

Exercise 1: A new type of mixing (8 points)

Consider the Lagrangian

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} (\partial_\mu S_1)^2 + \frac{1}{2} (\partial_\mu S_2)^2 + \alpha (\partial_\mu S_1) (\partial^\mu S_2) \\ & + \frac{1}{2} [(\partial_\mu \varphi)^2 - m^2 \varphi^2] + g\varphi S_1^2. \end{aligned} \quad (1)$$

Which range is allowed for the parameter  $\alpha$ ?

Determine the decay rates of the field  $\varphi$ .

Hint: one has to make an appropriate transformation in order to 'eliminate' the mixing term  $\alpha (\partial_\mu S_1) (\partial^\mu S_2)$ .

Exercise 2: Decays of a scalar particle in vector particles (7 = 3 + 4 points)

Consider the free Lagrangian

$$\mathcal{L}_0 = \frac{-1}{4} \rho_{\mu\nu}^2 + \frac{m_\rho^2}{2} \rho_\mu^2 + \frac{1}{2} [(\partial_\mu \varphi)^2 - m^2 \varphi^2] \quad (2)$$

where  $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$ ;  $\rho_\mu$  describes a vector field, while  $\varphi$  describes a scalar field.

Determine the decay width  $\varphi \rightarrow \rho\rho$  generated by the following interaction Lagrangians:

1.

$$\mathcal{L}_1 = g\varphi \rho_\mu \rho^\mu. \quad (3)$$

2.

$$\mathcal{L}_1 = g\varphi (\partial_\mu \rho_\nu) (\partial^\mu \rho^\nu). \quad (4)$$

Exercise 3: Decay of the Higgs particles into two photons (5 points)

Consider the Lagrangian

$$\mathcal{L} = \frac{-1}{4} F_{\mu\nu}^2 + \frac{1}{2} [(\partial_\mu H)^2 - m_H^2 H^2] + gH F_{\mu\nu} F^{\mu\nu} \quad (5)$$

where  $H$  represents the Higgs field;  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $A_\mu$  is the photon field.

Evaluate  $\Gamma_{H \rightarrow \gamma\gamma}$ .

Hint: use the following polarization sum for the photon:

$$\sum_{a=1,2} \varepsilon_\mu^a \varepsilon_\nu^a = -g_{\mu\nu}. \quad (6)$$