not{k}_1 + m)(k_2 - m)\right]$$

$$= \frac{g^2}{4m}\left(\mathrm{Tr}[K_1 K_2] - m^2\, \mathrm{Tr}[1_2]\right)$$

$$= \frac{g^2}{4m^2}\left[4(K_1 K_2) - 4m^2\right] =$$

$$= \frac{g^2}{4m^2}\left[\cancel{4}^{\,2}\, \frac{M_H^2 - 2m^2}{\cancel{2}\,1} - 4m^2\right] =$$

$$= \frac{g^2}{4m^2}\left[2M_H^2 - 8m^2\right] = \frac{g^2}{2m^2}\left[M_H^2 - 4m^2\right]$$

— Άρα:

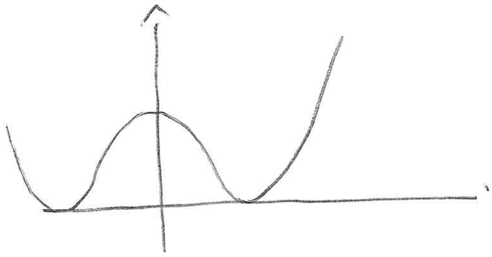
$$\Gamma_{H \to \bar{\varphi}\varphi} = \frac{\sqrt{\frac{M_H^2}{4} - m^2}}{8\pi M_H^2}\,(4\cancel{m^2})\,\frac{g^2}{2\cancel{m^2}}\left[M_H^2 - 4m^2\right]$$

$$= \frac{\sqrt{\frac{M_H^2}{4} - m^2}}{8\pi M_H^2}\, 2g^2 \cdot \frac{4}{4}\left(\frac{M_H^2}{4} - m^2\right)$$

$$\boxed{\Gamma_{H \to \bar{\varphi}\varphi} = \frac{\left(\frac{M_H^2}{4} - m^2\right)^{3/2}}{\pi M_H^2}\, g^2\, \theta(M_H - 2m)}$$

The peculiar case of the Higgs boson:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda}{4}(H^2 - v^2)^2$$



$$H \mapsto v + H$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda}{4}\left(\cancel{v^2} + H^2 + 2vH - \cancel{v^2}\right)^2 = \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda}{4}\cdot 4 v^2 H^2 - \frac{\lambda}{4} H^4 - \lambda v H$$

$$\boxed{M_H^2 = 2\lambda v^2}$$

$$\begin{cases} v = 246 \text{ GeV} \\ M_H = 125 \text{ GeV} \end{cases}$$

The other important point is that the masses as well are generated by the Higgs field.

For instance, for the electron we have the unique interaction

$$\mathcal{L}_e = \bar{\psi} i \partial\!\!\!/ \psi - \lambda_e H \bar{\psi}\psi$$ original interaction

But then, $H \mapsto v + H$ we get a mass

$$\left(\mathcal{L}_{full} = \bar{\psi} i \partial\!\!\!/ \psi - \lambda_e H \bar{\psi}\psi + \frac{1}{2}(\partial_\mu H)^2 - \frac{\lambda}{4}(H^2 - v^2)^2 \right)$$

$$\mathcal{L}_e = \bar{\psi} i \partial\!\!\!/ \psi - \lambda_e v \bar{\psi}\psi - \lambda_e H \bar{\psi}\psi$$

$\lambda_e v = m_e$

$\lambda_e = \frac{m_e}{v} = -g$

$$\Gamma_{H\mapsto e^- e^+} = \frac{\left(\frac{M_H^2}{4} - m_e^2\right)^{3/2}}{\pi M_H^2}\left(\frac{m_e}{v}\right)^2 .$$

$M_H = 125$ GeV

$v = 246$ GeV

$m_e = 0.5 \cdot 10^{-3}$ GeV

$= 2.13 \cdot 10^{-11}$ GeV very small

For μ = Just replace $m_e \mapsto m_\mu$.

$$\Gamma_{H \mapsto \mu^- \mu^+} = 9.32 \cdot 10^{-7} \text{ GeV} \ldots$$

much larger....

μ^--collider for the future?

Unstable particle... $\tau = 2.197 \cdot 10^{-6}$ sec $\boxed{\mu^- \mapsto e^- \bar{\nu}_e \nu_\mu}$

(This is huge for strong decay purposes)

Time dilation if the μ^- are very fast

—

$$\Gamma_{H \mapsto \tau^- \tau^+} = 0.00026 \simeq 2.6 \cdot 10^{-4} \text{ GeV} = 0.26 \text{ MeV} \ldots$$

already visible..

—

$$\Gamma_{H \mapsto \bar{\nu} \nu} \cong 0 \,!!!! \ldots$$

because $m_\nu \simeq 0$!!!

For the quarks the same story... but quarks carry color.

* quarks carry color

* mass uncertain (hadronisation)

$m_b =$

$\lambda H(\bar{\psi}_{b,R}\psi_{b,R} + \bar{\psi}_{b,G}\psi_{b,G} + \bar{\psi}_{b,B}\psi_{b,B})$

Ergo:

$$\Gamma_{H\to\bar{b}b} = \underbrace{3}_{N_c} \frac{\left(\frac{M_H^2}{4} - m_b^2\right)^{3/2}}{\pi M_H^2}\left(\frac{m_b}{v}\right)^2 = 0.00432 \text{ GeV} = 4.32 \text{ MeV}$$

Dominant!

($\Gamma_{H\to\bar{s}s} \simeq 2.8 \cdot 10^{-6}$ GeV... and even smaller for u, d)

$\Gamma_{H\to\bar{c}c} = 0.000397 \text{ GeV} \simeq 0.39 \text{ MeV}$

(it contributes a bit....
small, but just as T
not completely negligible)

There are also decays into WW and ZZ which we do not study here..

beauty of the standard model!!!!

($\Gamma_{H \to \bar{t}t} = 0$... but the coupling is indeed very large....)
if the H had a mass $> 2 m_t$, it would decay very fast).

Higgs spectral function:

- renormalization, Källen-Lehmann and gauge invariance, renormalization
- anyway BW is good for this mass.
- what next? prediction of Wetterich....