

# Allgemeine Relativitätstheorie mit dem Computer

*VORLESUNGSREIHE  
JOHANN WOLFGANG GOETHE UNIVERSITÄT  
29. JUNI, 2023*

*MATTHIAS HANAUSKE*

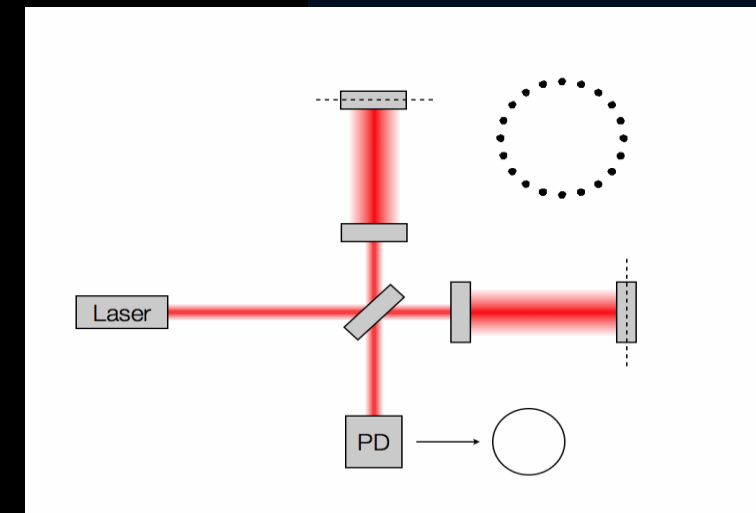
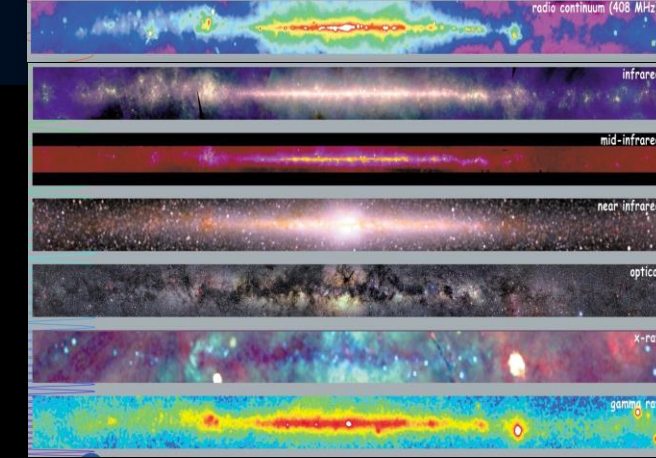
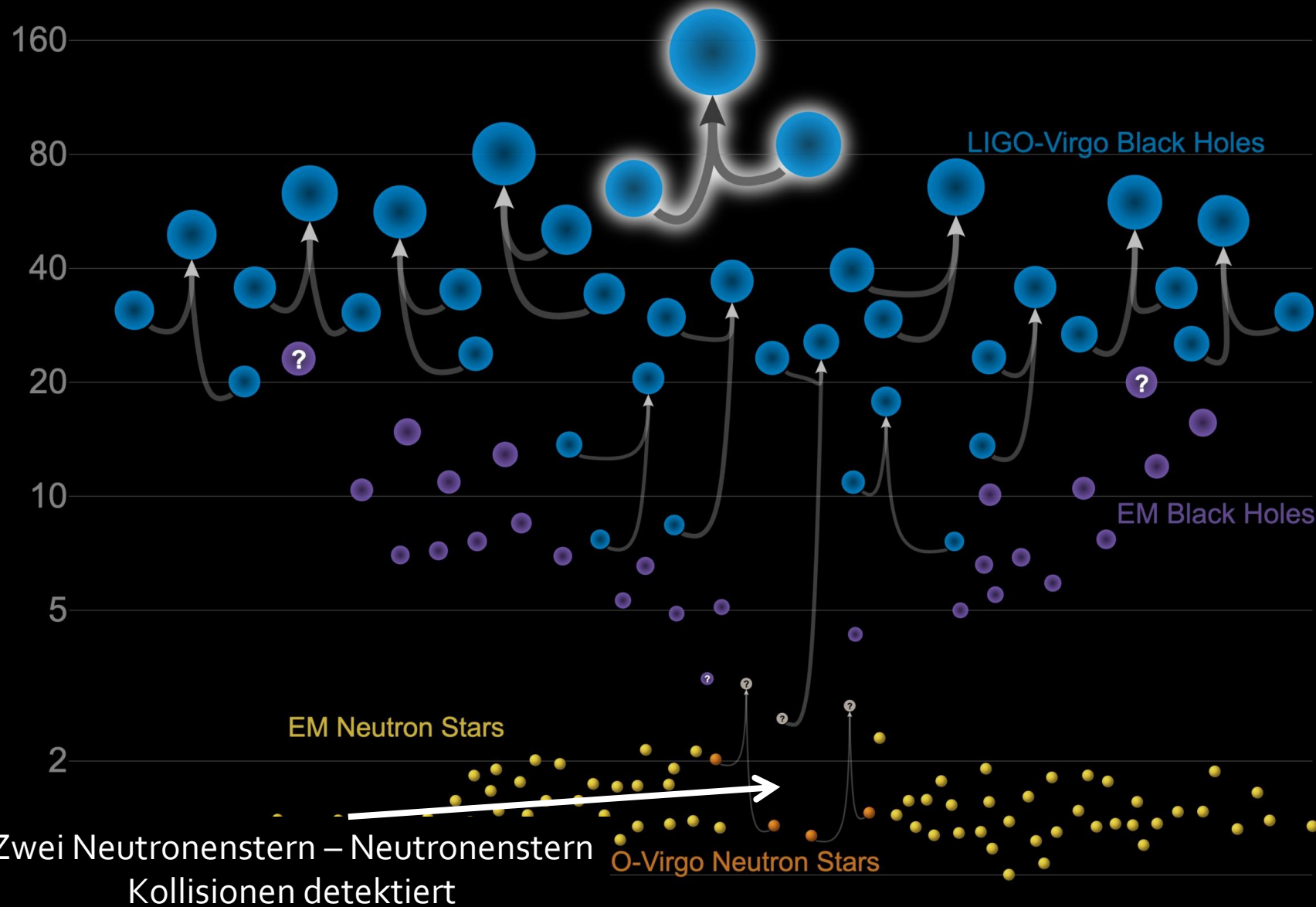
*FRANKFURT INSTITUTE FOR ADVANCED STUDIES  
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10. Vorlesung

# Gravitational Waves (GW): The new way of looking at our universe

Es ist so als ob die Menschheit eine neue wundersame Brille hat, ein neues Sinnesorgan, mit denen sie zuvor unbeobachtbare Ereignisse in unserem Universum wahrnehmen kann.

*Im Jahre 2015 gelang die erste Detektion einer Gravitationswelle*



# Das lang ersehnte Ereignis GW170817

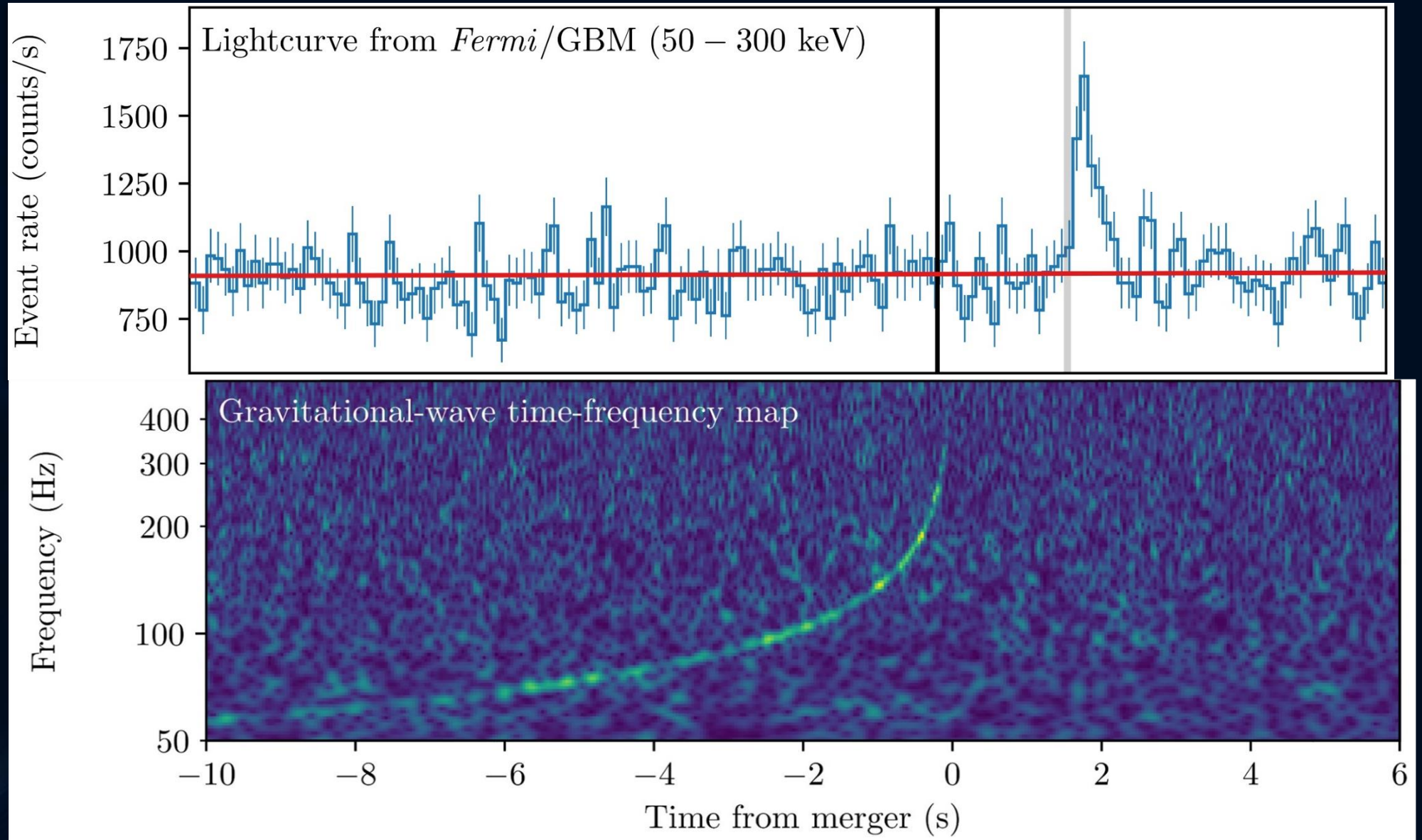
	Low-spin priors ( $ \chi  \leq 0.05$ )	High-spin priors ( $ \chi  \leq 0.89$ )
Primary mass $m_1$	$1.36-1.60 M_\odot$	$1.36-2.26 M_\odot$
Secondary mass $m_2$	$1.17-1.36 M_\odot$	$0.86-1.36 M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio $m_2/m_1$	$0.7-1.0$	$0.4-1.0$
Total mass $m_{\text{tot}}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\text{rad}}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_L$	$40^{+8}_{-14}$ Mpc	$40^{+8}_{-14}$ Mpc
Viewing angle $\Theta$	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	$\leq 700$	$\leq 700$
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	$\leq 800$	$\leq 1400$

17. August 2017

Gravitationswelle einer  
Neutronenstern Kollision gemessen!

# Die gemessene Gravitationswelle und der darauf folgende hochenergetische Lichtblitz

Der von dem  
Gammastrahlen  
Detektor FERMI  
gemessene  
Gammastrahlen  
Ausbruch  
(1.7 Sekunden später)



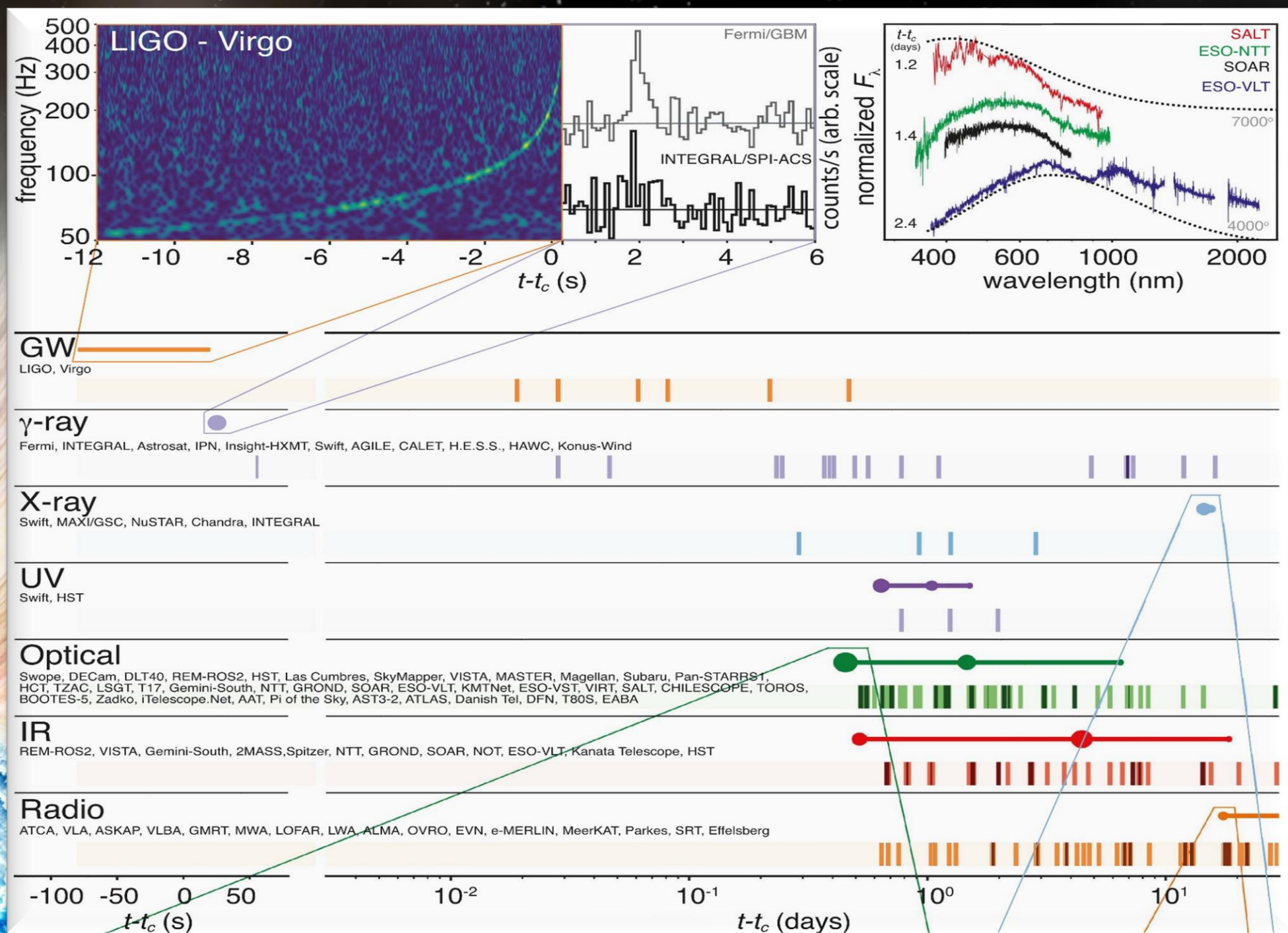
Die von dem  
Gravitationswellen  
Detektor LIGO  
detektierte  
Frequenz der  
Gravitationswelle



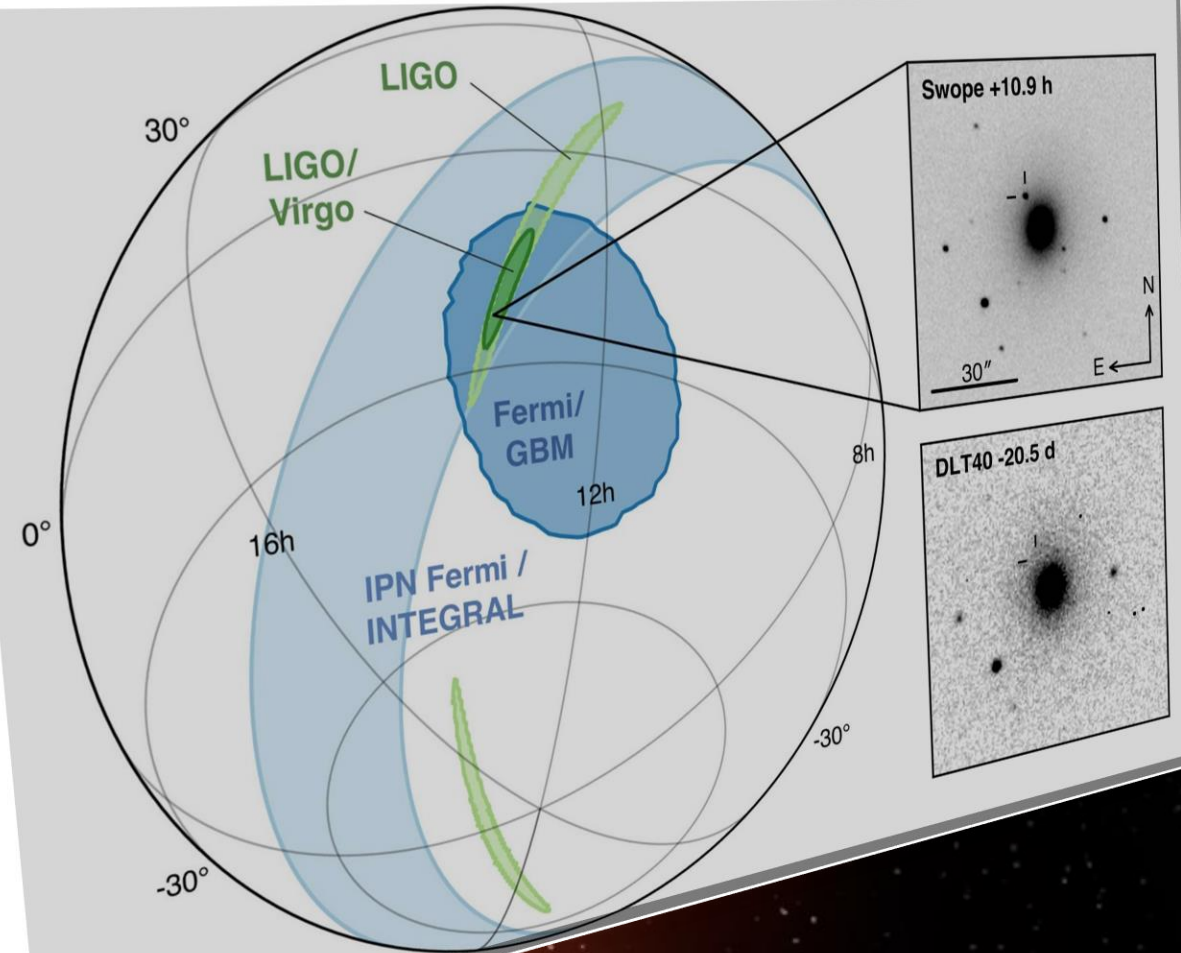
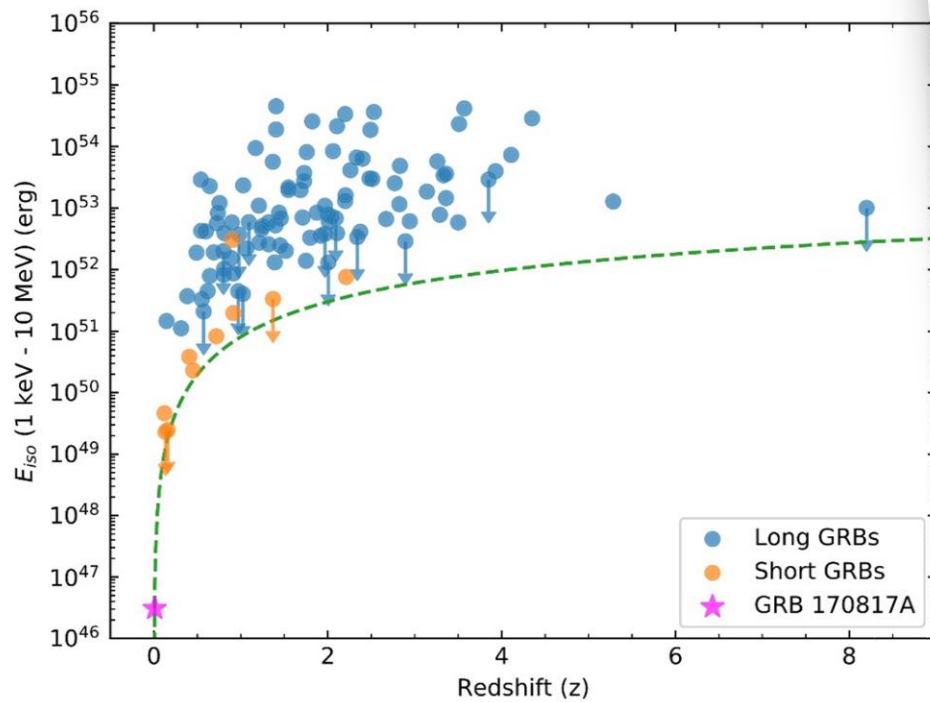
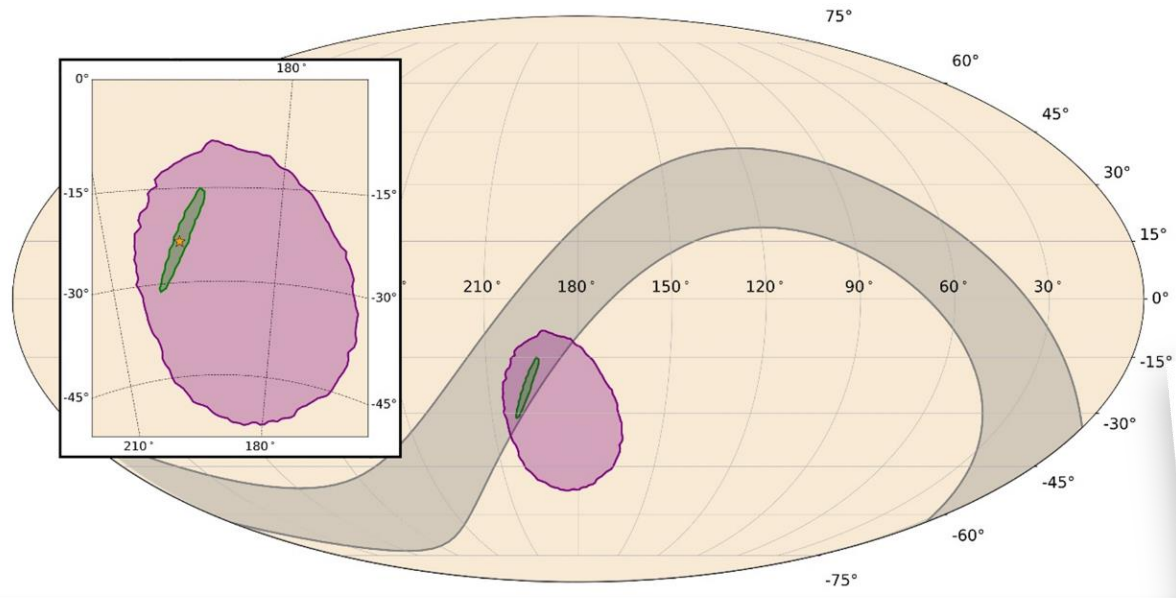
# GW170817

Tage, Wochen und Monate später detektierten weltweit unterschiedliche Teleskope (radio, infrarot, optische,...) eine Nachstrahlung dieser Neutronenstern Kollision (eine so genannte Kilonova)

Multi-Messenger Observations of a Binary Neutron Star Merger, LIGO and Virgo Collaborations together with 50 teams of electromagnetic and neutrino astronomers, *Astrophys. J. Lett.* 848, L12 (2017)

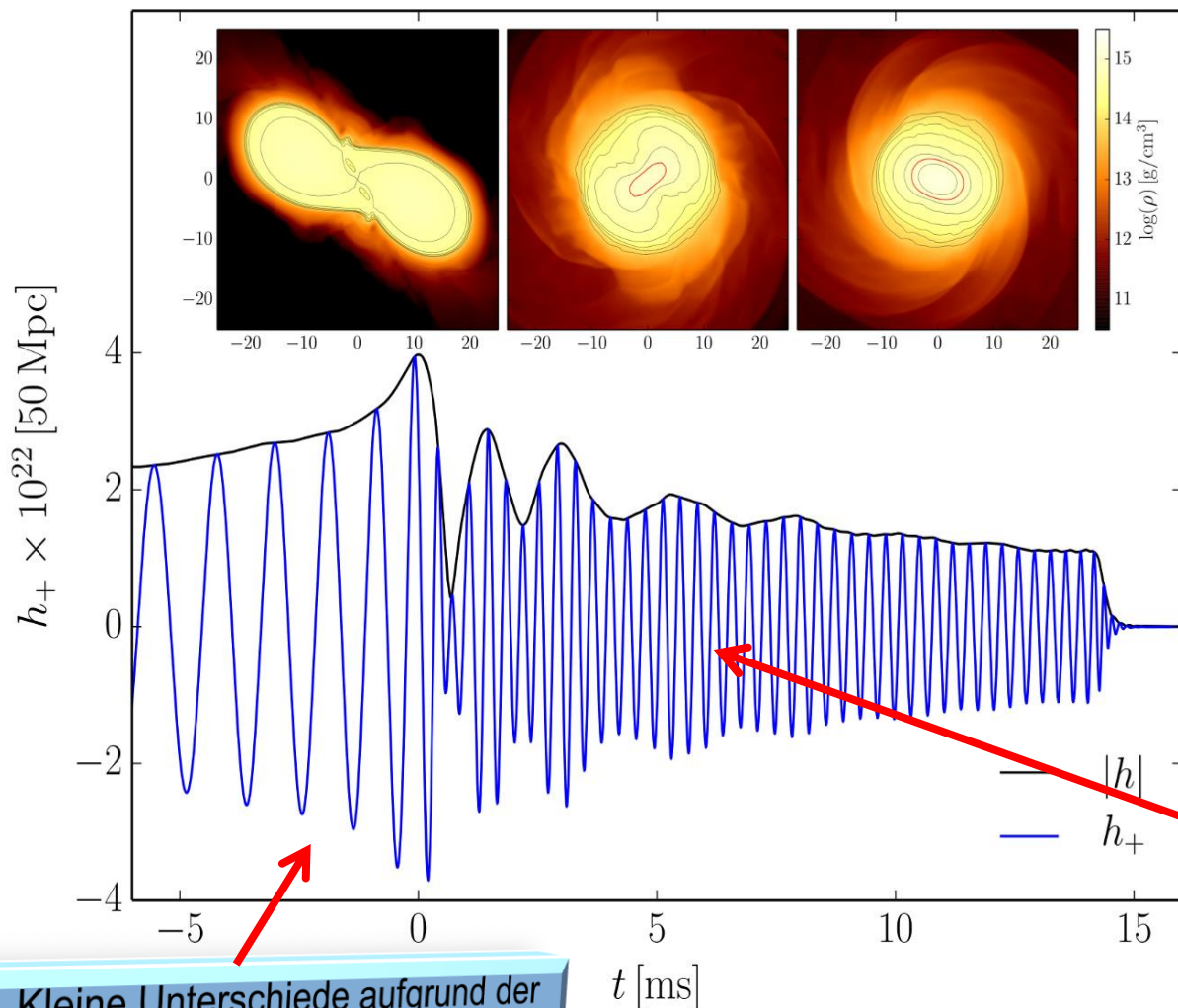


# GW170817, GRB170817A Localisation and unusual dimness of GRB

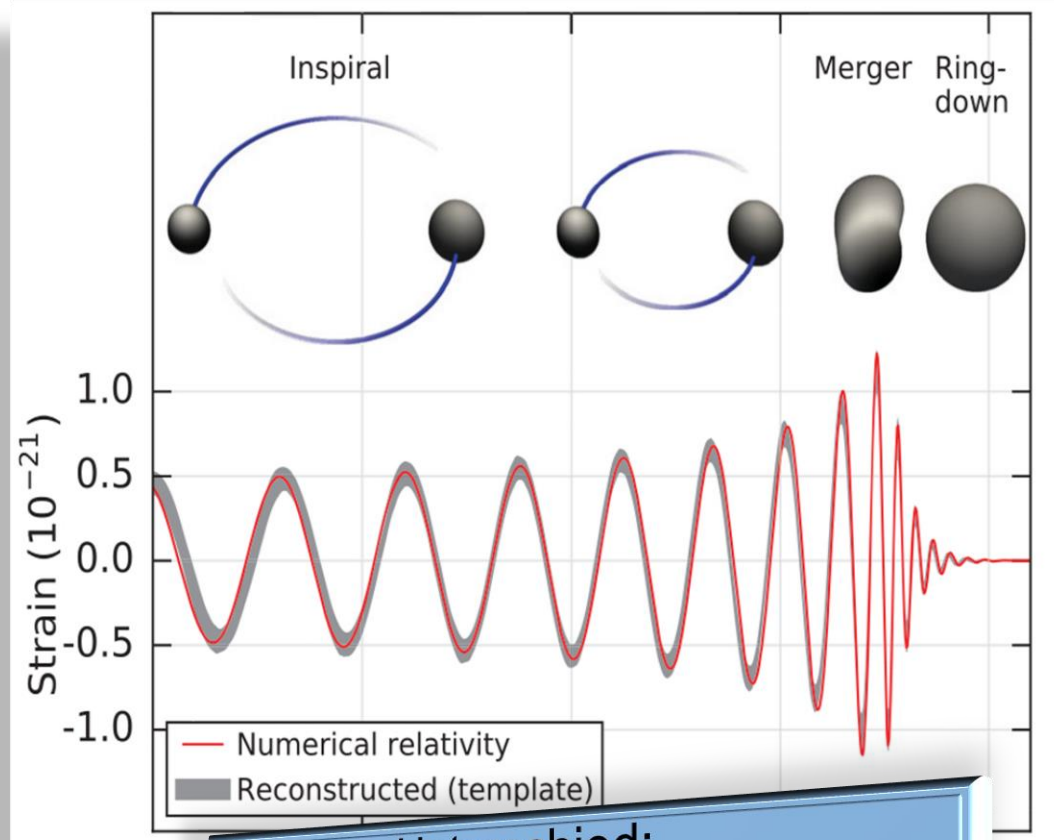


# Gravitationswellen von Neutronenstern Kollisionen

## Neutronenstern Kollision (Simulation)



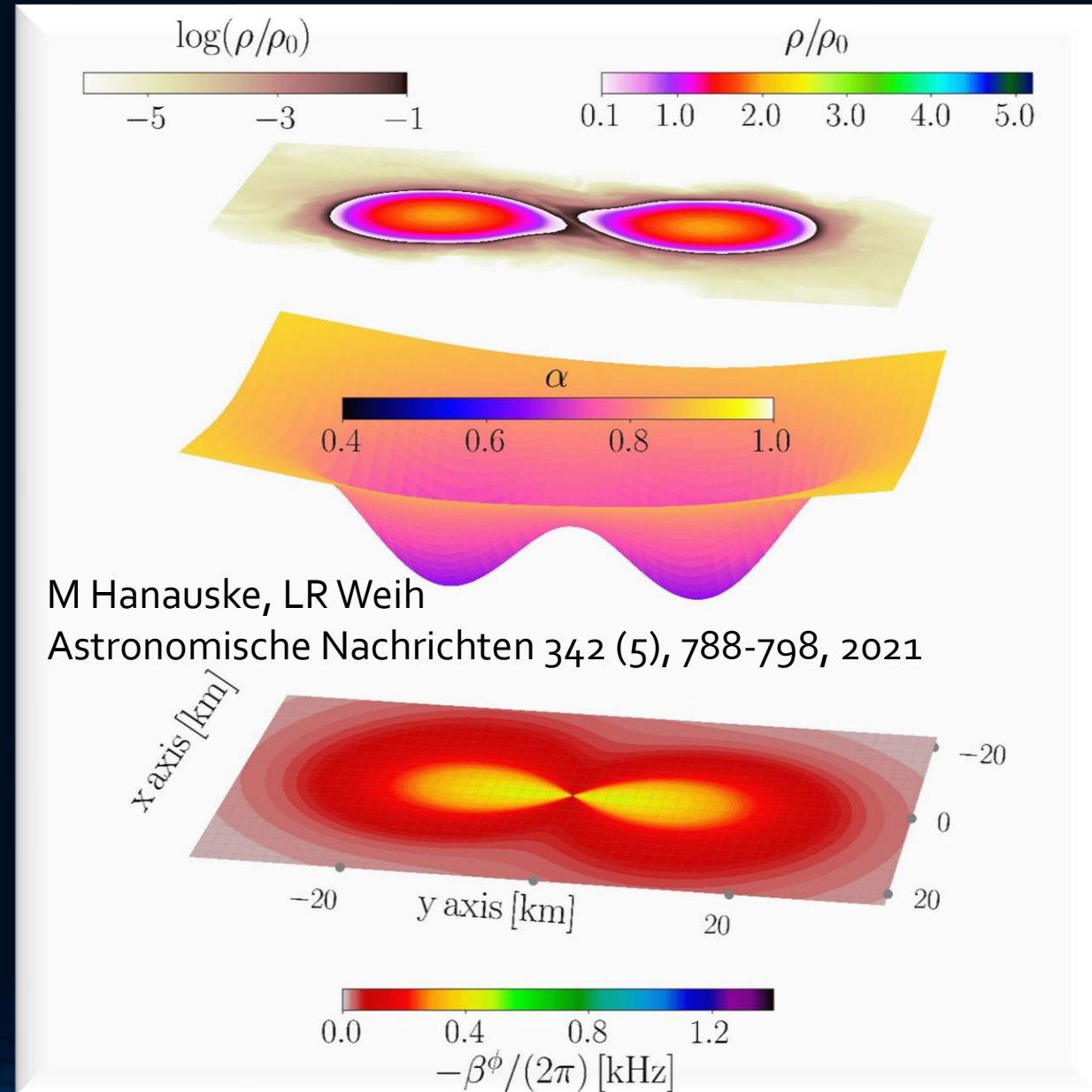
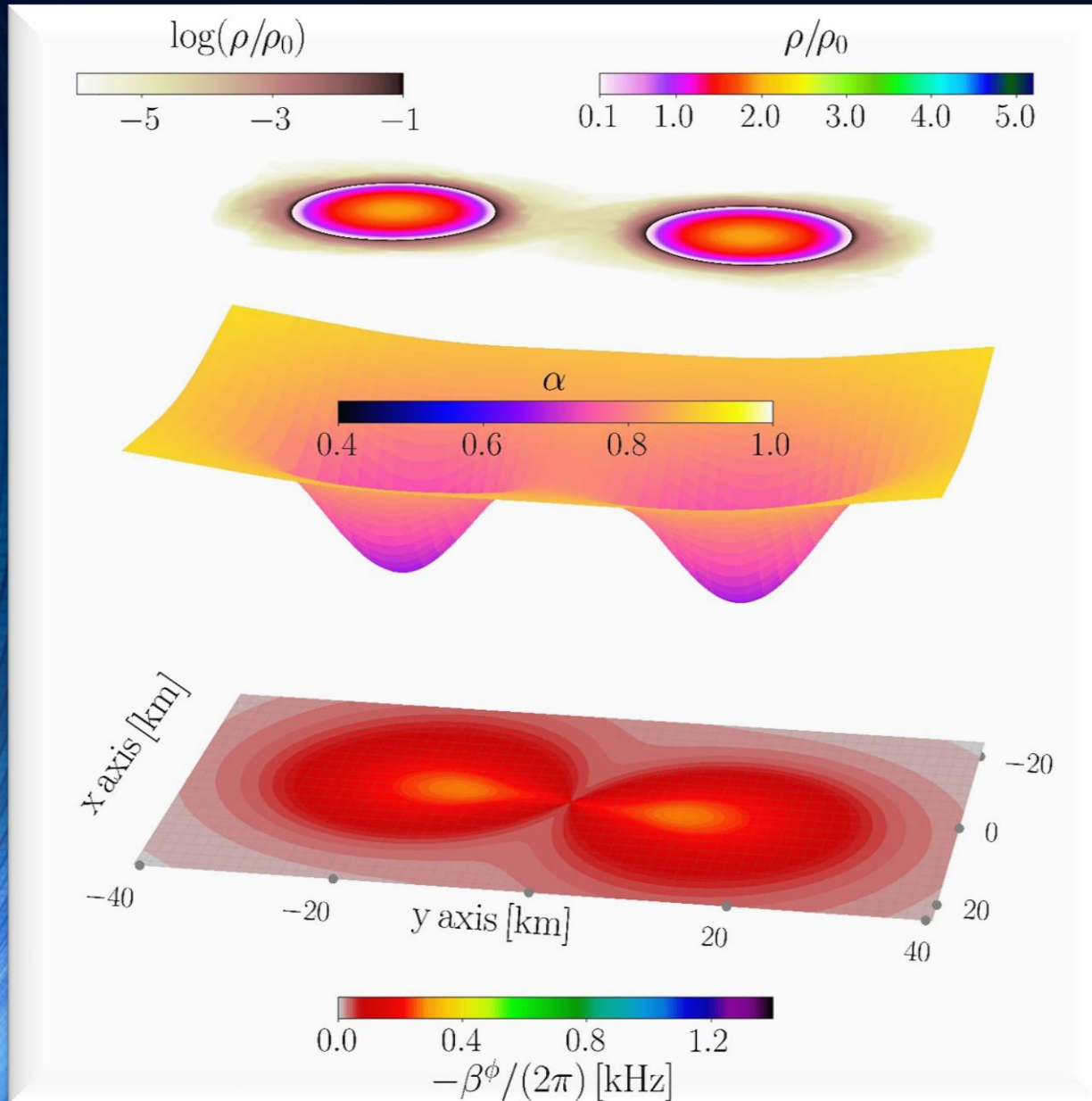
## Kollision zweier schwarzer Löcher



Unterschied:  
Bei Neutronenstern Kollisionen  
gibt es oft eine  
**Post-Kollisionsphase**

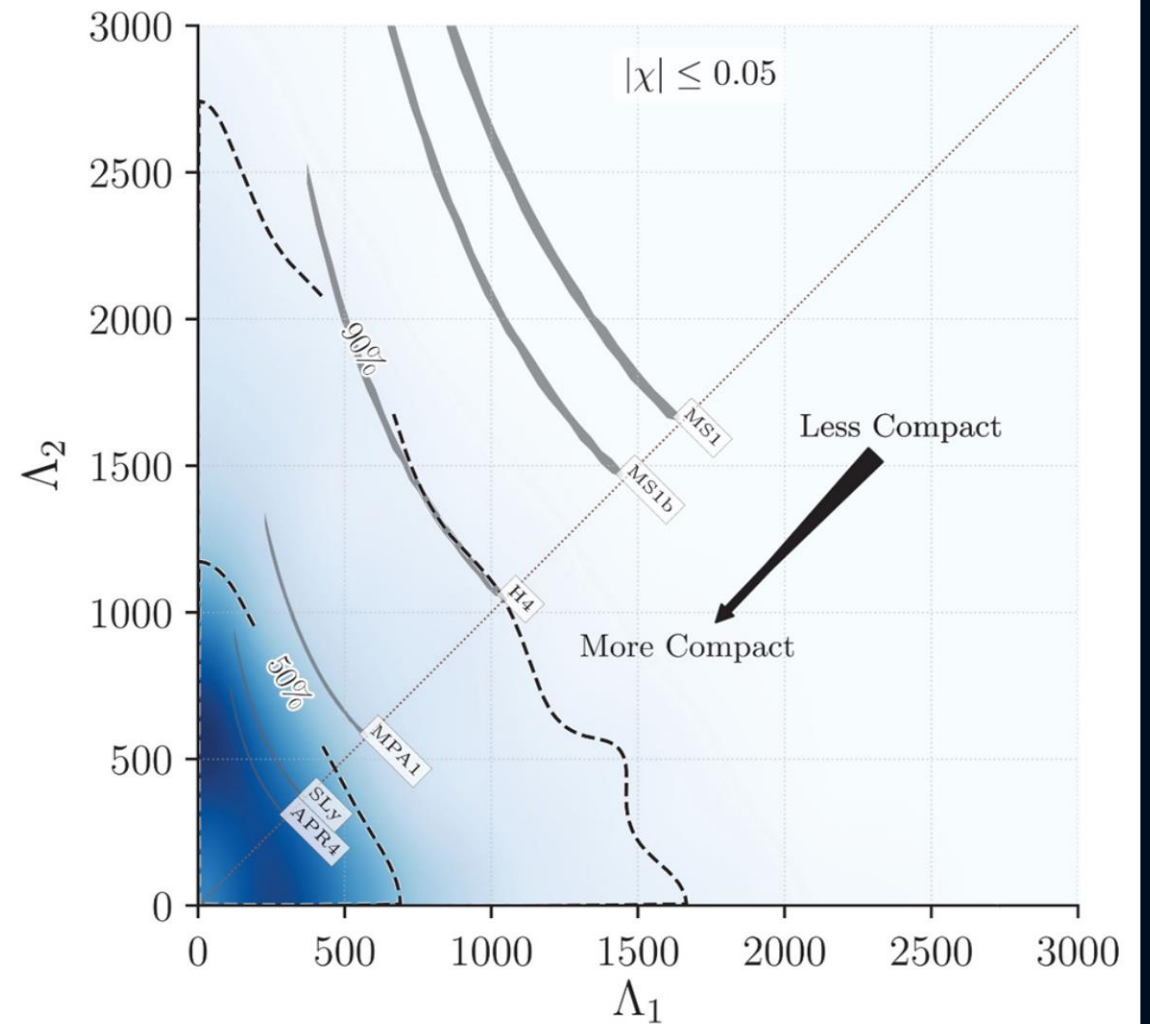
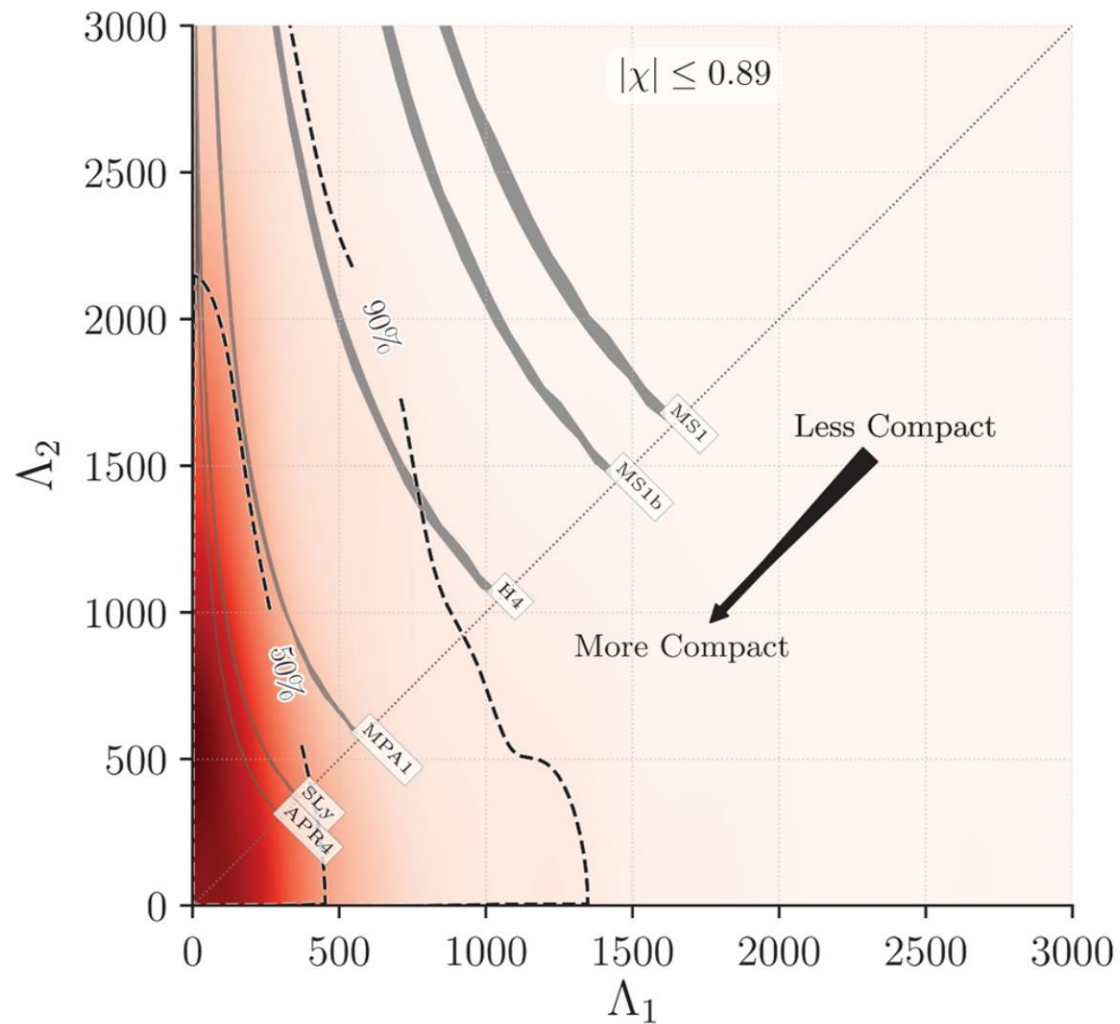
Kleine Unterschiede aufgrund der  
Verformbarkeit der Neutronensterne

# The late inspiral phase (density, lapse and shift)

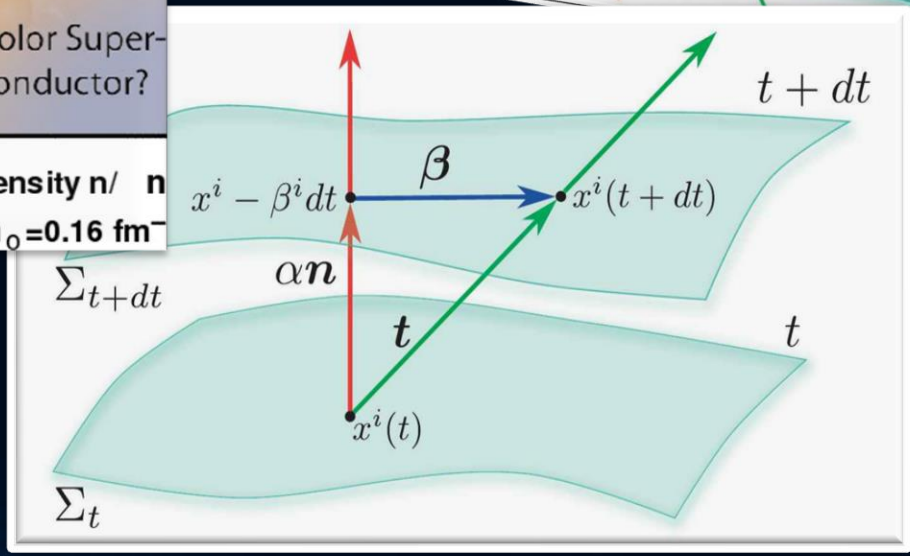
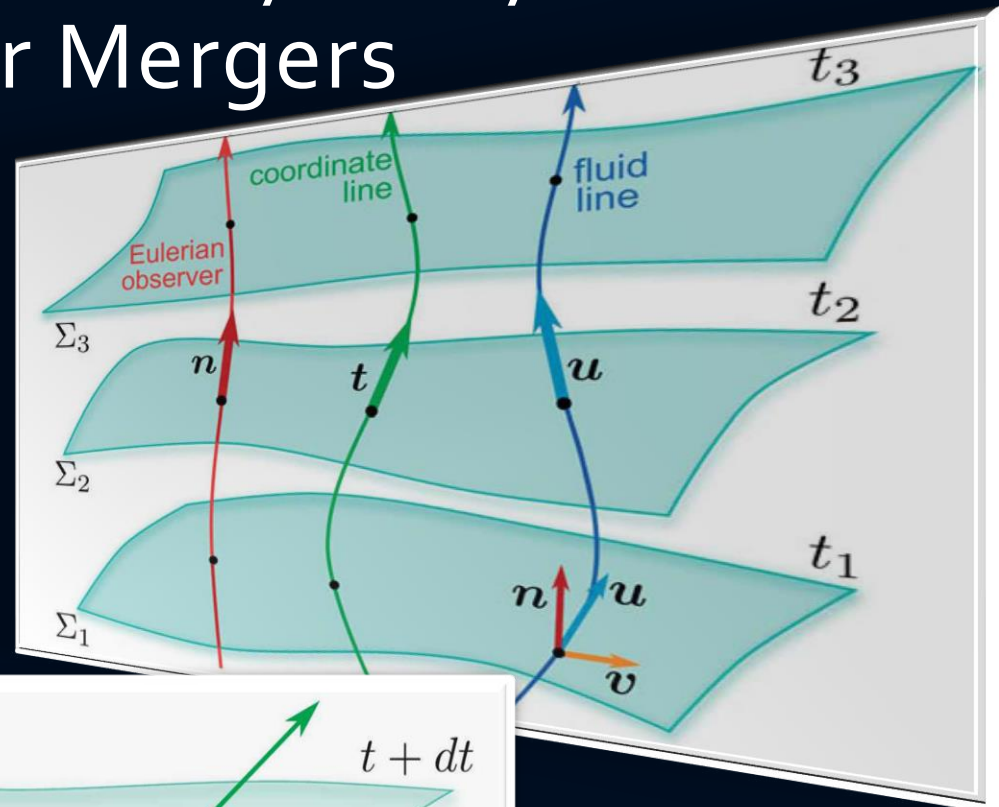
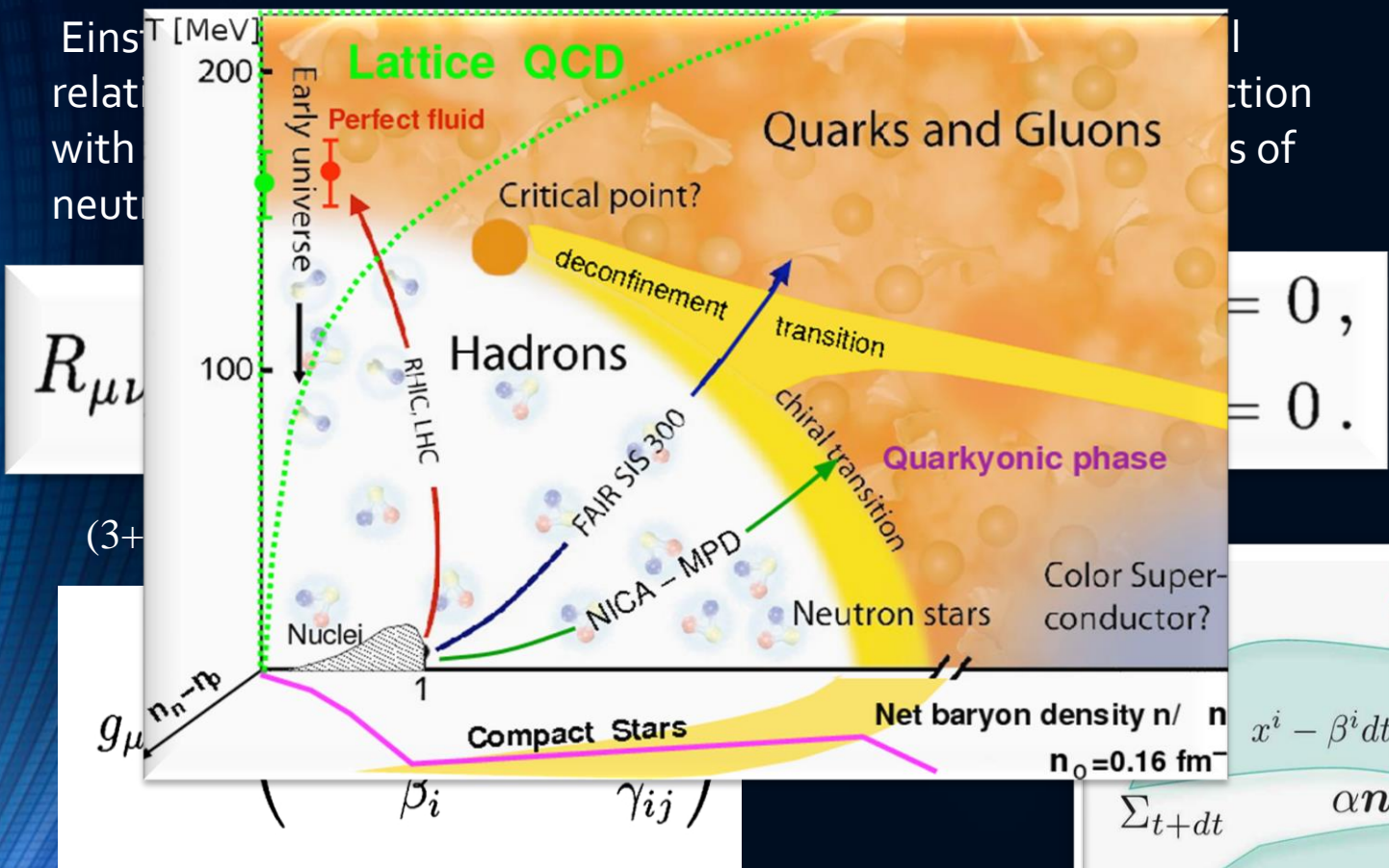




# GW170817: Tidal Deformability Restrictions on the Equation of State (EOS) (for high and low spin assumption)

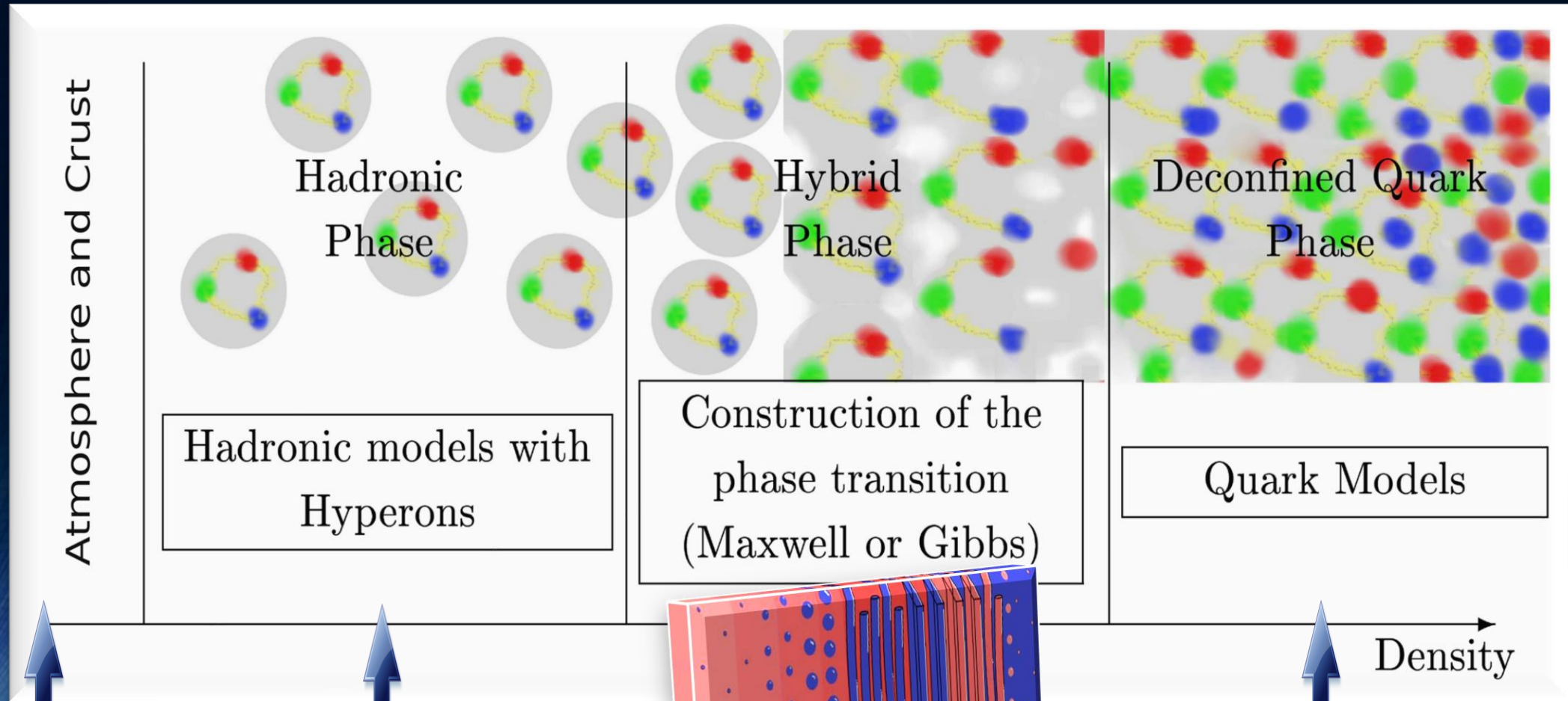


# Numerical Relativity and Relativistic Hydrodynamics of Binary Neutron Star Mergers



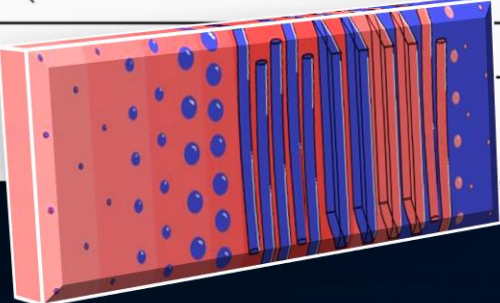
$$d\tau^2 = \alpha^2(t, x^j) dt^2 \quad x^i_{t+dt} = x^i_t - \beta^i(t, x^j) dt$$

# Das Innere von hybriden Sternen



Äußere Eisenkruste

Neutronen und einige Protonen, Elektronen vielleicht auch Hyperonen



Phasenübergang vielleicht Pasta-Struktur

Befreite reine Quarkphase

Density

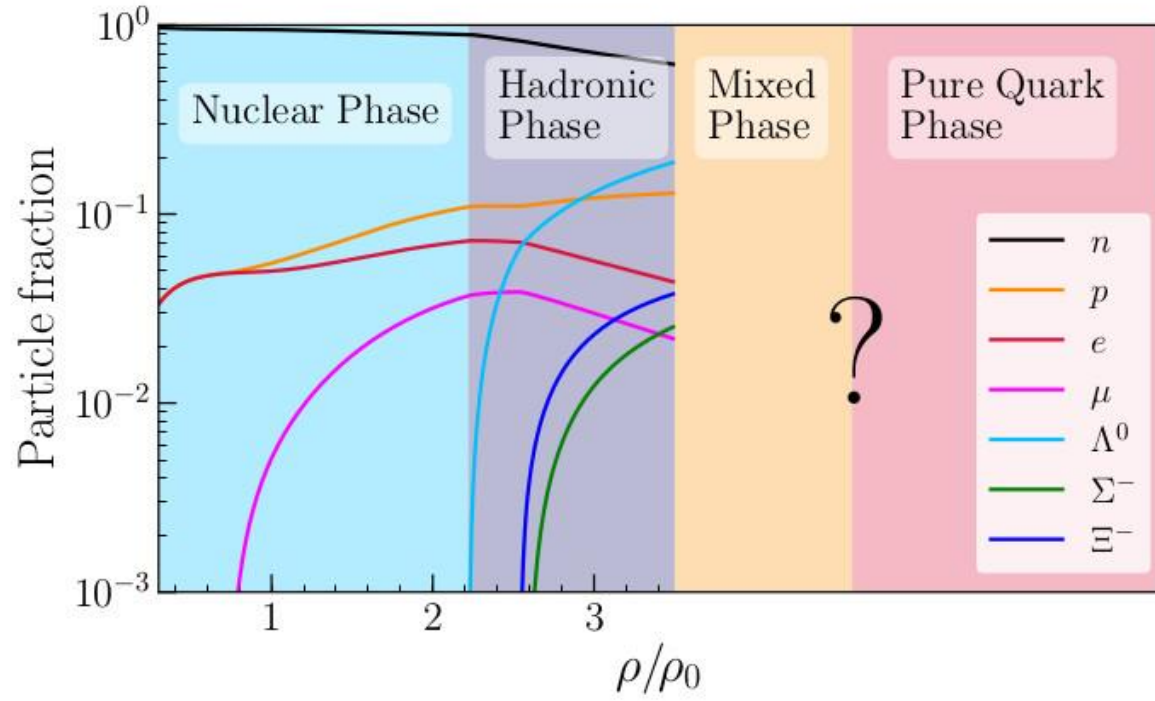
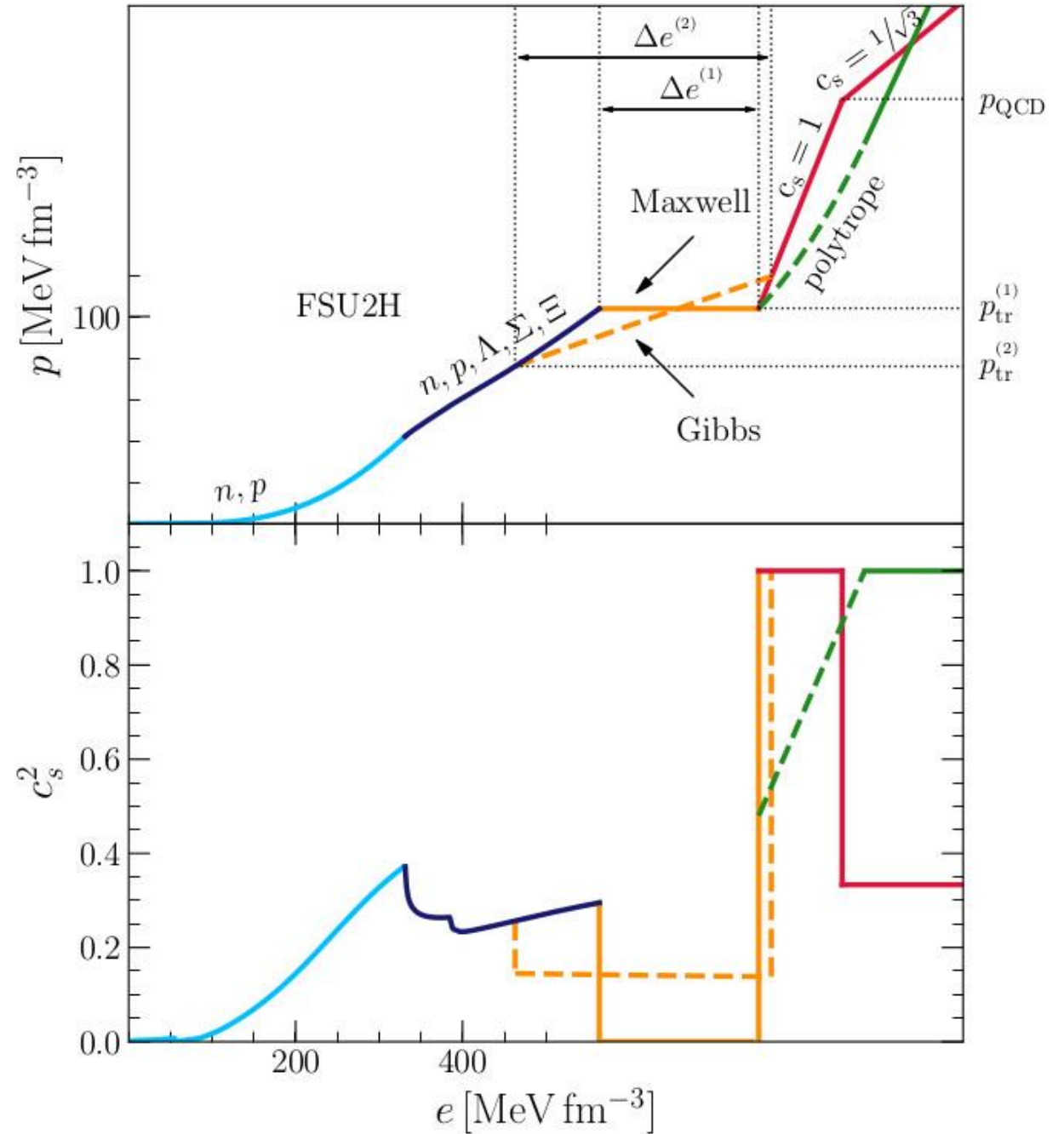


FIG. 1. Particle fractions as functions of the baryonic density for the FSU2H model [69, 70] up to the point where the HQPT is implemented, giving rise to a phase of deconfined quark matter which can be separated from the nuclear (or hadronic) phase by a mixed phase of hadrons and quarks. We note that the actual fractions of nucleons/hyperons and quarks  $u, d, s$  in the mixed and quark phases cannot be determined with the parametrizations used in this work.



# The Hadron-Quark Phase Transition and the Third Family of Compact Stars (Twin Stars)

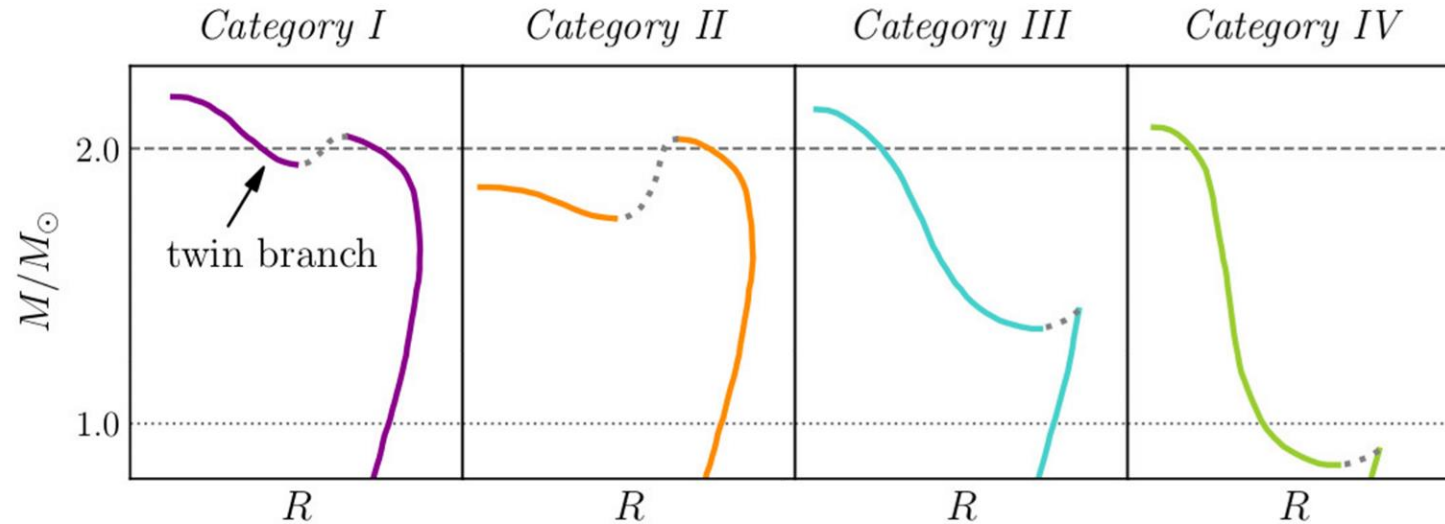
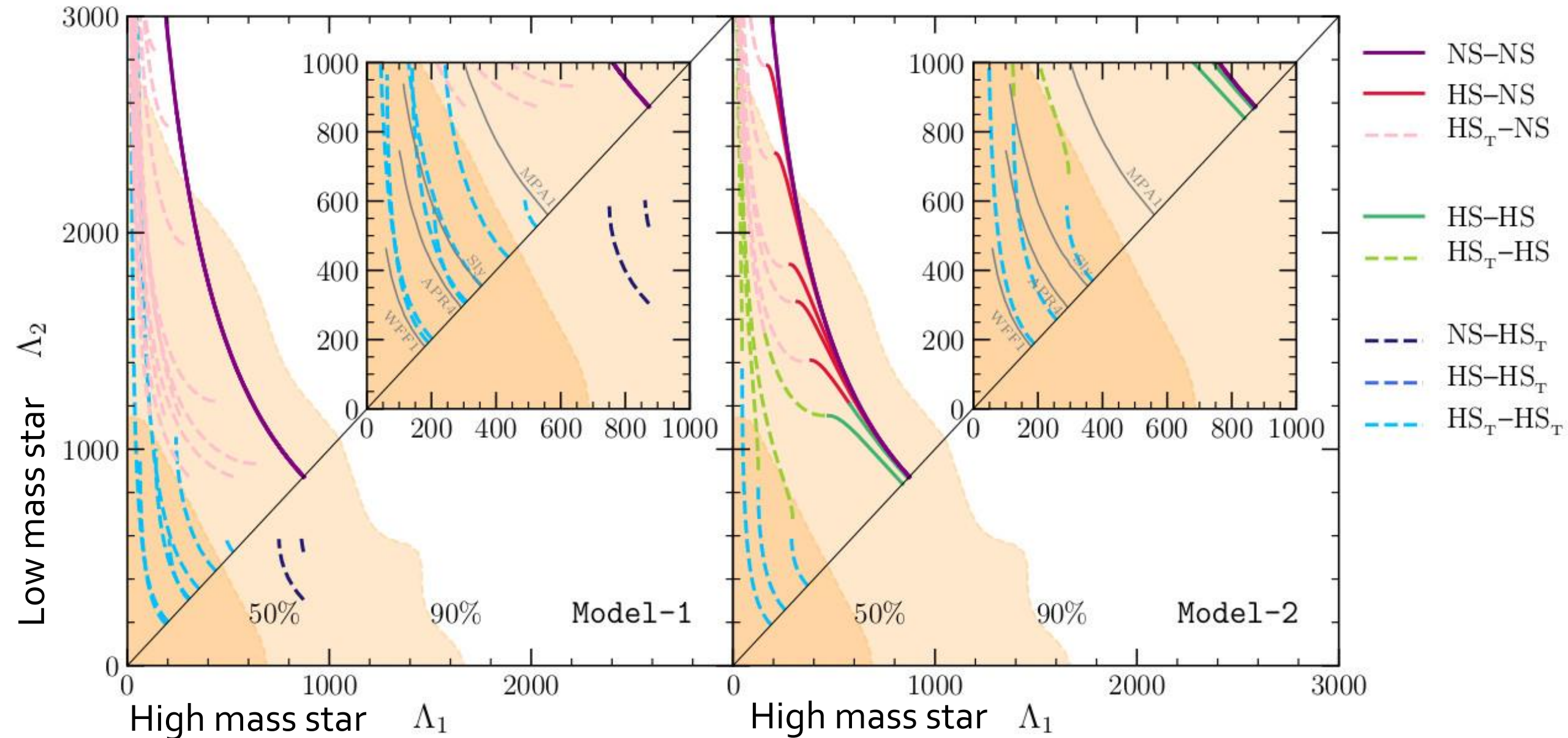


FIG. 3. Schematic behaviour of the mass–radius relation for the twin-star categories *I–IV* defined in the text. Note the appearance of a “twin” branch with a mixed or pure-quark phase; the twin branch has systematically smaller radii than the branch with a nuclear or hadronic phase. The colors used for these categories will be employed also in the subsequent figures.

- Glendenning, N. K., & Kettner, C. (1998). Nonidentical neutron star twins. *Astron. Astrophys.*, 353(LBL-42080), L9.
- Sarmistha Banik, Matthias Hanauske, Debades Bandyopadhyay and Walter Greiner, Rotating compact stars with exotic matter, *Phys.Rev.D* 70 (2004) p.12304
- I.N. Mishustin, M. Hanauske, A. Bhattacharyya, L.M. Satarov, H. Stöcker, and W. Greiner, Catastrophic rearrangement of a compact star due to quark core formation, *Physics Letters B* 552 (2003) p.1-8
- M.Alford and A. Sedrakian, Compact stars with sequential QCD phase transitions. *Physical review letters*, 119(16), 161104 (2017)..
- D.Alvarez-Castillo and D.Blaschke, High-mass twin stars with a multipolytrope equation of state. *Physical Review C*, 96(4), 045809 (2017) .
- A. Ayriyan, N.-U. Bastian, D. Blaschke, H. Grigorian, K. Maslov, D. N. Voskresensky, How robust is a third family of compact stars against pasta phase effects?, arXiv:1711.03926 [nucl-th]





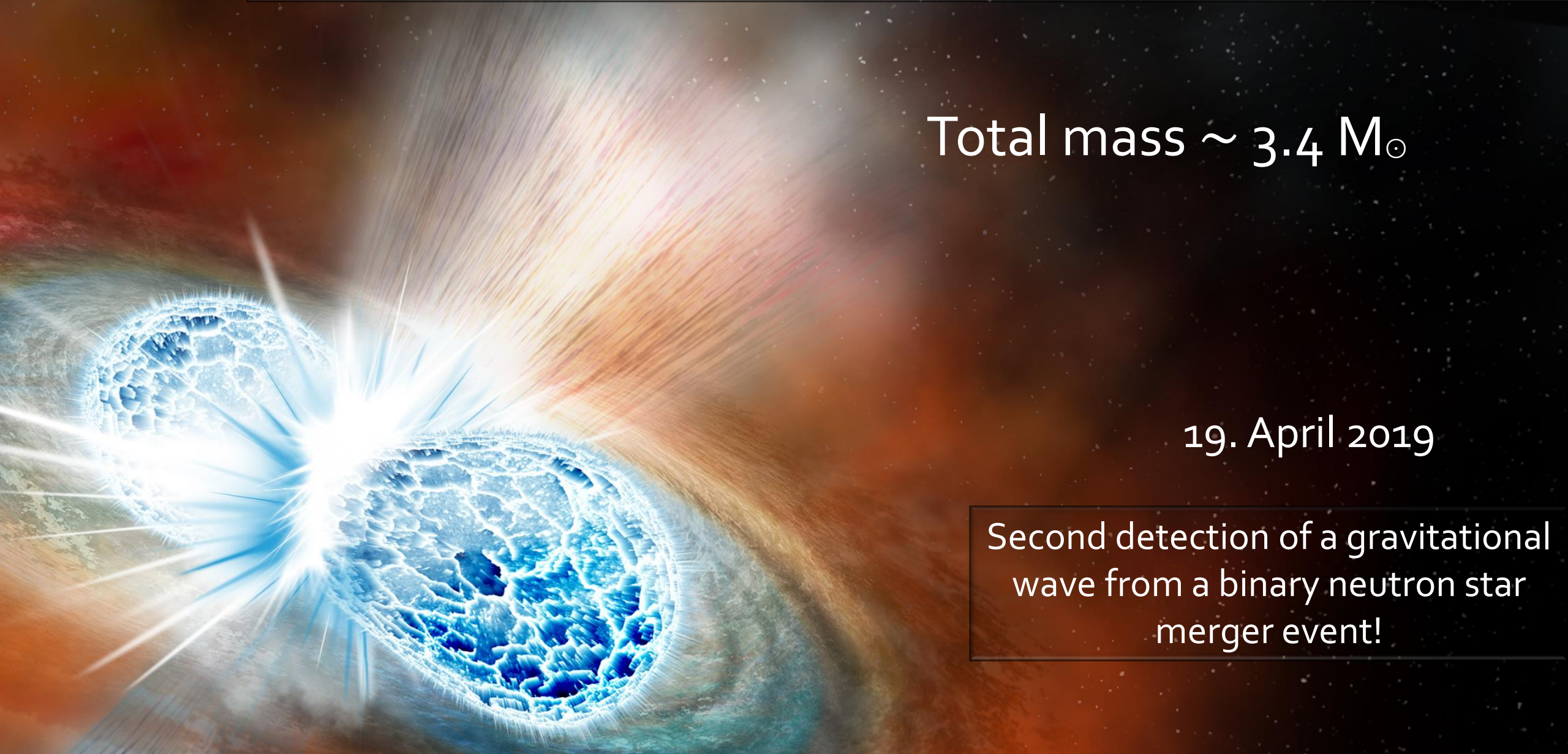
In a binary hybrid star merger the two masses of the individual stars can be different ( $q < 1$ ). As a result, the tidal deformability and the stars composition can be different. In this plot the total mass of the binary system has been fixed to the measured chirp mass of GW170817 ( $M = 1.188 M_{\text{solar}}$ ) and the different curve show results for EOSs of Category III.

# The second event: GW190425

Total mass  $\sim 3.4 M_{\odot}$

19. April 2019

Second detection of a gravitational wave from a binary neutron star merger event!



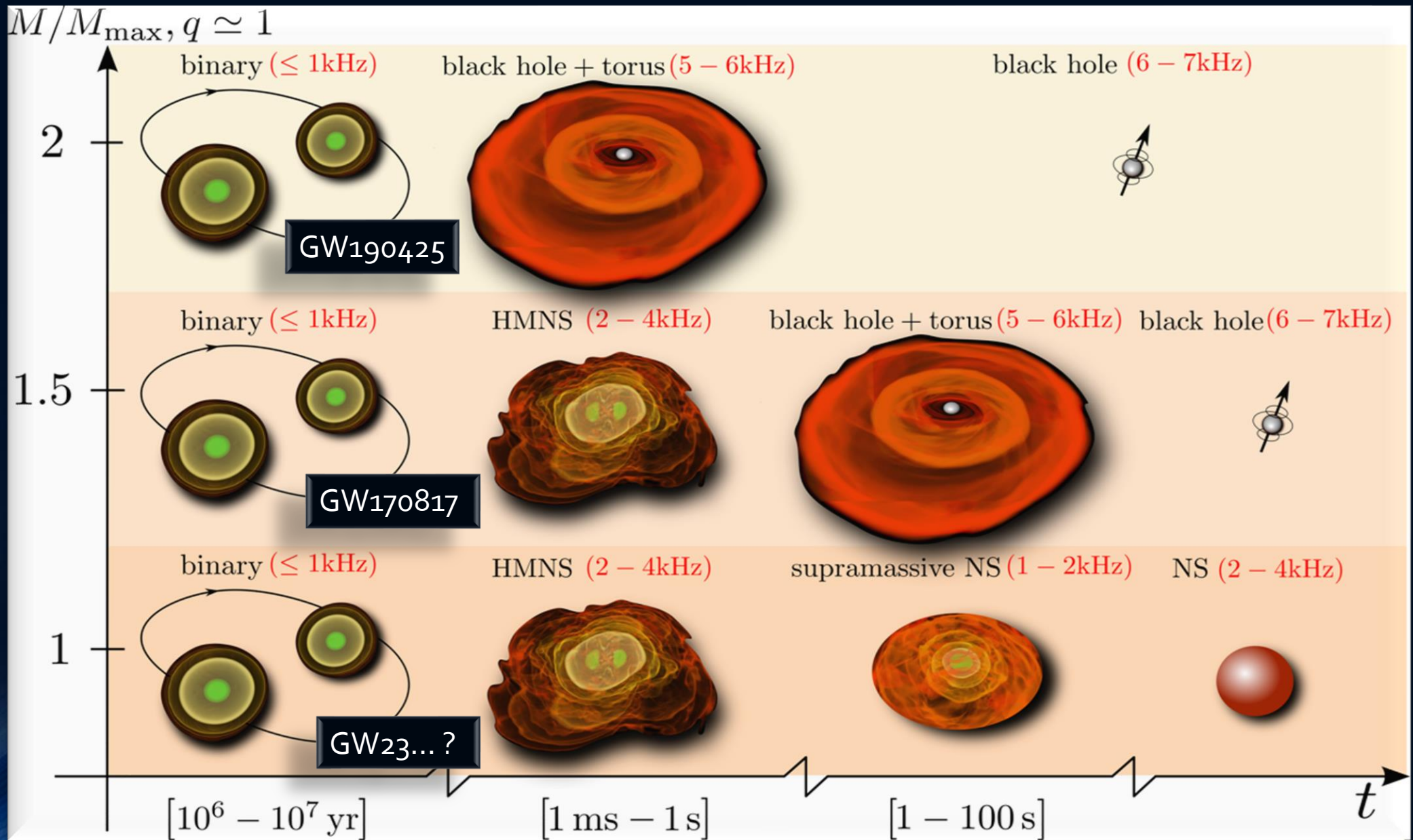


# Was geschieht wenn zwei Neutronensterne miteinander kollidieren?

Zwei sehr massive Neutronensterne

Zwei mittelschwere Neutronensterne

Zwei leichte Neutronensterne



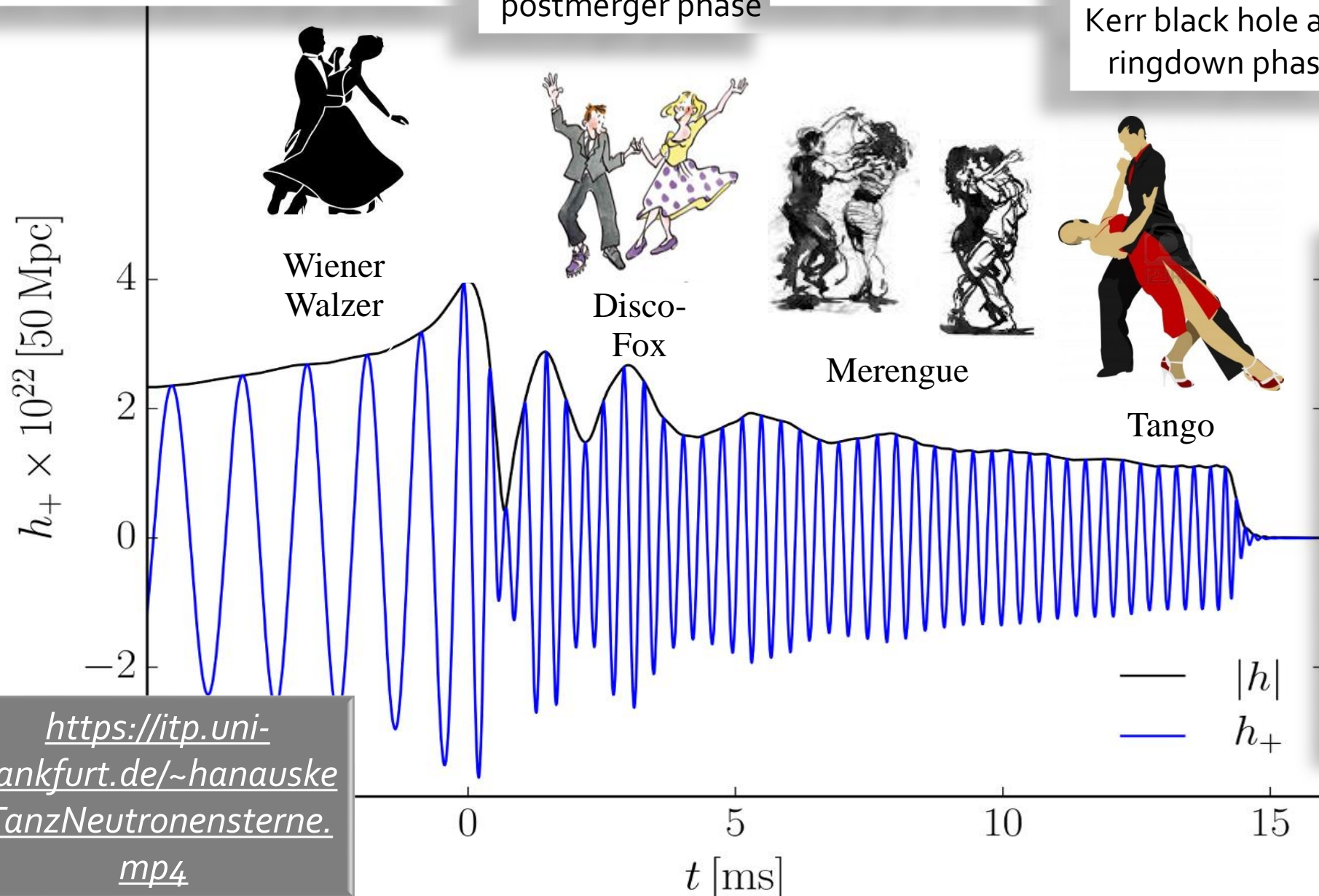
# The different Phases of a Binary Compact Star Merger Event

Late inspiral and merger phase

Transient early postmerger phase

Postmerger phase

Collapse to the Kerr black hole and ringdown phase



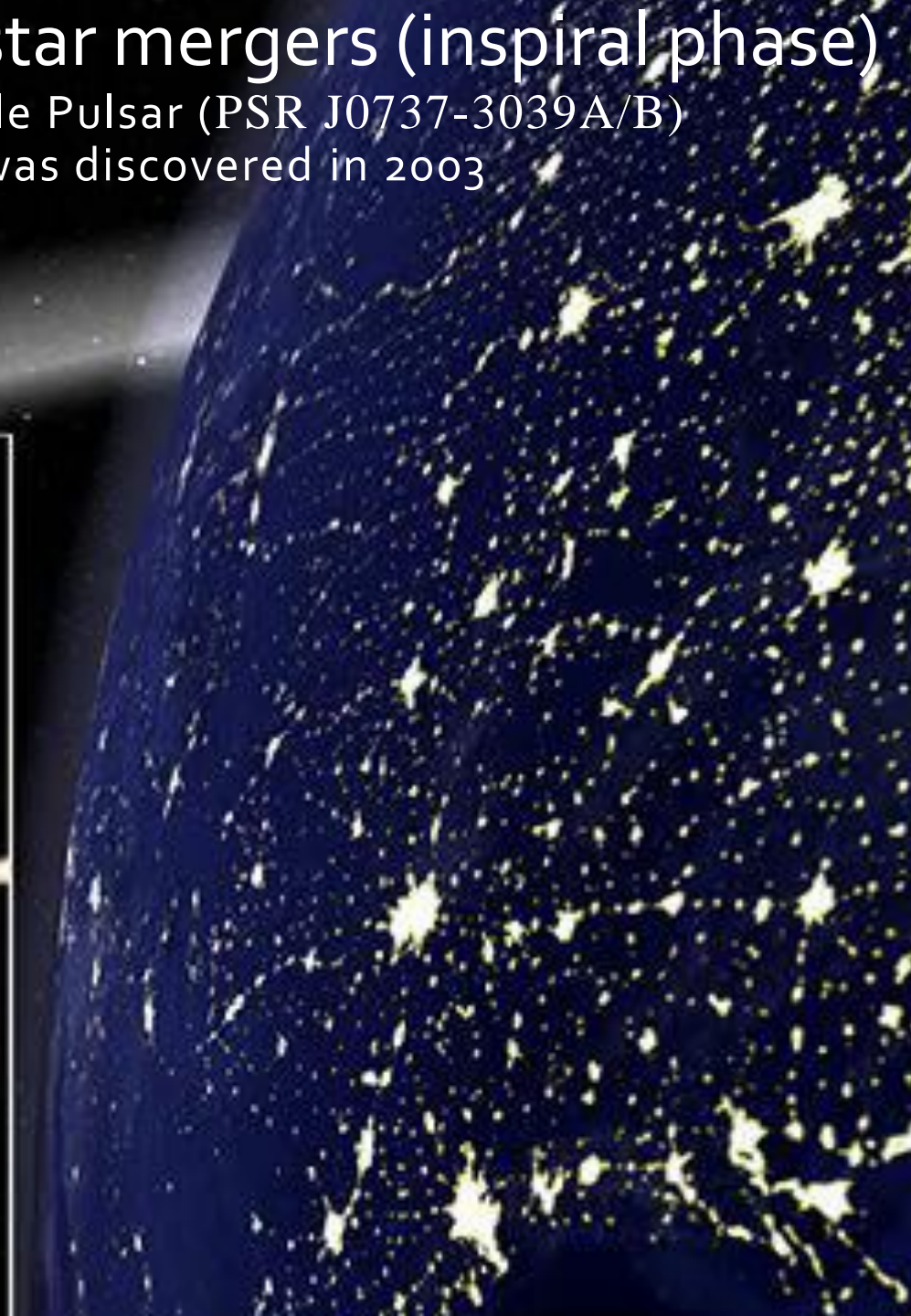
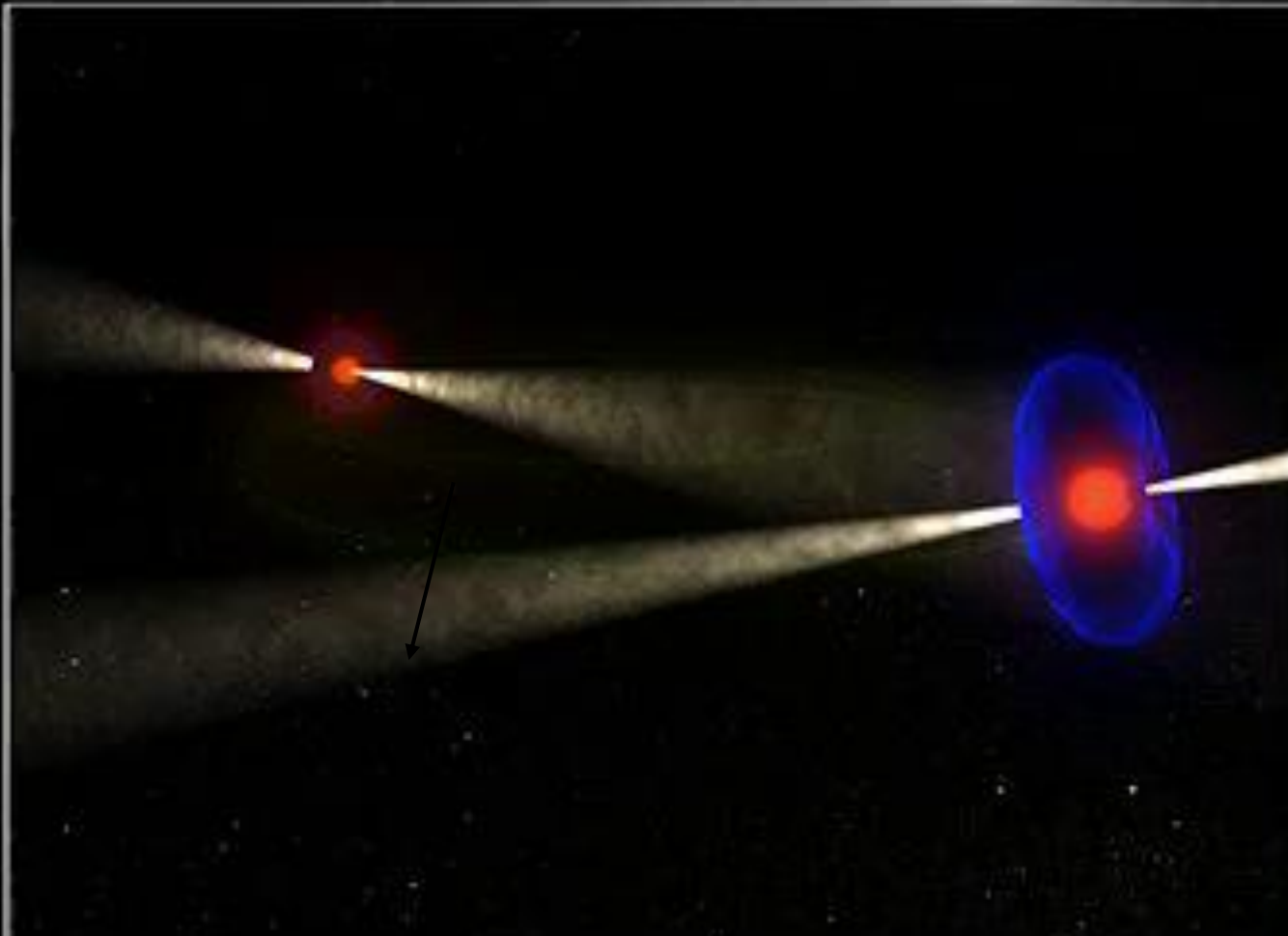
*Why exactly these dances?  
Details in*

"Binary Compact Star Mergers and the Phase Diagram of Quantum Chromodynamics", Matthias Hanauske and Horst Stöcker, Discoveries at the Frontiers of Science, 107-132; Springer, Cham (2020)

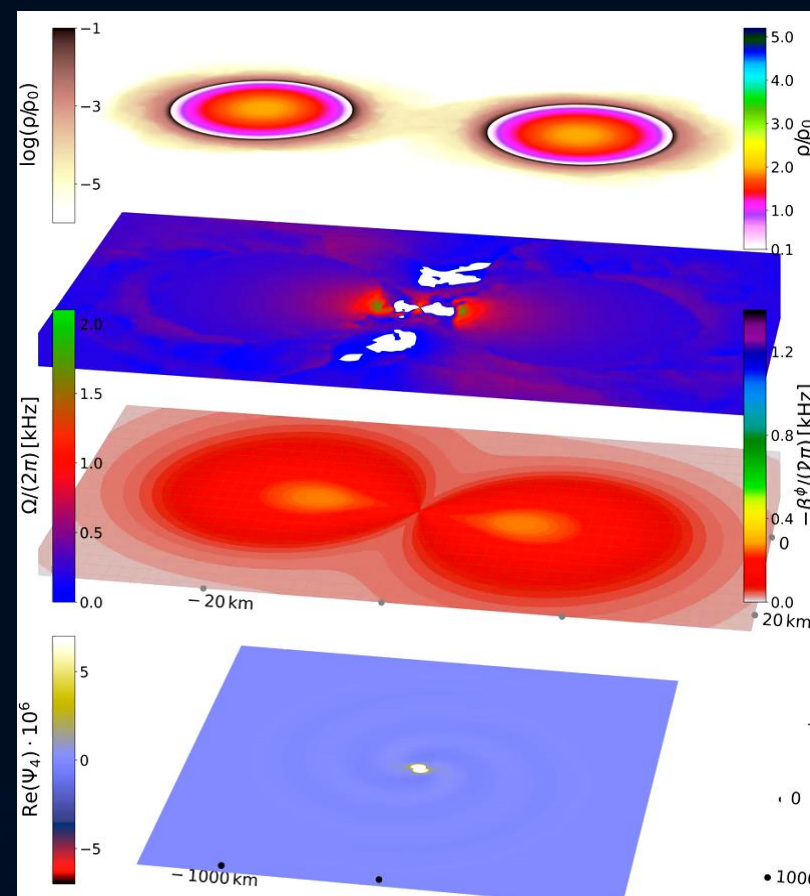
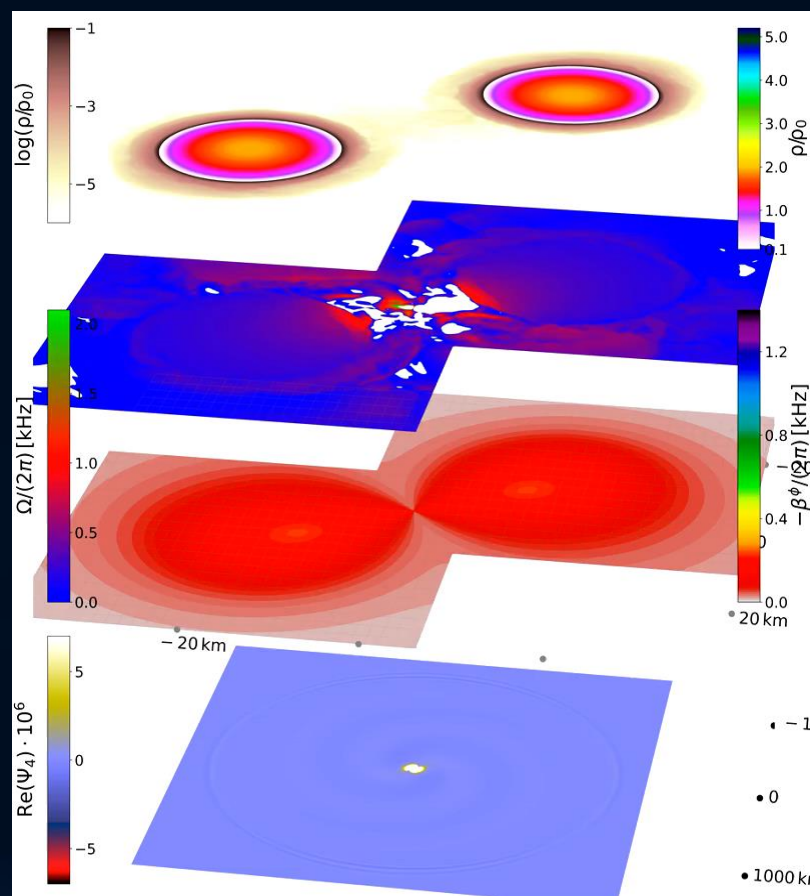
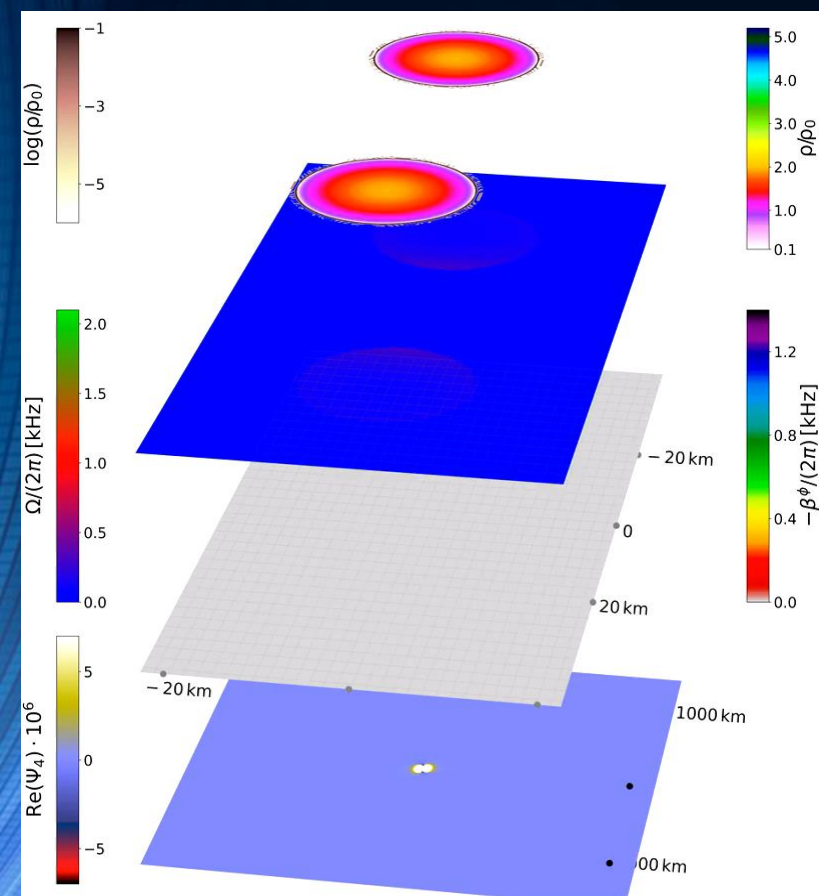
<https://itp.uni-frankfurt.de/~hanauske/TanzNeutronensterne.mp4>

# Binary neutron star mergers (inspiral phase)

The Double Pulsar (PSR J0737-3039A/B)  
was discovered in 2003



# Binary neutron star mergers (the late inspiral phase)



# Computer Simulation of a neutron star merger

*Credits: Cosima Breu, David Radice und Luciano Rezzolla*



**Log of density**

8.5 14



$\lg(\rho)$  [g/cm<sup>3</sup>]

**Temperature**

0 50



T [MeV]

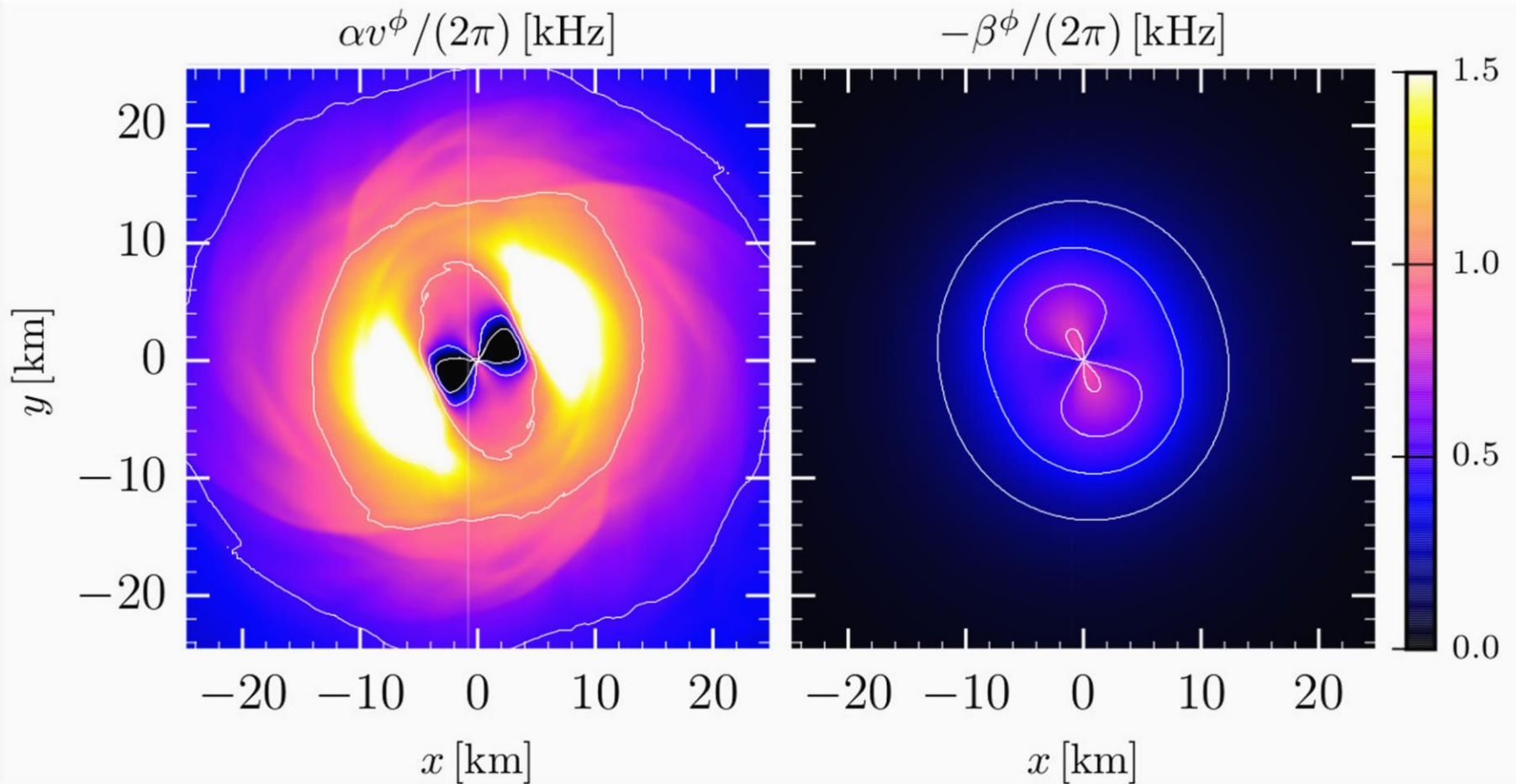
# The Angular Velocity in the (3+1)-Split

The angular velocity  $\Omega$  in the (3+1)-split is defined as a function of the lapse function  $\alpha$ , the  $\phi$ -component of the fluid velocity  $v^\phi$  of the fluid (spatial projection of the fluid velocity), and the lapse function  $\alpha$ .

$$\Omega(x, y, z, t) = \frac{\alpha v^\phi}{2\pi} - \frac{\beta^\phi}{2\pi}$$

Angular velocity  
 $\Omega$

Lapse



Focus: Inner core of the

M. Shibata, K. Taniguchi, and K. Uryu, Phys. Rev. D 71, 084021 (2005)

M. Shibata and K. Taniguchi, Phys. Rev. D 73, 064027 (2006)

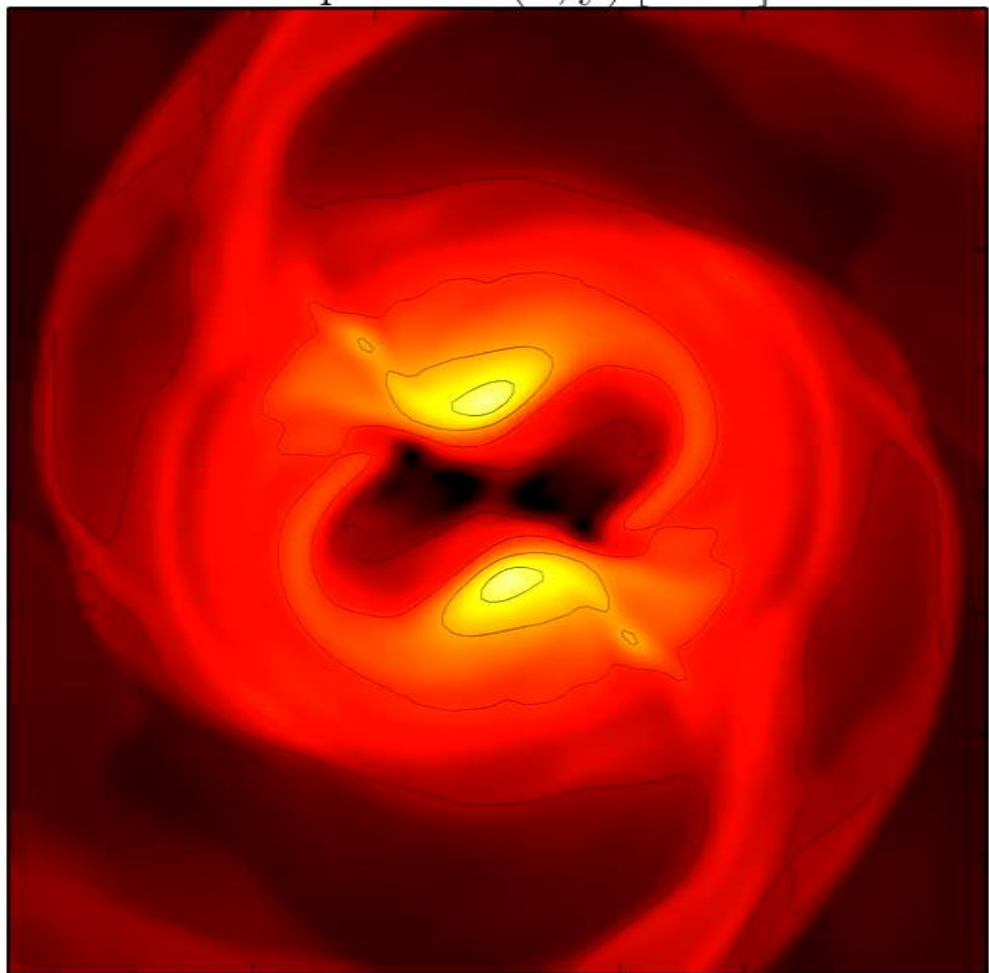
F. Galeazzi, S. Yoshida and Y. Eriguchi, A&A 541, p. A156 (2012)

W. Kastaun and F. Galeazzi, Phys. Rev. D 91, p. 064027 (2015)



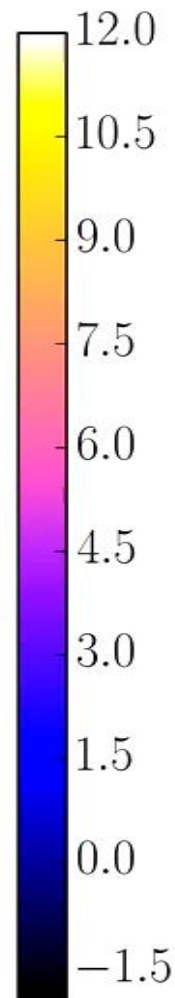
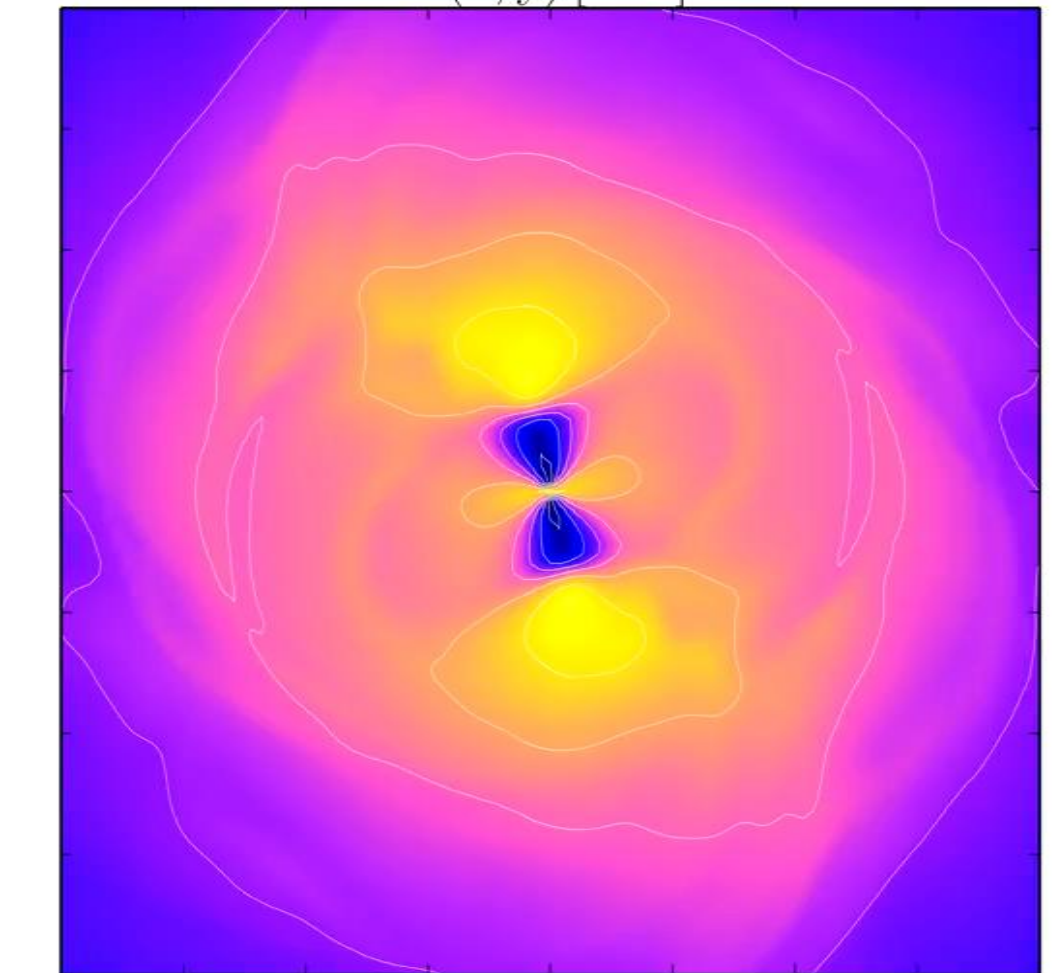
# Temperature

Temperature(x, y) [MeV]



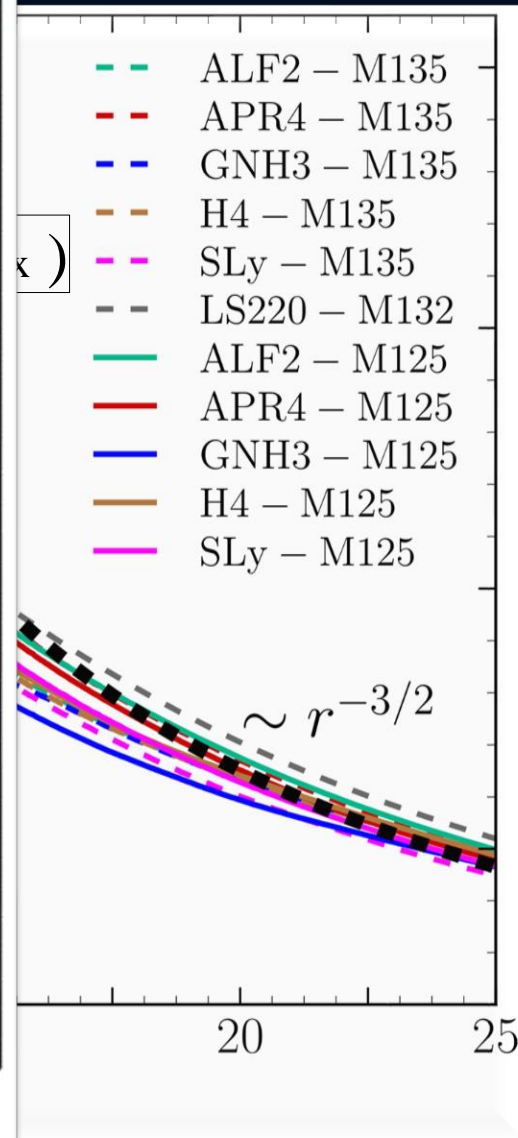
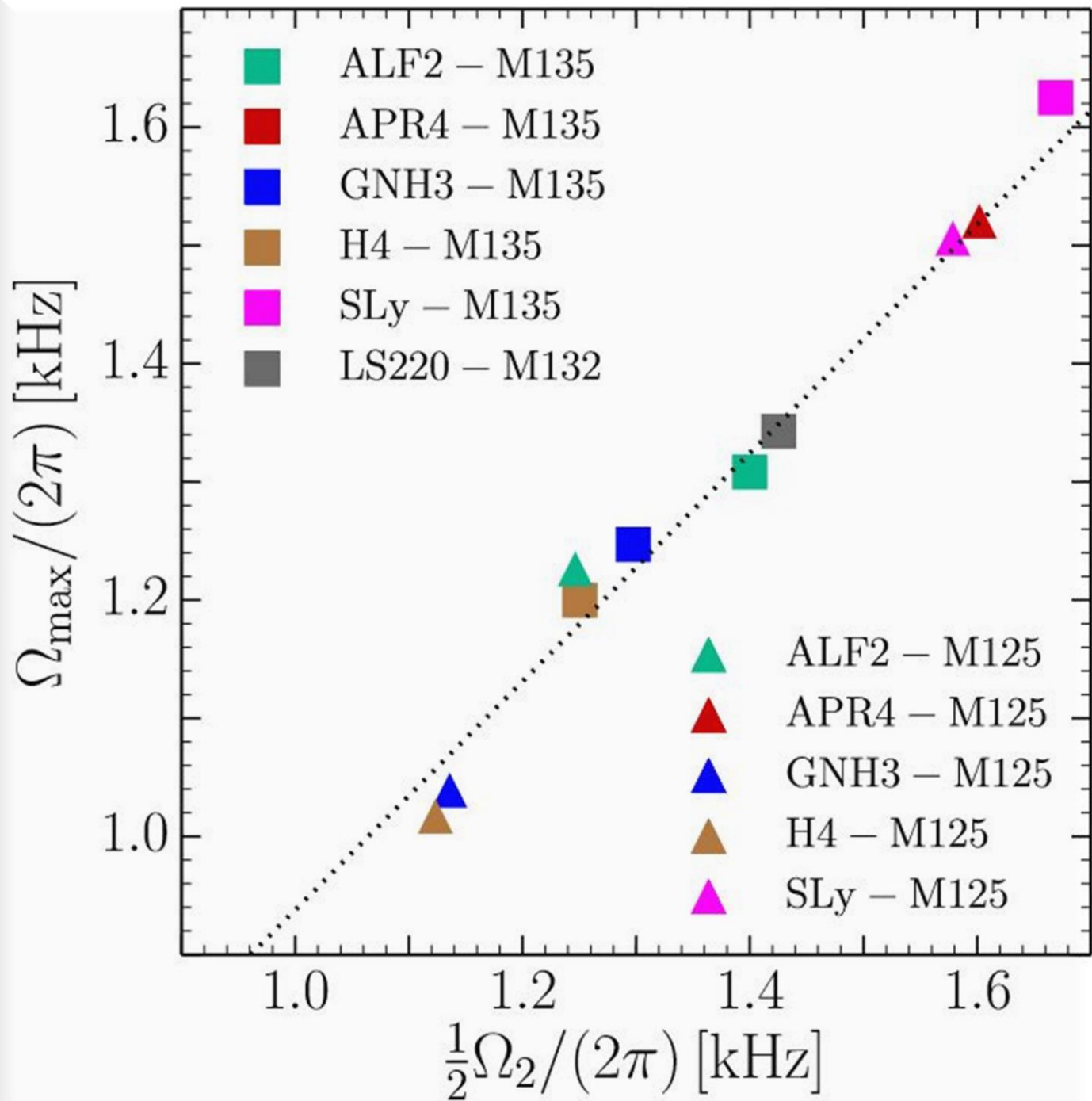
# Angular Velocity

$\Omega(x, y)$  [kHz]



EOS: LS200 , Mass:  $1.32 M_{\text{solar}}$  , simulation with Pi-symmetry

# files of the HMNSs



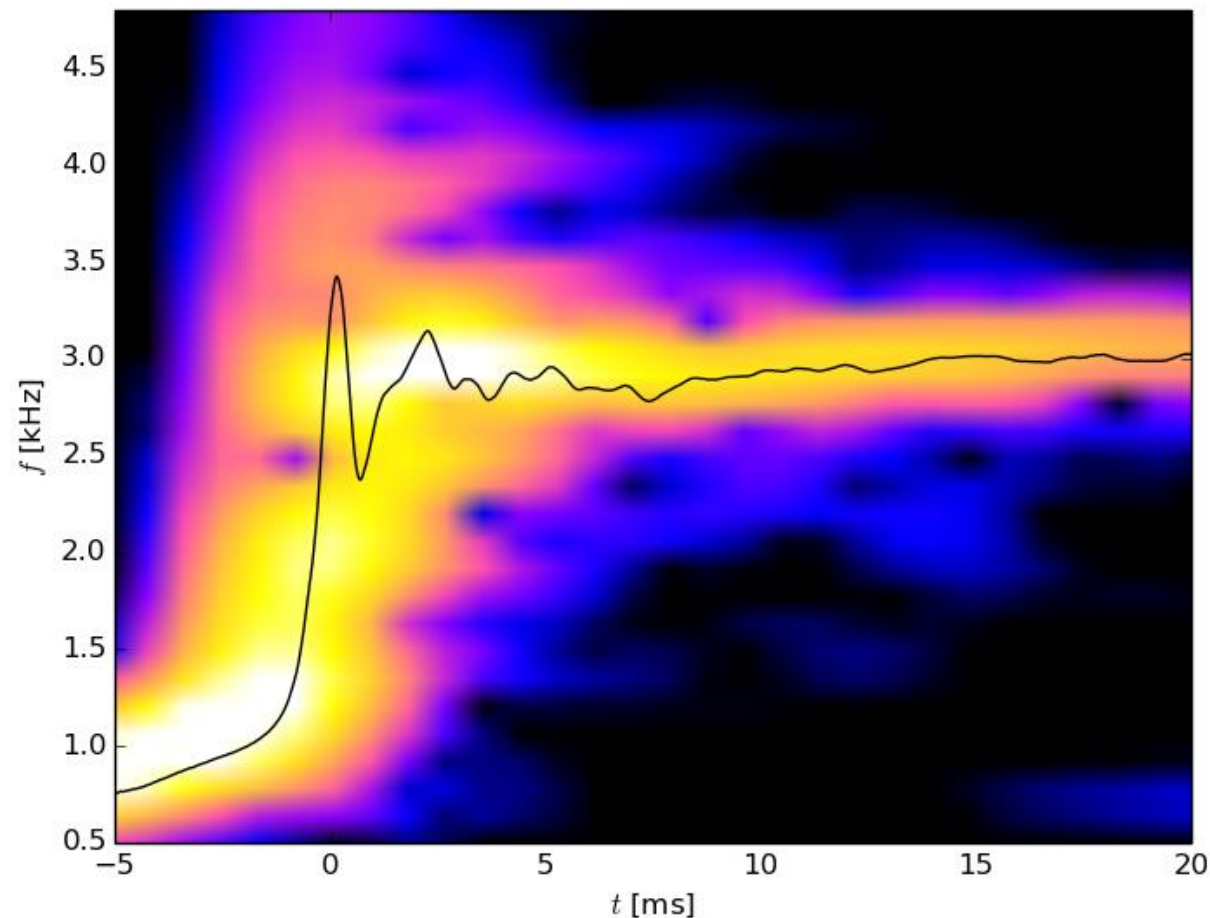
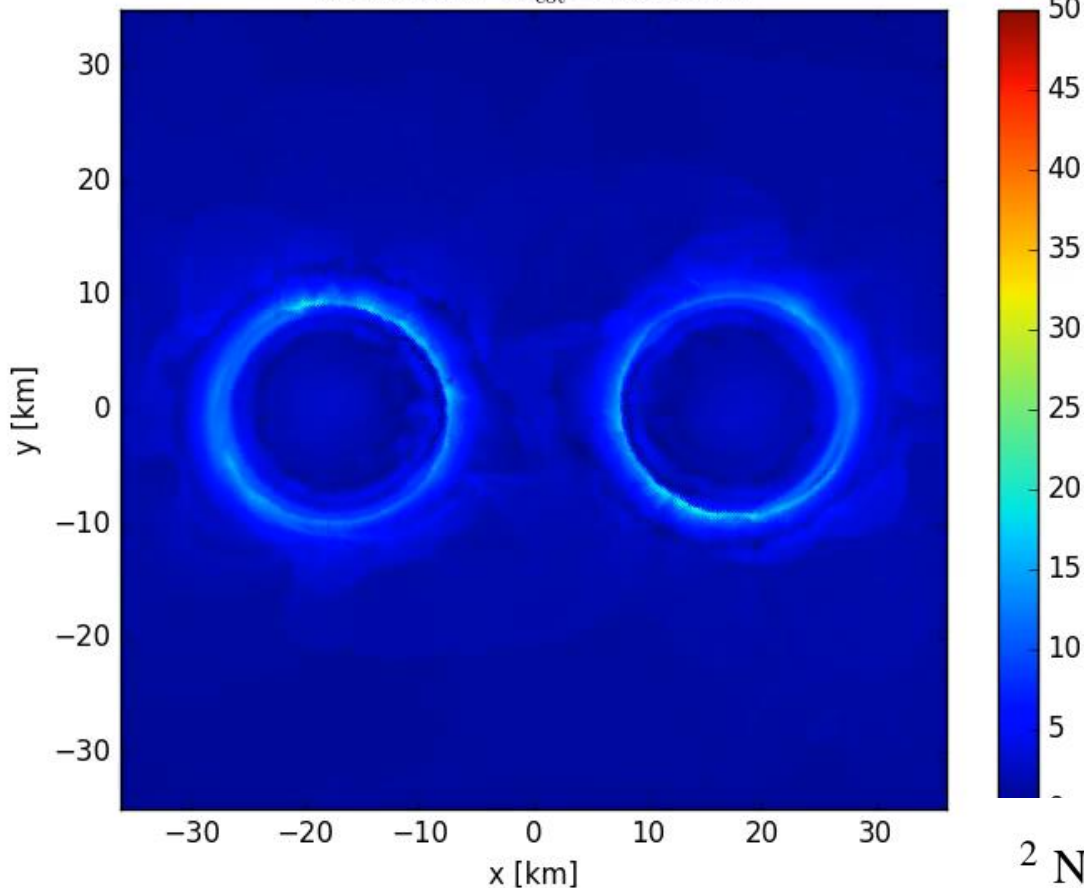
Soft EoSs:  
Sly  
APR4

Stiff EoSs:  
GNH3  
H4



# The Co-Rotating Frame

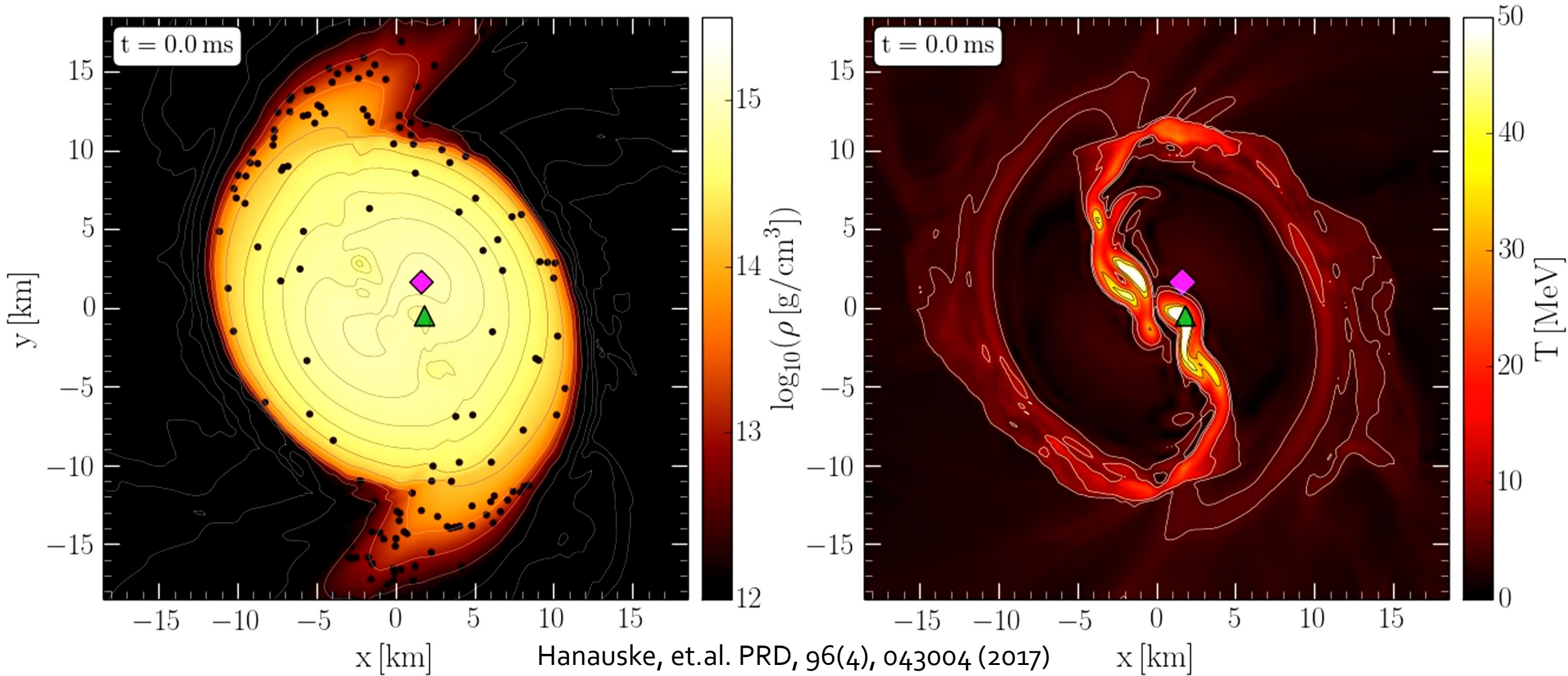
$t = 9.58\text{ms}$   $\Omega_{\text{cot}} = 387.20\text{Hz}$



- <sup>2</sup> Note that the angular-velocity distribution in the lower central panel of Fig. 10 refers to the corotating frame and that this frame is rotating at half the angular frequency of the emitted gravitational waves,  $\Omega_{\text{GW}}$ . Because the maximum of the angular velocity  $\Omega_{\text{max}}$  is of the order of  $\Omega_{\text{GW}}/2$  (cf. left panel of Fig. 12), the ring structure in this panel is approximately at zero angular velocity.

Simulation and movie has been produced by Luke Bovard

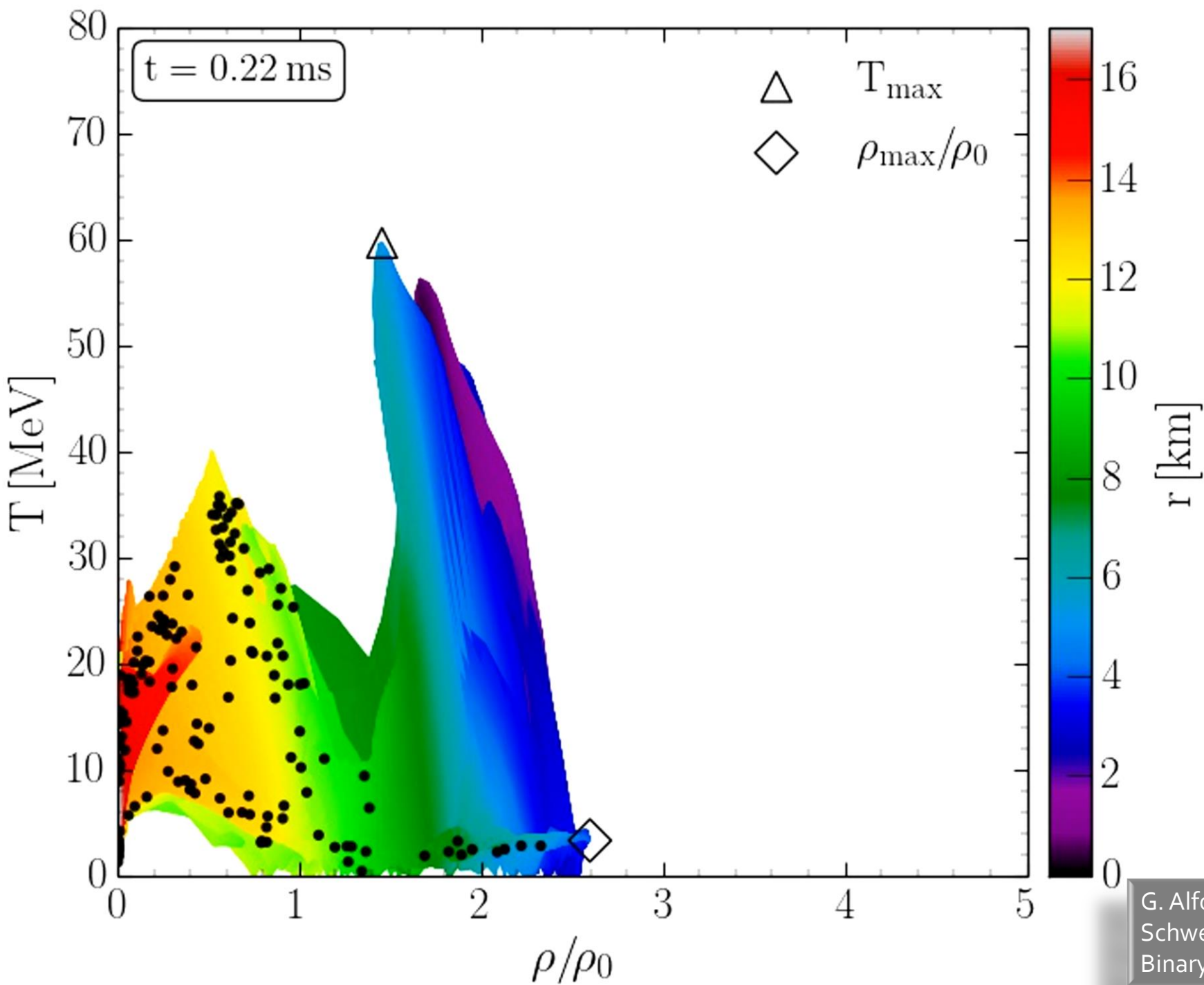
# Density and Temperature Evolution inside the HMNS



Rest mass density on the equatorial plane

Temperature on the equatorial plane

# Binary Neutron Star Mergers in the QCD Phase Diagram

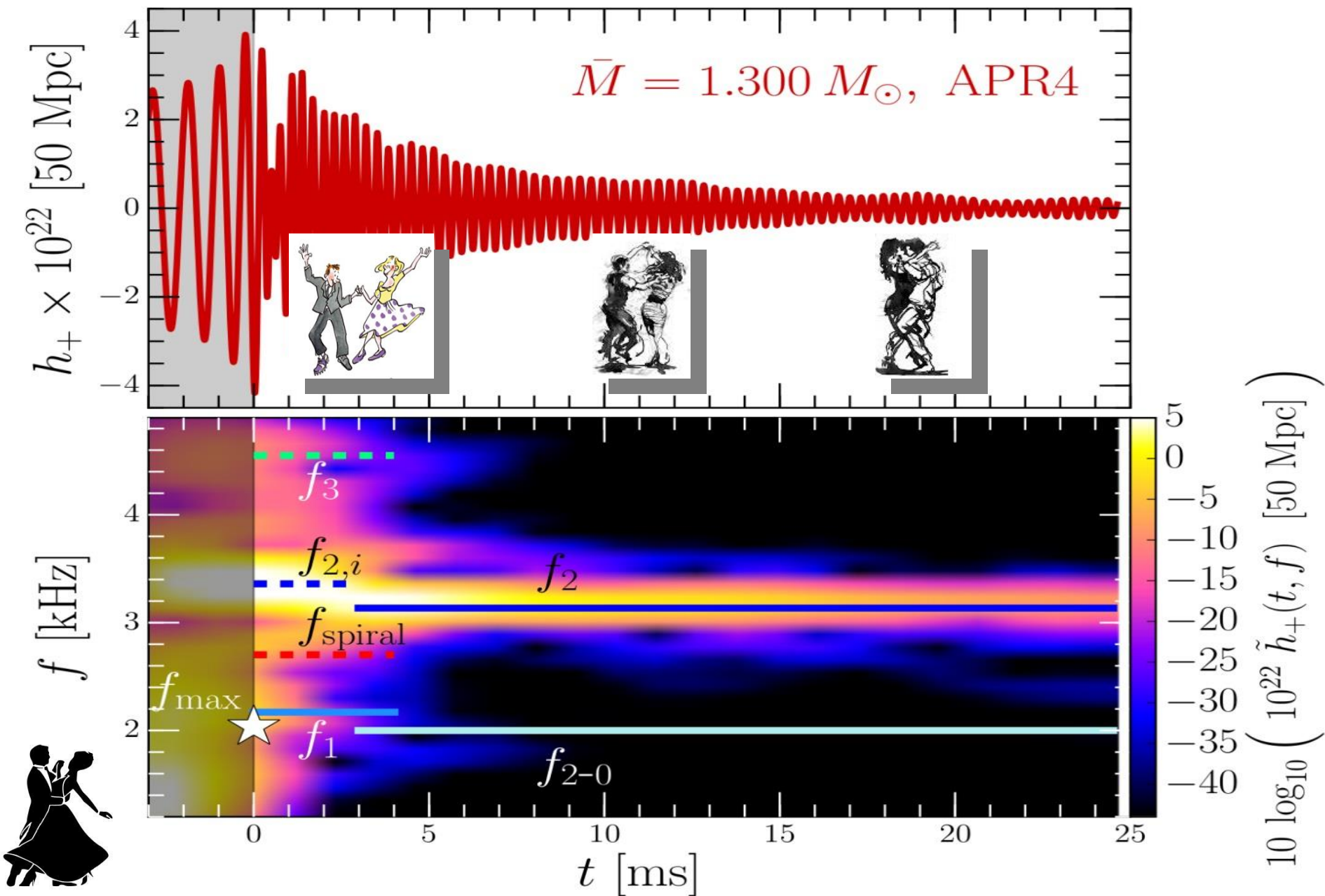


Evolution of hot and dense matter inside the inner area of a hypermassive neutron star simulated within the LS220 EOS with a total mass of  $M_{\text{total}} = 2.7 M_{\odot}$  in the style of a  $(T-\rho)$  QCD phase diagram plot

The color-coding indicates the radial position  $r$  of the corresponding  $(T-\rho)$  fluid element measured from the origin of the simulation  $(x, y) = (0, 0)$  on the equatorial plane at  $z = 0$ .

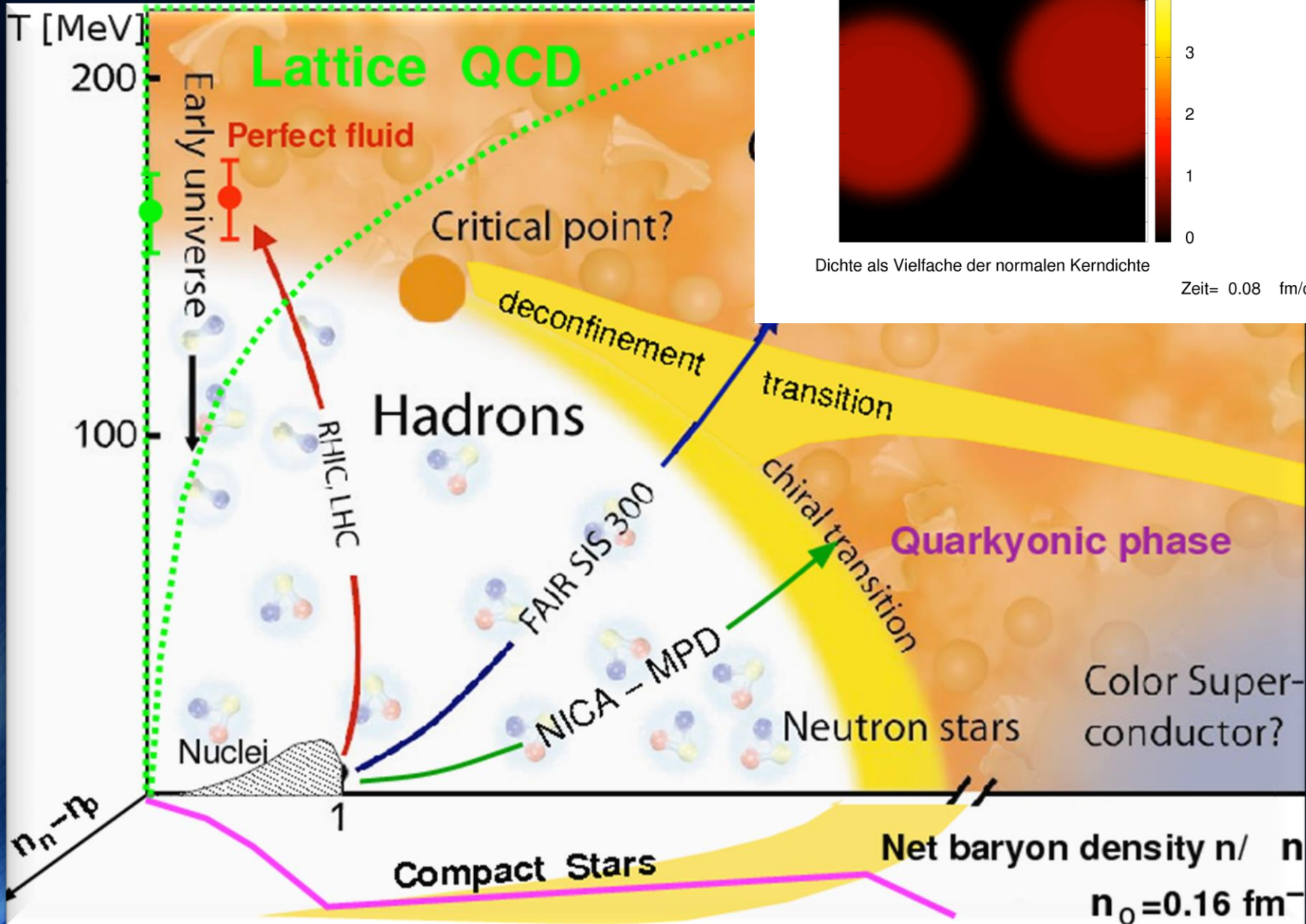
The open triangle marks the maximum value of the temperature while the

# The different Phases during the Postmergerphase of the HMNS



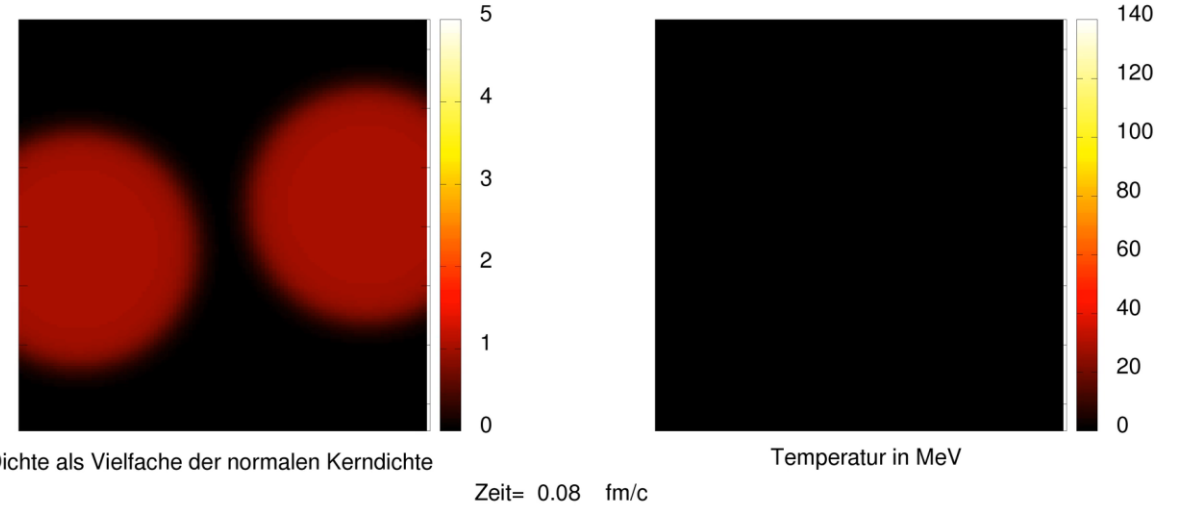
# The Hadron-Quark Phase Transition

The QCD Phase Diagram

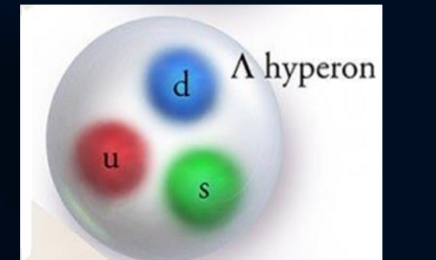
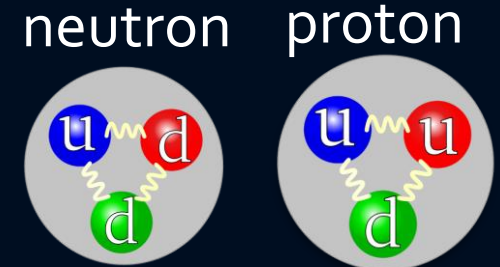


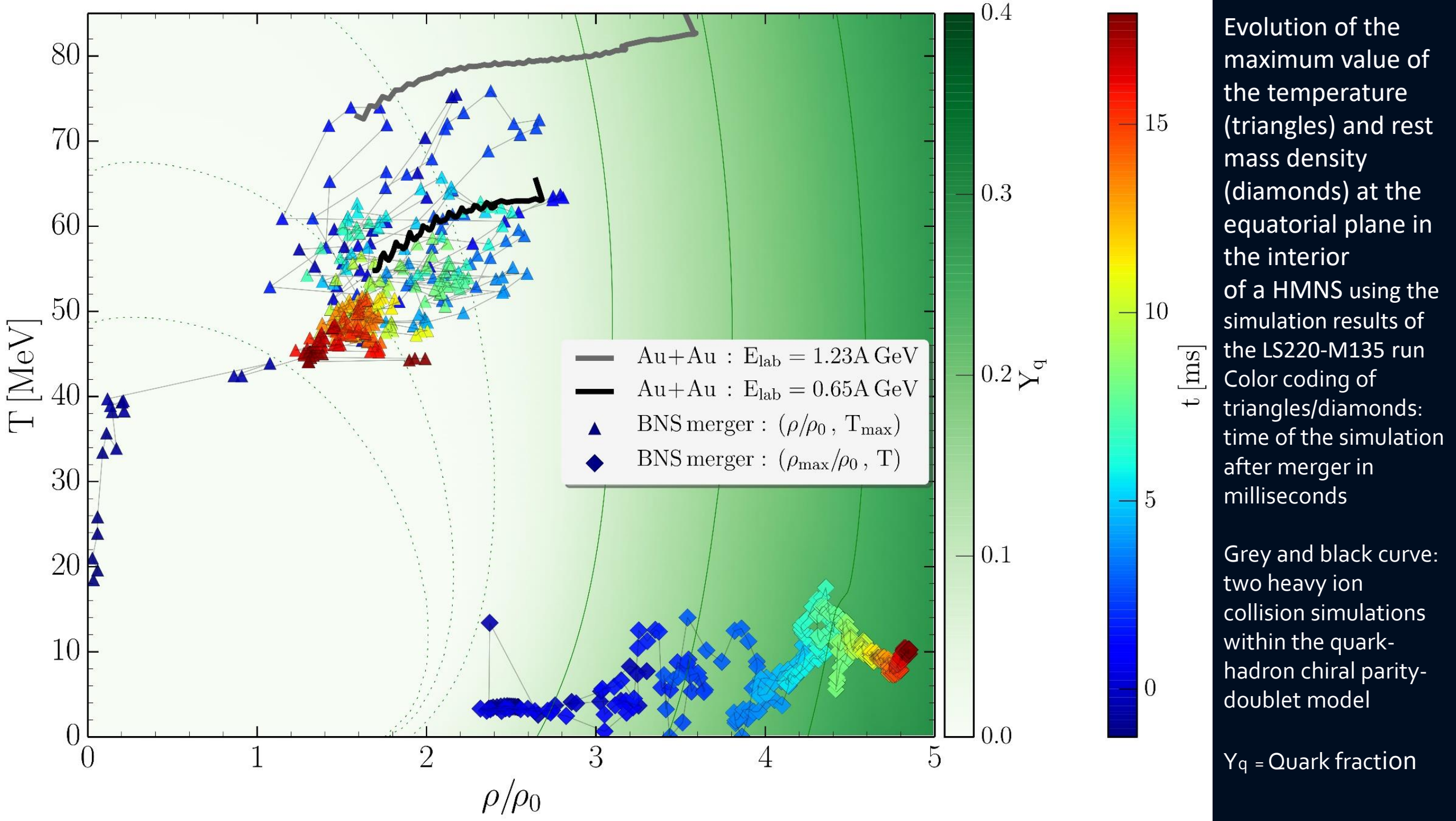
Gold+Gold Kollision am GSI: Helmholtz Zentrum für Schwerionenforschung / HADES Experiment

Am FAIR Beschleuniger: noch höhere Strahlintensität



Credits:  
Jan Steinheimer



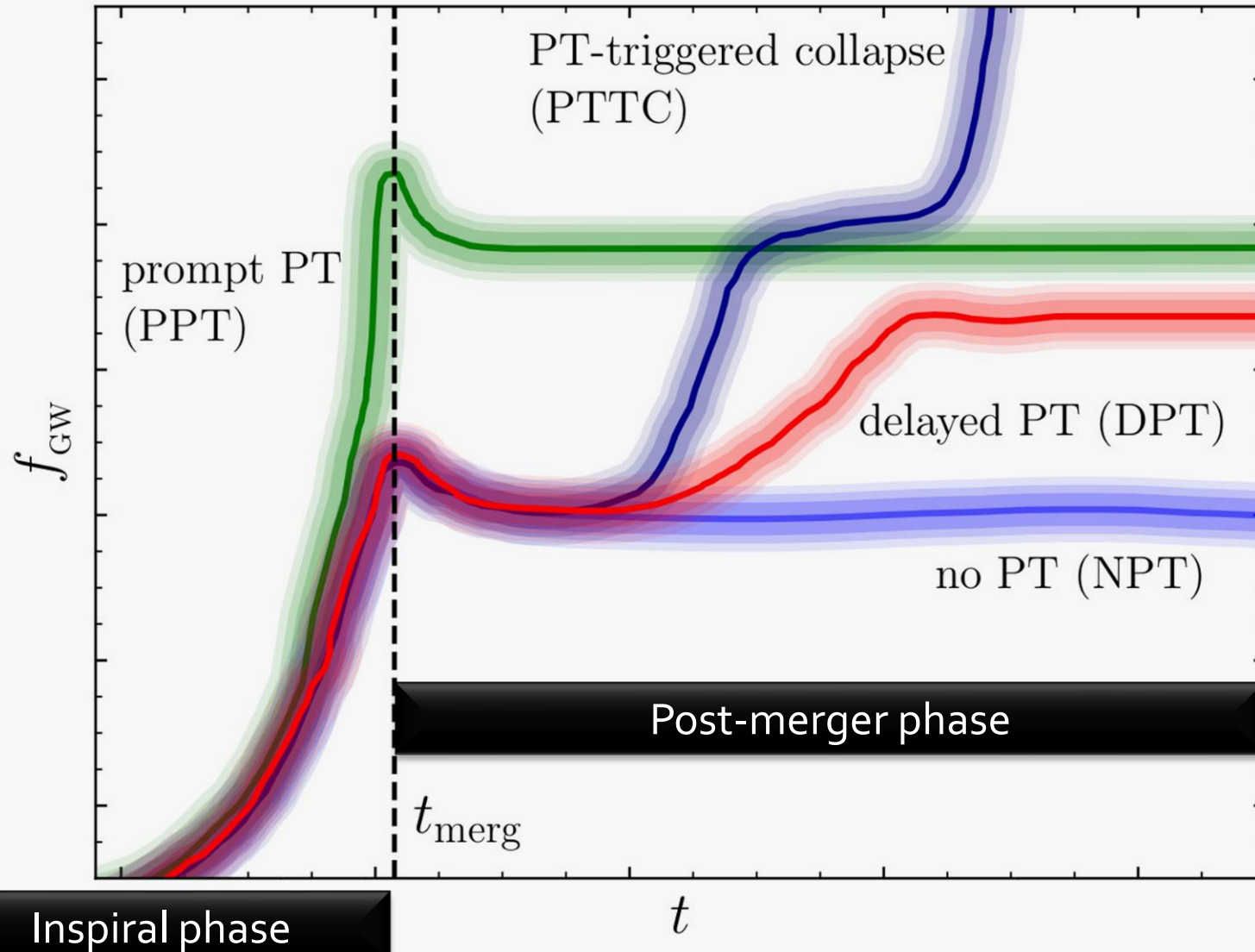


# Can we detect the quark-gluon plasma with gravitational waves?

- Gravitational-wave signatures of the hadron-quark phase transition in compact star mergers
  - Signatures within the late inspiral phase (premerger signals)
    - Constraining twin stars with GW170817; G Montana, L Tolós, M Hanauske; *Physical Review Letters* 123 (10), 103009 (2019)
  - Signatures within the post-merger phase evolution
    - **Phase-transition triggered collapse scenario**  
Signatures of quark-hadron phase transitions in general-relativistic neutron star mergers; P. Papenfort, V Dexheimer, M Hanauske, S Schramm, H Stöcker, L. Rezzolla; *Physical Review Letters* 123 (10), 101101 (2019)
    - **Delayed phase transition scenario**  
Postmerger Gravitational-Wave Signatures of Phase Transitions in Binary Neutron Star Mergers; L. Rezzolla; *Physical Review Letters* 124 (17), 171103 (2020)
    - **Prompt phase transition scenario**  
Identifying a first-order phase transition in neutron-star mergers through gravitational waves; M. C. Miller, M. A. Zilhao, M. Hanauske, M. Reuter, M. Rezzolla, M. C. Miller, M. A. Zilhao, M. Hanauske, M. Reuter, M. Rezzolla; *Physical Review Letters* 123 (10), 101101 (2019)

YES  
WE  
CAN

# Post-merger gravitational-wave signatures of phase transitions in binary compact star mergers

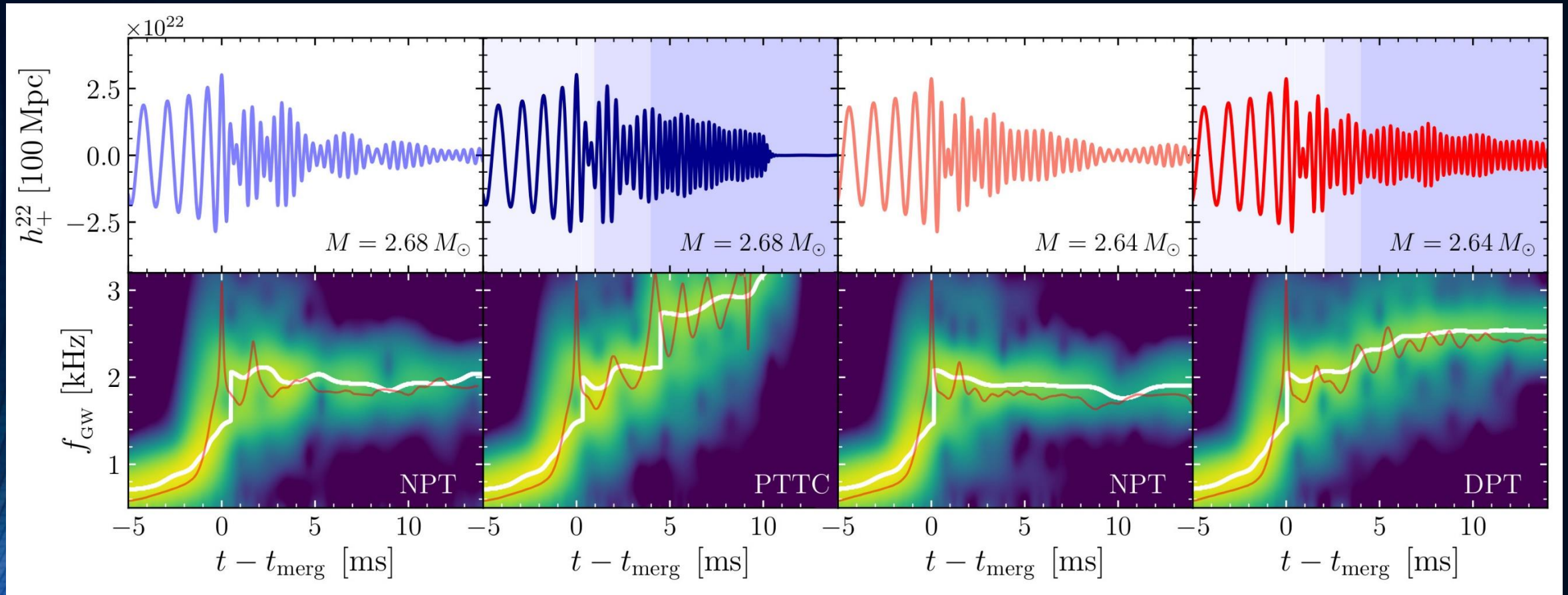


Postmerger Gravitational-Wave Signatures of Phase Transitions in Binary Mergers;  
LR Weih, M Hanauske, L Rezzolla;  
Physical Review Letters 124 (17),  
171103 (2020)

Schematic overview of the instantaneous gravitational wave frequency and how its evolution can be used to classify the different scenarios associated with a hadron-quark phase transition.

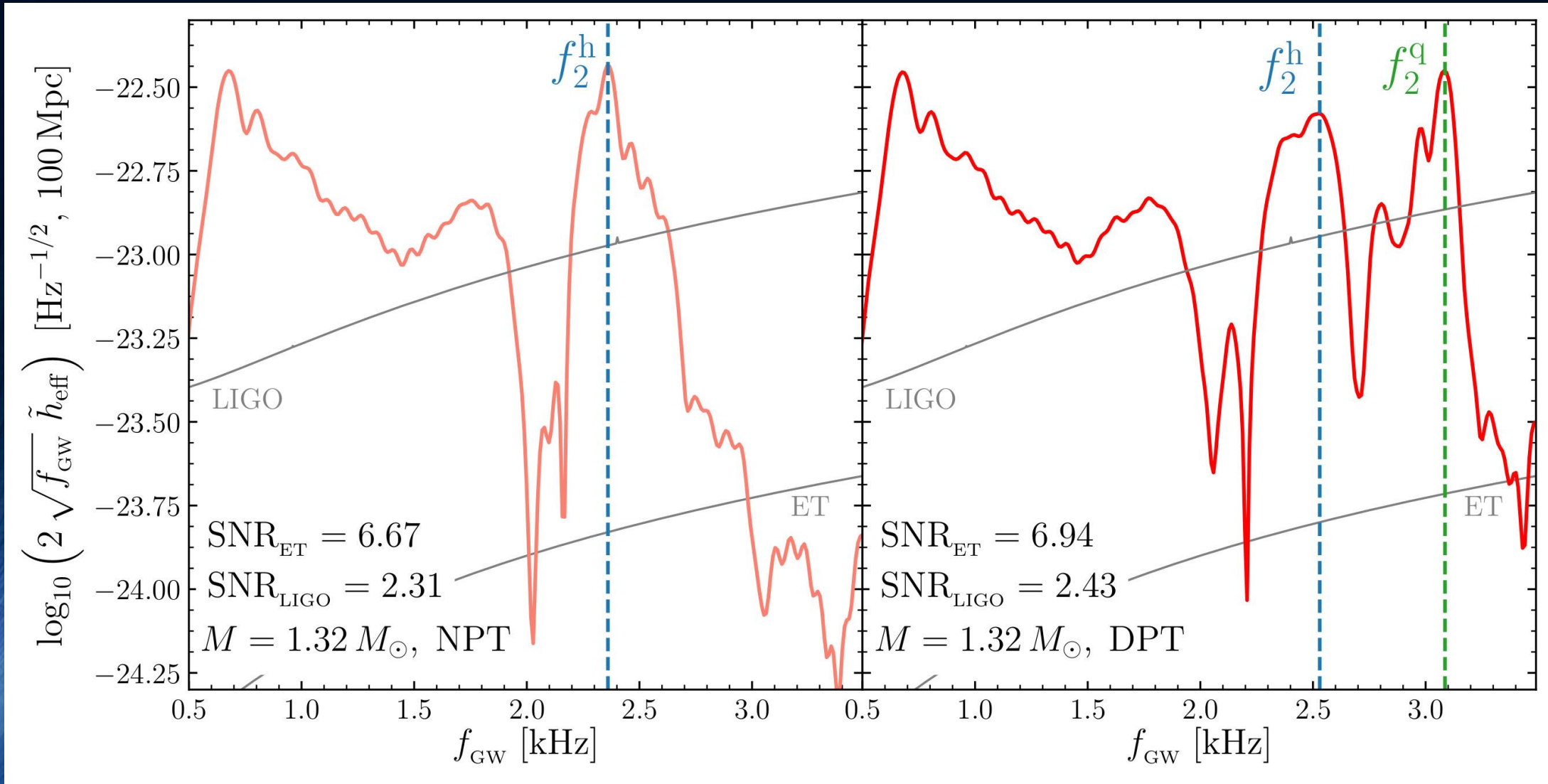


# Postmerger Gravitational-Wave Signatures of Phase Transitions in Binary Mergers; LR Weih, M Hanauske, L Rezzolla; Physical Review Letters 124 (17), 171103 (2020)



Strain  $h_+$  (top) and its spectrogram (bottom) for the four BNSs considered. In the top panels the different shadings mark the times when the HMNS core enters the mixed and quark phases the NPT models are always purely hadronic. In the bottom panels, the white lines trace the maximum of the spectrograms, while the red lines show the instantaneous gravitational-wave frequency.

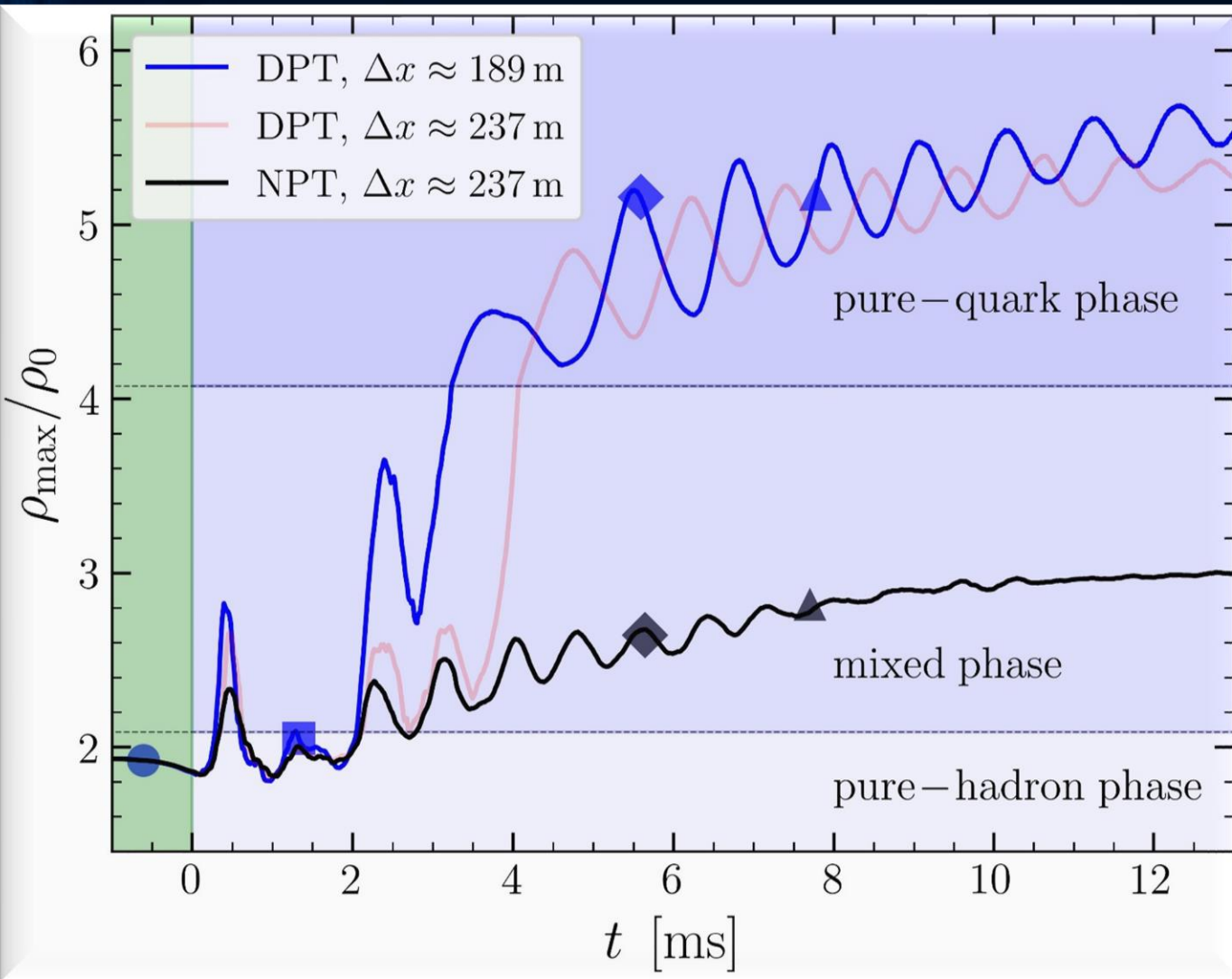
# How to detect the hadron-quark phase transition with gravitational waves



Total gravitational wave spectrum (left NPT, right DPT), PRL 124, 171103 (2020)

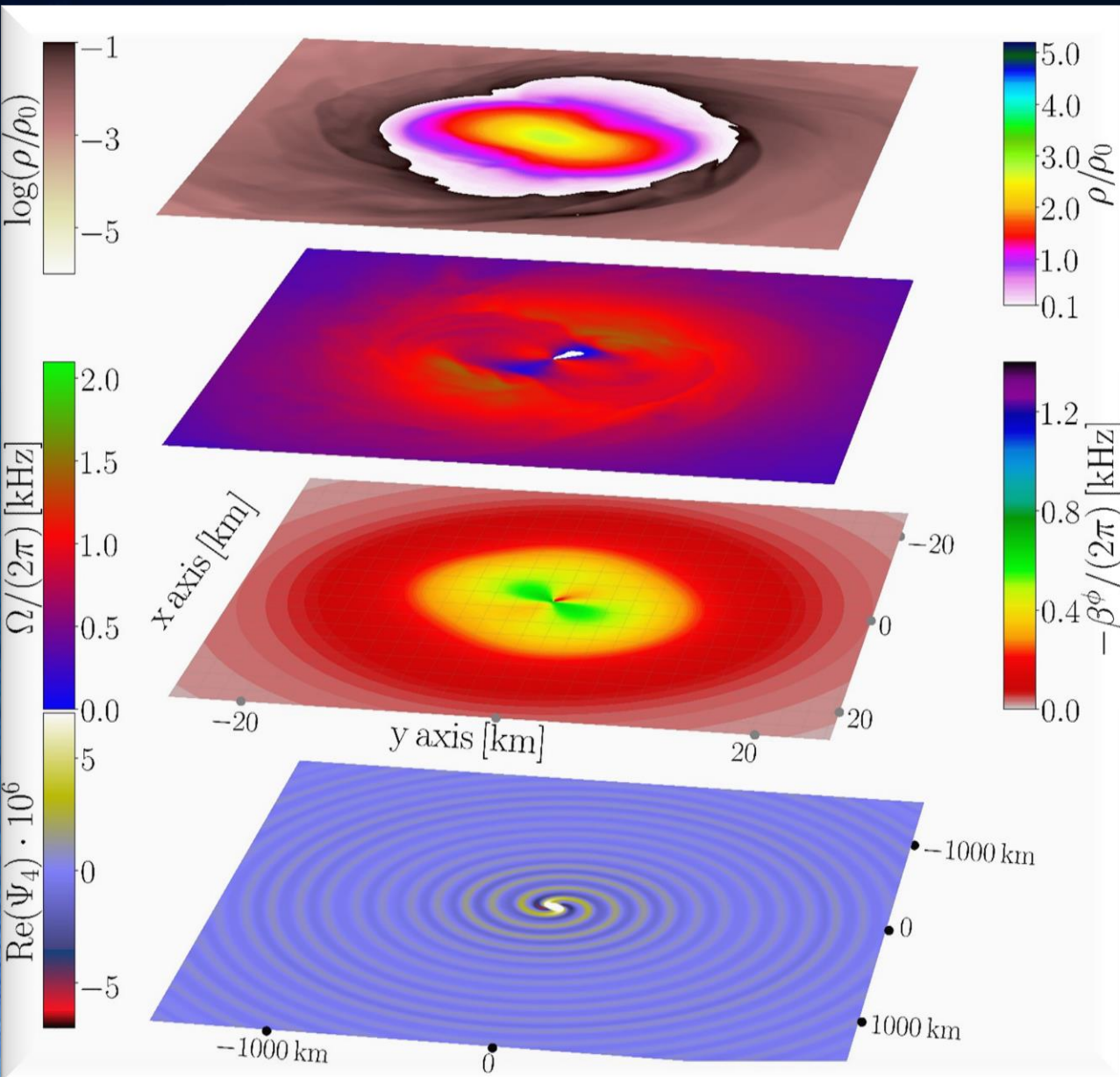
# Signatures within the post-merger phase evolution

## DPT: Delayed phase transition scenario

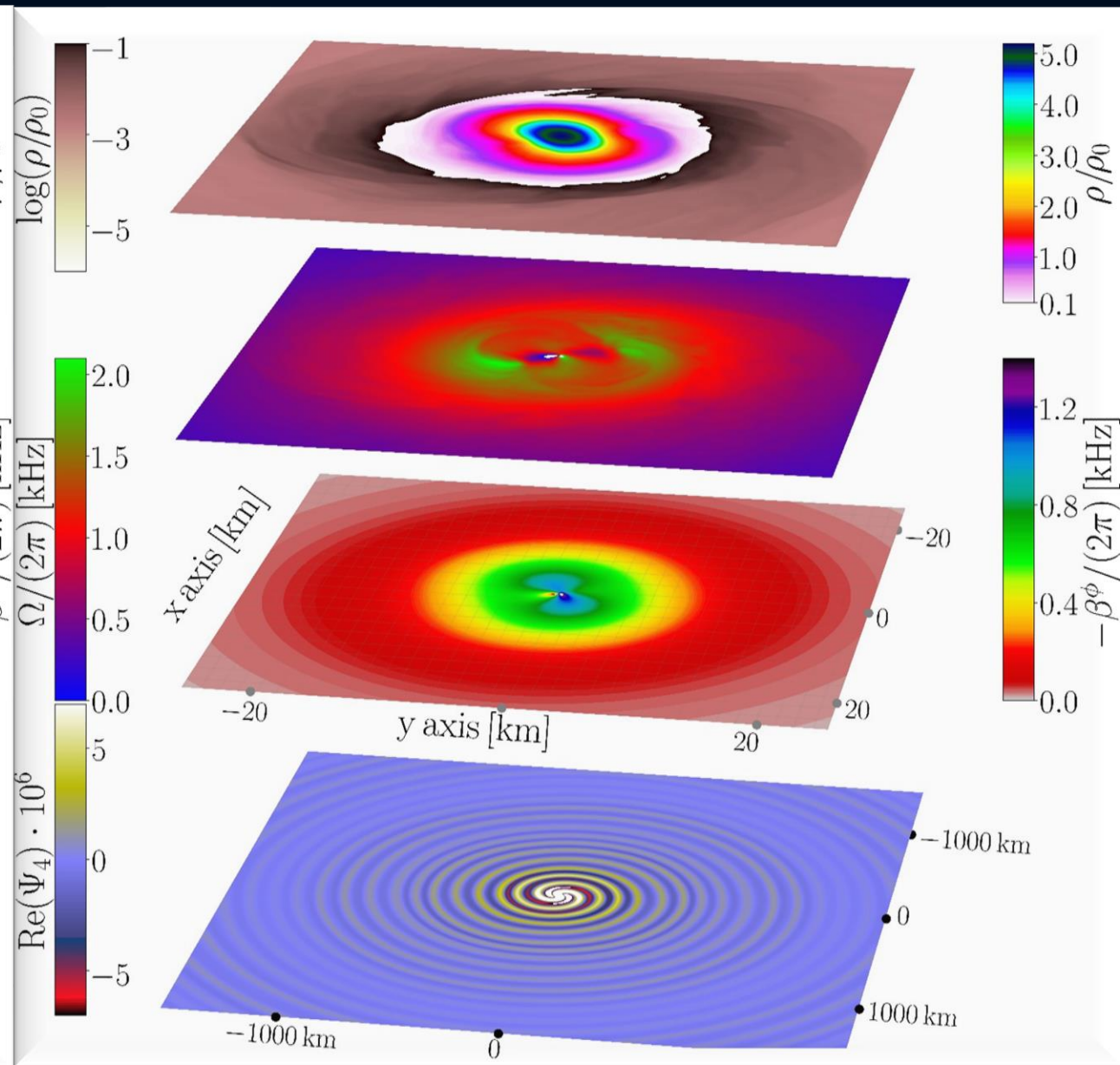


Maximum value of the rest-mass density vs time for three binary neutron star simulations. Black curve without a phase transition (NPT) and blue/red with a Gibbs-like hadron-quark phase transition (DPT: standard/low resolution). Blue-shaded regions mark the different phases of the EOS (mixed phase and pure-quark phase).

# Without Phase Transition



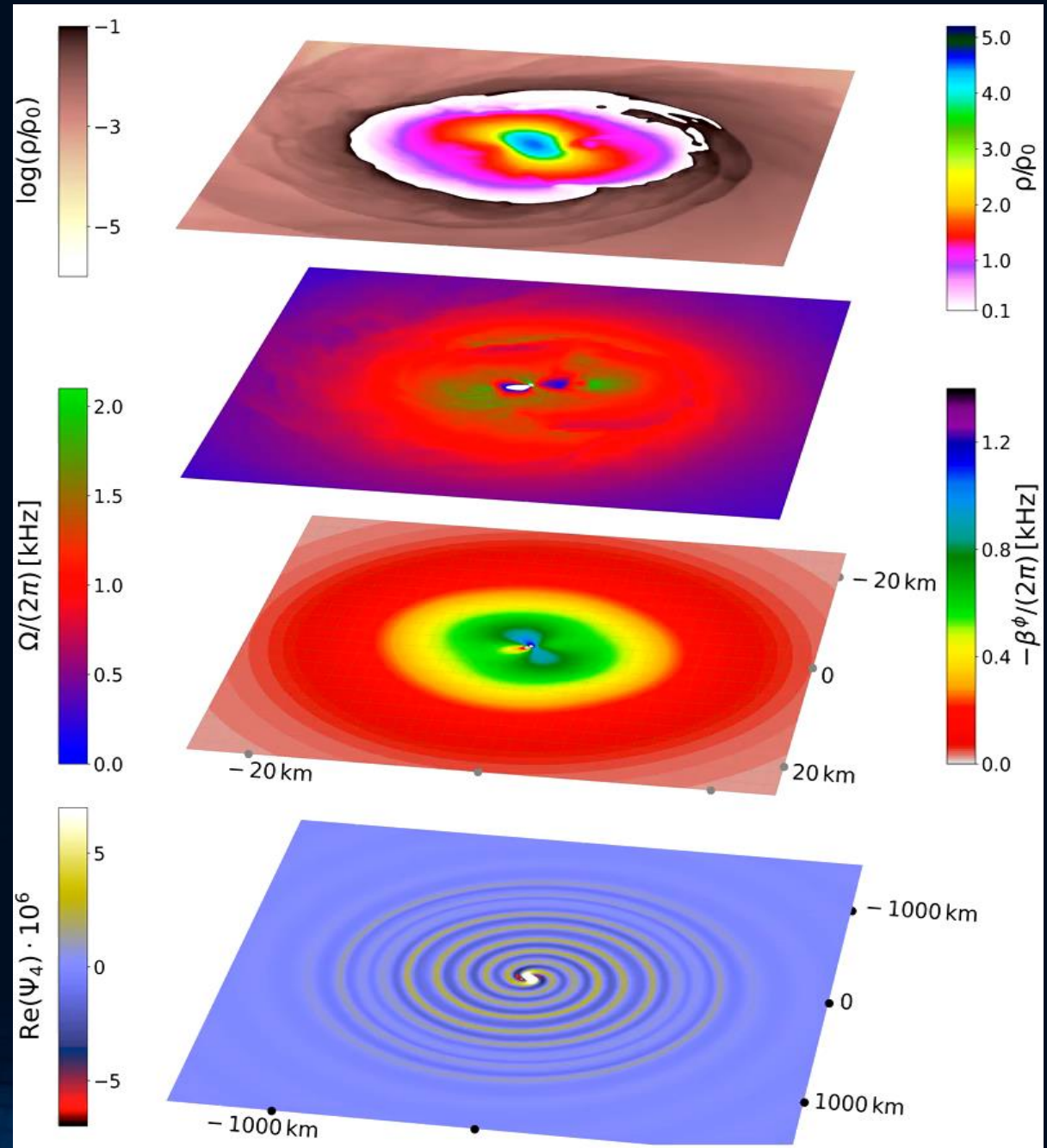
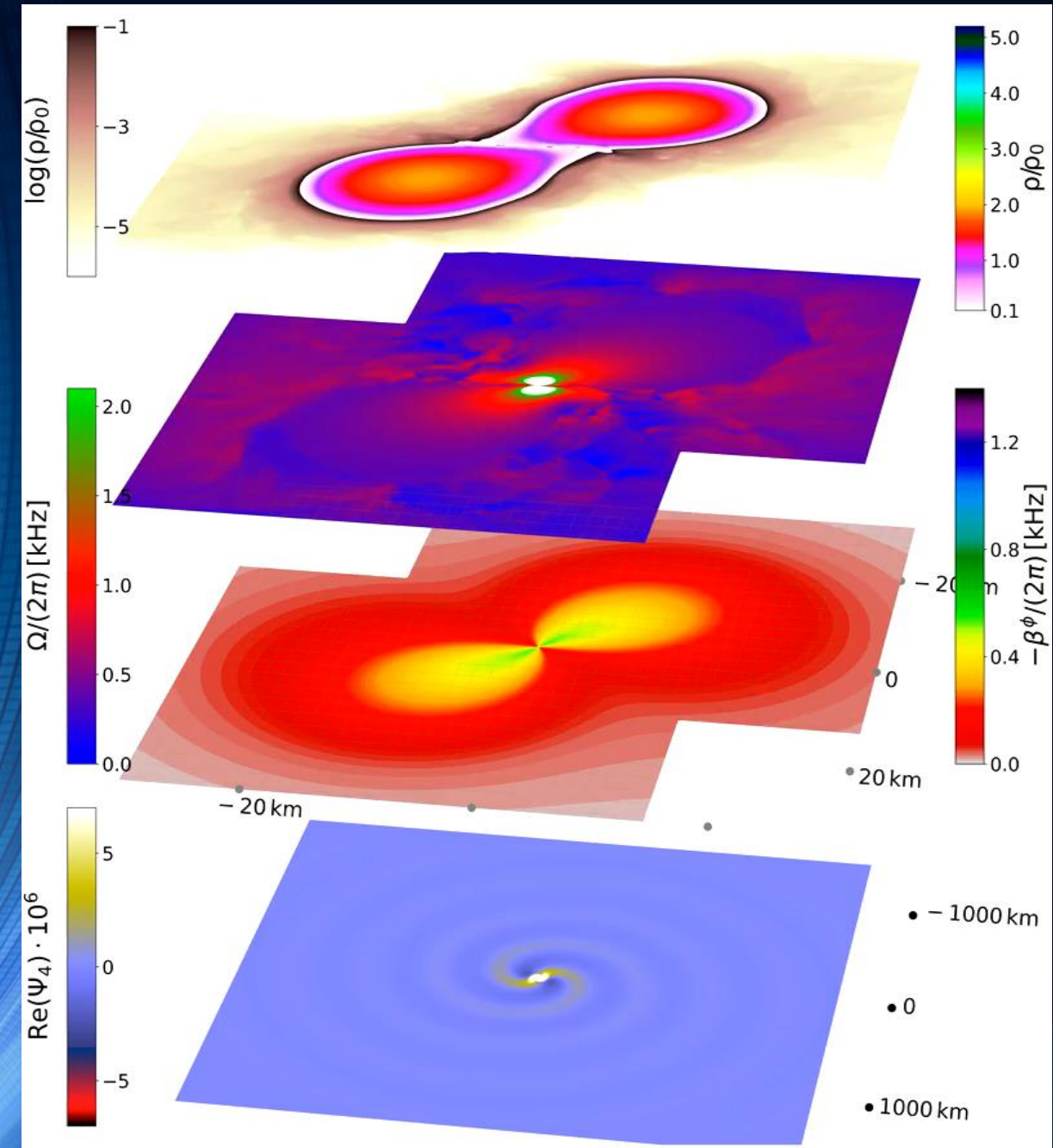
# With Phase Transition



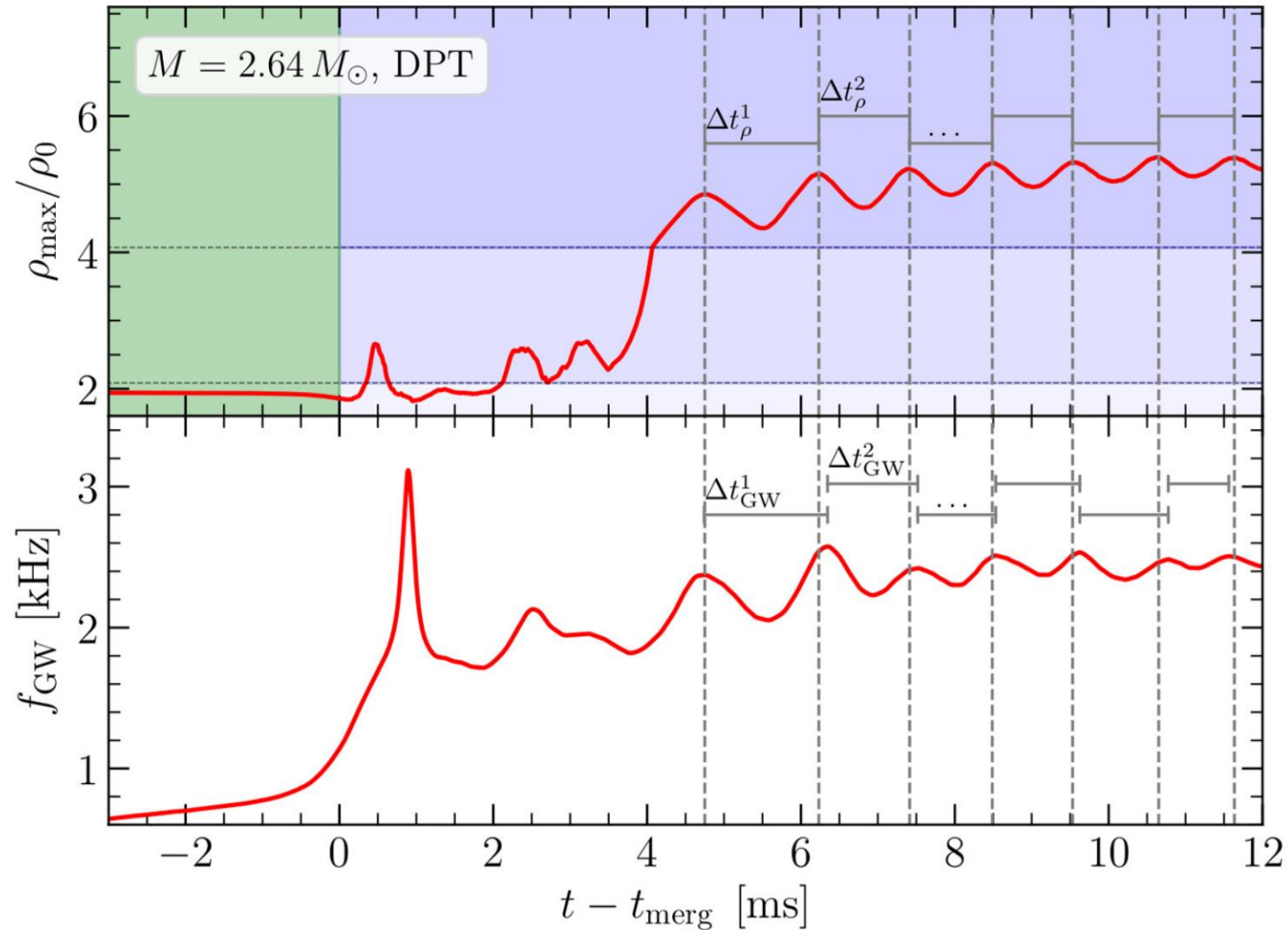
t=0 .. 2 ms

Post-Merger Phase

t=2 ... 7 ms

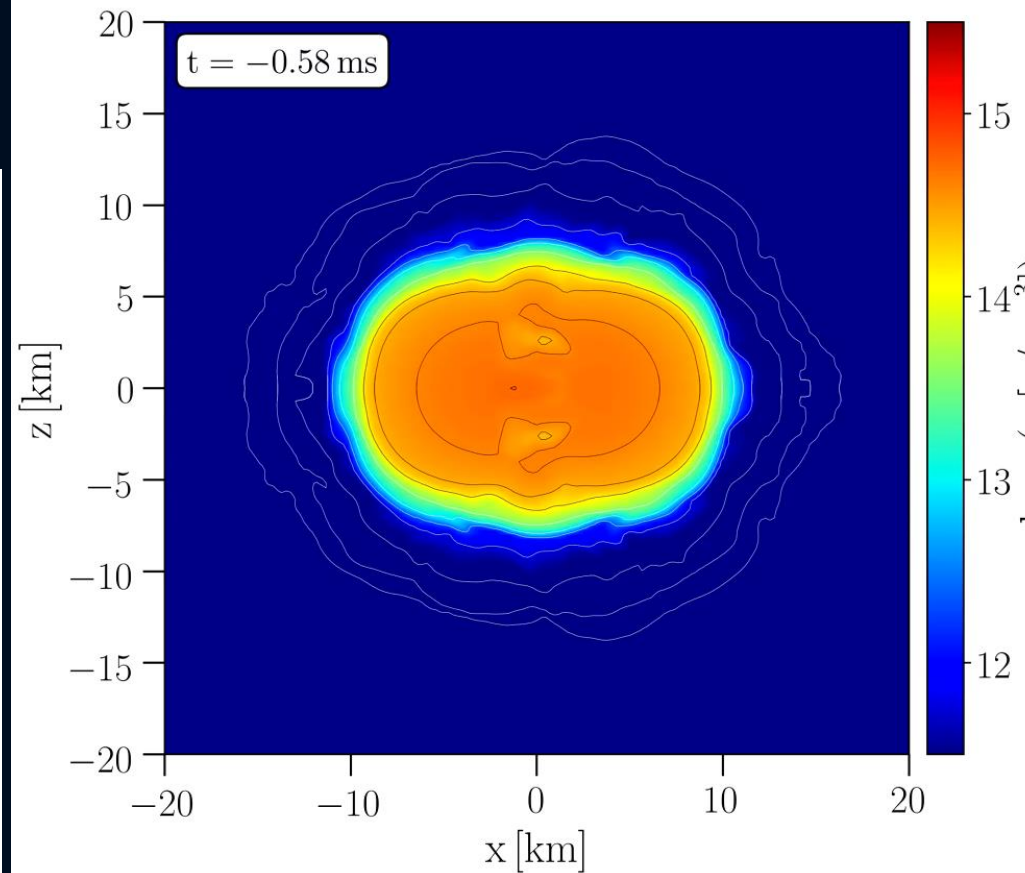
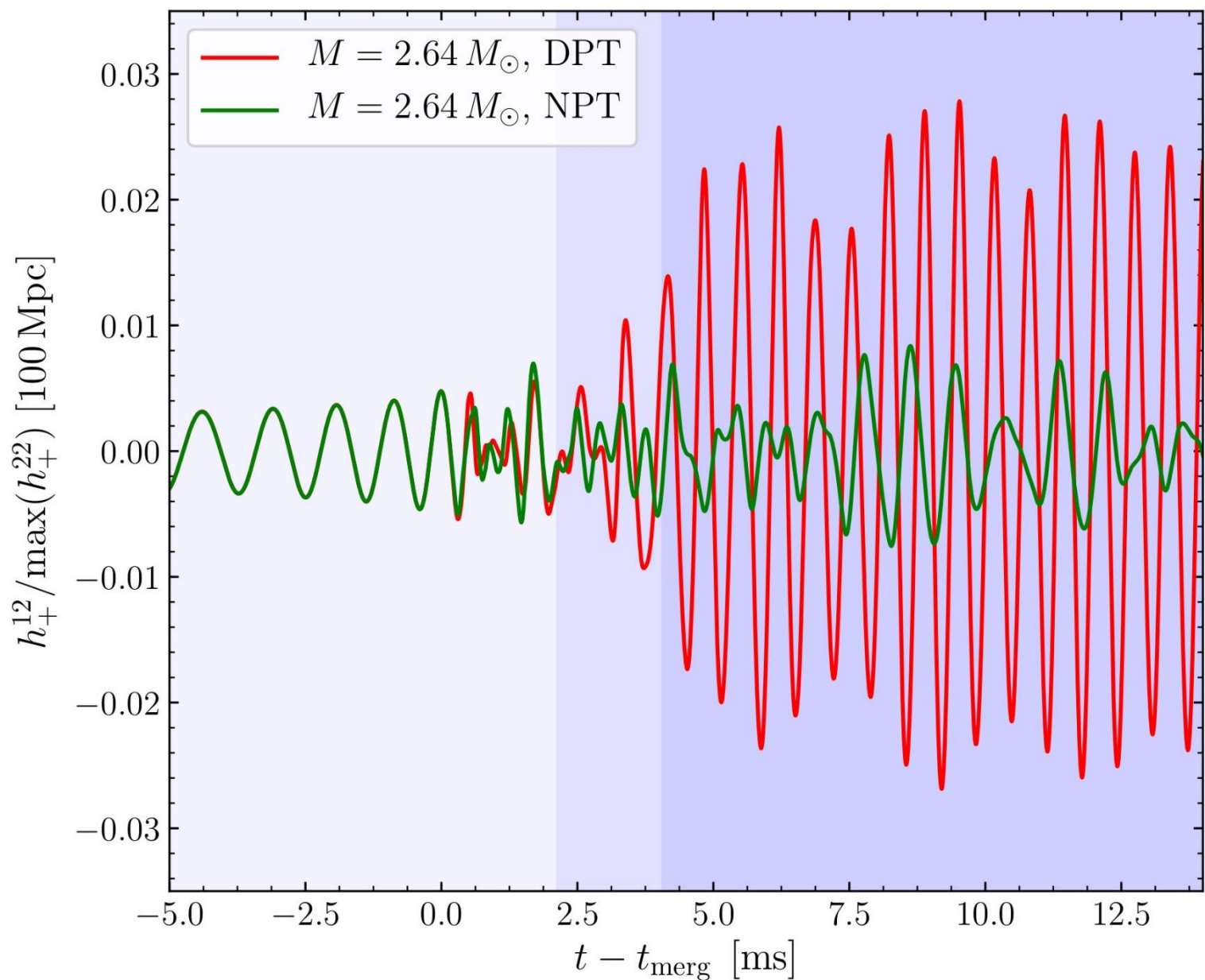


M. Hanauske, L. Weih, H. Stöcker and L. Rezzolla  
*Metastable hypermassive hybrid stars as neutron-star merger remnants*  
The European Physical Journal Special Topics: 1-8 (2021)



**Fig. 3.** *Top:* Same as Fig. 1. *Bottom:* Instantaneous GW frequency. The time intervals  $\Delta t_{\rho}^i$  and  $\Delta t_{\text{GW}}^i$  between consecutive peaks in the top and bottom panel, respectively, are marked by horizontal grey lines. The average difference between the two different types of peaks is less than 5%.

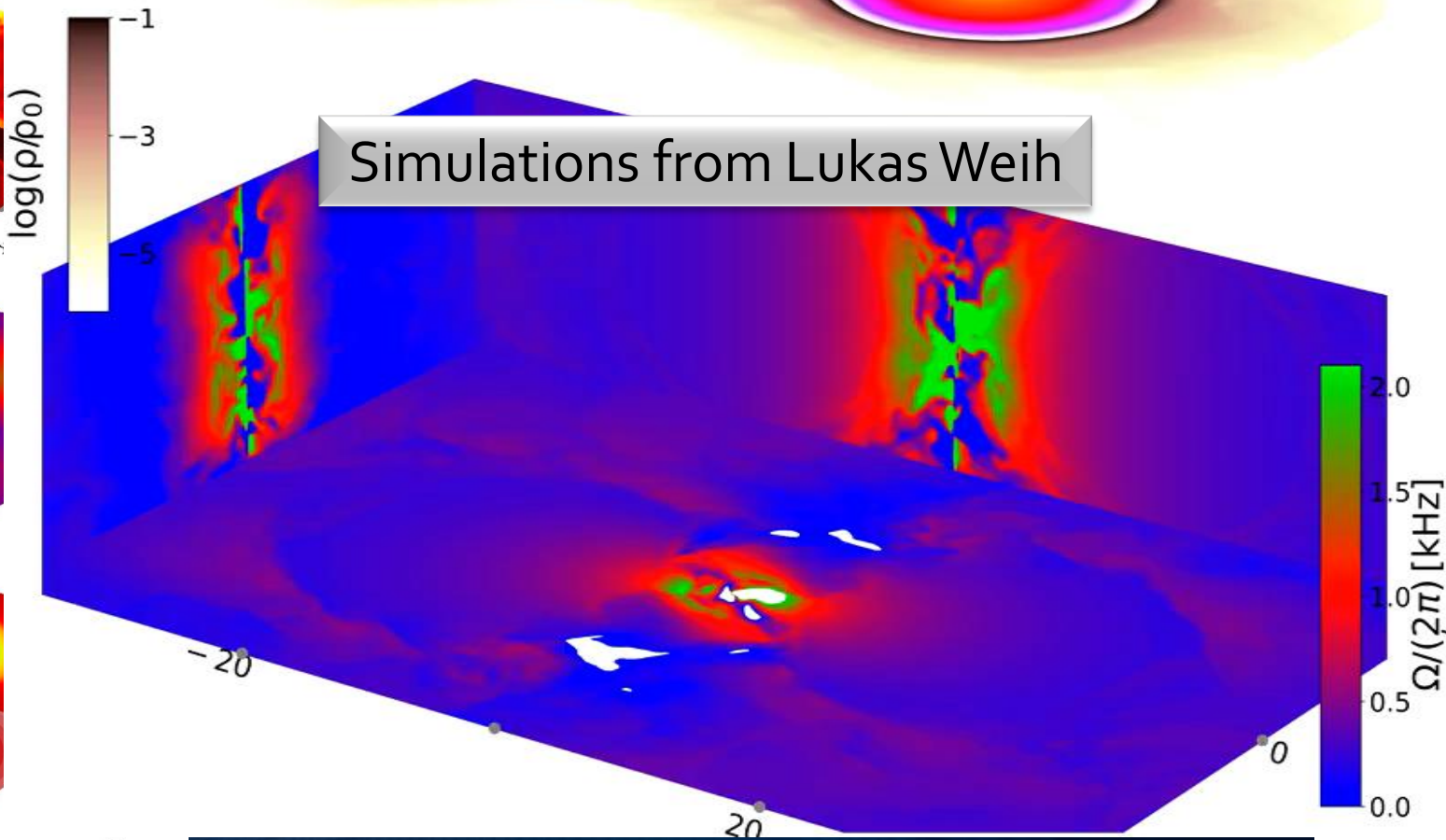
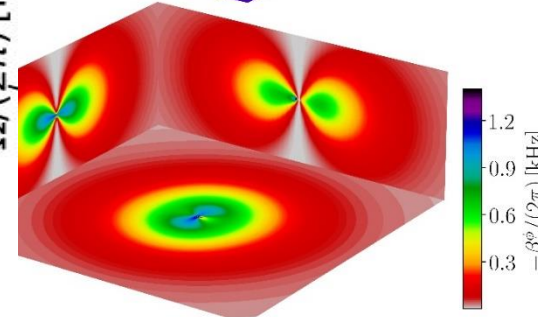
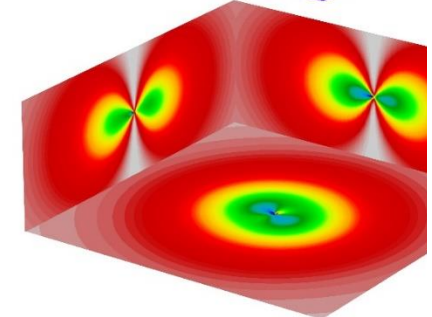
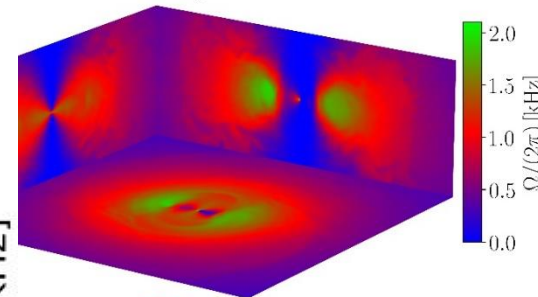
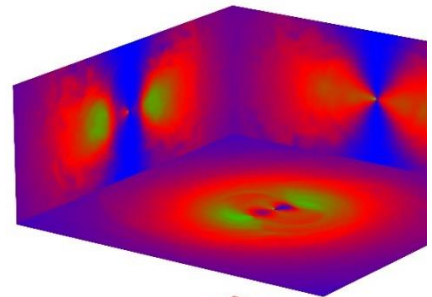
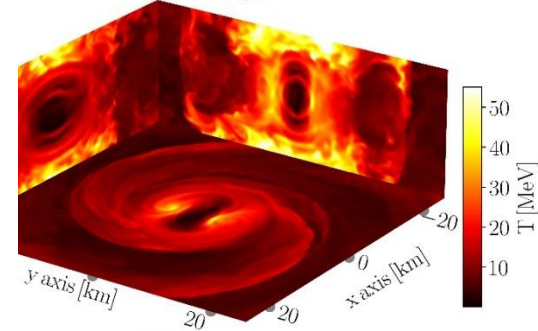
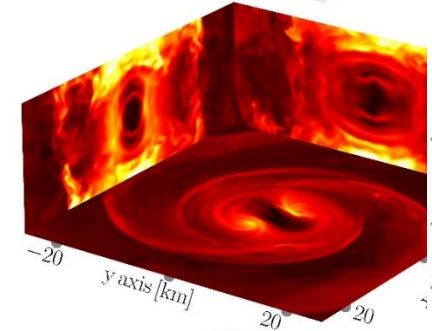
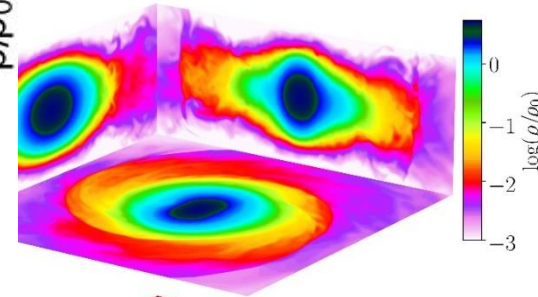
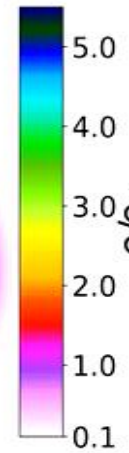
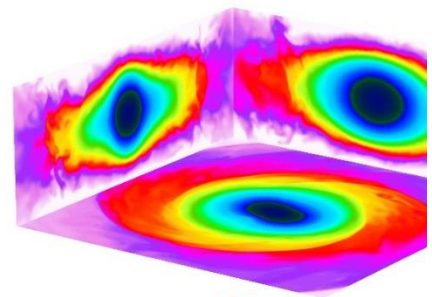
# Difference in the $h_+^{12}$ – gravitational wave mode



Due to the large  $m=1$  mode of the emitted gravitational wave in the DPT case, a qualitative difference to the NPT scenario might be observable in future by focusing on the  $h_+^{12}$  – gravitational wave mode during the post-merger evolution.

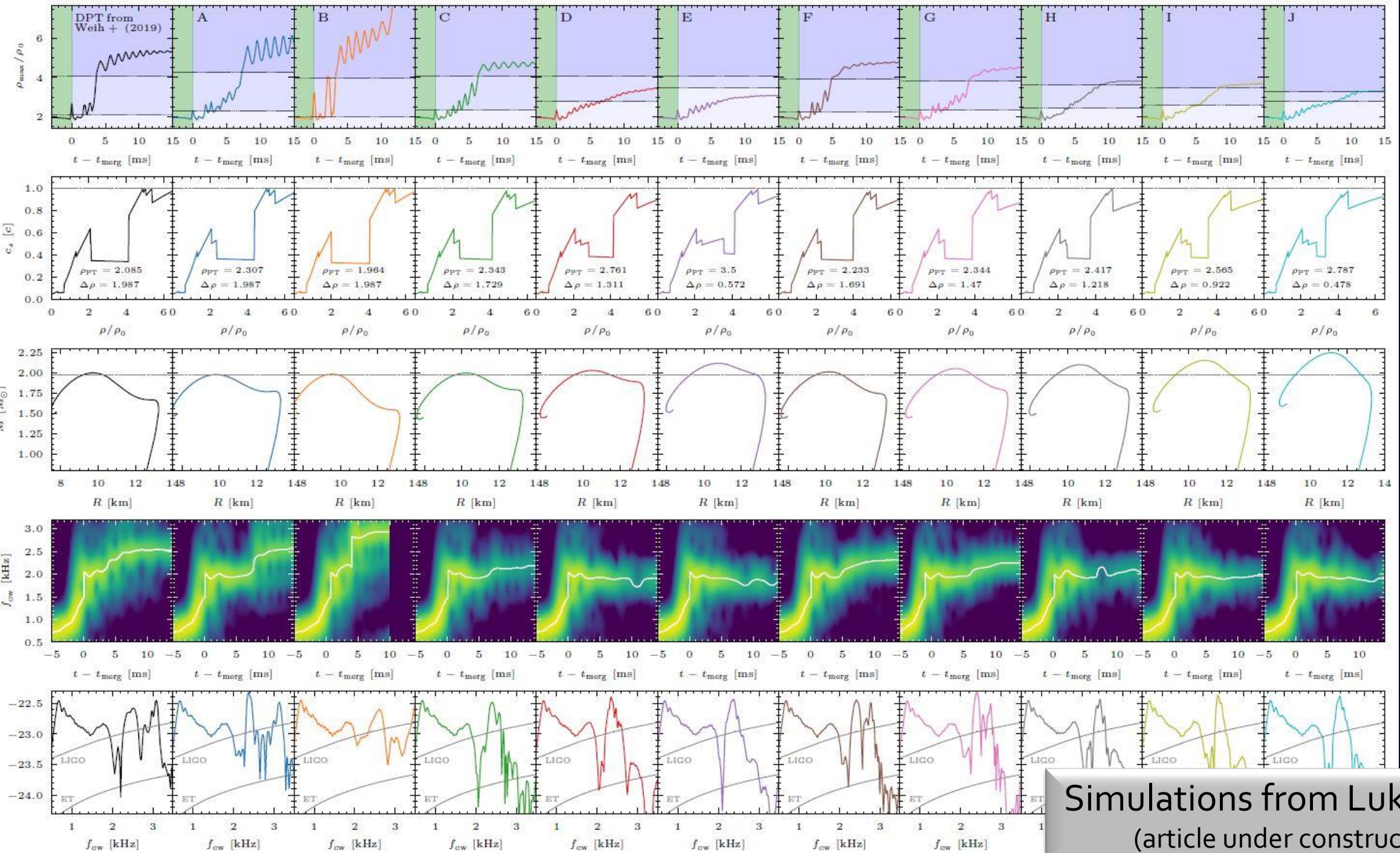
*With Phase Transition  
on the xz- and yz-plane*

Simulations from Lukas Weih



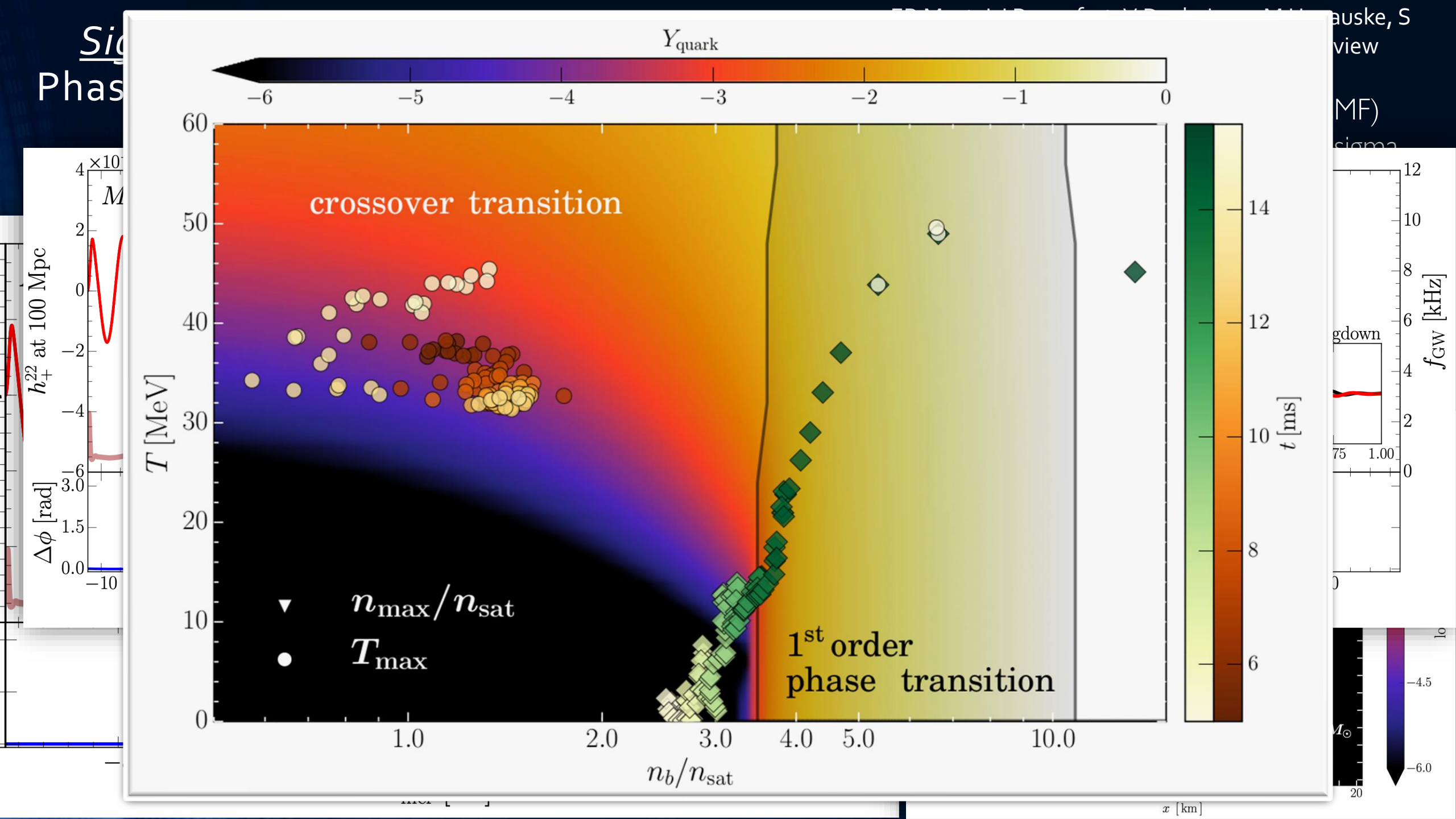


# Analysis of phase transitions of impact

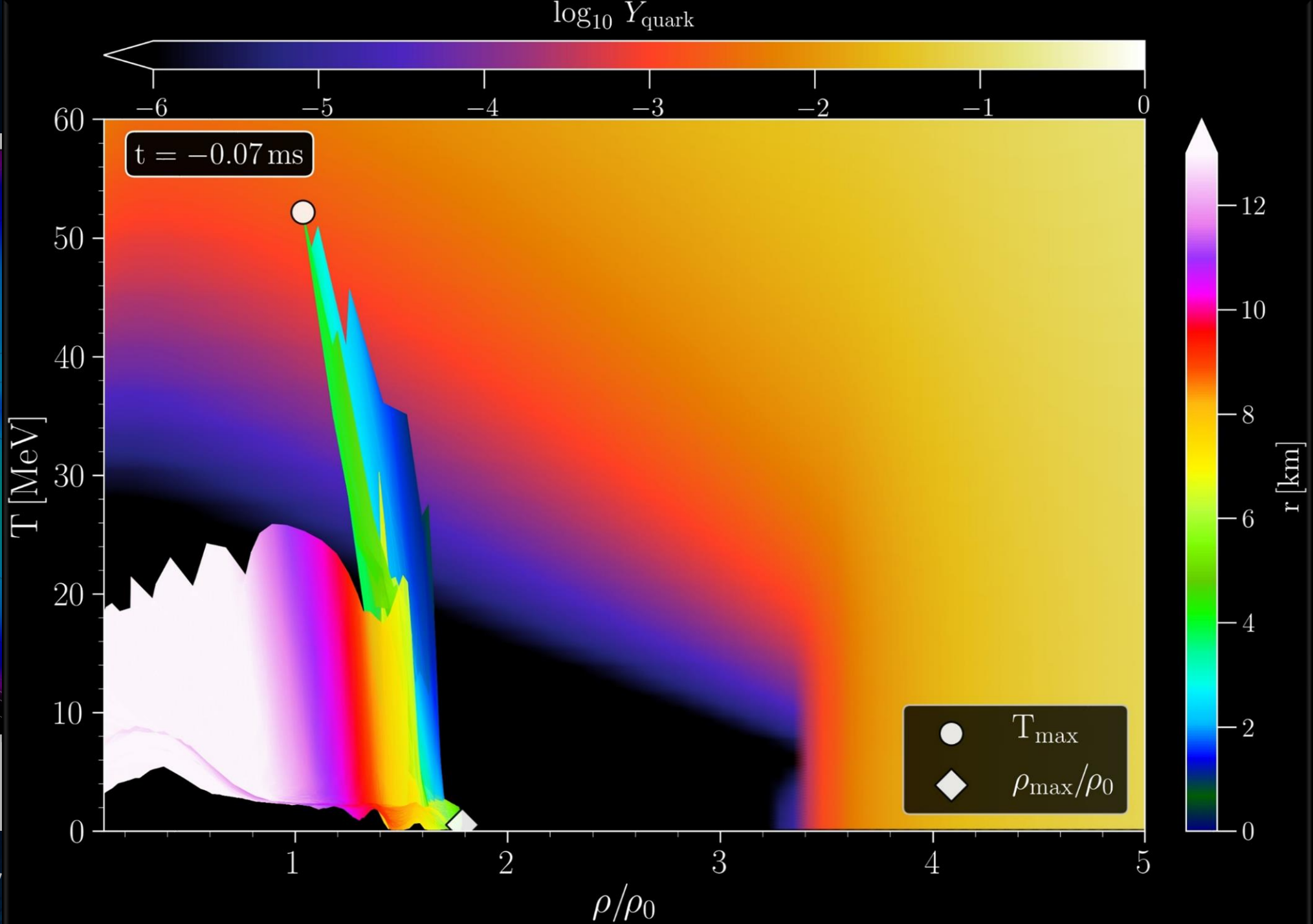
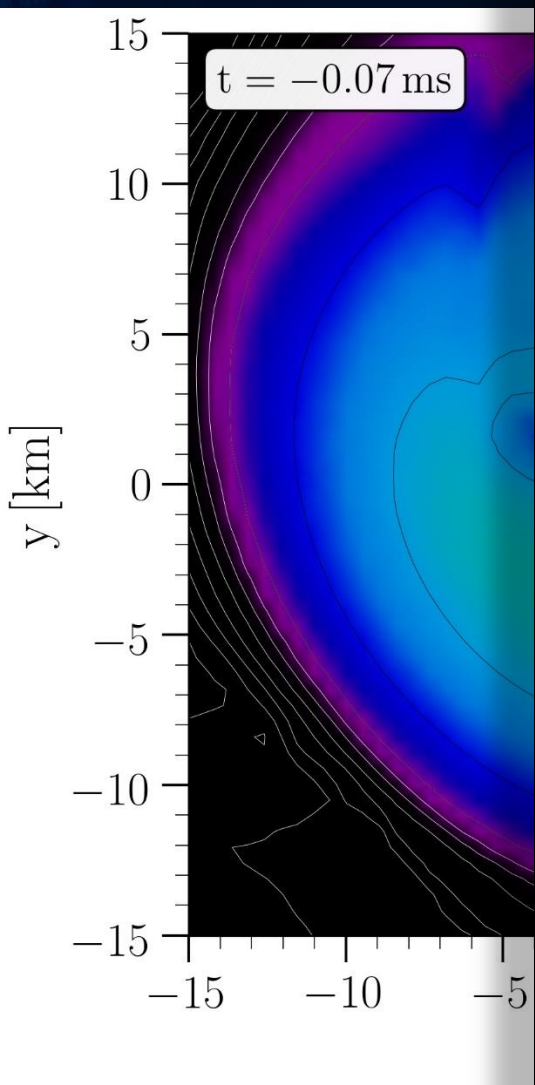


Simulations from Lukas Weih  
(article under construction)



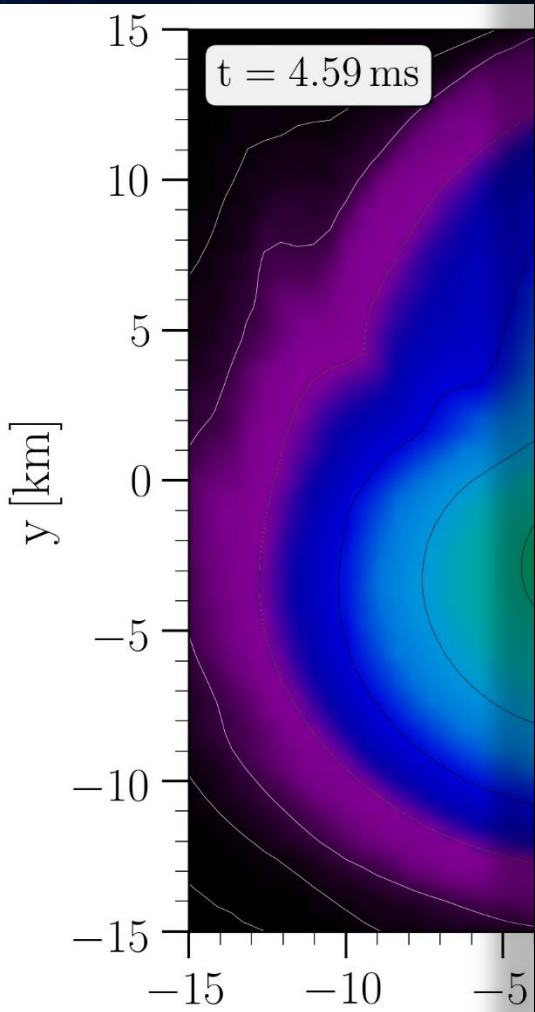


# Merger Phase

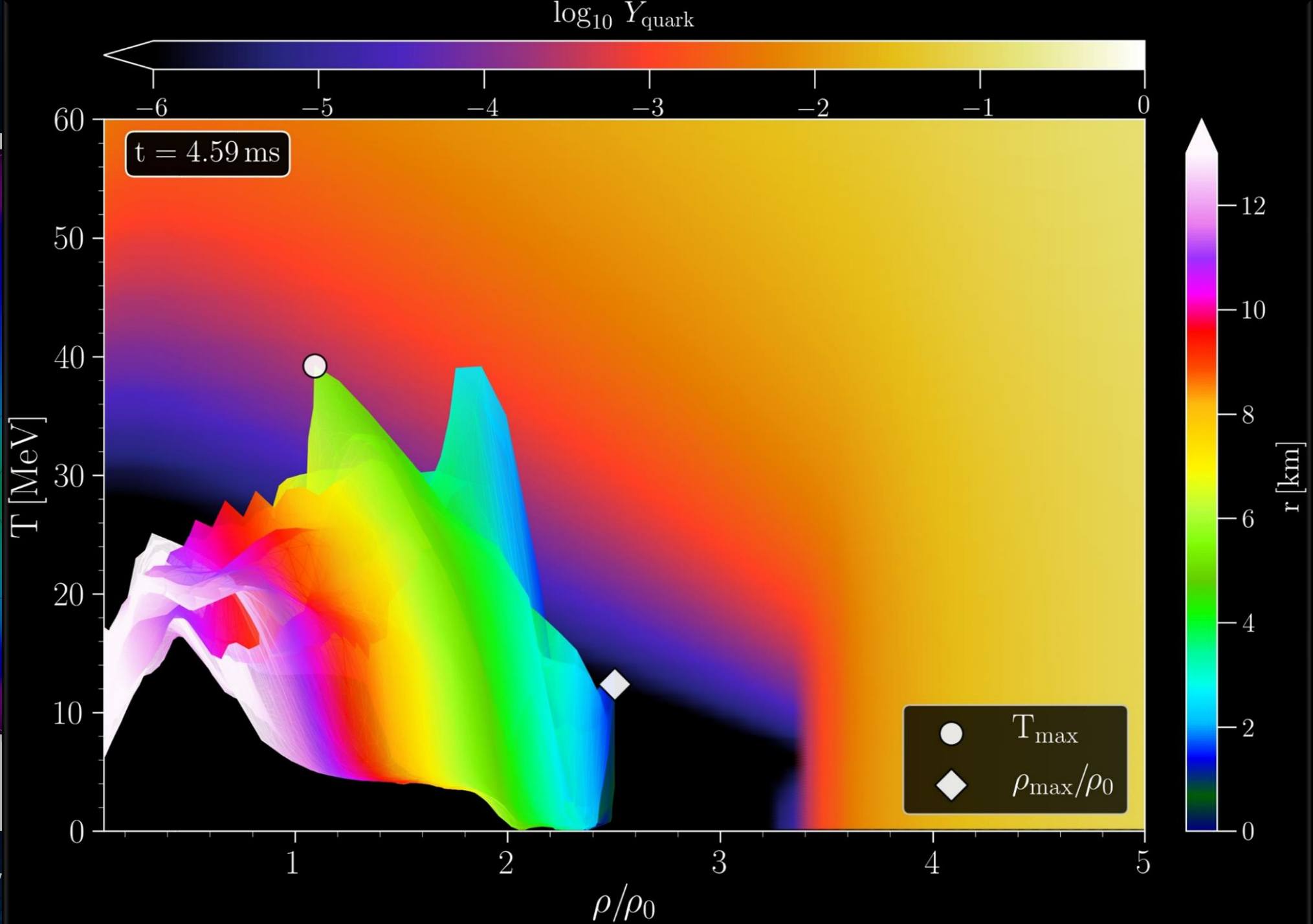


Rest mass density

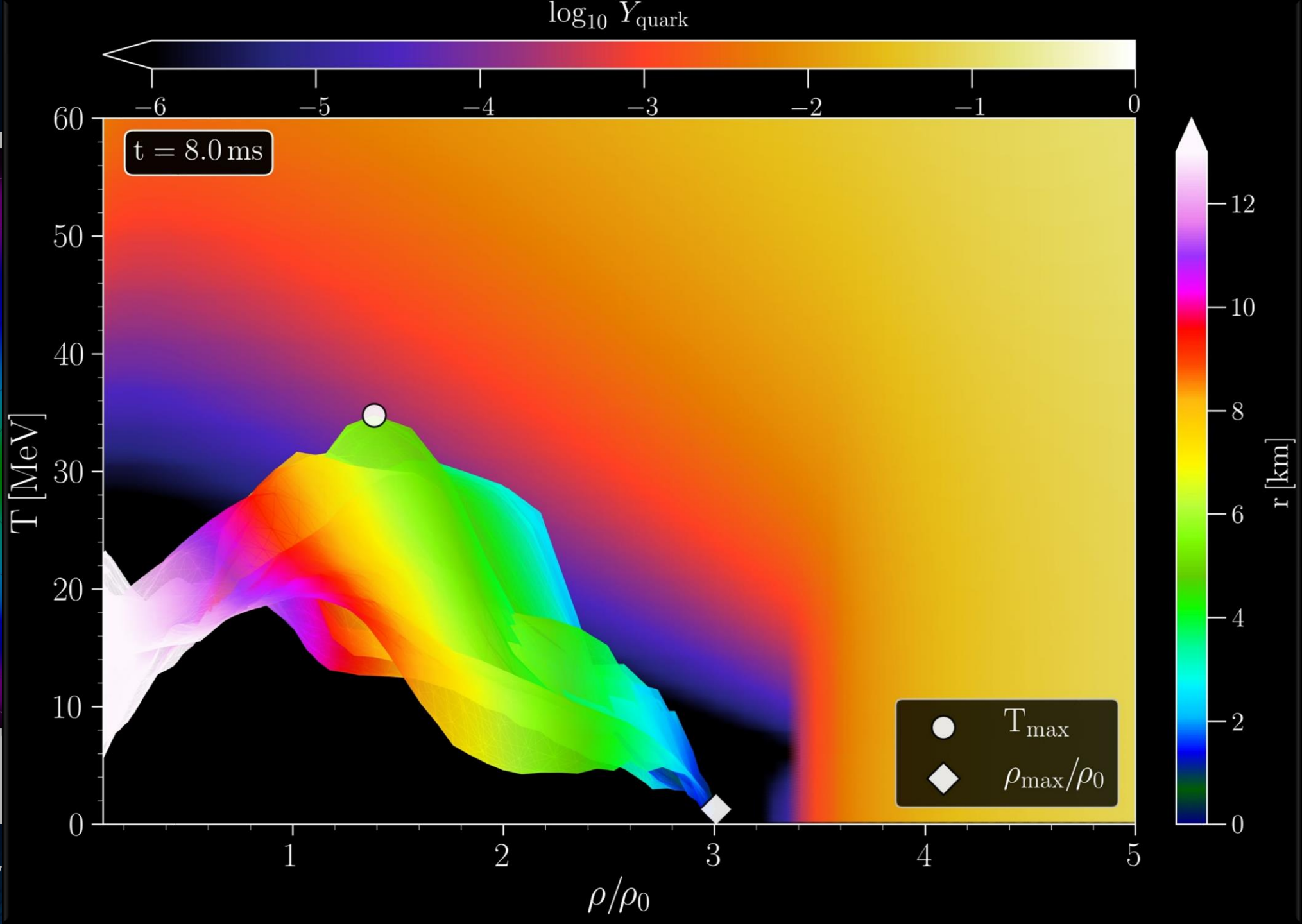
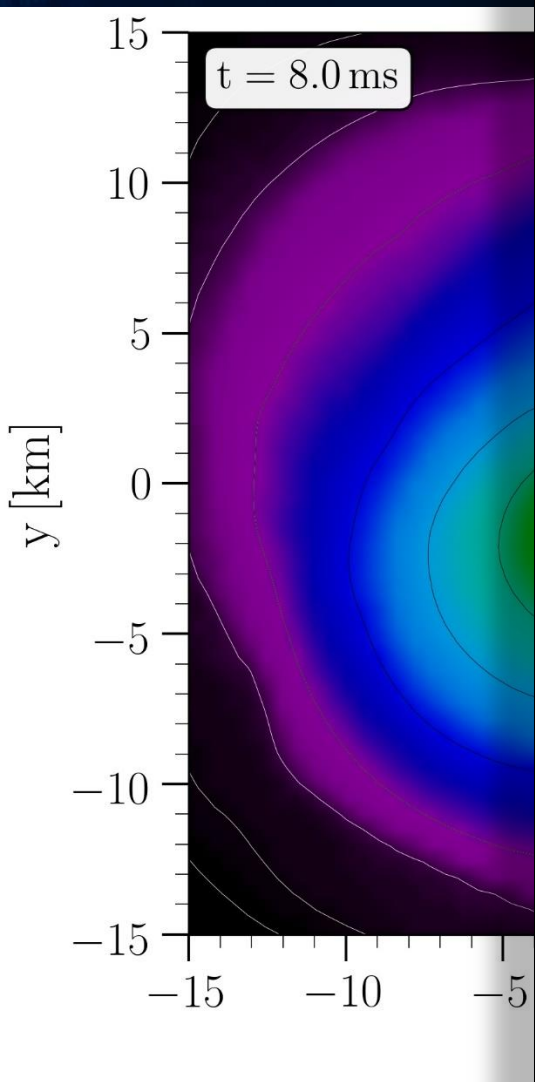
# Post Merger Phase



Rest mass density

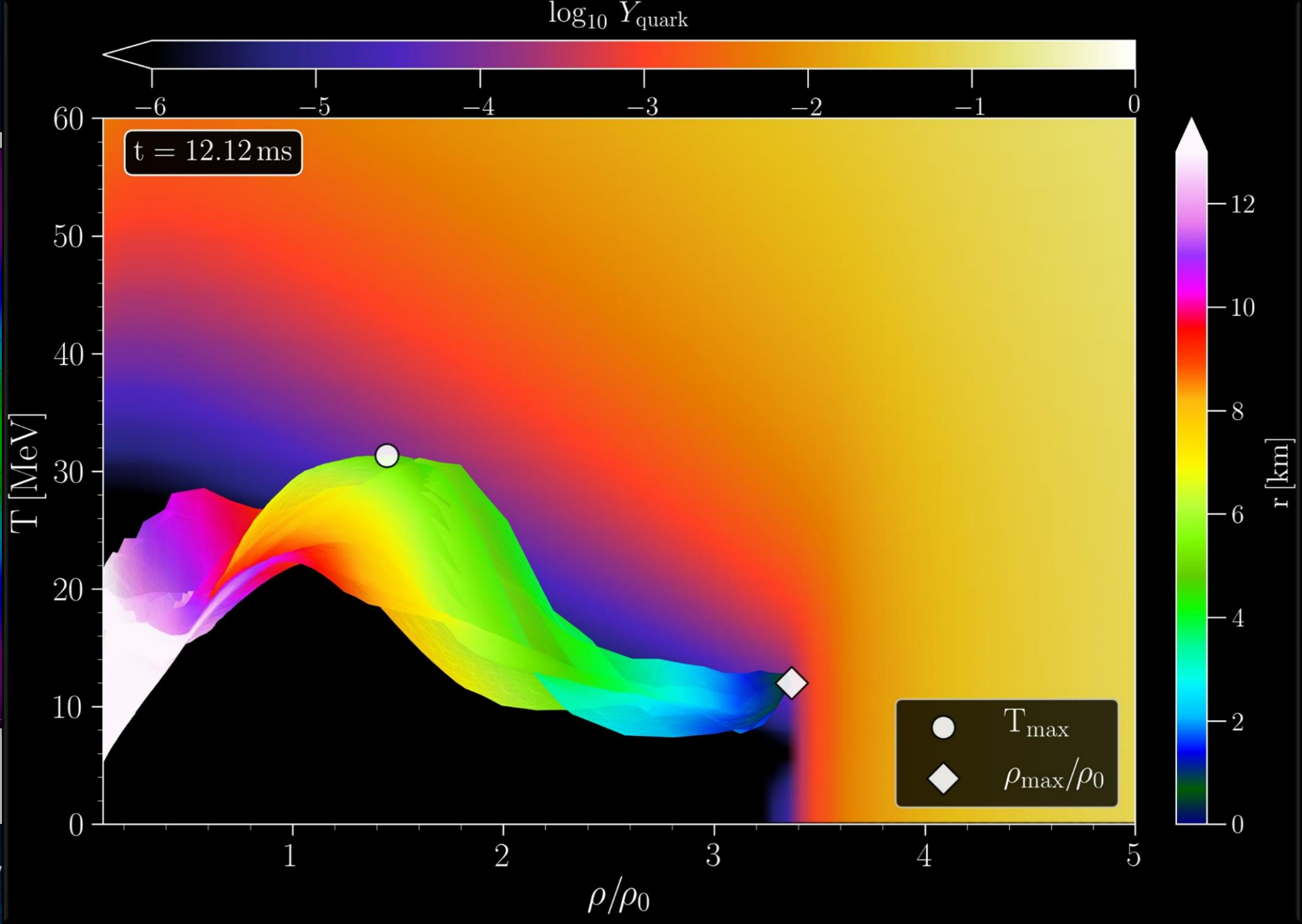
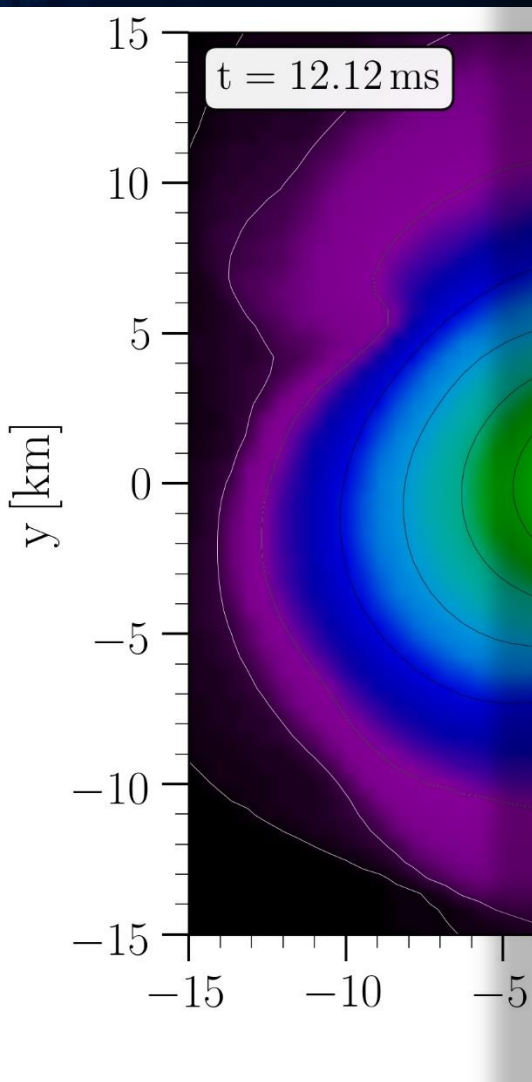


# Merger Phase



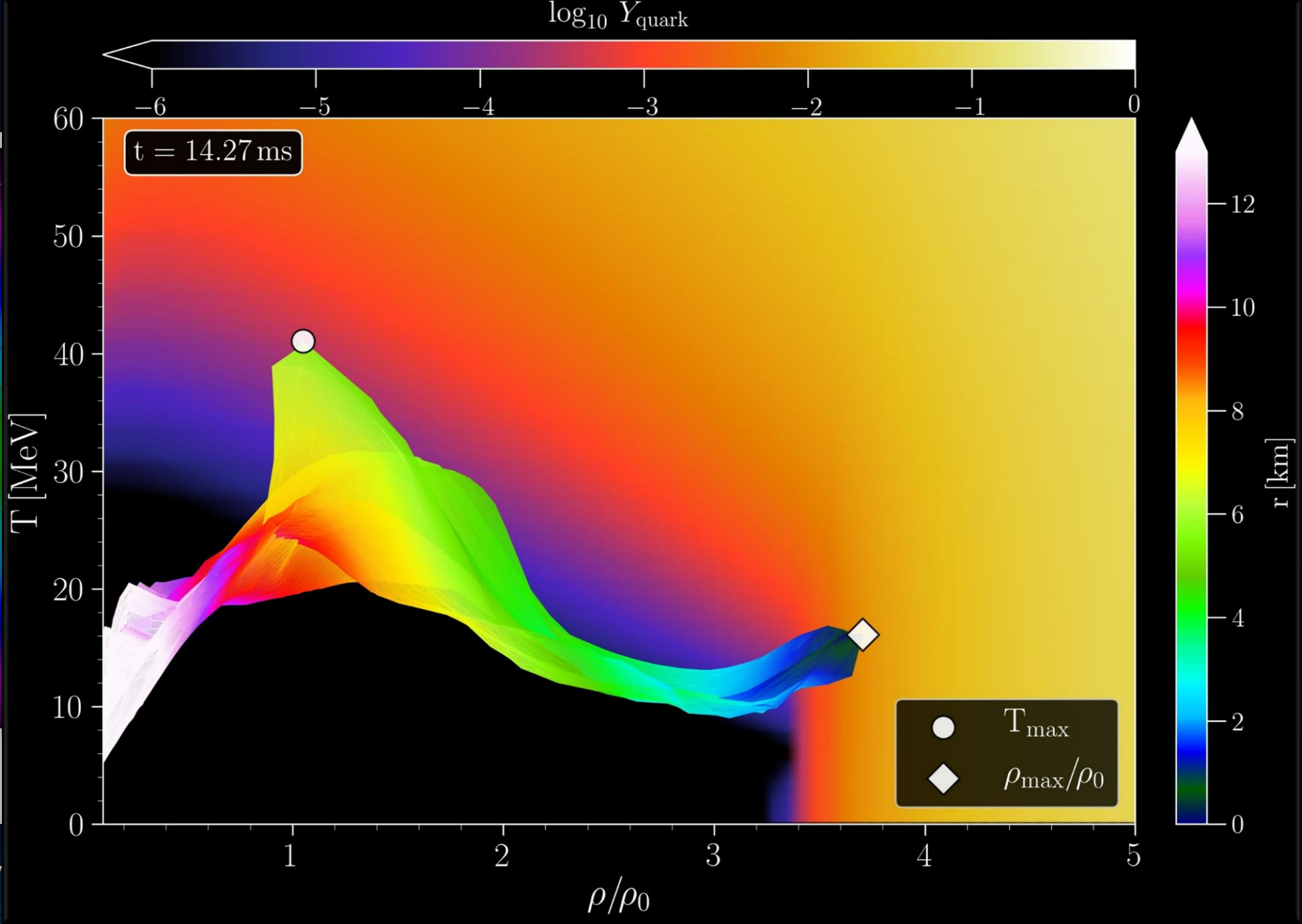
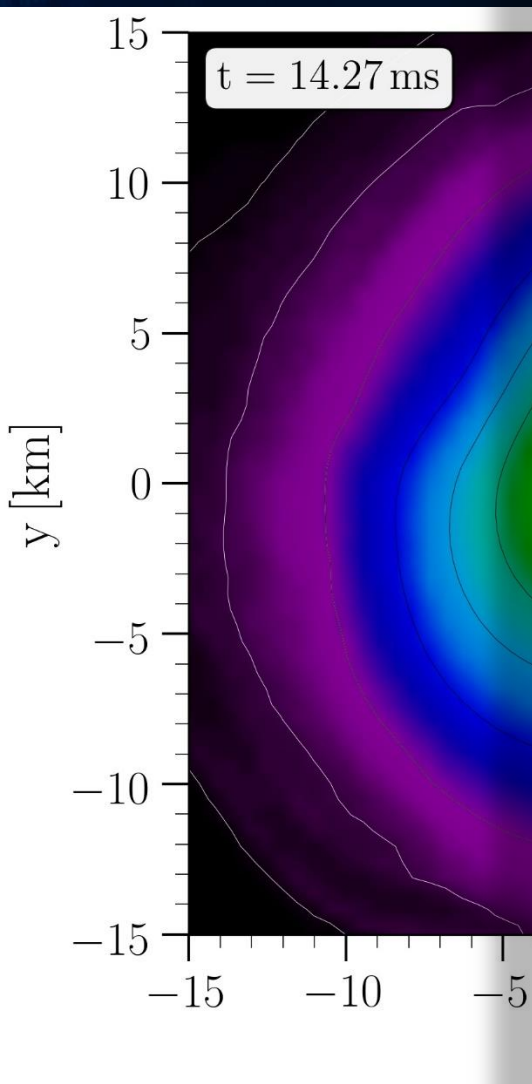
Rest mass density

# Merger Phase



Rest mass density

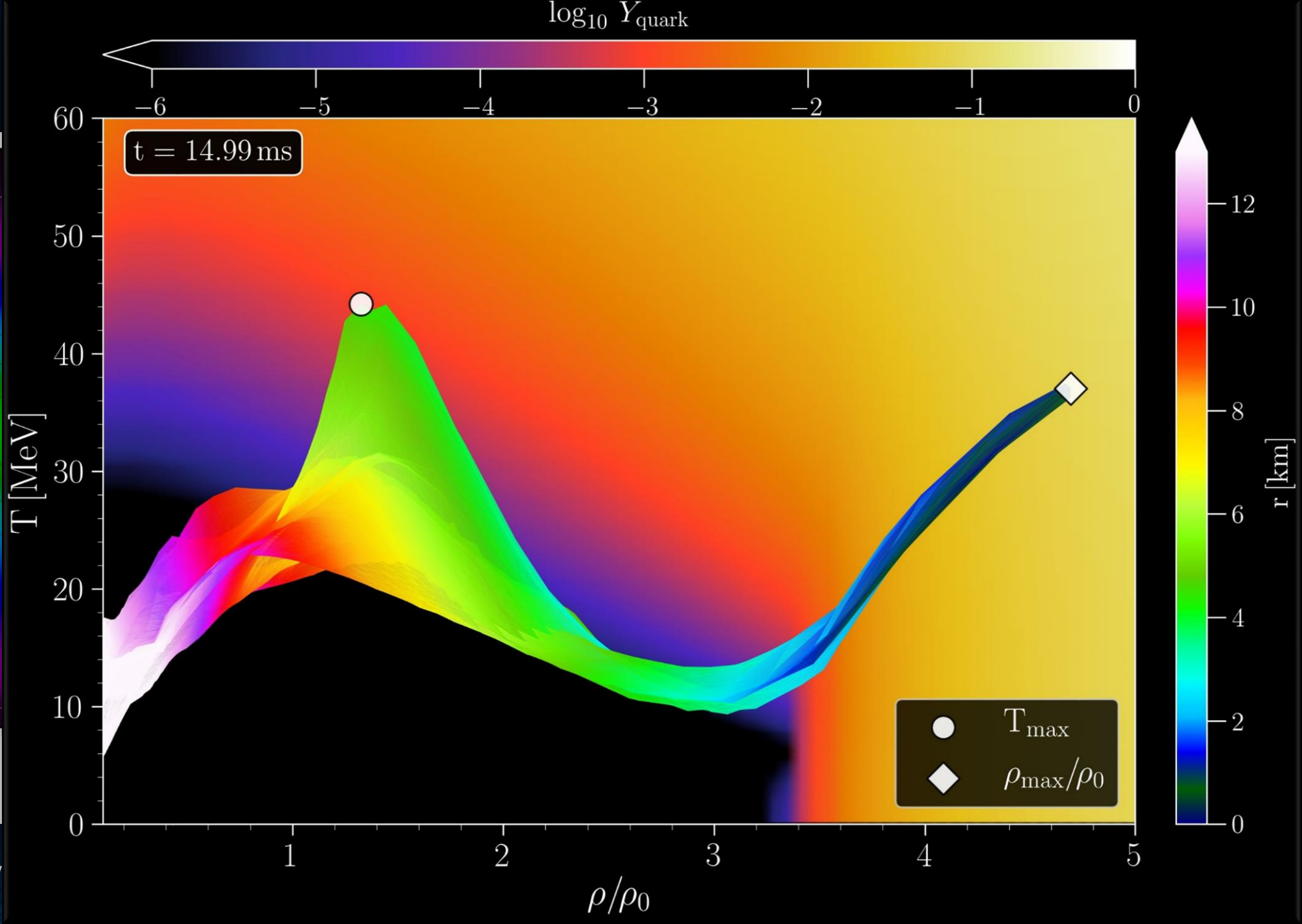
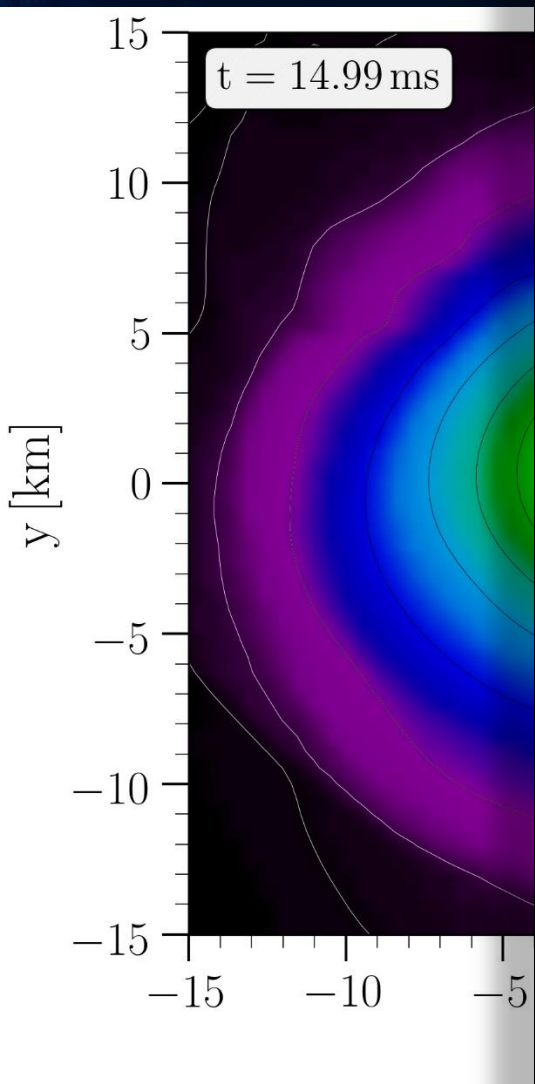
# Merger Phase



Rest mass density

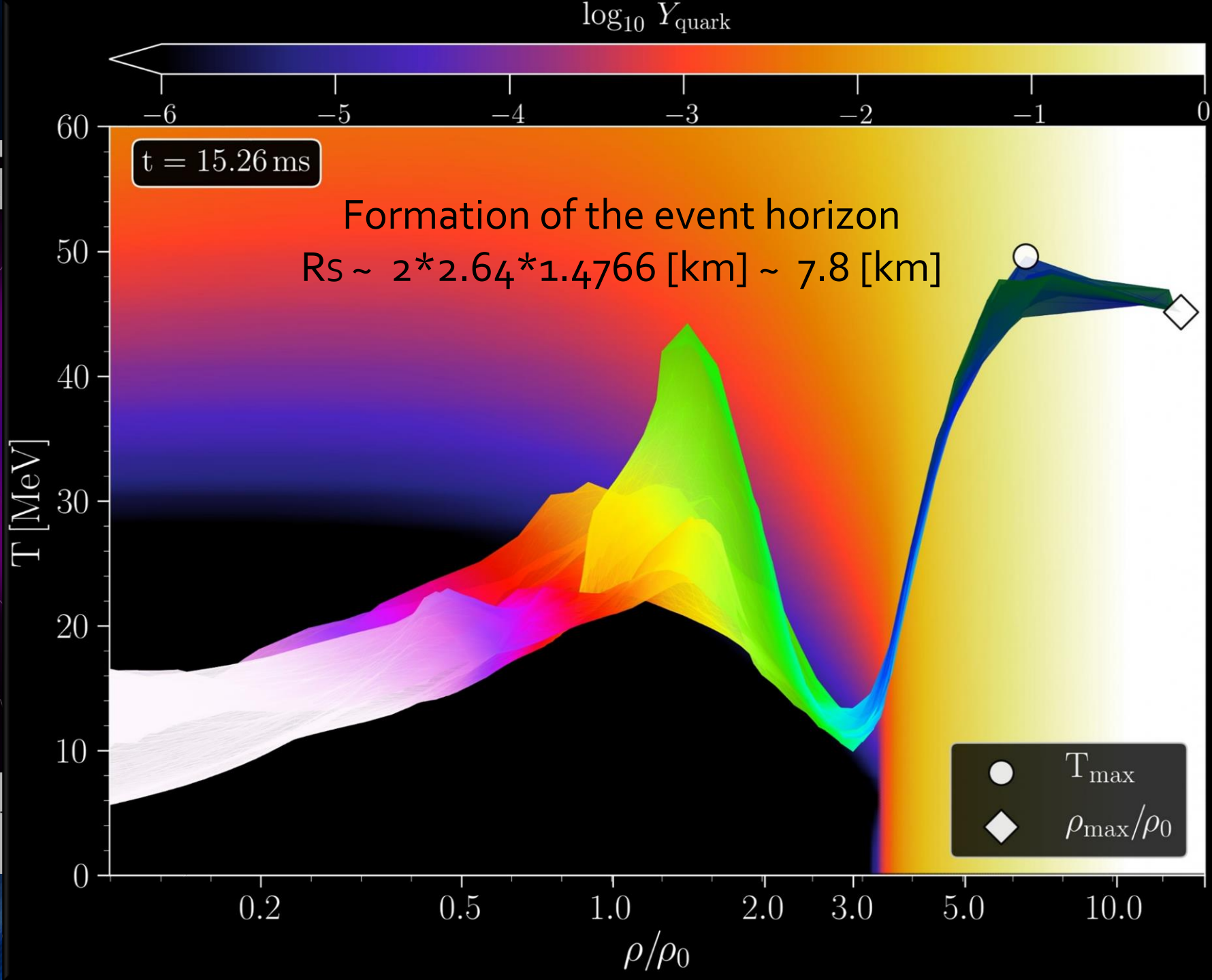
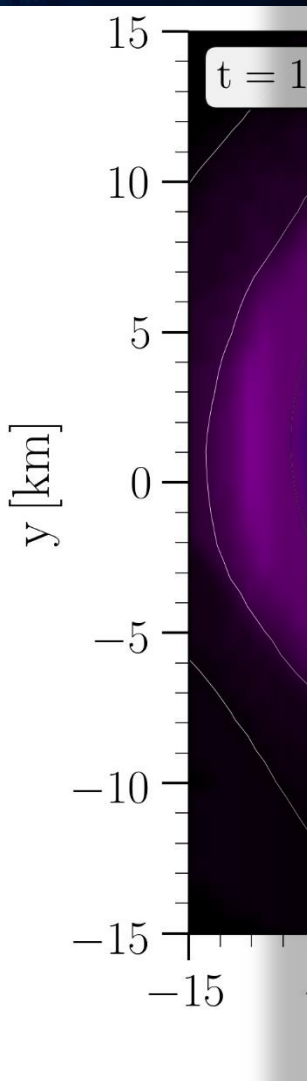


# Merger Phase

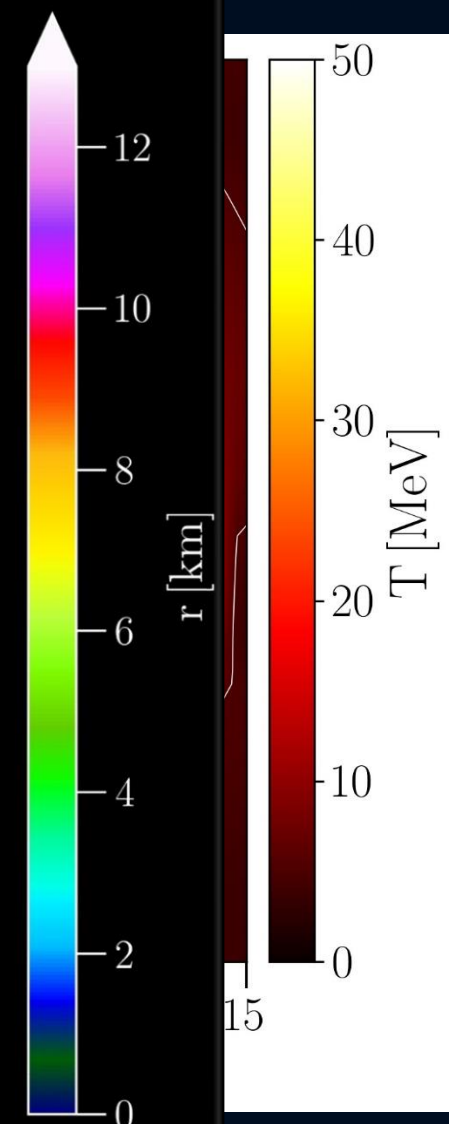


Rest mass density

The last sim



(HMHS)



Rest mas

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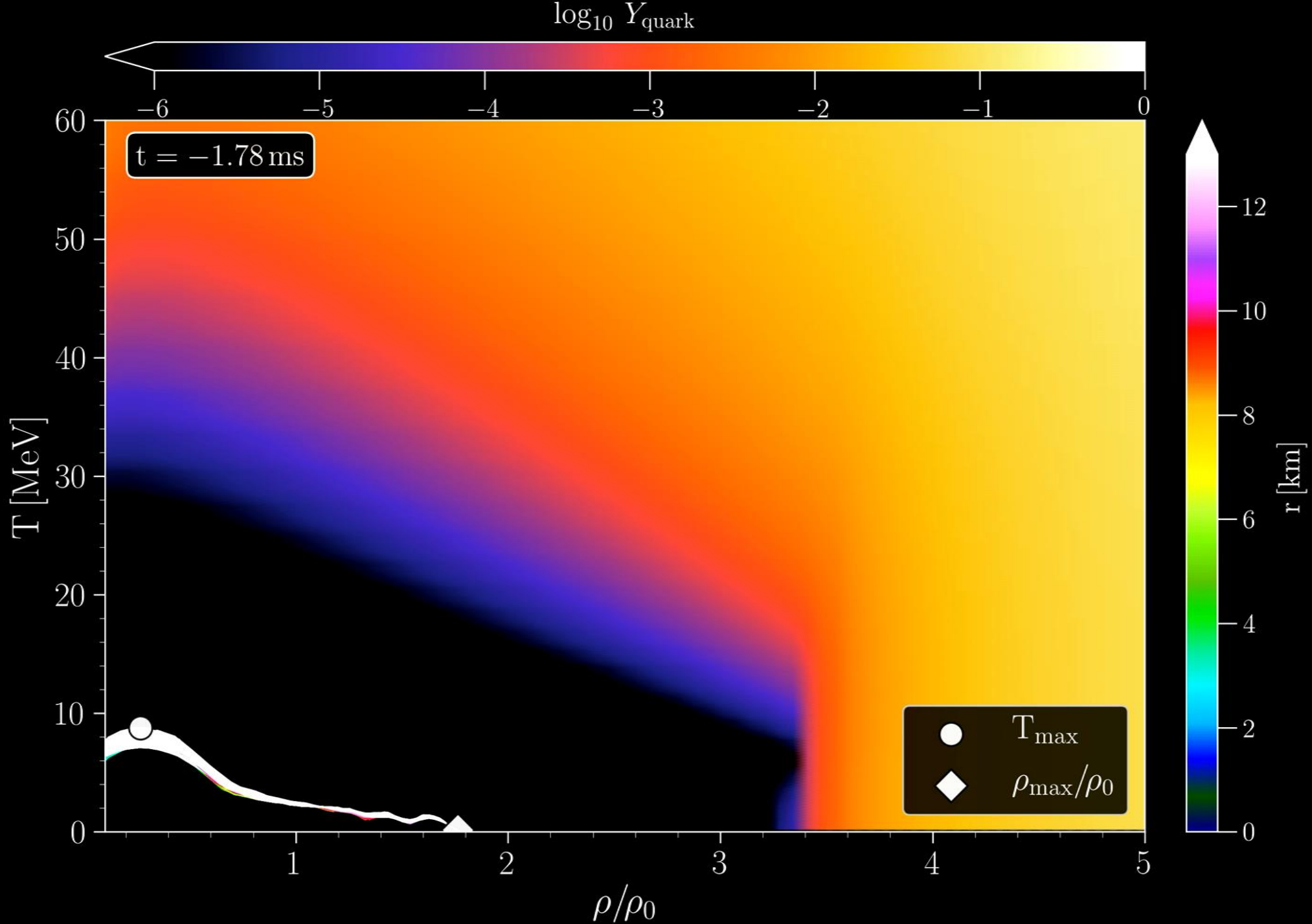
# Phase-transition triggered collapse scenario

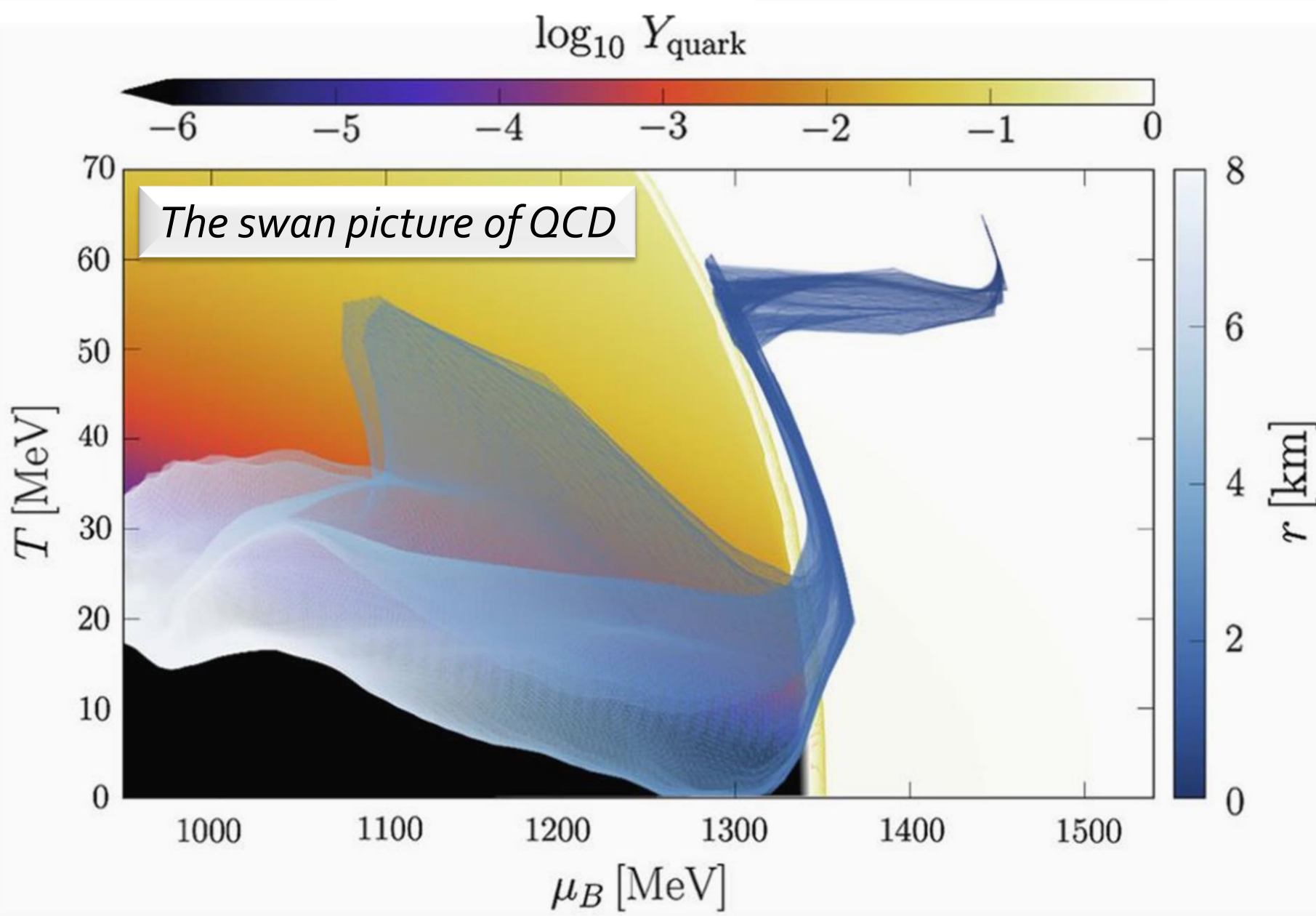
*Signatures of quark-hadron phase transitions in general-relativistic neutron-star mergers*

ER Most, LJ Papenfort, V Dexheimer, M Hanauske, S Schramm, H Stöcker and L. Rezzolla

Physical review letters 122 (6), 061101 (2019)

Density-Temperature-Composition dependent EOS within the CMFo model.





$r=2m$     $r=0$     $r=2m$

Figure 1. Spherically symmetrical collapse in the usual Schwarzschild co-ordinates.

E. Most, J. Papenfort,  
V. Dexheimer, M. Hanauske,  
H. Stöcker and L. Rezzolla,  
*On the deconfinement phase  
transition in neutron-star  
mergers*  
The European Physical Journal  
A 56 (2), 1-11 (2020)

A. Motornenko, M. Hanauske,  
L. Weih, J. Steinheimer and  
H. Stöcker, *MAGIC: Matter in  
Astrophysics, Gravitational  
Waves, and Ion Collisions.* 原子  
核物理评论, 37(3), 272-282  
(2020)

The last picture  
what an outside  
observer sees is the  
frozen picture  
of a dying swan

Self-drawn space-time diagram by R. Penrose (1965)

R. Penrose in Rivista del Nuovo Ci

# General Relativity in the Theater of the Absurd



*A Report to an Academy*



MG16  5-10 JULY 2021  
SIXTEENTH MARCEL GROSSMANN MEETING  
ON RECENT DEVELOPMENTS IN THEORETICAL AND EXPERIMENTAL GENERAL RELATIVITY, ASTROPHYSICS AND RELATIVISTIC FIELD THEORIES

**VIRTUAL MEETING**  
websites: <http://www.icra.it/mg16/> email: [mg16@icranet.org](mailto:mg16@icranet.org)  
<https://indico.icranet.org/event/1/>  
6:30-19:30 CENTRAL EUROPEAN SUMMER TIME

50TH ANNIVERSARY OF  
"INTRODUCING THE BLACK HOLE" 

