

Exercise 1: Tree-level decays (15 points = 3 + 3 + 3 + 3 + 3)

Evaluate the decay rate(s) of the unstable state S when the full Lagrangian reads

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \quad (1)$$

whereas \mathcal{L}_0 is the free Lagrangian reads

$$\mathcal{L}_0 = \frac{1}{2} \left[(\partial_\mu S)^2 - M_0^2 S^2 \right] + \frac{1}{2} \left[(\partial_\mu \varphi_1)^2 - m_1^2 \varphi_1^2 \right] + \frac{1}{2} \left[(\partial_\mu \varphi_2)^2 - m_2^2 \varphi_2^2 \right] \quad (2)$$

and the interaction term \mathcal{L}_1 takes the following forms:

1.

$$\mathcal{L}_1 = gS\varphi_1\varphi_2 . \quad (3)$$

2.

$$\mathcal{L}_1 = g_1S\varphi_1^2 + g_2S\varphi_2^2 + gS\varphi_1\varphi_2. \quad (4)$$

3.

$$\mathcal{L}_1 = gS(\partial_\mu\varphi_1)(\partial^\mu\varphi_2) . \quad (5)$$

4.

$$\mathcal{L}_1 = g(\partial_\mu S)\varphi_1(\partial^\mu\varphi_2) . \quad (6)$$

5.

$$\mathcal{L}_1 = gS\varphi_1\varphi_2 + hS(\partial_\mu\varphi_1)(\partial^\mu\varphi_2). \quad (7)$$

Which condition must be met in order that in this case the decay rate $\Gamma_{S \rightarrow \varphi_1\varphi_2}$ vanishes?

(Determine in all the previous cases the dimension of the coupling constants g and h).

Exercise 2: Mixing (5 points)

Consider the Lagrangian

$$\mathcal{L}_0 = \frac{1}{2} \left[(\partial_\mu S_1)^2 - M_0^2 S_1^2 \right] + \frac{1}{2} \left[(\partial_\mu S_2)^2 - M_0^2 S_2^2 \right] + \alpha S_1 S_2 . \quad (8)$$

Determine the physical fields \tilde{S}_1 and \tilde{S}_2 . Which is the value of the mixing angle? Then, calculate their decay width when

$$\mathcal{L}_1 = gS_1\varphi^2 \quad (9)$$

whereas the field φ has mass m .