DECAYS IN QFT - WS 2012/2013

Sheet 2 **2/11/2012**

Exercise 1: Oscillations with an external electric field (15 points = 3 + 2 + 4 + 4 + 2)

A two-level system is described by the Hamiltonian

$$H = H_0 + H_1 \tag{1}$$

with

$$H_0 = M_0 |S\rangle \langle S| + \omega |\omega\rangle \langle\omega| \ , \tag{2}$$

$$H_1 = g\cos(\alpha t) \left(|S\rangle \langle\omega| + |\omega\rangle \langle S|\right) \ . \tag{3}$$

1. Make the following Ansatz

$$|s(t)\rangle = a(t)e^{-iM_0 t} |S\rangle + r(t)e^{-i\omega t} |\omega\rangle \ . \tag{4}$$

   (Note, we have simply factorized out the phases. This trick is typical for the so-called interaction picture).

   Determine the first-order differential equations for $a(t)$ and $r(t)$ which must be fulfilled, in order that the state $|s(t)\rangle$ is a solution of the Schrödinger equation

$$i\frac{d}{dt} |s(t)\rangle = H |s(t)\rangle \ . \tag{5}$$

   Use the following convention: $\beta = M_0 - \omega > 0$, $\Delta = \beta - \alpha$.

2. In the Eqs. derived in the previous point there is $\cos(\alpha t)$. Rewrite it as $\cos(\alpha t) = \frac{1}{2}\left(e^{i\alpha t} + e^{-i\alpha t}\right)$. Then, two terms: $e^{i(\beta-\alpha)t}$ and $e^{i(\beta+\alpha)t}$. Neglect the latter. Why is this approximation valid for $\beta \simeq \alpha$.

3. The system of first order of diff. Eqs. in (1) can be solved using the simplification in (2). By a further time-derivative one can recast the equation for $a(t)$ as a second-order differential equation of the type

$$\ddot{a} + A\dot{a} + Ba = 0 \tag{6}$$

   whereas A and B are some (complex) numbers. Solve the equation for $a(t)$. A similar second-order equation holds for $r(t)$, which can be solved similarly.

4. Determine the specific solution for the boundary condition $a(0) = 1$, $r(0) = 0$ (that is, $|s(0)\rangle = |S\rangle$). Determine also $p(t) = |a(t)|^2$.

5. Discuss the case $\Delta = \beta - \alpha = 0$.

Exercise 2: Entanglement with the apparatus (5 points)

Consider a double-slit experiment. Let y be the axis along the screen. The wave function coming from the first slit is $\psi_1(y) = f(y^2)e^{-i\omega(y+\delta/2)}$ and the wave function coming from the second slit is $\psi_2(y) = f(y^2)e^{-i\omega(y-\delta/2)}$, whereas

$$\delta = \frac{d}{L}y \tag{7}$$

whereas d is the distance between the two slits and L the distance of the slit place to the y axis. (The quantity $f(y^2)$ assures the correct normalization of the wave functions, but is not important here).

Now, the particle goes through both slits and the full wave function is given by

$$\psi(y) = \frac{1}{\sqrt{2}} \left(\psi_1(y) + \psi_2(y)\right) \tag{8}$$

the probability on the screen is given by

$$\frac{1}{2} \left|\psi_1(y) + \psi_2(y)\right|^2 \tag{9}$$

which gives the usual interference fringes. (Verify this!).
Now, let us put a detector just before the two slits which can measures where the particles goes. $\Phi_1(\alpha_i)$ is the wave function of the detector when it measures the electron passing though the first slit; the coordinates α_i with $i = 1, \ldots .N$ refers to the position of all the atoms of the detectors, that is N is a very large number. (Of course, there are many degenerate possibilities for such a configuration in which the detector shows "slit 1„). Similarly, $\Phi_2(\alpha_i)$ is the wave function of the detector when it measures the electron passing though the first hole.
Now, in presence of the detector, the full wave function is given by

$$\psi_{tot}(y) = \frac{1}{\sqrt{2}} \left(\psi_1(y)\Phi_1(\alpha_i) + \psi_2(y)\Phi_2(\alpha_i)\right) \ . \tag{10}$$

Do we see interference on the screen? If not, why not?