

Exercise 1: $f : \mathbb{R} \rightarrow \mathbb{C}$ (10 points = 5+5)

Consider the function $f : \mathbb{R} \rightarrow \mathbb{C}$

$$f(x) = \sqrt{x^2 - 4m^2} \ln \left[\frac{\sqrt{x^2 - 4m^2} + x}{\sqrt{x^2 - 4m^2} - x} \right], \quad (1)$$

whereas m is a real number.

1. Determine and plot $\text{Re}[f(x)]$.
2. Determine and plot $\text{Im}[f(x)]$.

Exercise 2: $f : M \rightarrow \mathbb{C}$ (10 points)

The complex function

$$f(z) = \sqrt{z(z-1)}, \quad (2)$$

where $z \in \mathbb{C}$, is a multivalued function defined on the manifold M . Study $f(z)$. In particular, determine the manifold M and which is the value of the function on the different sheets. Discuss also the different possibilities for the branch cut.