Vorlesung 11

Allgemeine Relativitätstheorie mit dem Computer

PC-Pool Raum 01.120   Johann Wolfgang Goethe Universität   27. Juni, 2016

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Allgemeines

Ort und Zeit:

PC-Pool Raum 01.120, immer Montags von 16.15 bis 17.45 Uhr
Zusätzlicher, freiwilliger Übungstermin 15.00 bis 16.15 Uhr

Vorlesungs-Materialien und *Lon Capa* Online-Lernplatform:

http://th.physik.uni-frankfurt.de/~hanauske/VARTC/

http://lon-capa.server.uni-frankfurt.de/

Mündliche Prüfungen im September

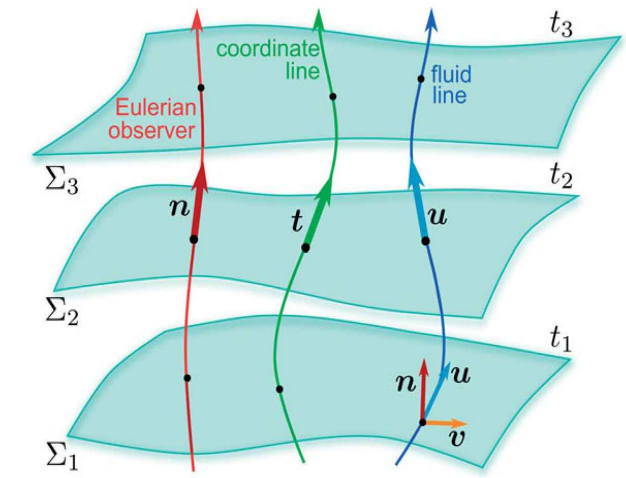
Plan für die heutige Vorlesung:

Teil III: Computersimulationen mit dem Einstein-Toolkit (ET)

- Wiederholung: Die Einsteingleichung im (3+1)-Split
- Ausführung einer Simulation (Parameter-Datei, Starten einer Simulation)
- Visualisierung der Simulationsergebnisse mit Mathematica und Python
- Migration eines instabilen Neutronensterns
- Kollaps eines Neutronensterns zum schwarzen Loch

Relativistic Hydrodynamics and Numerical General Relativity

A realistic numerical simulation of a twin star collapse, a merger of two compact stars or a collapse to a black hole, needs to go beyond a static, spherically symmetric TOV-solution of the Einstein- and Hydrodynamical equations.

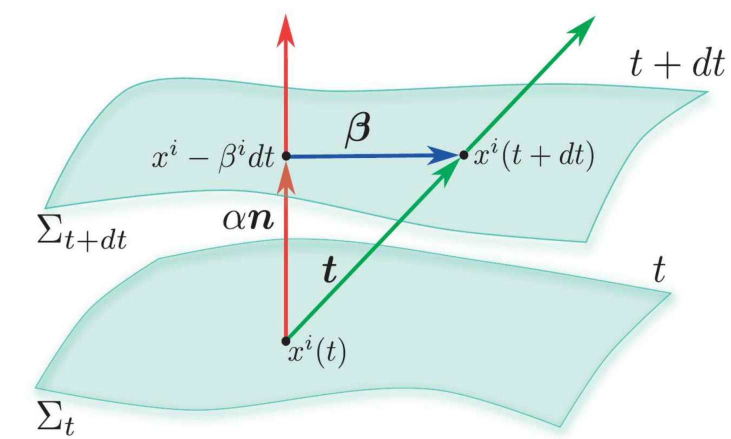


$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

$$\nabla_\mu (\rho u^\mu) = 0 \,, \qquad \nabla_\nu T^{\mu\nu} = 0 \,.$$

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_i \beta^i & \beta_i \\ \beta_i & \gamma_{ij} \end{pmatrix}$$

(3+1) decomposition of spacetime



$$d\tau^2 = \alpha^2(t, x^j) dt^2 \qquad x^i_{t+dt} = x^i_t - \beta^i(t, x^j) dt$$

All figures and equations from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

The ADM equations

The ADM (Arnowitt, Deser, Misner) equations come from a reformulation of the Einstein equation using the (3+1) decomposition of spacetime.

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \mathscr{L}_{\boldsymbol{\beta}} \gamma_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$

$$\partial_t K_{ij} = -D_i D_j \alpha + \beta^k \partial_k K_{ij} + K_{ik} \partial_j \beta^k + K_{kj} \partial_i \beta^k + \alpha \left({}^{(3)}R_{ij} + K K_{ij} - 2K_{ik} K^k{}_j \right) + 4\pi\alpha \left[\gamma_{ij} (S - E) - 2S_{ij} \right]$$

Time evolving part of ADM

$$D_j (K^{ij} - \gamma^{ij} K) = 8\pi S^i \qquad {}^{(3)}R + K^2 - K_{ij} K^{ij} = 16\pi E$$

Constraints on each hypersurface

Three dimensional covariant derivative

$$D_\nu := \gamma^\mu{}_\nu \nabla_\mu = (\delta^\mu_\nu + n_\nu n^\mu) \nabla_\mu$$

Three dimensional Riemann tensor

$${}^{(3)}R^\mu{}_{\nu\kappa\sigma} = \partial_\kappa {}^{(3)}\Gamma^\mu_{\nu\sigma} - \partial_\sigma {}^{(3)}\Gamma^\mu_{\nu\kappa} + {}^{(3)}\Gamma^\mu_{\lambda\kappa} {}^{(3)}\Gamma^\lambda_{\nu\sigma} - {}^{(3)}\Gamma^\mu_{\lambda\sigma} {}^{(3)}\Gamma^\lambda_{\nu\kappa}$$

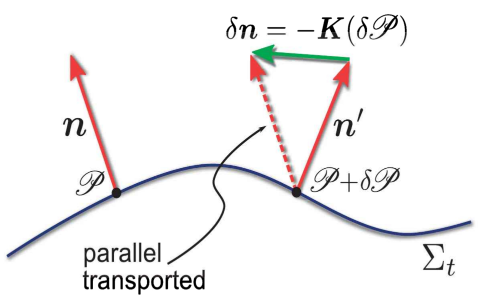
$${}^{(3)}\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} \gamma^{\alpha\delta} \left(\partial_\beta \gamma_{\gamma\delta} + \partial_\gamma \gamma_{\delta\beta} - \partial_\delta \gamma_{\beta\gamma} \right)$$

Spatial and normal projections of the energy-momentum tensor:

$$S_{\mu\nu} := \gamma^\alpha{}_\mu \gamma^\beta{}_\nu T_{\alpha\beta} \,,$$
$$S_\mu := -\gamma^\alpha{}_\mu n^\beta T_{\alpha\beta} \,,$$
$$S := S^\mu{}_\mu \,,$$
$$E := n^\alpha n^\beta T_{\alpha\beta} \,,$$

Extrinsic Curvature:

$$K_{\mu\nu} := -\gamma^\lambda{}_\mu \nabla_\lambda n_\nu$$



All figures and equations from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

From ADM to BSSNOK

Unfortunately the ADM equations are only weakly hyperbolic (mixed derivatives in the three dimensional Ricci tensor) and therefore not "well posed". It can be shown that by using a conformal traceless transformation, the ADM equations can be written in a hyperbolic form. This reformulation of the ADM equations is known as the BSSNOK (Baumgarte, Shapiro, Shibata, Nakamuro, Oohara, Kojima) formulation of the Einstein equation. Most of the numerical codes use this (or the CCZ4) formulation.

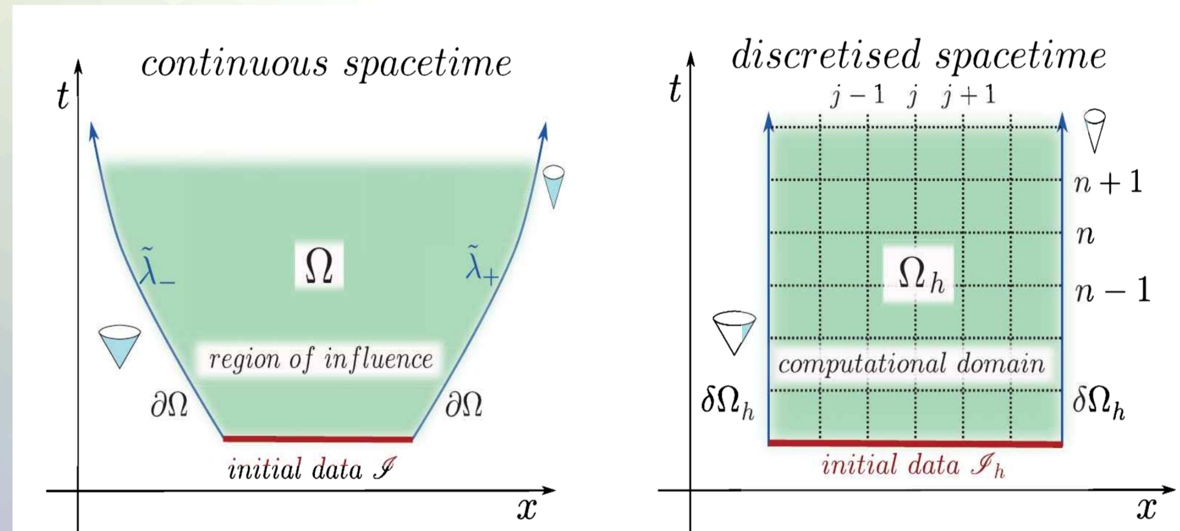
The 3+1 Valencia Formulation of the Relativistic Hydrodynamic Equations

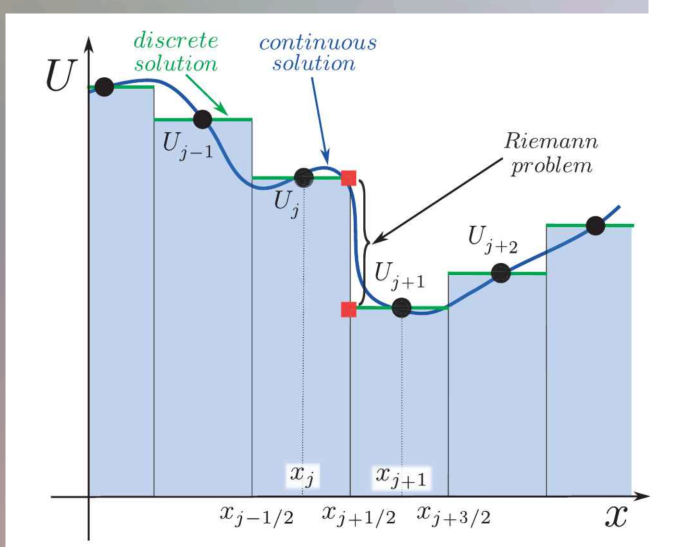
$$\nabla_\mu(\rho u^\mu) = 0\,,$$
$$\nabla_\nu T^{\mu\nu} = 0\,.$$

To guarantee that the numerical solution of the hydrodynamical equations (the conservation of rest mass and energy-momentum) converge to the right solution, they need to be reformulated into a conservative formulation. Most of the numerical “hydro codes” use here the 3+1 Valencia formulation.

Finite difference methods

Discretisation of a hyperbolic initial value boundary problem.





High resolution shock capturing methods (HRSC methods) are needed, when Riemann problems of discontinuous properties and shocks needs to be evolved accurately

All figures from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

The Einstein Toolkit

einstein toolkit

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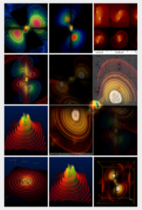
Welcome

The Einstein Toolkit Consortium is developing and supporting open software for relativistic astrophysics. Our aim is to provide the core computational tools that can enable new science, broaden our community, facilitate interdisciplinary research and take advantage of emerging petascale computers and advanced cyberinfrastructure.

Please read our pages about the Einstein Toolkit, its governance, and how to get started with the toolkit for more information.



EinsteinToolkit@Flickr

Download

November 2014: We are pleased to announce the tenth release (code name "Herschel") of the Einstein Toolkit, an open, community developed software infrastructure for relativistic astrophysics.

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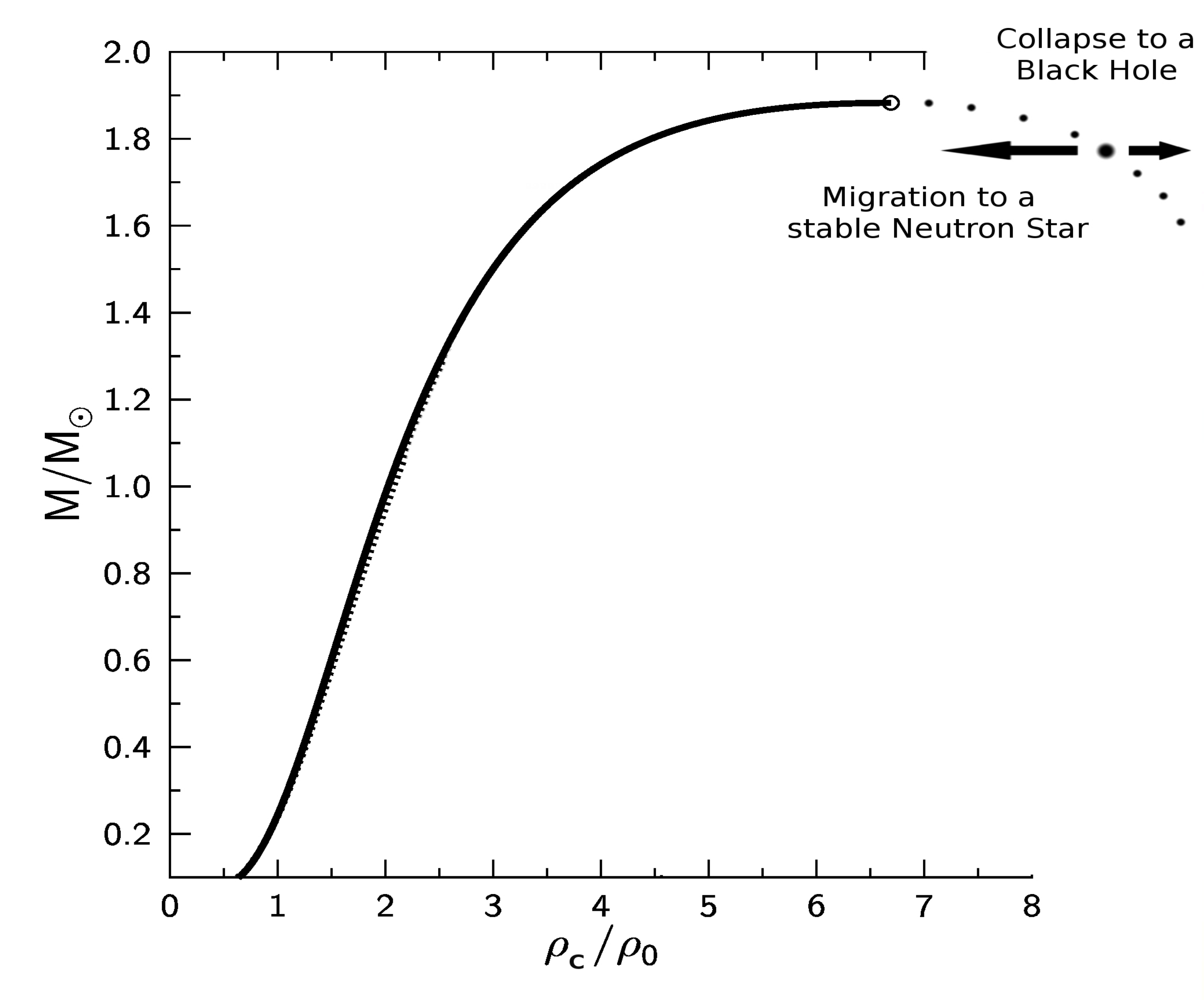
Kollaps eines Neutronensterns zum schwarzen Loch



Kollaps eines Neutronensterns zum schwarzen Loch