

Allgemeine Relativitätstheorie mit dem Computer

*PC-POOL RAUM 01.120
JOHANN WOLFGANG GOETHE UNIVERSITÄT
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9.+10. Vorlesung

Plan für die heutige Vorlesung

- Einführung in Teil III
- Der (3+1)-Split der Einsteingleichung
- Das Einstein Toolkit (ET)
 - Installation des ET mit Simfactory (Laptop, Fuchs-Cluster)
 - Die “.par”-Datei einer Simulation
 - “create” and “submit” einer Testsimulation auf dem Fuchs-Cluster
 - Beispiel-Simulation: Radiale Oszillation eines Neutronensterns mit dem ET
 - Visualisieren der Ergebnisse einer Simulation mit Gnuplot, Mathematica und Python
- Vorlesungsprojekte (einzeln oder in Gruppen)

Die letzten beiden Vorlesungstermine werden vertreten
(bin auf der GR22 Konferenz in Valencia)

Numerical Relativity and Relativistic Hydrodynamics of Binary Neutron Star Mergers

A realistic numerical simulation of a twin star collapse, a merger of two compact stars or a collapse to a black hole needs to go beyond a static, spherically symmetric TOV-solution of the Einstein- and hydrodynamical equations.

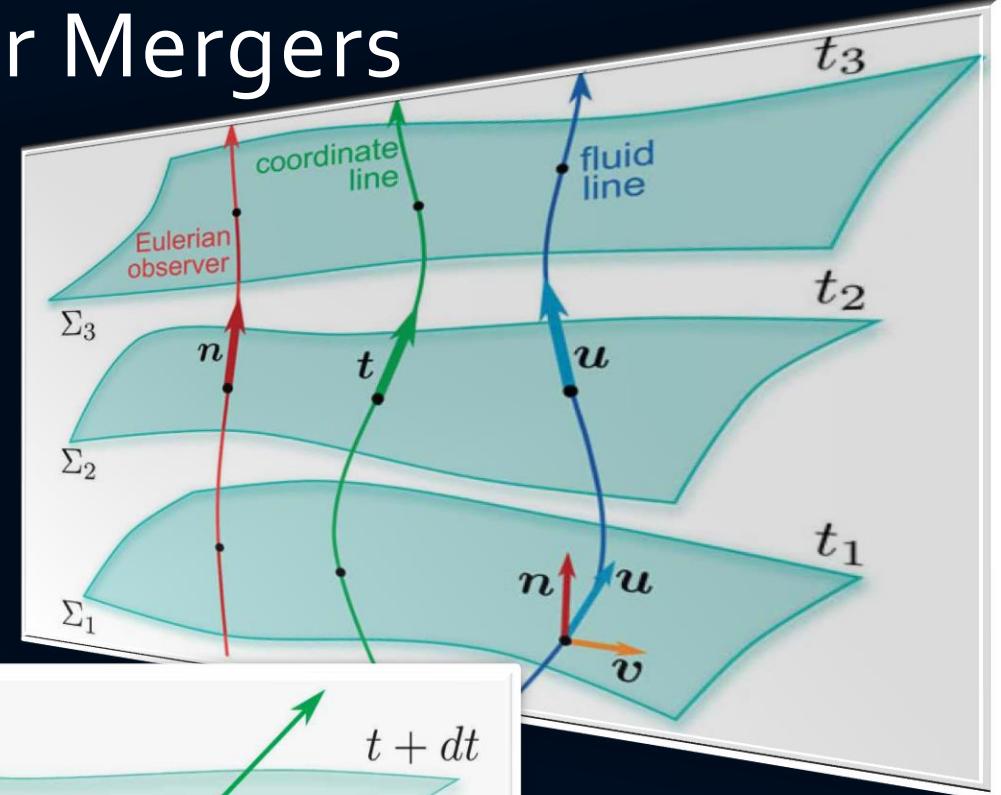
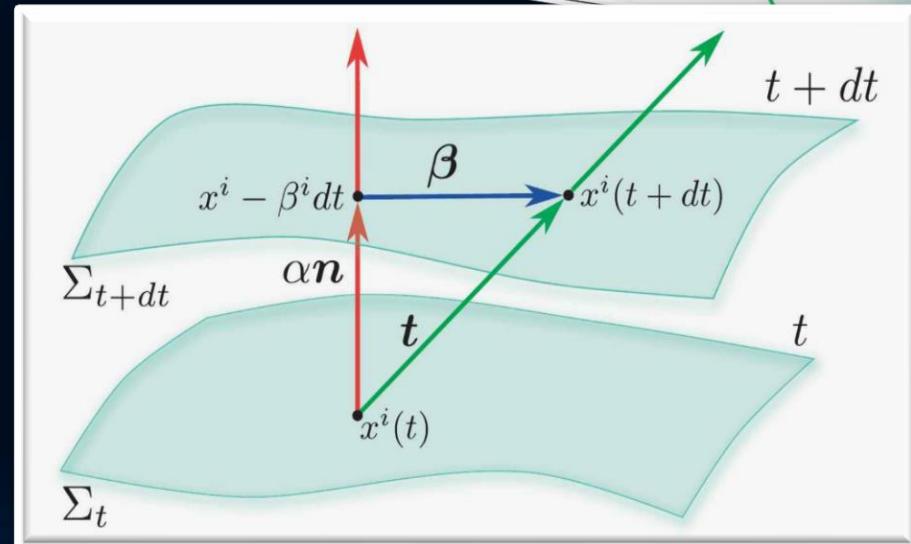
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$$

$$\begin{aligned}\nabla_\mu(\rho u^\mu) &= 0, \\ \nabla_\nu T^{\mu\nu} &= 0.\end{aligned}$$

(3+1) decomposition of spacetime

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_i \beta^i & \beta_i \\ \beta_i & \gamma_{ij} \end{pmatrix}$$

$$d\tau^2 = \alpha^2(t, x^j) dt^2 \quad x^i_{t+dt} = x^i_t - \beta^i(t, x^j) dt$$



The ADM equations

The ADM (Arnowitt, Deser, Misner) equations come from a reformulation of the Einstein equation using the (3+1) decomposition of spacetime.

$$\begin{aligned}\partial_t \gamma_{ij} &= -2\alpha K_{ij} + \mathcal{L}_\beta \gamma_{ij} \\ &= -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i\end{aligned}$$

$$\begin{aligned}\partial_t K_{ij} &= -D_i D_j \alpha + \beta^k \partial_k K_{ij} + K_{ik} \partial_j \beta^k + K_{kj} \partial_i \beta^k \\ &\quad + \alpha \left({}^{(3)}R_{ij} + K K_{ij} - 2K_{ik} K^k_j \right) + 4\pi \alpha [\gamma_{ij} (S - E) - 2S_{ij}]\end{aligned}$$

Time evolving part of ADM

$$D_j (K^{ij} - \gamma^{ij} K) = 8\pi S^i$$

$${}^{(3)}R + K^2 - K_{ij} K^{ij} = 16\pi E$$

Constraints on each hypersurface

Three dimensional covariant derivative

$$D_\nu := \gamma^\mu_\nu \nabla_\mu = (\delta^\mu_\nu + n_\nu n^\mu) \nabla_\mu$$

Three dimensional Riemann tensor

$${}^{(3)}R^\mu_{\nu\kappa\sigma} = \partial_\kappa {}^{(3)}\Gamma^\mu_{\nu\sigma} - \partial_\sigma {}^{(3)}\Gamma^\mu_{\nu\kappa} + {}^{(3)}\Gamma^\mu_{\lambda\kappa} {}^{(3)}\Gamma^\lambda_{\nu\sigma} - {}^{(3)}\Gamma^\mu_{\lambda\sigma} {}^{(3)}\Gamma^\lambda_{\nu\kappa}$$

$${}^{(3)}\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} \gamma^{\alpha\delta} (\partial_\beta \gamma_{\gamma\delta} + \partial_\gamma \gamma_{\delta\beta} - \partial_\delta \gamma_{\beta\gamma})$$

Spatial and normal projections of the energy-momentum tensor:

$$S_{\mu\nu} := \gamma^\alpha_\mu \gamma^\beta_\nu T_{\alpha\beta},$$

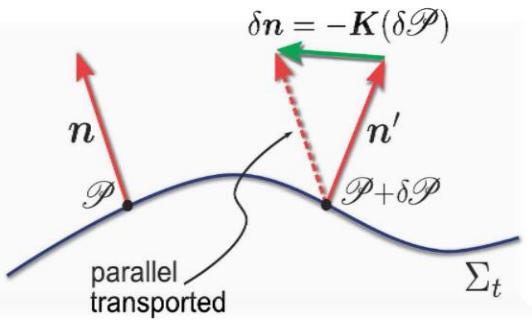
$$S_\mu := -\gamma^\alpha_\mu n^\beta T_{\alpha\beta},$$

$$S := S^\mu_\mu,$$

$$E := n^\alpha n^\beta T_{\alpha\beta},$$

Extrinsic Curvature:

$$K_{\mu\nu} := -\gamma^\lambda_\mu \nabla_\lambda n_\nu$$



From ADM to BSSNOK

Unfortunately the ADM equations are only weakly hyperbolic (mixed derivatives in the three dimensional Ricci tensor) and therefore not "well posed". It can be shown that by using a conformal traceless transformation, the ADM equations can be written in a hyperbolic form. This reformulation of the ADM equations is known as the BSSNOK (Baumgarte, Shapiro, Shibata, Nakamuro, Oohara, Kojima) formulation of the Einstein equation. Most of the numerical codes use this (or even better the CCZ4) formulation.

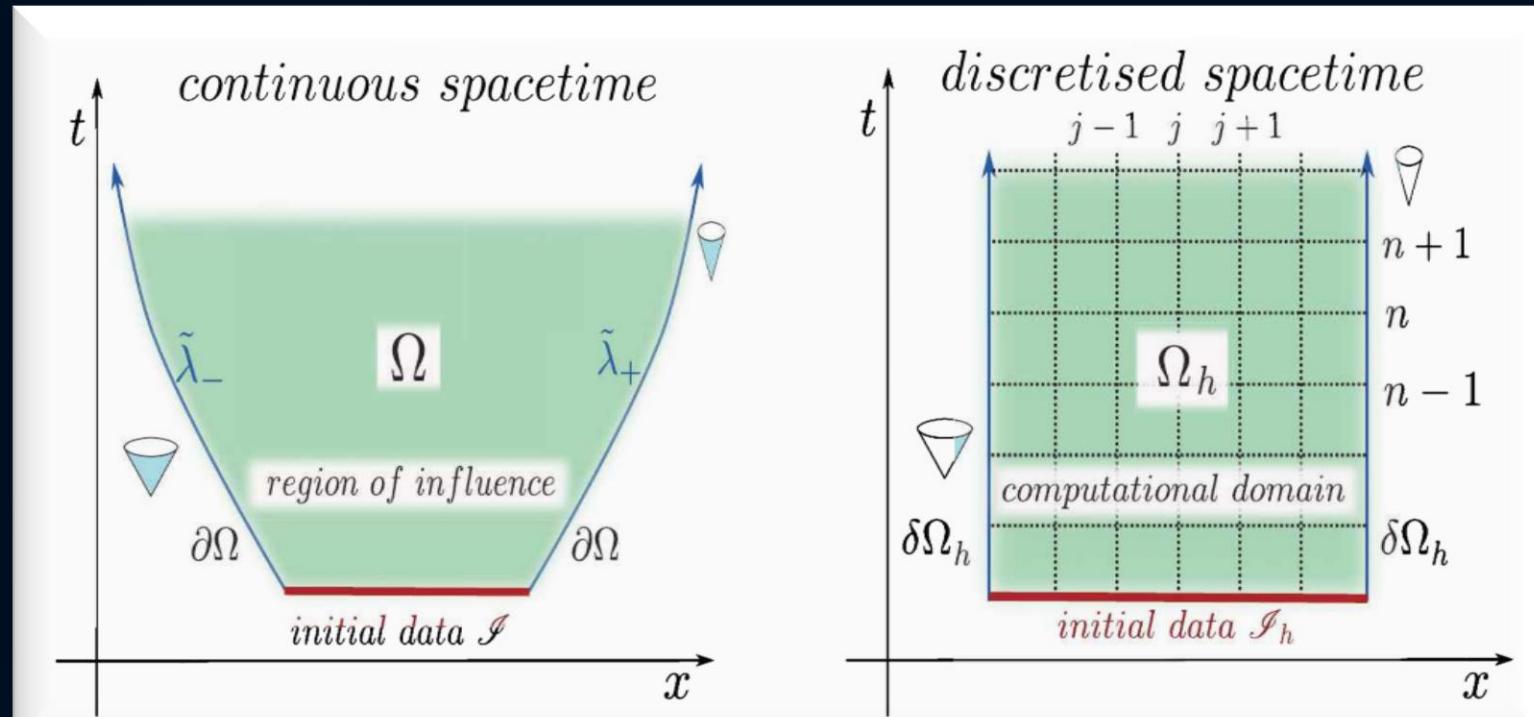
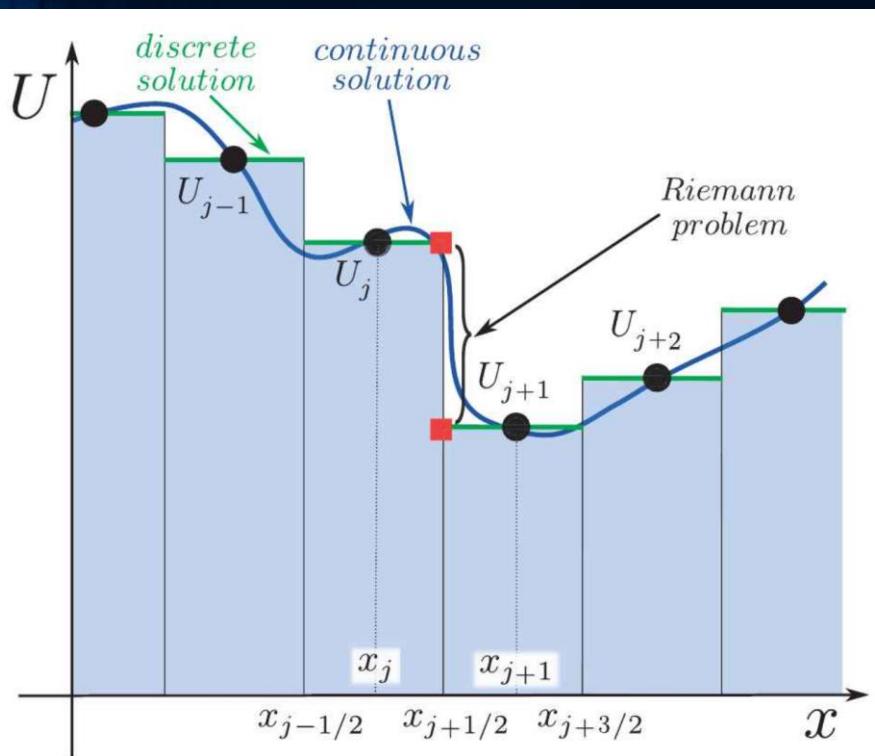
The 3+1 Valencia Formulation of the Relativistic Hydrodynamic Equations

$$\begin{aligned}\nabla_\mu(\rho u^\mu) &= 0, \\ \nabla_\nu T^{\mu\nu} &= 0.\end{aligned}$$

To guarantee that the numerical solution of the hydrodynamical equations (the conservation of rest mass and energy-momentum) converge to the right solution, they need to be reformulated into a conservative formulation. Most of the numerical "hydro codes" use here the 3+1 Valencia formulation.

Finite difference methods

Discretisation of a hyperbolic initial value boundary problem.



High resolution shock capturing methods (HRSC methods) are needed, when Riemann problems of discontinuous properties and shocks needs to be evolved accurately.

Gauge Conditions

On each spatial hypersurface, four additional degrees of freedom need to be specified: A slicing condition for the lapse function and a spatial shift condition for the shift vector need to be formulated to close the system. In an optimal gauge condition, singularities should be avoided and numerical calculations should be less time consuming.

Bona-Massó family of slicing conditions:

$$\partial_t \alpha - \beta^k \partial_k \alpha = -f(\alpha) \alpha^2 (K - K_0)$$

“1+log” slicing condition:

$$f = 2/\alpha \quad \text{where } f(\alpha) > 0 \text{ and } K_0 := K(t=0)$$

“Gamma-Driver” shift condition:

$$\partial_t \beta^i - \beta^j \partial_j \beta^i = \frac{3}{4} B^i,$$

$$\partial_t B^i - \beta^j \partial_j B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$$

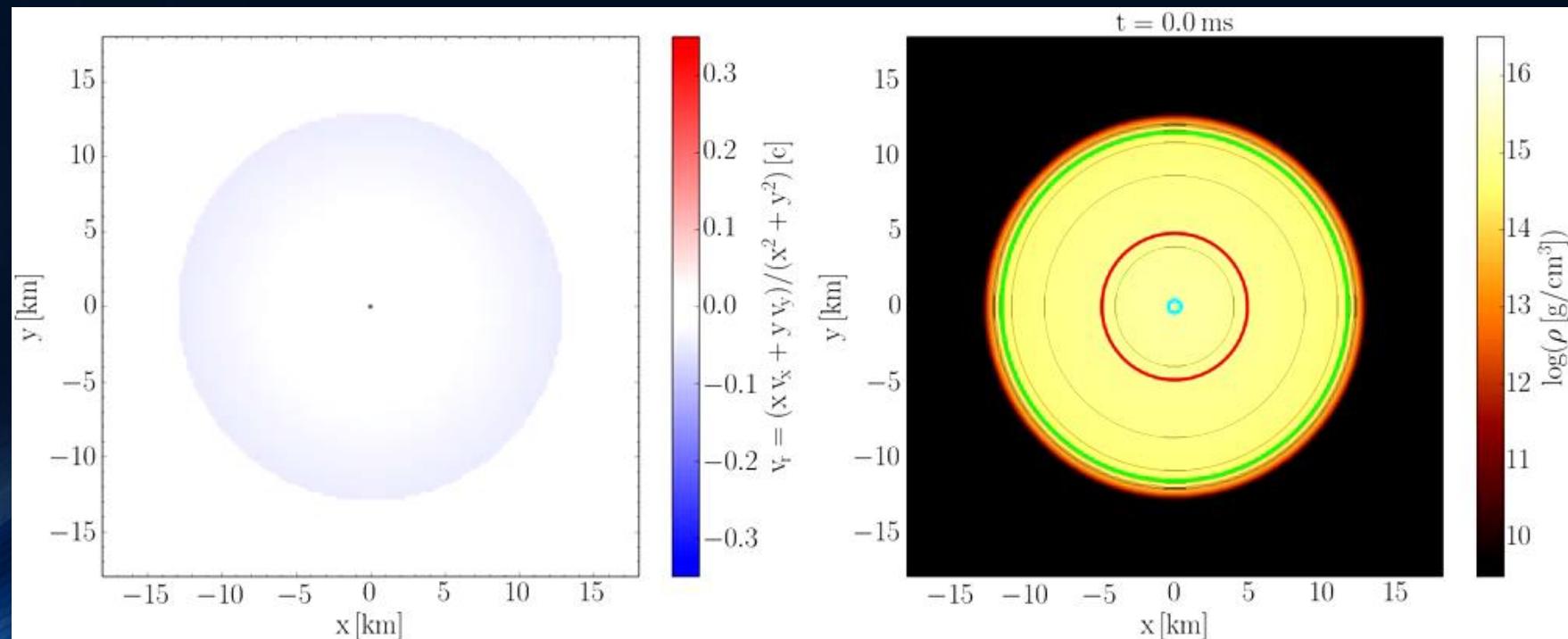
Teil III

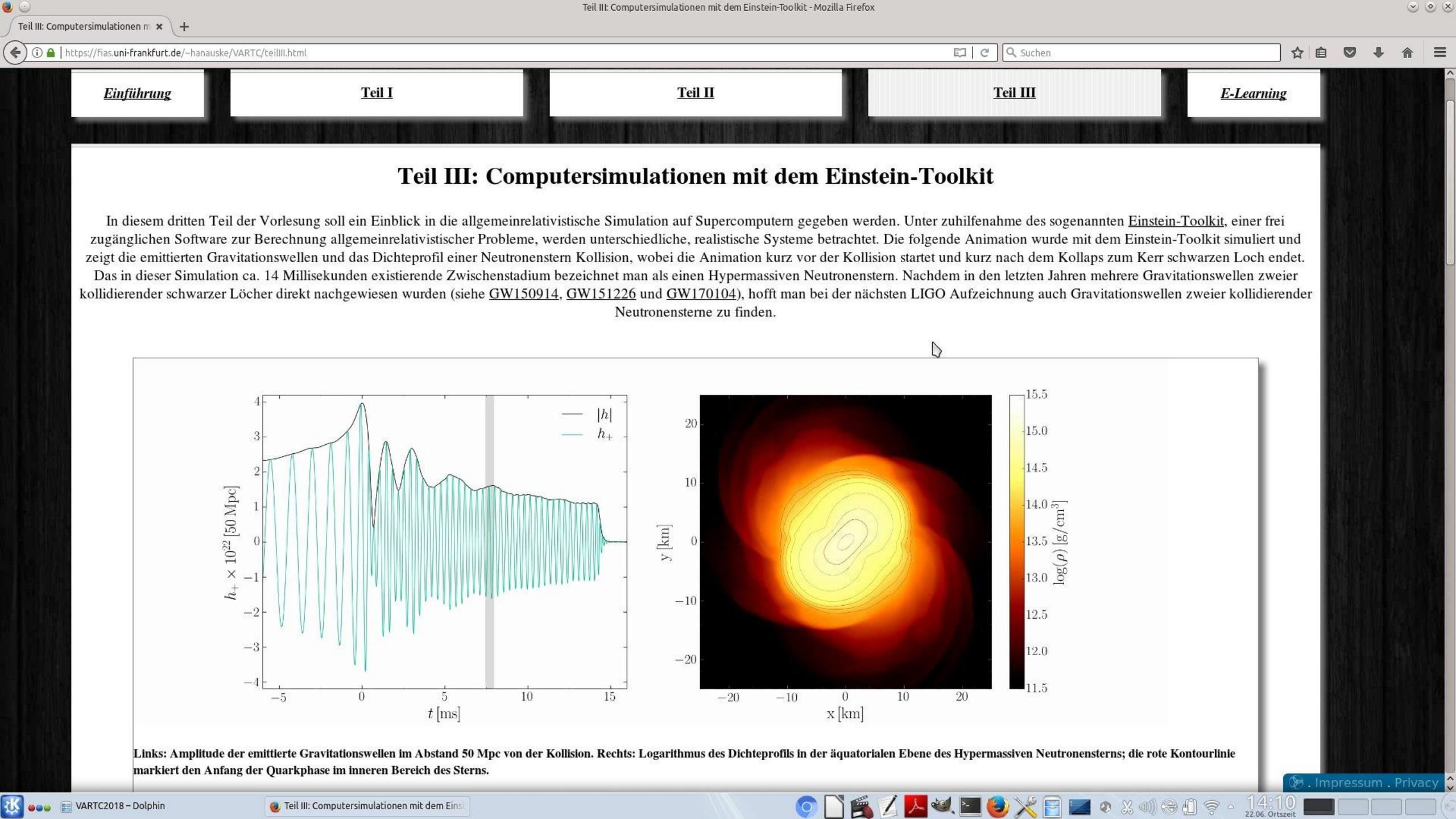
Inhalte des Teil III:

- How to download and build (compile) the Einstein Toolkit
- How to run a test simulation
- Run and visualize (Mathematica or Python) one of the following problems
 - Migration of an unstable neutron star to a stable configuration
 - Collapse of an unstable neutron star to a black hole
 - Binary neutron star mergers
 - Collapse of a neutron star to a quark star (twin star collapse)



einstein
toolkit





Teil III: Computersimulationen mit dem Einstein-Toolkit - Mozilla Firefox

Teil III: Computersimulationen mit dem Einstein-Toolkit - Mozilla Firefox

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1) Die Einsteingleichung im (3+1)-Split

Die (3+1) Zerlegung der Raumzeit

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_i \beta^i & \beta_i \\ \beta_i & \gamma_{ij} \end{pmatrix}$$
$$x^i_{t+dt} = x^i_t - \beta^i(t, x^j) dt$$
$$d\tau^2 = \alpha^2(t, x^j) dt^2$$

Credit: L. Rezzolla, O. Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press (2013)

Um die zeitliche Entwicklung von komplizierten, allgemeinrelativistischen Systemen auf dem Computer zu simulieren muss die Einsteingleichung zunächst umformuliert werden. In der sogenannten (3+1)-Zerlegung der Raumzeit wird die vierdimensionale Mannigfaltigkeit der Raumzeit in dreidimensionale, raumartige Hyperflächen Σ_t zerlegt. Die Metrik der Raumzeit $g_{\mu\nu}$ (siehe nebenstehende Abbildung) besteht in dieser Zerlegung aus einer Lapse-Funktion α , aus einem Shift-Vektor β^i und aus einer rein raumartigen Metrik γ_{ij} ($\mu\nu=0,..,3$ und $i,j=1..3$). Die Lapse-Funktion α beschreibt den Unterschied zwischen der Koordinatenzeit t und der Eigenzeit τ und der Shift-Vektor β^i beschreibt wie stark ein Probekörper in der Raumrichtung "i" von der Struktur der Raumzeit mitgezogen wird, wenn er sich um einen infinitesimalen Zeitschritt dt bewegt. Setzt man diesen Ansatz in die Einsteingleichung ein, so gelangt man zu den sogenannten ADM-Gleichungen (nach Richard Arnowitt, Stanley Deser und Charles W. Misner), die ein System von Differentialgleichungen erster Ordnung darstellen. Um die Konvergenzeigenschaften numerischer Lösungen sicherzustellen wird im Einstein-Toolkit zusätzlich noch eine konforme, spurlose Transformation der Metrik durchgeführt und das System von Differentialgleichungen in eine hyperbolische Form gebracht. Diese Gleichungen werden dann, zusammen mit den hydrodynamischen Gleichungen, im Einstein-Toolkit numerisch gelöst.

2) Download und Kompilierung des Einstein-Toolkit

In diesem Unterpunkt werden die einzelnen Schritte beschrieben, wie man das frei erhältliche Simulationsprogramm Einstein-Toolkit installiert (für eine ausführliche Beschreibung siehe [Einstein-Toolkit](#)).

Impressum . Privacy

VARTC2018 – Dolphin

Teil III: Computersimulationen mit dem Einstein-Toolkit

14:11 22.06. Ortszeit



Einstein Toolkit



"The Einstein Toolkit Consortium is developing and supporting open software for relativistic astrophysics. Our aim is to provide the core computational tools that can enable new science, broaden our community, facilitate interdisciplinary research and take advantage of emerging petascale computers and advanced cyberinfrastructure."

- * Consortium: 94 members, 49 sites, 14 countries
- * Sustainable community model:
 - * 9 Maintainers from 6 sites: oversee technical developments, quality control, verification and validation, distributions and releases
 - * Whole consortium engaged in directions, support, development
 - * Open development meetings
 - * Governance model: still being discussed (looking at CIG, iPlant)

[HTTP://WWW.EINSTEINTOOLKIT.ORG](http://www.einsteintoolkit.org)

Das Einstein Toolkit: Weitere Informationen



einstein toolkit



Welcome

About the Toolkit

Members

Maintainers

Governance

Capabilities

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Releases

Tools

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Community Services

Wiki

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Seminars

Issue Tracker

Documentation

Tutorial for New Users

Citing

WELCOME

The Einstein Toolkit Consortium is developing and supporting open software for relativistic astrophysics. Our aim is to provide the core computational tools that can enable new science, broaden our community, facilitate interdisciplinary research and take advantage of emerging petascale computers and advanced cyberinfrastructure.

Please read our pages [about](#) the Einstein Toolkit, its [governance](#), and how to [get started](#) with the toolkit for more information.

Download

November 2014: We are pleased to [announce the tenth release](#) (code name "[Herschel](#)") of the Einstein Toolkit, an open, community developed software infrastructure for relativistic astrophysics.

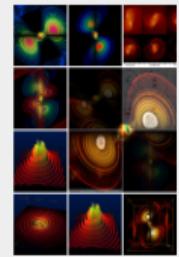
<https://www.youtube.com/watch?v=EO4d32ch6OI>

<https://www.youtube.com/watch?v=p5bq2iUO3DE>

https://www.youtube.com/watch?v=MNpyd_o0MT4

<https://www.youtube.com/watch?v=Qg6PwRI2uS8>

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EinsteinToolkit@Flickr

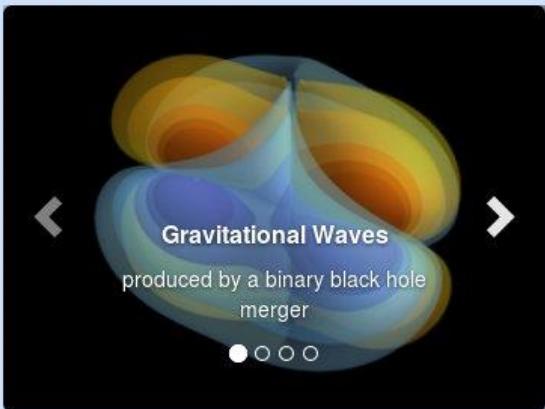
Das Einstein Toolkit



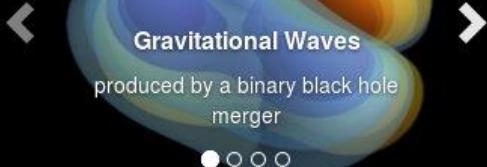
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The Einstein Toolkit



Gravitational Waves
produced by a binary black hole
merger



Einstein Toolkit School and Workshop

Join us at the North American Einstein Toolkit School and Workshop at NCSA, at the University of Illinois at Urbana-Champaign from July 31 to August 4 2017.

This meeting is open to anyone interested in numerical relativity and computational astrophysics and cosmology and in particular to Einstein toolkit users.

The first three days will be dedicated to a school useful for new users of the Einstein Toolkit followed by a two day long workshop open to developers interested in the Einstein Toolkit.

Registration closes July 17, 2017.

[More information](#)

Gallery

About

The Einstein Toolkit is a community-driven software platform of core computational tools to advance and support research in relativistic astrophysics and gravitational physics.

[About](#)

Download

We provide a convenient method to get all of the Einstein Toolkit with just a few commands, and explain the whole process.

[Download](#)

Documentation

A lot of the documentation within the Einstein Toolkit is generated from comments in the source code, and more can be found on the Einstein Toolkit Wiki or other documents. We provide links to guides, tutorials and references.

[Documentation](#)

Contribute

The Einstein Toolkit would not exist without numerous contributions from its community. It is easy to learn how you can contribute as well.

[Contribute](#)

Das Einstein Toolkit: Download



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Download & Requirements

The Einstein Toolkit is hosted on many different machines around the world. We provide a script called [GetComponents](#) to simplify downloading the toolkit. This page just describes how to download the toolkit - you may also be interested in the [Tutorial for New Users](#) which leads you through these steps and more on the Queen Bee supercomputer, or in a simpler [tutorial](#) for setup on a typical Linux box.

Users of the Einstein Toolkit are encouraged to [register](#) which also signs up for the [users mailing list](#).

Main Toolkit

Citations

The development of production level scientific software, such as the components of the Einstein Toolkit, represents the academic output of researchers. These scientific contributions should be acknowledged and respected on par with those solely based in theory or experiment. Please review our [Citation Policy](#).

Current release: Payne-Gaposchkin (released on December 16th, 2016)

This is the recommended version of the toolkit for most users. See the [release notes](#) for more information.

Note: OSX users cannot use the 'subversion' client shipped by Apple. In that case install subversion either from homebrew or macports.

Enter the directory on your machine in which you would like to download the ET (for example, your home directory), and type the commands listed below. This will create a directory called Cactus in which the components of the Einstein Toolkit are downloaded.

```
curl -kLO https://raw.githubusercontent.com/gridaphobe/CRL/ET_2016_11/GetComponents  
chmod a+x GetComponents  
. ./GetComponents --parallel https://bitbucket.org/einsteintoolkit/manifest/raw/ET_2016_11/einsteintoolkit.th
```

A tarball of the release is also available [here](#), but using GetComponents is the preferred method to obtain the code. Use the tarball only if there is no way to use GetComponents (which should almost never be the case).

ET-Download auf dem Fuchs-Cluster

```
[prakti1@login02.csc ~]$ cd ET-2016-11/
[prakti1@login02.csc ET-2016-11]$ curl -kLO https://raw.githubusercontent.com/gridaphobe/CRL/ET_2016_11/GetComponents
 % Total    % Received % Xferd  Average Speed   Time   Time   Time  Current
      Dload  Upload Total Spent   Left Speed
100 99330  100 99330     0     0  486k    0 --::-- --::-- --::-- 30.9M
[prakti1@login02.csc ET-2016-11]$ chmod a+x GetComponents
[prakti1@login02.csc ET-2016-11]$ ./GetComponents --parallel https://bitbucket.org/einsteintoolkit/manifest/raw/ET_2016_11/einsteintoolkit.th
-----
Checking out module: par
  from repository: https://bitbucket.org/einsteintoolkit/einsteinexamples.git
    into: Cactus
  into: Cactus
    as: flesh
-----
Checking out module: COPYRIGHT
  from repository: https://bitbucket.org/cactuscode/cactus.git
    into: Cactus
    as: flesh
-----
Checking out module: doc
  from repository: https://bitbucket.org/cactuscode/cactus.git
    into: Cactus
    as: flesh
-----
Checking out module: lib
  from repository: https://bitbucket.org/cactuscode/cactus.git
    into: Cactus
    as: flesh
-----
Checking out module: ./utils
  from repository: https://bitbucket.org/cactuscode/utilities.git
    into: Cactus
    as: utils
-----
Checking out module: Makefile
  from repository: https://bitbucket.org/einsteintoolkit/pittnullcode.git
    into: Cactus/arrangements
-----
Checking out module: PITNullCode/SphericalHarmonicDecomp
  from repository: https://bitbucket.org/einsteintoolkit/pittnullcode.git
    into: Cactus/arrangements
-----
from repository: https://bitbucket.org/einsteintoolkit/pittnullcode.git
  into: Cactus/arrangements
-----
Checking out module: EinsteinInitialData/IDConstraintViolate
  from repository: https://bitbucket.org/einsteintoolkit/einsteininitialdata.git
    into: Cactus/arrangements
-----
Checking out module: ./CoreDoc
  from repository: https://bitbucket.org/cactuscode/coredoc.git
    into: Cactus/arrangements/CactusDoc
    as: CoreDoc
-----
268 components checked out successfully.
0 components updated successfully.
Time Elapsed: 18 minutes, 5 seconds
[prakti1@login02.csc ET-2016-11]$ █
```

Das Einstein Toolkit: Setup mit SimFactory

```
[prakti1@login02.csc Cactus]$ ./simfactory/bin/sim setup --machine fuchs
```

Here we will define some necessary Simulation Factory defaults.

```
Determining local machine name: login02.cm.cluster
Creating machine login02.cm.cluster from generic: machine login02.cm.cluster [/home/agmisc/prakti1/ET-2016-11/Cactus/repos/simfactory2/mdb/machine]
enter value for key user [prakti1]:
enter value for key email [prakti1]:
enter value for key allocation []:
enter value for key sourcebasedir (the parent directory containing the Cactus sourcetree) [/home/agmisc/prakti1/ET-2016-11]:
enter value for key basedir (the location of simfactory simulations) [/home/agmisc/prakti1/simulations]:
```

would you like to enter key/value pairs for a specific machine? [Y/N*]:

-----SUMMARY-----:

```
[default]
user      = prakti1
email     = prakti1
allocation =
sourcebasedir = /home/agmisc/prakti1/ET-2016-11
basedir    = /home/agmisc/prakti1/simulations
```

-----END SUMMARY-----:

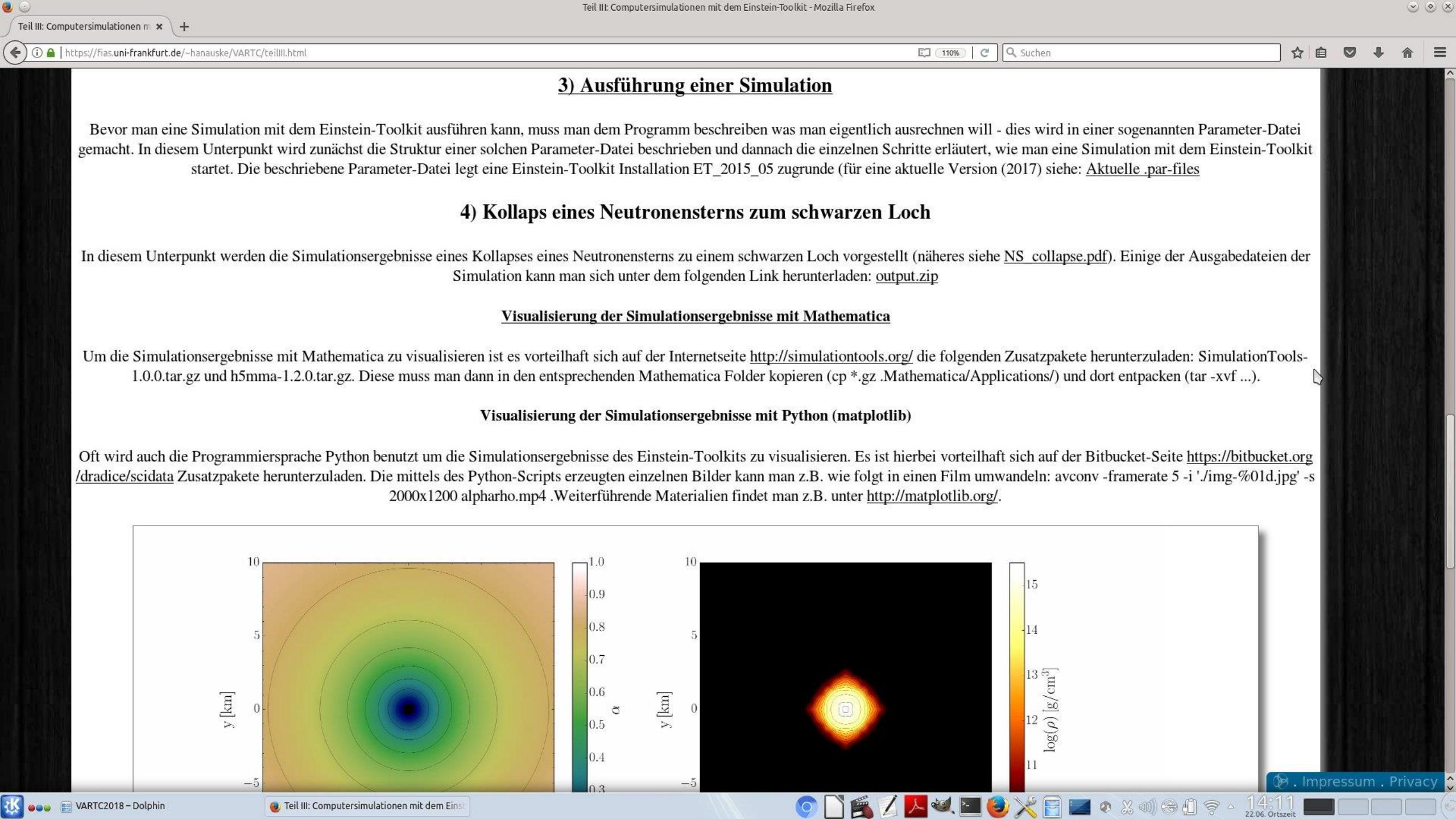
Save contents [Y*/N]:

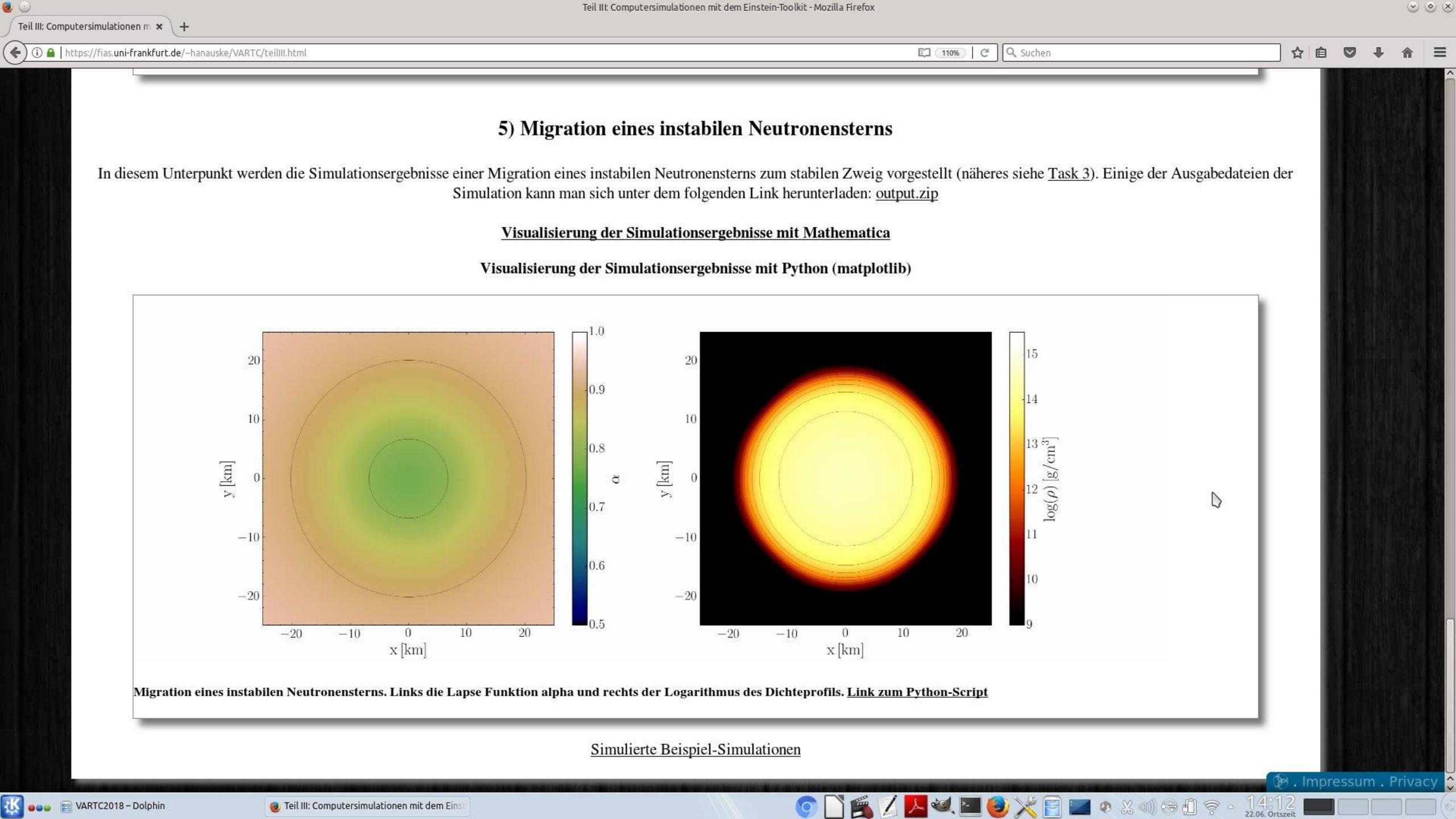
Contents successfully written to /home/agmisc/prakti1/ET-2016-11/Cactus/repos/simfactory2/etc/defs.local.ini

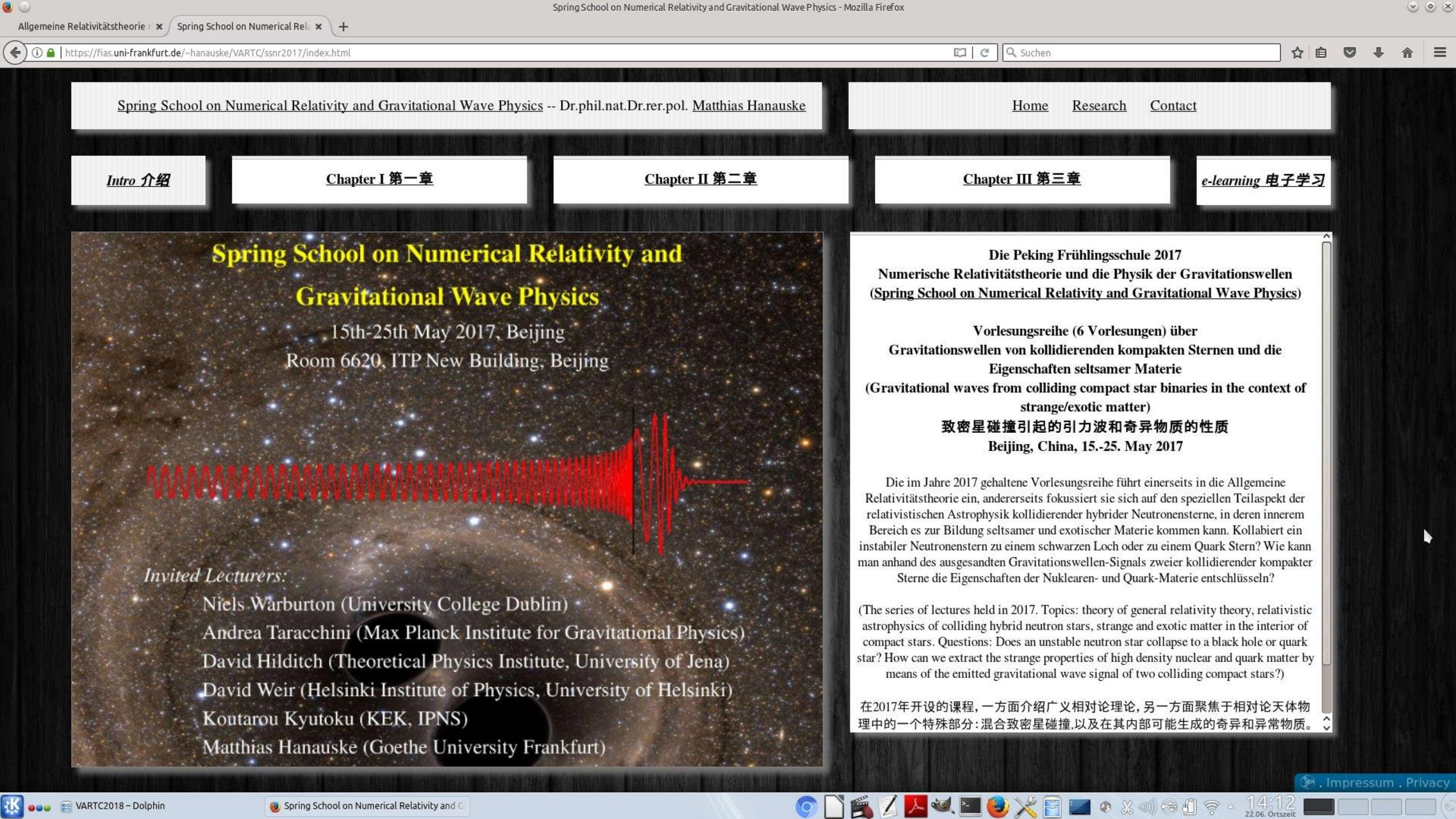
```
[prakti1@login02.csc Cactus]$ █
```

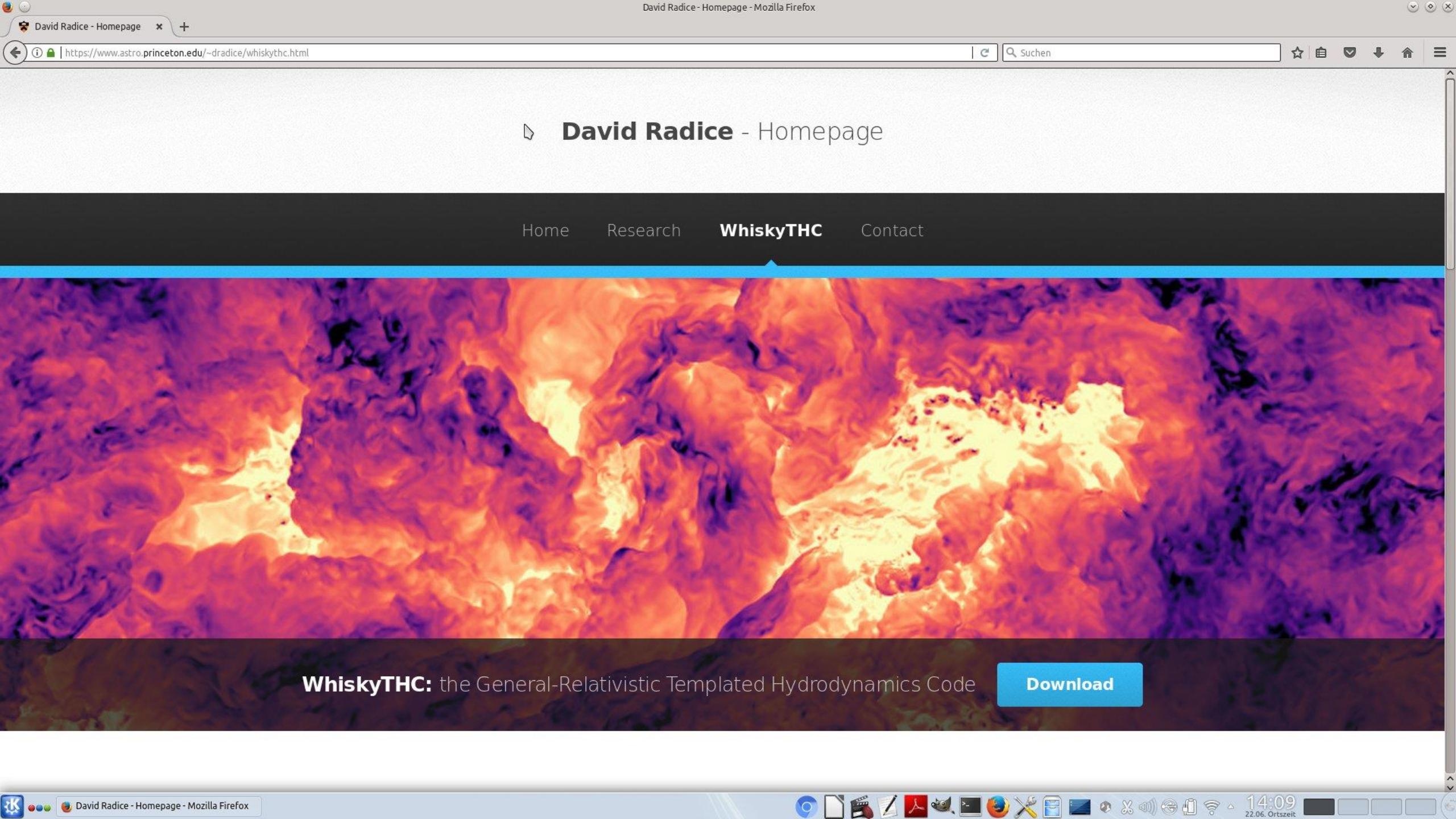
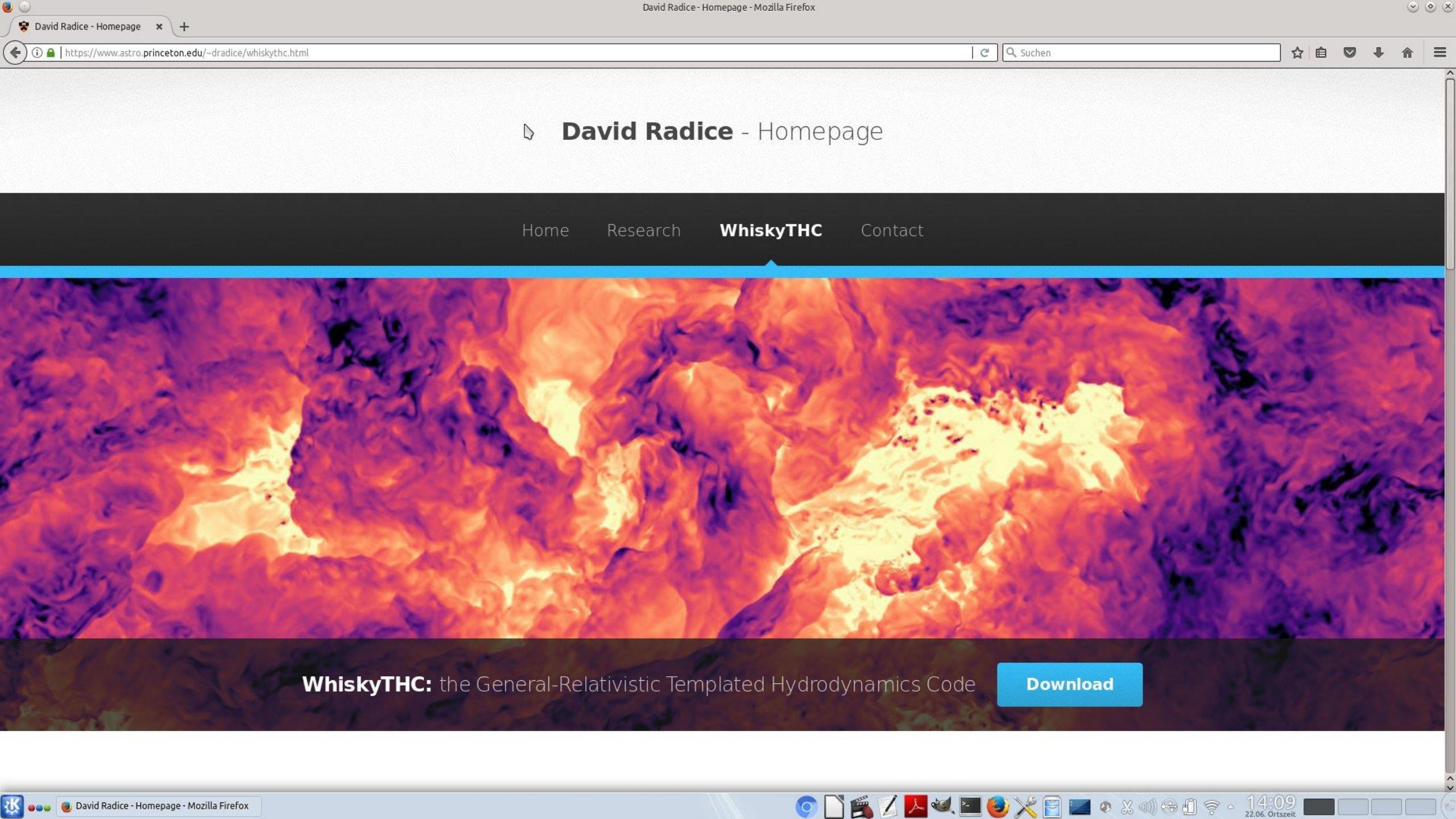
Das Einstein Toolkit: Kompilierung

```
[prakti1@login02.csc Cactus]$ ./simfactory/bin/sim build et --thornlist ./manifest/einstein toolkit.th --machine fuchs
Using configuration: et
Reconfiguring et
Writing configuration to: /home/agmisc/prakti1/ET-2016-11/Cactus/configs/et/OptionList
Cactus - version: 4.2.3
Reconfiguring et.
Using configuration options from configure line
  Setting fds to '4,5 -j --'
End of options from configure line
Adding configuration options from '/home/agmisc/prakti1/ET-2016-11/Cactus/configs/et/OptionList'...
  Setting VERSION to '2015-05-16'
  Setting CPP to 'cpp'
  Setting FPP to 'cpp'
  Setting CC to '/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/composer_xe_2013_sp1.3.174/bin/intel64/icc'
  Setting CXX to '/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/composer_xe_2013_sp1.3.174/bin/intel64/icpc'
  Setting F77 to '/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/composer_xe_2013_sp1.3.174/bin/intel64/ifort'
  Setting F90 to '/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/composer_xe_2013_sp1.3.174/bin/intel64/ifort'
  Setting CPPFLAGS to '-DCCTK_DISABLE_OMP_COLLAPSE -DCCTK_DISABLE_RESTRICT'
  Setting FPPFLAGS to '-DCCTK_DISABLE_OMP_COLLAPSE -traditional -DCCTK_DISABLE_RESTRICT'
  Setting CFLAGS to '-g -traceback -msse3 -align -std=c99 -U_STRICT_ANSI_'
  Setting CXXFLAGS to '-g -traceback -msse3 -align -std=c++11 -D_builtin_fmaxf=fmaxf -D_builtin_fmaxl=fmaxl -D_builtin_fmi
  Setting F77FLAGS to '-g -traceback -msse3 -align -pad -safe-cray_ptr'
  Setting F90FLAGS to '-g -traceback -msse3 -align -pad -safe-cray_ptr'
  Setting C_LINE_DIRECTIVES to 'yes'
  Setting F_LINE_DIRECTIVES to 'yes'
  Setting LDFLAGS to '-Wl,--export-dynamic -Wl,-rpath,/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/composer_xe_2013_sp1.3.174/lib/intel64 -Wl,-rpath,/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/composer_xe_2013_sp1.3.174/ipp/lib/intel64 -Wl,-rpath,/cm/shared/apps/intel/composer_xe/2013_sp1.3.174/tbb/lib/intel64/gcc4.4'
  Setting BEGIN_WHOLE_ARCHIVE_FLAGS to '-Wl,--whole-archive'
```









David Radice - Homepage

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WhiskyTHC: the General-Relativistic Templated Hydrodynamics Code

[Download](#)

Mögliche Vorlesungsprojekte

- Teil I: Simulationen und Berechnungen in Maple
 - Weiterführende Themen der Kerr-Metrik
 - Kosmologie und die Robertson-Walker Metrik
 - Alternative Gravitationstheorien
- Teil II: C++ oder Python
 - Die Masse-Radius Beziehung von Zwillingssternen
 - Geodätengleichung mit C++ oder Python lösen (nichtrotierendes schwarzes Loch)
 - Geodätengleichung mit C++ oder Python lösen (rotierendes schwarzes Loch)