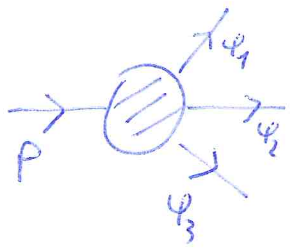
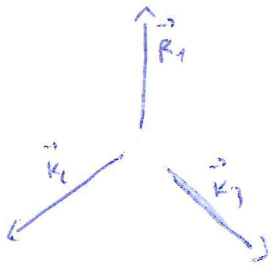
Unstable particle P which decays into 3 particles: $\varphi_1, \varphi_2, \varphi_3$

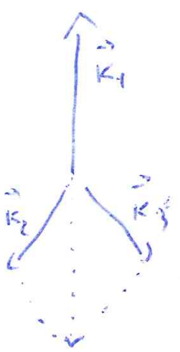
$$P \mapsto \varphi_1 \varphi_2 \varphi_3$$



There are much more possibilities w.r.t. the two-body decay.

For instance:





$$\boxed{\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = \vec{0}}$$

Three-body decays take place very often in particle physics.

A very "famous" example is the decay of the neutron.

$$n \mapsto p + e^- + \bar{\nu}_e$$

$\tau = 881.5 \pm 1.5$ sec ; $P(t) = e^{-t/\tau}$ ($\tau \sim 14$ min... this is very long if compared to strong decays)

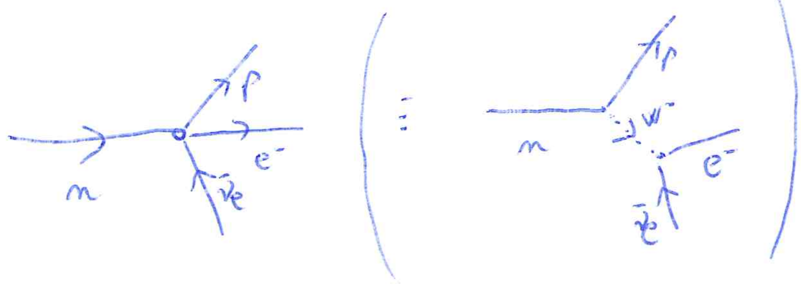
$$\Delta E = m_n - m_p - m_e = 0.782 \text{ MeV}$$

$\tau \sim 14$ min is long...

It is interesting that very energetic neutrons may be emitted in the center of our galaxy and reach us.

In order this to be possible a 'huge' dilatation factor is needed.

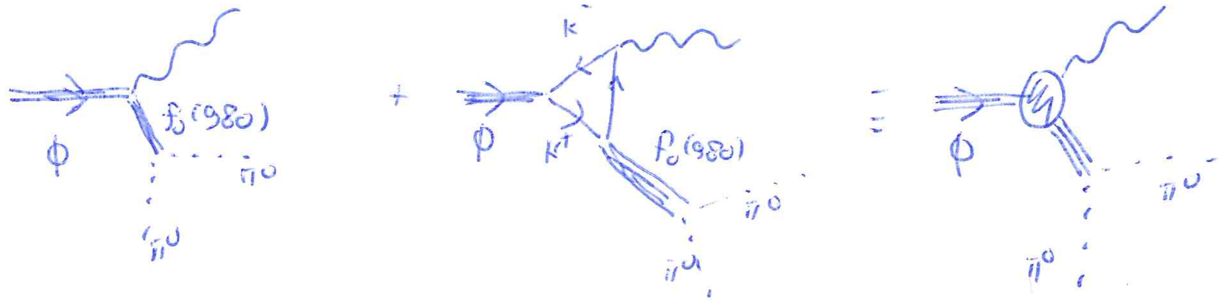
$\rightsquigarrow$ See ex. 1 of sheet 9.



but FAPP it is a point-like four-leg interaction

(iii) $\phi \mapsto \gamma \pi^0 \pi^0$

$\phi = \bar{s}s$

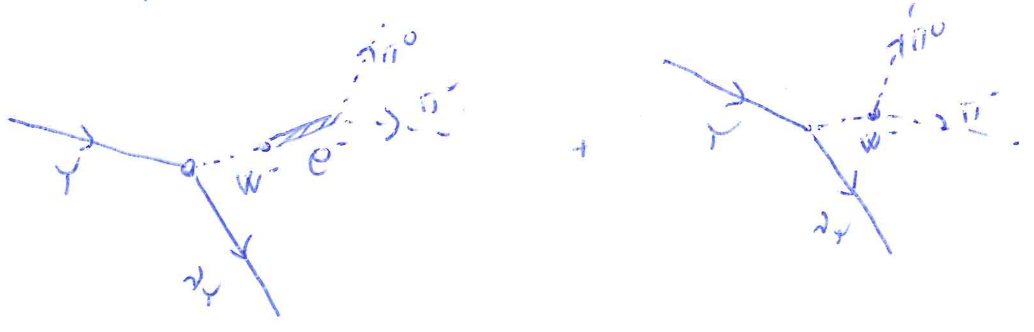


Studied in Frascati: $e^+e^- \rightarrow \leftarrow$ collide and produce a ϕ meson almost at rest. The ϕ subseq. decays.

Kloe detector → photons are measured. Each $\pi^0 \mapsto \gamma\gamma$.

Then, a 5γ process is measured.

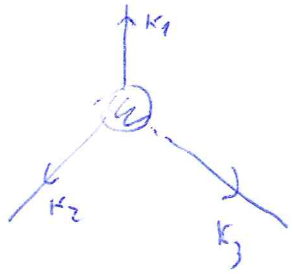
(iv) $\tau^- \mapsto \nu_\tau \pi^0 \pi^-$



Aleph experiment; spectral function of ρ^- (very well measured).

$$P \mapsto \varphi_1 + \varphi_2 + \varphi_3$$

$$P = (M_P, \vec{0}).$$



General formula for $S \mapsto \varphi_1 + \varphi_2 \ldots + \varphi_N$

$$d\Gamma = \frac{(2\pi)^4}{2M_P} |M|^2 \, \delta\left(P - \sum_{i=1}^{N} k_i\right) \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2} \cdots \frac{d^3k_N}{(2\pi)^3 2E_N}$$

nr of free variables:

$$3N$$

(Note: in terms of four-vectors there are $4N$ nr, but the conditions $k_i^2 = m_i^2$ eliminate 1 d.o.freedom per particle).

Then, we have the conservation of energy-momentum:

$$P = \sum_{i=1}^{N} k_i \quad \begin{cases} \mapsto E = E_1 + E_2 + \ldots + E_N \\ \mapsto 0 = \vec{k}_1 + \vec{k}_2 + \ldots + \vec{k}_N \end{cases}$$

$\boxed{n = 3N - 4}$ $\mapsto$ in the end, one has to perform an integral over $3N-4$ variables.

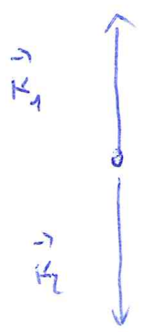
Recall: $\underline{\underline{N=2}}$

$$d\Gamma = \frac{|-i\mathcal{M}|^2}{32\pi^2}\,\frac{|\vec{k}_1|}{M^2}\,d\Omega$$

$3 \cdot 2 - 4 = 2$ variables

$$\boxed{\Gamma = \frac{|\vec{k}_1|}{8\pi M^2}\,|-i\mathcal{M}|^2}$$

This is valid as long as $|-i\mathcal{M}|^2$ doesn't depend on θ and φ: this is the case if the particles are without spin or if we average over it!



once you fix the "direction of $\vec{k}_1$" no more freedom is there.

$$\mathcal{L}_{int} = g\, e_\mu \partial^\mu \pi\, \pi$$

if we say that, for instance, $\varepsilon_\mu(p) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, then $|-i\mathcal{M}|^2$ depends on θ and φ...

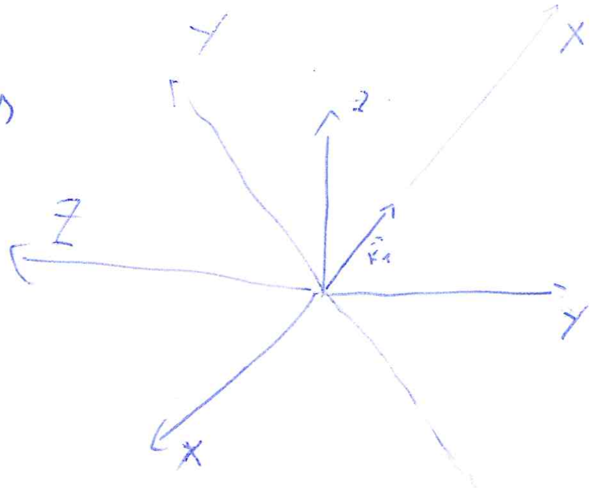
$$|-i\mathcal{M}|^2 = g^2 \sin^2\theta \cos^2\varphi$$

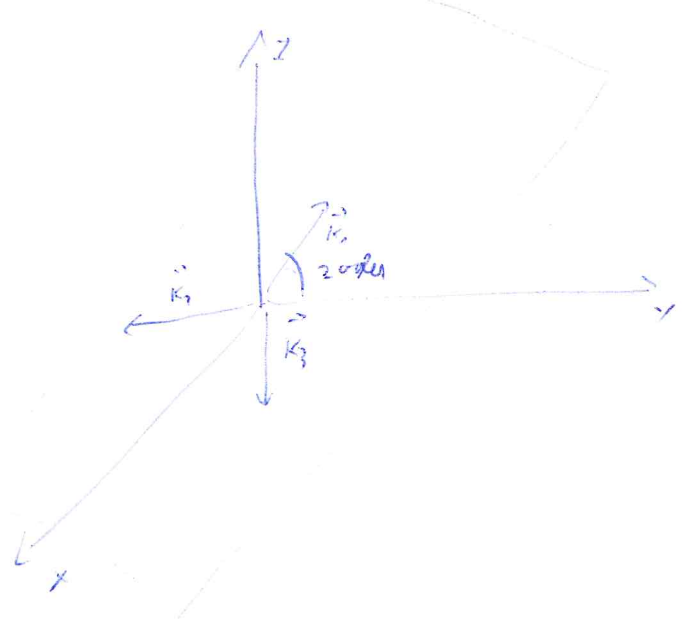
Back to 3-body decay

$3 \cdot 3 - 4 = 5$ variables

Let us take as variables (as convention):

- $E_1 \equiv$ energy of the first particle
- $E_3 \equiv$ energy of the third particle
- 3 Euler angles
  α, β, γ





$$d\Gamma = \frac{1}{(2\pi)^5}\frac{1}{16M}\left|-i\mathcal{M}\right|^2 dE_1\, dE_2\, \underbrace{d\alpha\, d(\cos\beta)\, d\gamma}_{(0,2\pi)\ \left(-\frac{\pi}{2},\frac{\pi}{2}\right)\ (0,2\pi)}$$

Let us perform the integration over the Euler-angles:

$$d\Gamma = \frac{2\pi \cdot 2\pi \cdot 2}{(2\pi)^5\, 16M}\left|-i\mathcal{M}\right|^2 dE_1\, dE_3$$

$$\boxed{d\Gamma = \frac{1}{(2\pi)^3\, 8M}\left|-i\mathcal{M}\right|^2 dE_1\, dE_3}$$

Lorentz-invariant Kynematic variables

$$\boxed{m_{ij}^2 = (K_i + K_j)^2}.$$

m_{ij}^2 do not depend on the ref. frame...
$m_{ij}^2 = (K_i + K_j)^2 = (K_i^* + K_j^*)^2$ for $SR \mapsto SR^*$.

$$m_{12}^2 = (K_1+K_2)^2 = (P-K_3)^2 = P^2 + K_3^2 - 2PK_3 = M^2 + m_3^2 - 2ME_3$$

Ergo:

$$d(m_{12}^2) = -2M\,dE_3$$

$$\boxed{m_{12} \longleftrightarrow E_3}$$

Similarly:

$$m_{23}^2 = (K_2+K_3)^2 = (P-K_1)^2 = M^2 + m_1^2 - 2ME_1$$

$$dm_{23}^2 = -2M\,dE_1$$

it then follows:

$$\boxed{d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|-i\mathcal{M}|^2\,dm_{12}^2\,dm_{23}^2}$$

Exp. and theoretical physicists work on this quantity...

$|-i\mathcal{M}|^2$ is a function of m_{12}^2 and m_{23}^2 (if the particles are scalar and if an average over spin states has been performed).

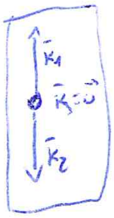
The full decay width is given by

$$\Gamma = \int_{D} dm_{12}^2 \, dm_2^2 \; \frac{1}{(2\pi)^3 \, 32 M^3} \, \left| -i \overline{\mathcal{M}} \right|^2$$

↳ Dalitz region

It is crucial to determine D.

$$m_{12}^2 = M^2 + m_3^2 - 2 M E_3$$



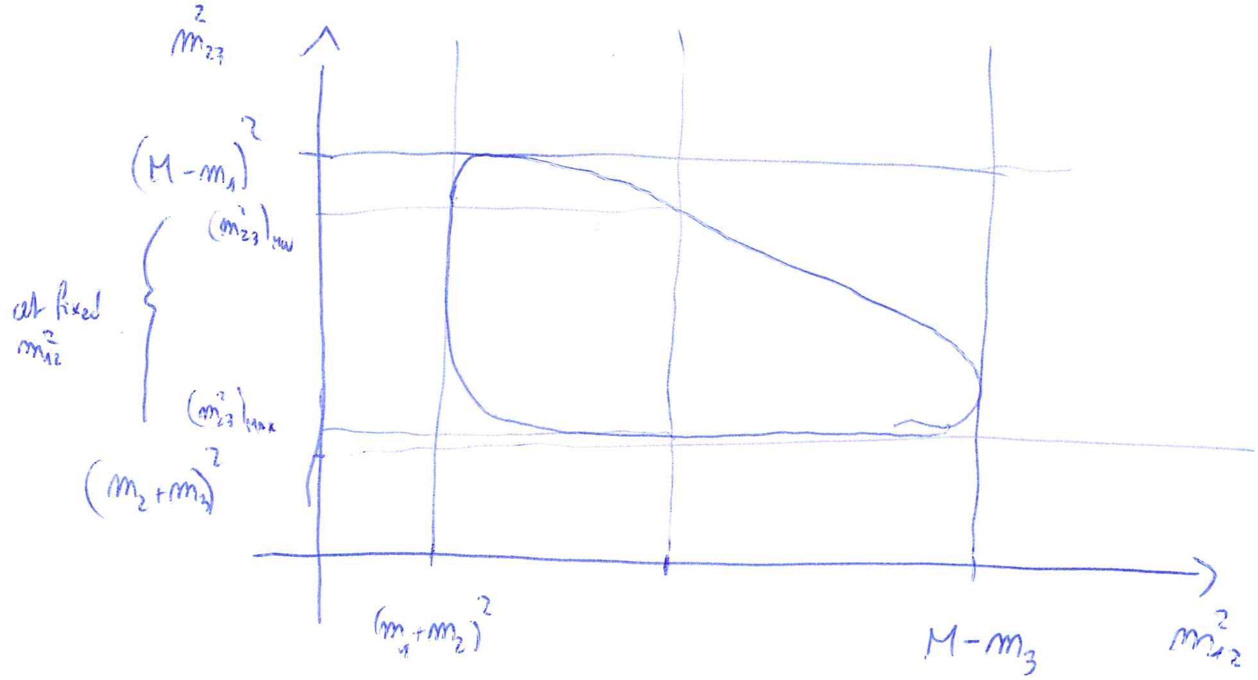
$$E_3^{min} = m_3 \qquad (m_{12}^2)^{MAX} = M^2 + m_3^2 - 2 M m_3 = (M - m_3)^2$$

$$m_{12}^2 = (k_1 + k_2)^2 =$$

$$(m_{12}^2)^{MIN} = (m_1 + m_2)^2$$

Similarly:

$$(m_2 + m_3)^2 \leq m_{23}^2 \leq (M - m_1)^2$$



The internal of the 'square' is a necessary condition: D is included in it.

D has the "approximate form" of a triangle with curved angles....

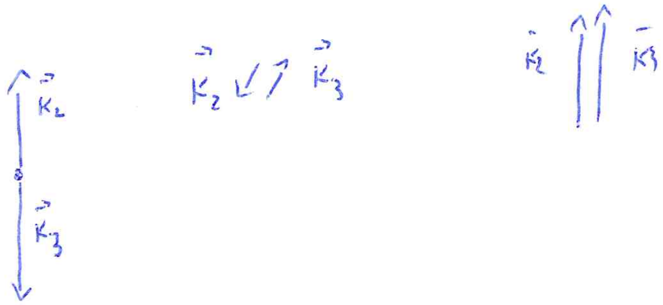
How to get it?

At fixed m_{12}^2 we can determine the values

$(m_{23}^2)_{MAX}$ and $(m_{23}^2)_{MIN}$

at fixed m_{12}^2.

In order to determine $(m_{23}^2)_{MAX,\,MIN}$ we have the following possibility:



$$(m_{23}^2)_{MAX} = (K_2+K_3)^2 = (K_2^*+K_3^*)^2 =$$

in R.F. of m_{12}

$$= (E_2^*+E_3^*) - \left(\sqrt{E_2^{*2}-m_2^2} - \sqrt{E_3^{*2}-m_3^2}\right)^2 \qquad \uparrow\downarrow \text{ Antiparallel}$$

$$(m_{23}^2)_{MIN} = (E_2^*+E_3^*) - \left(\sqrt{E_2^{*2}-m_2^2} + \sqrt{E_3^{*2}-m_3^2}\right)^2 \qquad \uparrow\uparrow \text{ Parallel}$$

and:

$$\begin{cases} E_2^* = \dfrac{m_{12}^2 - m_1^2 + m_2^2}{2m_{12}} \\[2ex] E_3^* = \dfrac{M^2 - m_{12}^2 - m_3^2}{2m_{12}} \end{cases}$$

Namely:

$$(m_{12}, \vec{0}) = K_1^* + K_2^*$$

$$(m_{12}, \vec{0}) - K_2^* = K_1^*$$

$$m_{12}^2 + m_2^2 - 2\, m_{12} E_2^* = m_1^2 \;\rightarrow\; E_2^* = \frac{m_{12}^2 + m_2^2 - m_1^2}{2 m_{12}}$$

$$P = \underbrace{(m_{12}, \vec{0})}_{K_1^* + K_2^*} + K_3^*$$

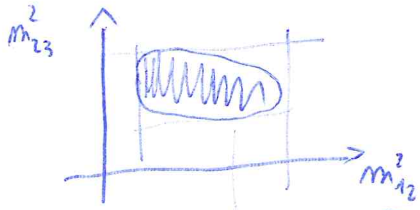
Ergo:

$$M^2 = m_{12}^2 + m_3^2 + 2 m_{12} E_3^*$$

$$\boxed{E_3^* = \frac{M^2 - m_{12}^2 - m_3^2}{2 m_{12}}}$$

In the case of $n \to p + e^- + \bar{\nu}_e$ the interaction is local....

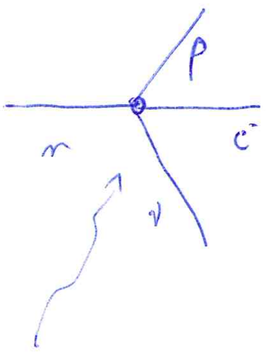
It then follows that $|-i\mathcal{M}|^2$ is a constant which does not depend



uniformly filled!!!
No structure...
all the "points"
are equally probable.

Scalar theory:

$$\mathcal{L} = g\, S\, \varphi_1 \varphi_2 \varphi_3 \qquad |-i\mathcal{M}|^2 = g^2 .$$



constant....

$$\mathcal{L} = -\frac{(G_F V_{ud})}{\sqrt{2}} \bar{\psi}_e \gamma_\mu (1-\gamma_5) \psi_\nu \cdot \bar{\psi}_N \underbrace{\Gamma}_{\gamma^\mu (1 - g_A \gamma^5)} \psi_N$$

$$\boxed{\left(m_{12}^R\right)^{MAX} = (M - m_\nu)^2} \mapsto \left(m_{12}^{MAX}\right) = M - m_\nu$$

We can indeed write:

$$m_{12}^2 = (E_1+E_2)^2 - \underbrace{(\vec{k}_1+\vec{k}_2)^2}_{\vec{0}} \equiv (E_1+E_2)^2 \quad ; \quad m_{12}^{max} = E_1 + E_2 \simeq$$

Consider the case in which the neutrino is emitted at rest.

$$E_1 + E_2 + m_\nu = M_n$$

$$E_1 \cong M_p$$

$$E_2 = E_2 - m_e + m_e = T_e + m_e$$

$$T_2^{MAX} \cong M_n - M_p - m_\nu - m_e$$

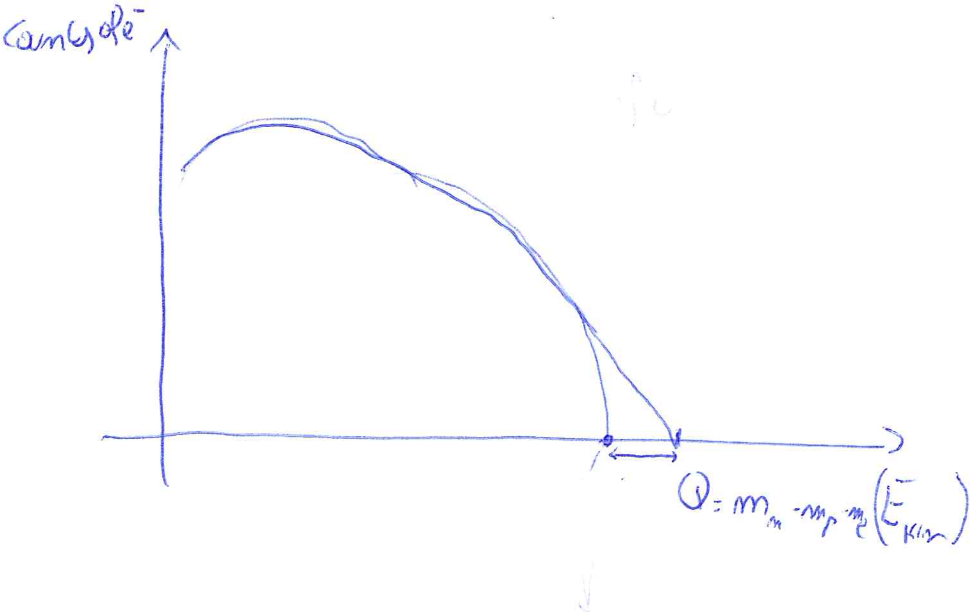
"measure the fastest possible e^- ... their kinetic energy tells us something about the mass of the ν ....

$\rightsquigarrow$ exercise about this.

(Determine the exact kinetic energy $\mapsto$ ex.)

$$T_2^{max} = 0.782\,\text{MeV} - m_\nu$$

One measures exactly that $\to$ $m_\nu < 13$ KeV!



If a ν has a mass $m_\nu \neq 0$, the maximal energy

is $m_n - m_p - m_e - m_\nu$

(in principle measurable!)

Karls. Tritium Neutrino

The same principle is used by the Katrin experiment in Karlsruhe $Q = 18.6 \text{ keV} = M_T - M$

$\delta E \simeq 1 \text{ eV}$

^3_1H

if $m_\nu \gtrsim 0.35$ eV

it can be measured by this experiment.

(present limit by 2.3 eV)

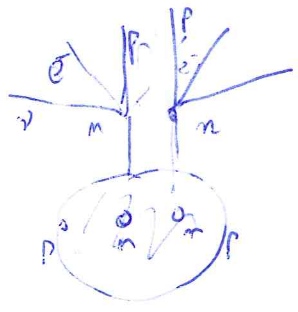
$T \mapsto {}^3He + e^- + \bar{\nu}_e$

$\mapsto + e^- + \bar{\nu}_e$

Last but not least, the ν-less double-β-decay:

- it involves also ν
- it is a "3-body decay as well

A double β decay is rather peculiar.
It is relevant in those cases when a single decay is not allowed.



Two "n" undergo at the same time

$$ {}^{A}_{Z}X \longmapsto {}^{A}_{Z+2}X + 2\bar{\nu}_e + 2e^- $$

$\boxed{\text{5-body decay}}$

$$ {}^{A}_{Z}X \longmapsto {}^{A}_{Z-2}X + 2\nu_e + 2e^+ $$

$$ \boxed{{}^{106}_{48}Cd \longmapsto {}^{106}_{46}Pd + 2e^+ + 2\nu_e} $$

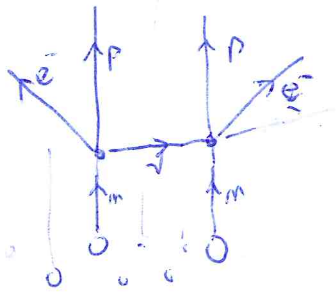
Huge lifetimes: $T_{1/2} \sim 0.04 \cdot 10^{21}\ y$

(longer than the life of the Universe)

${}^{76}_{32}Ge$ also decays as such.. $\boxed{{}^{76}_{32}Ge \longmapsto {}^{76}_{34}Se \quad \sim 1.78 \cdot 10^{21}\ y}$

ν-less double β decay.

if ν's are Majorana particles.



→ Virtual ν ...
possible only for Majorana
very improbable....

$$^{A}_{Z}X \longmapsto {}^{A}_{Z+2}X + 2e^-$$

Exactly in the case of the Ge, it was claimed that the ν-less double β decay has been seen.

$(10^{25}\ y\ ?)$ $m_\nu \simeq 0.3-0.6\ eV$.

H.d-Moscow collab.

$\tau_{1/2} \simeq 2.3 \cdot 10^{25}\ y$.

Various experiments are working on it.

Cuore / Cuoricino (Gran Sasso).