

Exercise 1: Neutron extended lifetime (4 points)

Consider a ultrarelativistic neutron with energy $E \gg m_n$ w.r.t. to the reference frame of the Earth (m_n is the neutron mass).

Determine the expression for the half-life of the neutron in our reference frame. Calculate also its numerical value for the following energies: $E = 1$ TeV, $E = 100$ TeV, $E = 10^{18}$ eV.

Exercise 2: Kynetic energy (6 points = 4 + 2)

1. Consider a two-body decay $S \rightarrow \varphi_1 + \varphi_2$, in which the particles S, φ_1, φ_2 have masses M, m_1, m_2 , respectively.

Determine the kynetic energies T_1 and T_2 of the outgoing particles. Show that, in the limit $m_1 \ll m_2 \simeq M, T_1 \gg T_2$.

2. In the β -decay of the neutron consider the situation in which a neutrino is emitted at rest. Determine the kynetic energy of the outgoing electron.

Exercise 3: Dalitz-plot and 3-body decay (10 points = 2 + 4 + 4)

We study a three-body decay. A particle P and mometum $p = (M, \vec{0})$ decays into three particles φ_1, φ_2 , and φ_3 with masses m_1, m_2, m_3 and momenta k_1, k_2, k_3 . The quantities m_{ij}^2 are defined as $m_{ij}^2 = (k_i + k_j)^2$.

1. Suppose that m_{12}^2 and m_{23}^2 are known. Which is the value of m_{13}^2 ? (Hint: calculate the sum $m_{12}^2 + m_{23}^2 + m_{13}^2$).
2. Determine the form of the Dalitz plot in the case in which $m_1 = m_2 = m_3 = 0$.
3. Consider the case in which the three outgoing particles are identical: $\varphi_1 = \varphi_2 = \varphi_3 = \varphi$. Determine the overall effect of having three identical particles. (Hint: study the degeneracy of the the decay amplitudes and the neccessary prefactors. No need to perform explicit calculations here.)