DECAYS IN QFT - WS 2012/2013

Sheet 11 **1/2/2013**

Exercise 1: Tilted Mexican hat (12 points = 2 + 4 + 3 + 3)

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - V(\sigma, \pi) \tag{1}$$

where the potential reads

$$V(\sigma, \pi) = \frac{\lambda}{4}\left(\sigma^2 + \pi^2 - F^2\right)^2 - \varepsilon\sigma \ . \tag{2}$$

1. The minimum of the potential is realized for $(\sigma, \pi) = (\phi, 0)$. Determine the equation which defines ϕ. (It is an equation of third order; no need to solve it exactly.)
2. Perform the shift $\sigma \to \sigma + \phi$ and determine the masses M_σ and M_π. In particular, show that $M_\pi^2 = \varepsilon/\phi$. (In order to show it use the result of ex. 1.1).
3. Determine the numerical value of F, ε and λ when $\phi = f_\pi = 92.4$ MeV, $M_\sigma = 1$ GeV, $M_\pi = 139$ MeV.
4. Determine the mathematical expression of the decay $\Gamma_{\sigma \to \pi\pi}$ and determine its numerical value by using the input of ex. 1.3.

Exercise 2: Polar coordinates (8 points = 4 + 4)

Consider the Lagrangian of Eq. (1) and perform the transformation to polar coordinates:

$$\sigma = \rho \cos\varphi, \ \pi = \rho \sin\varphi \ . \tag{3}$$

1. Determine the Lagrangian in terms of the new fields ρ and φ.
2. Calculate the decay $\rho \to \varphi\varphi$ for the case $\varepsilon = 0$. Compare it to the result in Cartesian coordinates.