DECAYS IN QFT - WS 2012/2013

Sheet 8 **21/12/2012**

Exercise 1: Polarization vector (4 points)

Proove that the polariation vectors $\varepsilon^a_\mu(k)$ of a massive vector field with mass m and momentum k fullfills the following relation:

$$T_{\mu\nu}(k) = \sum_{a=1,2,3} \varepsilon^a_\mu(k)\varepsilon^a_\nu(k) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2} \ . \tag{1}$$

Hint: the tensor $T_{\mu\nu}(k)$ can be written as

$$T_{\mu\nu}(k) = A g_{\mu\nu} + B k_\mu k_\nu \ . \tag{2}$$

Use the properties $k^\mu \varepsilon^a_\mu(k) = 0$ and $\varepsilon^a_\mu(k)\varepsilon^{b,\mu}(k) = -\delta^{ab}$ to determine A and B.

Exercise 2: Decays of a vector particle into fermion-antifermion pair (8 points)

Consider the Lagrangian

$$\mathcal{L} = \frac{-1}{4} Z^2_{\mu\nu} + \frac{m_Z^2}{2} Z^2_\mu + \bar{\psi}\left[i\gamma^\mu\partial_\mu - m\right]\psi + g Z_\mu \bar{\psi}\gamma^\mu\psi \ , \tag{3}$$

where $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$; Z_μ describes a vector field, while ψ describes a fermion field.
Determine the decay width $Z \to \bar{\psi}\psi$.

Exercise 3: Decay of a scalar field into two fermions (8 points)

Consider the free Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}\left[(\partial_\mu D)^2 - m_H^2 D^2\right] + + \bar{\psi}\left[i\gamma^\mu\partial_\mu - m\right]\psi \tag{4}$$

where D represents a scalar field and ψ a fermion field. The interaction Lagrangian is given by

$$\mathcal{L}_1 = igD\left(\psi^t C\gamma^5\psi - \bar{\psi}C\gamma^5\bar{\psi}^t\right) \tag{5}$$

where C is the conjugation matrix and g a real coupling constant.
Determine the decay of D into two fermions.