

Exercise 1: Breit-Wigner (6 points = 1 + 3 + 2)

Consider an unstable state  $|S\rangle$  described by the spectral function

$$d_S(x) = \frac{N}{(x - M)^2 + \Gamma^2/4} \quad (1)$$

1. Determine  $N$ .
2. Determine the Zeno time  $\tau_Z$ . Why is the result expected? Recall that:  $\tau_Z = \left(\langle H^2 \rangle_S - \langle H \rangle_S^2\right)^{-1/2}$ .
3. Determine through an explicit calculation the survival probability  $a(t)$ .

Exercise 2: Breit-Wigner with cutoff (6 points = 2 + 4 + 0)

Consider an unstable state  $|S\rangle$  described by the spectral function

$$d_S(x) = \frac{N}{(x - M)^2 + \Gamma^2/4} \theta(\Lambda^2 - (x - M)^2) \quad (2)$$

1. Determine  $N$ .
2. Determine the Zeno-time  $\tau_Z$ . Comment the result.

Exercise 3: Something is wrong (4 points)

Consider the following identities:

$$e = e^{1+2\pi i} = (e^{1+2\pi i})^{1+2\pi i} = e^{1+4\pi i-4\pi^2} = e^{1-4\pi^2} \rightarrow e^{-4\pi^2} = 1???? \quad (3)$$

What did it go wrong?

Exercise 4: Change of coordinates (4 points)

Consider the Lagrangian

$$L_x = \frac{1}{2}m\dot{x}^2 - V(x, t) . \quad (4)$$

Perform the change of coordinate  $y = x - \frac{1}{2}at^2$ . In term of the new variable  $y$  the system is described by the new Lagrangian

$$L_y = \frac{1}{2}m\dot{y}^2 - U(y, t) . \quad (5)$$

Determine  $U(y, t)$ .