DECAYS IN QFT - WS 2012/2013

Sheet 1 **26/10/2012**

Exercise 1: Time-evolution (12 points = 3 + 4 + 2 + 3)

A two-level system is described by the Hamiltonian

$$H = H_0 + H_1 \tag{1}$$

with

$$H_0 = M_0 \left|S\right\rangle \left\langle S\right| + \omega \left|\omega\right\rangle \left\langle \omega\right| \ , \tag{2}$$

$$H_1 = g \left(\left|S\right\rangle \left\langle \omega\right| + \left|\omega\right\rangle \left\langle S\right|\right) \ . \tag{3}$$

1. Determine the eigenvalues and eigenvectors of H. ('Repetita juvant'; see the script if help is needed). Use the convention:

$$\begin{pmatrix} \left|E_1\right\rangle \\ \left|E_2\right\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \left|\omega\right\rangle \\ \left|S\right\rangle \end{pmatrix} \ . \tag{4}$$

2. Determine the state

$$\left|s(t)\right\rangle = e^{-iHt} \left|S\right\rangle = a(t) \left|S\right\rangle + r(t) \left|\omega\right\rangle \tag{5}$$

   for each time t.

3. Plot $p(t) = \left|a(t)\right|^2$ for $\theta = 0$, $\theta = \pi/6$, $\pi/4$.

4. Discuss the case $M_0 = \omega$. Which is the value of the mixing angle?

Exercise 2: Complex mixing (8 points)

A two-level system is described by the Hamiltonian

$$H = H_0 + H_1 \tag{6}$$

with

$$H_0 = M_0 \left|S\right\rangle \left\langle S\right| + \omega \left|\omega\right\rangle \left\langle \omega\right| \ , \tag{7}$$

$$H_1 = z \left|S\right\rangle \left\langle \omega\right| + z^* \left|\omega\right\rangle \left\langle S\right| \ . \tag{8}$$

where z is a complex number.
Determine the eigenvalues and the eigenstates of the Hamiltonian H.