(Q)ED IN STRONG LASER FIELDS

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KOLLOQUIUM ZUR STRUKTUR UND DYNAMIK DER ELEMENTAREN MATERIE

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EXTREME FIELDS WITH PLYMOUTH UNIVERSITY

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Outline

1. Introduction
2. Classical Dynamics
    1. Charges and Fields
    2. Charges and Lasers
3. Strong Field QED
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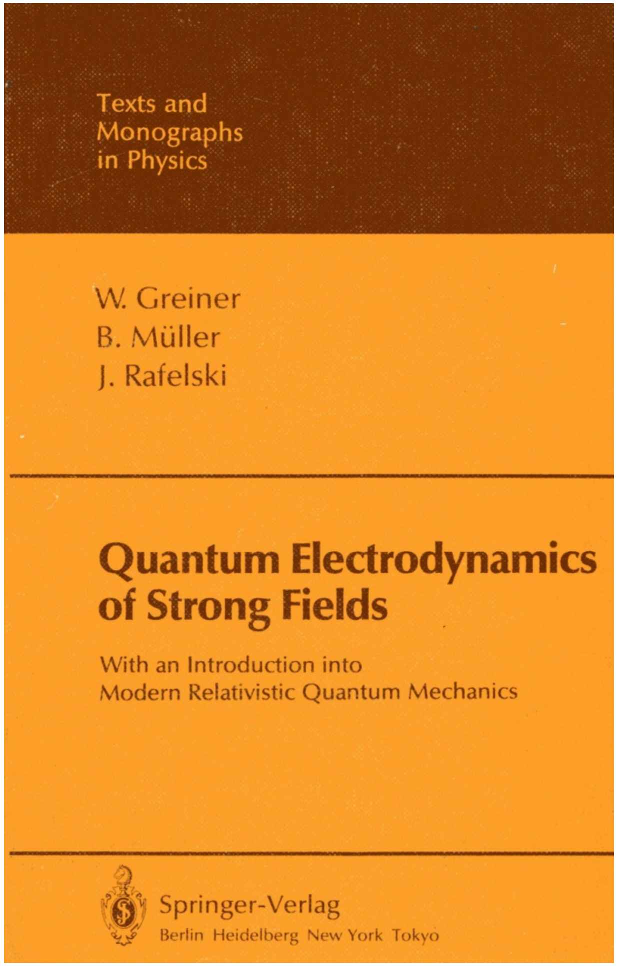
    2. Trees
    3. Loops
4. Conclusion and Outlook

Introduction

Context: (strong) external fields

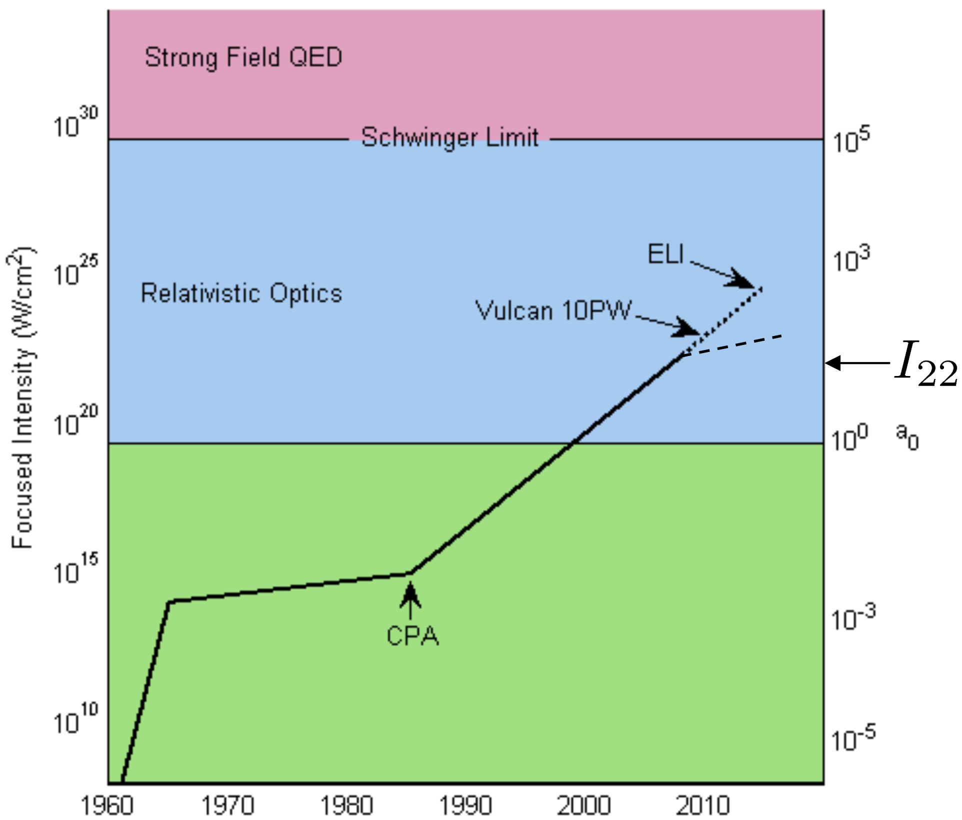
- Roads to 'New Physics':
- Explore new regimes in parameter space, e.g.
  - Resolution/energy (LHC, …, ?)
  - Temperature/density/pressure (FAIR, …)
  - External conditions: boundaries (Casimir effect)
  - Magnitude of external field, e.g.
    - Gravitational: e.g. ultracold neutrons (peV!)
    - Electromagnetic: E, B and combinations thereof
    - e.g. **laser**
  - **NB:** may need to go beyond concept of 'external'

Some local history…



- Published May 1985
- Mostly constant and Coulomb fields
- But (p.290):
- *"Strong electric and magnetic fields in the laboratory can be produced by laser beams."*

Laser intensity: Time evolution



(adapted from Mourou, Tajima, Bulanov, RMP **78**, 2006)

- Important parameter: dim.less amplitude

$$a_0 \equiv \frac{eE\lambda\!\!\!^{-}_L}{mc^2} \sim I^{1/2}$$

- Energy gain of e^- per λ_L
- $a_0 \gtrsim 1$: e^- relativistic
- magnitude:

$$a_0 = 60\sqrt{I/I_{22}}\;\lambda/\mu\text{m}$$

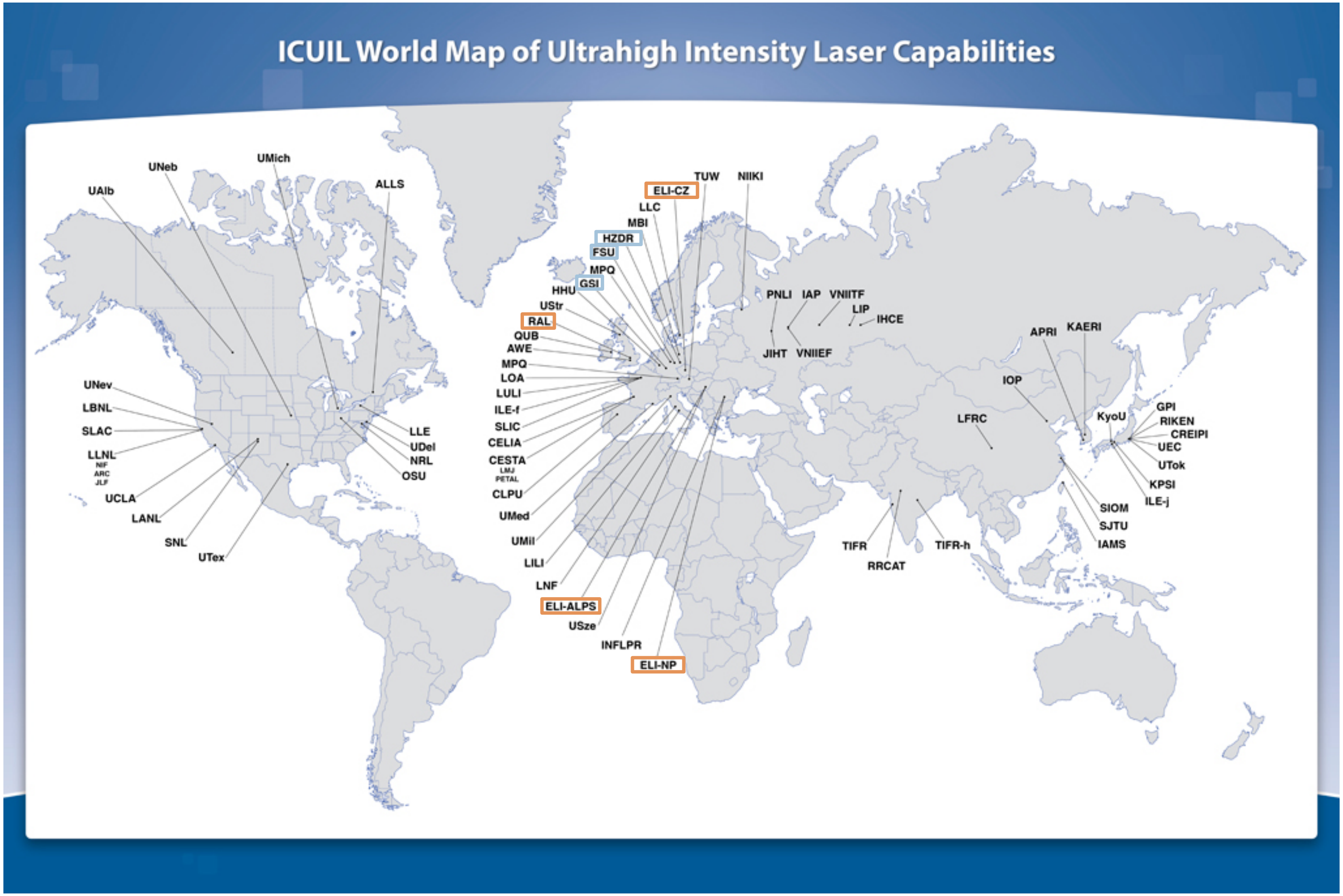
Regime of Extremes

- Current magnitudes:

Power	$P \gtrsim 10^{15}\,\mathrm{W} \equiv 1\,\mathrm{PW}$
Intensity	$I \gtrsim 10^{22}\,\mathrm{W/cm^2}$
Electric field	$E \gtrsim 10^{14}\,\mathrm{V/m}$
Magnetic field	$B \gtrsim 10^{10}\,\mathrm{G} \equiv 10^{6}\,\mathrm{T}$

- Fields **huge** but…
- … ***pulsed*** and ***alternating***

Facilities: World map



2 Laser Projects (of many)

Building (projected)

- CLF Vulcan 10 PW
  - 10^{23} Wcm^{-2}
  - Completion by 2016
  - Budget: 20 M£
- ELI ('4th pillar')
  - >100 PW (Exawatt ?)
  - >10^{25} Wcm^{-2}
  - Budget: several 100 M€
  - Decision in 2012 (?)

2. Classical Dynamics

2.1 Charges and Fields

Charge & field dynamics I

- Folklore: ‘accelerated charges radiate’
- Two-step procedure (cf. Jackson):

1. Solve Lorentz EoM in external field

$$m\dot{u}^\mu = eF^{\mu\nu}u_\nu \, , \quad F^{\mu\nu} = F^{\mu\nu}(x(\tau))$$

$\longrightarrow$ orbit: $\quad u^\mu = \dot{x}^\mu = \dot{x}^\mu(\tau) \, , \quad x^\mu = x^\mu(\tau)$

2. Calculate radiation spectrum from orbit

$$dP^0 \sim |j(k)|^2$$

$$j^\mu(k) = e \int d\tau \, u^\mu(\tau) \exp(ik \cdot x(\tau))$$

Charge & field dynamics II

- Used: **external field approximation**
  - ignore back reaction of radiation

$$A^\mu = A^\mu_{\text{ext}} + \Box^{-1} j^\mu \simeq A^\mu_{\text{ext}}$$

  - otherwise, two-step procedure fails – instead:
- Lorentz-Abraham-Dirac eq. (Lorentz 1892, Abraham 1905, Dirac 1938)

$$m\dot{u} = F + F_{\text{RR}} = F + \tau_0 \, \mathbb{P} \, m\ddot{u}$$

$$\mathbb{P}^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu \quad \rightarrow \quad u \cdot \dot{u} = 0$$

  - small time parameter $\tau_0 \equiv 3r_e/2c \simeq 10^{-23}$ s
- Reduction of order: Landau-Lifshitz eq.

$$m\dot{u} = F + F_{\text{RR}} = F + \tau_0 \, \mathbb{P} \, \dot{F} + O(\tau_0^2)$$

Solving the Lorentz EoM

- Lorentz EoM in general nonlinear as field strength orbit dependent
- Can be solved exactly for:
  - Constant fields (as EoM linear)
  - 'Univariate' fields, $F^{\mu\nu} = F^{\mu\nu}(\ell \cdot x(\tau))$, $\ell = const$
    - **Integrable:** four conservation laws

$$p_\ell \, , \quad \mathbf{p}_\perp \, , \quad p^2 = m^2$$

    - 'feeds through' to quantum theory (WKB?)
  - 'multivariate' fields: only very special cases
- Orbit type depends on invariant field character…

Field Invariants

- Lorentz and gauge invariant characterisation of vacuum fields using field strength tensor $F^{\mu\nu}$

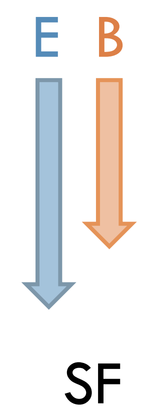
$$\mathcal{S} \equiv -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(E^2 - B^2) \qquad \text{(scalar)}$$

$$\mathcal{P} \equiv -\tfrac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \mathbf{E} \cdot \mathbf{B} \qquad \text{(pseudoscalar)}$$

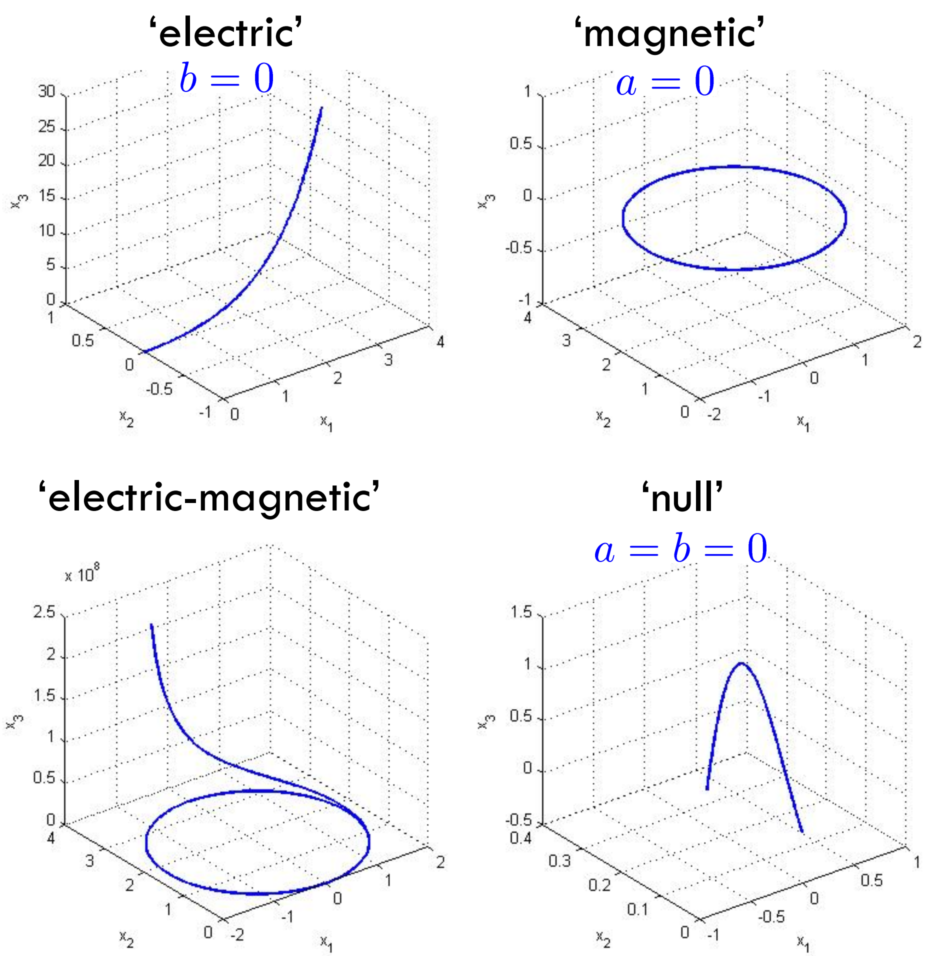
- Combine into invariant field variables

$$a \equiv \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} + \mathcal{S} \right)^{1/2} \overset{\text{SF}}{=} E$$

$$b \equiv \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} - \mathcal{S} \right)^{1/2} \overset{\text{SF}}{=} B$$



Constant field orbits: 4 cases (Taub 1948)



(C. Harvey, thesis 2009)

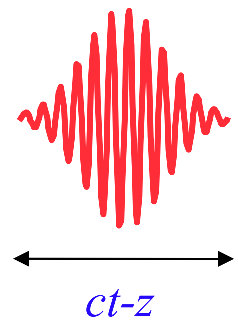
2. Classical Dynamics

2.2 Charges and Lasers

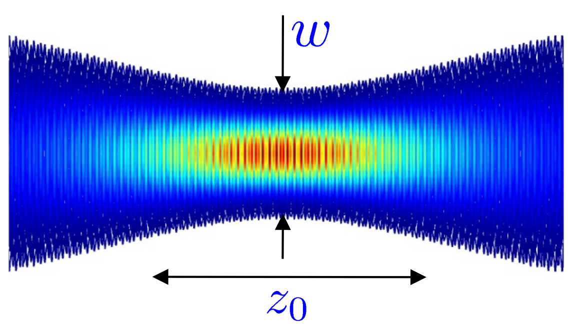
Modelling a laser: Overview

- In order of increasing complexity:
  - Plane wave
    - Infinite (IPW)

    - Pulsed (PPW)
      - Finite ct-z duration
      - Infinite transverse extension



  - Gaussian beam:
    - Finite time duration
    - Finite longitudinal extension z_0
    - Finite transverse waist w



Modelling a laser: Plane waves

- **Null** wave vector $k\,, \quad k^2 = 0$
- E.M. field: $F^{\mu\nu}$
  - univariate: $F^{\mu\nu} = F^{\mu\nu}(k \cdot x)\,, \quad k \cdot x \equiv \omega\, x^-$
  - transverse: $F^{\mu\nu} k_\nu = 0$
  - **Null:**

$$\boxed{\mathcal{S} = 0\,, \quad \mathcal{P} = 0\,, \quad F^3 = 0}$$

  - No intrinsic invariant scale – but note: $T \equiv F^2 \neq 0$
  - Invariants via probe momentum, e.g. $p_\mu T^{\mu\nu} p_\nu$
  - Integrals of motion: $k \cdot p \equiv \omega\, p^-\,,\ \mathbf{p}_\perp,\ p^2 = m^2$

Charge in PW: Dynamics

- Solution of EoM:
- rapid quiver motion (momentum $p(\tau)$)
- Charge acquires averaged **quasi-momentum**

$$q = \langle p \rangle = p_{\text{in}} + (m^2 a_0^2 / 2k \cdot p)\, k$$

- Longitudinal addition – consequence:
- **Effective mass** squared (= average effect!)

$$q^2 = m^2 (1 + a_0^2) \equiv m_*^2$$

- *The* basic intensity effect! (Sengupta 1951, Kibble 1964)
- But: elusive – never observed so far…

Charge in PW: Radiation

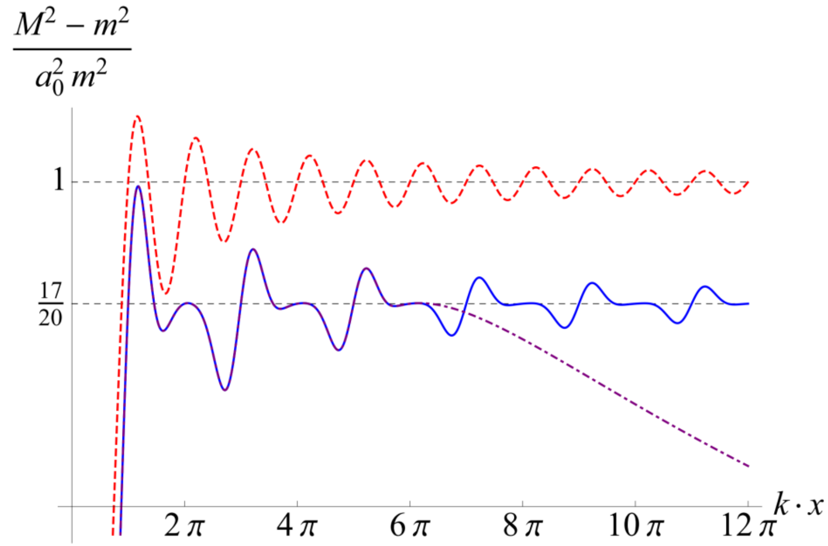
- Recall spectral density $dP^0 \sim |j(k')|^2$
  - For IPW: delta comb in frequency ω'
  - For PPW (N cycles): N-slit refraction pattern
- peak positions from modified Compton formula

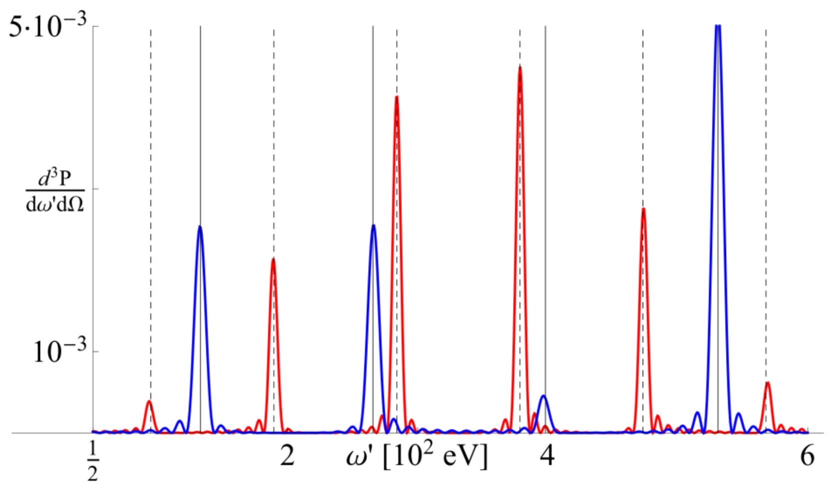
$$\frac{1}{\omega'_n} = \frac{1}{n\omega} + \frac{1}{m_*}(1 - \cos\theta)$$

(in average rest frame)

- Note:
  - Intensity dependence through $m \to m_*(a_0)$
  - Higher harmonics, $n > 1$
- Spectra: see below

Measuring the mass shift





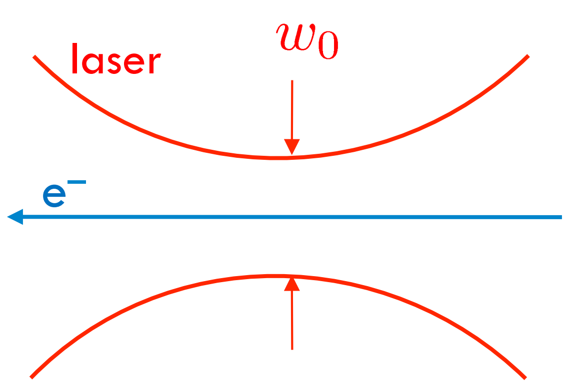
- Mass shift neither unique nor universal – depends on:
- pulse duration
  - gradually builds up with number of cycles/pulse
- pulse **shape**
  - Different quasi momenta
  - Different mass shifts
  - Different spectra
- Finite size effects!
- Observation: fine tuning!

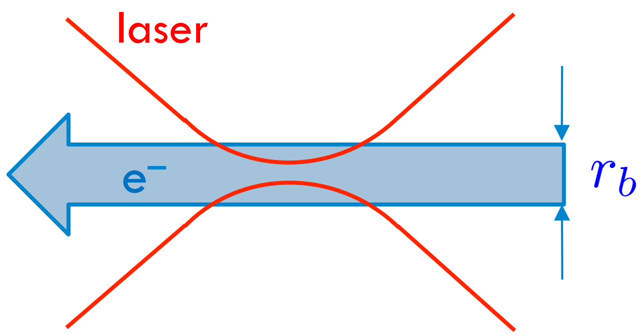
C. Harvey, TH, A. Ilderton, M. Marklund, arxiv:1203:6077

Finite Size Effects

Weakly focussed: $w_0 \gg r_b$

Strongly focussed: $w_0 < r_b$





$$a_0 = O(1)$$

$$a_0 \gg 1$$

PW results 'realistic'

PW results get modified

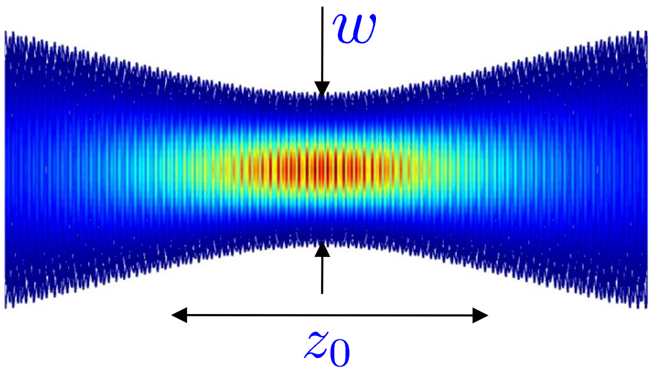
Modelling a laser: Gaussian beam

(Davis 1978, Narozhny et al. 2004)

- Finite geometry parameter:

$$\kappa \equiv w/z_0 \lesssim 1/2\pi$$

  - PW limit: $\kappa \to 0$



- Transverse fields:

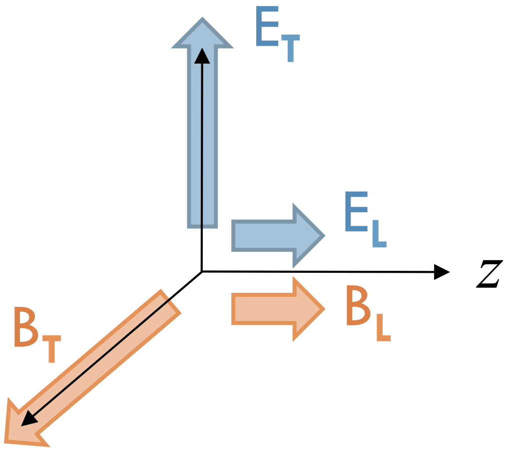
$$E_T = B_T \equiv E$$

- **Longitudinal** fields:

$$E_L, B_L \sim \kappa E$$

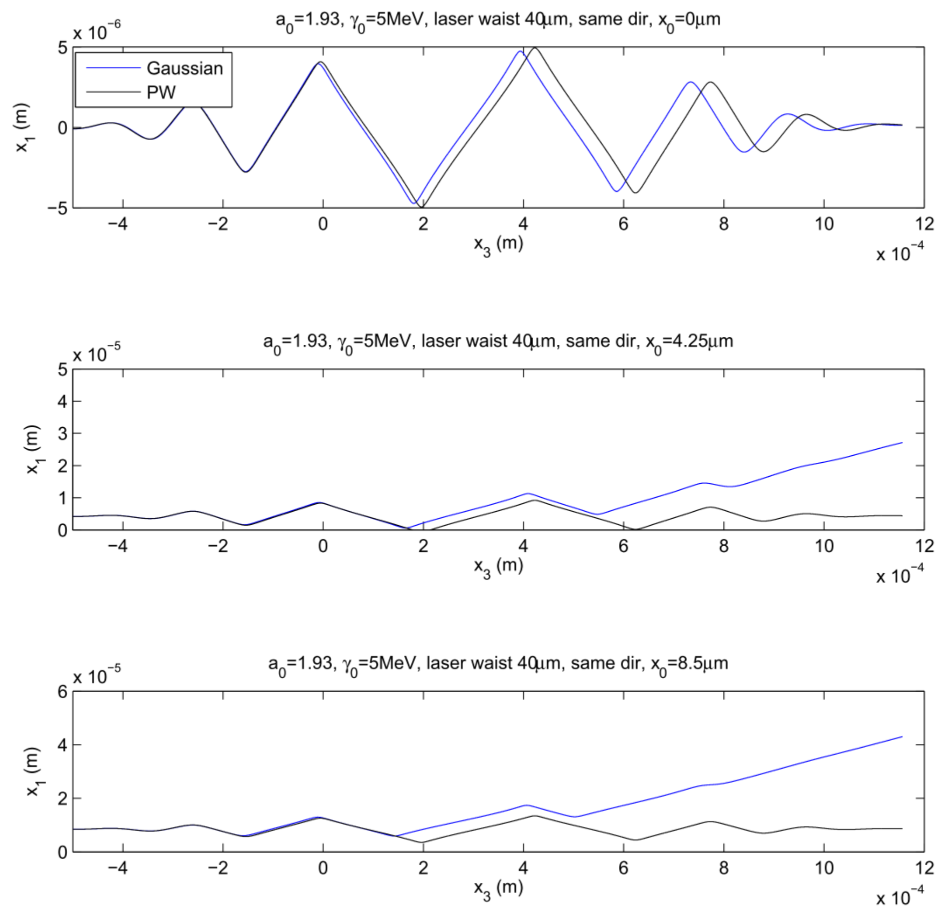
- Invariants **not null** but $O(\kappa^2)$:

$$\mathcal{S} = (E_L^2 - B_L^2)/2\ ,\ \mathcal{P} = \mathbf{E}_L \cdot \mathbf{B}_L$$



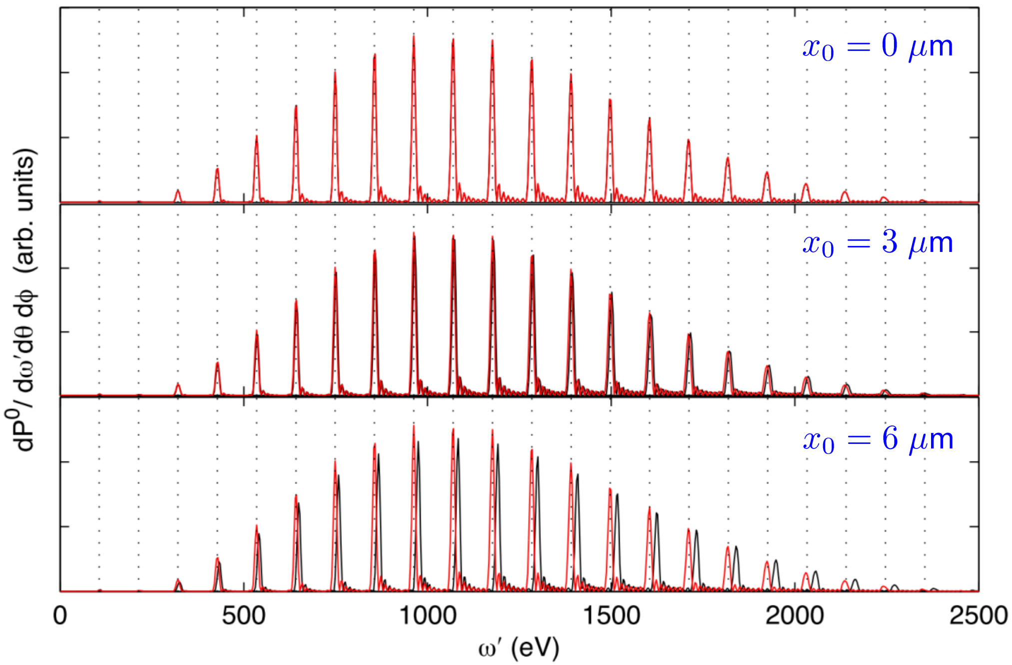
Charge in Gaussian beam

- $\mathbf{p}_{\perp} \neq const$!
- Main effect: ponderomotive (gradient) force tries to expel charges transversely (Kibble 1966)
- PW results hold when charge stays near laser axis
- Fine tuning!



Radiation in Gaussian beams

- REGAE @ DESY: 5 MeV e^-, $r = 5\ \mu$m, $w_0 = 40\ \mu$m
- Radiation spectra:



- Observation of mass-shift effects on radiation spectra seems feasible
- Spectrum = 'fingerprint' of beam shape
- Diagnostic tool?

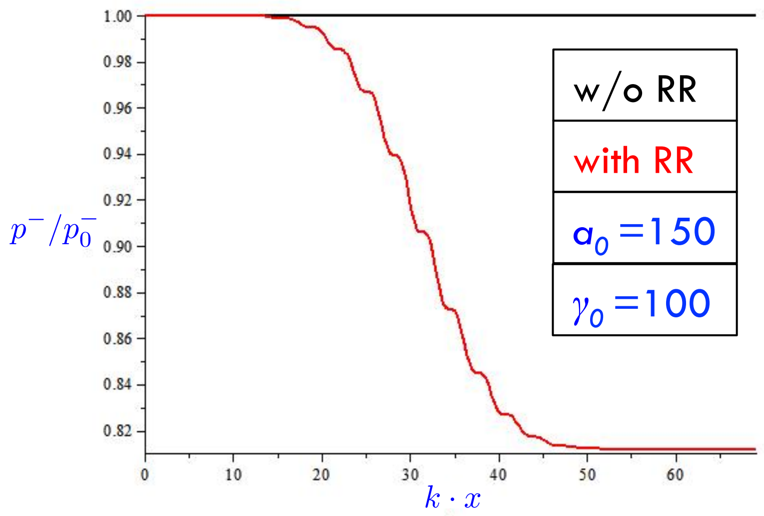
C. Harvey, TH, A. Ilderton, M. Marklund, arxiv:1203.6077

Charge in PW with back reaction

- Recall Landau-Lifshitz (LL) eq.

$$m\dot{u} = F + F_{\mathrm{RR}} = F + \tau_0 \mathbb{P}\dot{F} \qquad \tau_0 = 2\alpha/3m$$

- Analytic solution for PW (Di Piazza, LMP 2008)
- RR relevant when $k \cdot p_0/m^2 \simeq 1/\alpha = 137$
- Main feature: loss of conservation law, $p^- \neq const$



- Observation requires:
  - Long pulse duration
  - Large intensity
  - Large γ_0

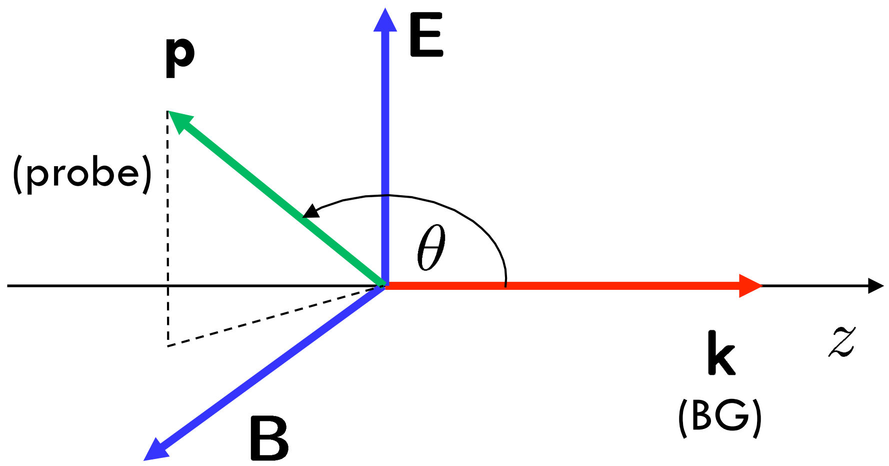
C. Harvey, TH, M. Marklund, PR D84 (2011) 116005

3. Strong Field QED

3.1 Basics

Kinematic invariants

- Sketch:



- Two basic invariants

$$I_1 \equiv k \cdot p \overset{\text{IRF}}{=} m\,\omega_0$$

$$I_2 \equiv (p_\mu T^{\mu\nu} p_\nu)^{1/2} \overset{\text{IRF}}{=} m\,E_0$$

- Lab momenta (head-on coll^n)

$$k = \omega(1, \mathbf{z})$$

$$p = m\gamma_e(1, -\beta\mathbf{z})$$

- Lab frame – IRF:

$$X_0 \equiv X\,e^{\zeta}$$

- Rapidity:

$$\zeta \equiv \gamma_e(1+\beta)$$

Dimensionless parameters

- Measure E in units of QED electric field

$$E_S \equiv m^2c^3/e\hbar = 1.3 \times 10^{18}\ \text{V/m}$$ (Sauter 1931)

- Def. dim.less field and frequency (as 'seen' by probe)

$$\epsilon_0 \equiv E_0/E_S \qquad \nu_0 \equiv \omega_0/m$$

- In terms of invariants

$$\nu_0 = I_1/m^2$$

$$\epsilon_0 = eI_2/m^3$$

$$a_0 = eI_2/mI_1 = \epsilon_0/\nu_0$$

- Quantum regime $\epsilon_0 \gtrsim 1\,, \quad \nu_0 \gtrsim 1$

Quantum integrability

- Exact solution of Dirac eq. in **PW** (Volkov 1931)

$$\Psi_p \equiv \exp(iS[A])\,\Gamma(k,p;A)\,u_p$$

Dirac matrix Dirac spinor

- with classical Hamilton-Jacobi action

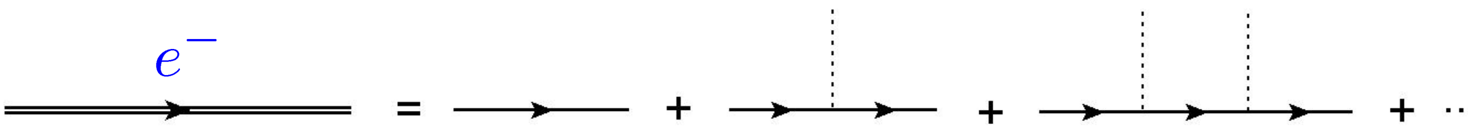
$$S[A] \equiv -p\cdot x - \int_{-\infty}^{x^-} dy^- \, \frac{2e\,p\cdot A - e^2 A^2}{p^-}$$

- **NB**: no analogue available for Gaussian beams etc.

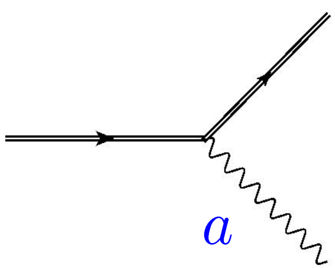
Strong-field QED: Feynman rules

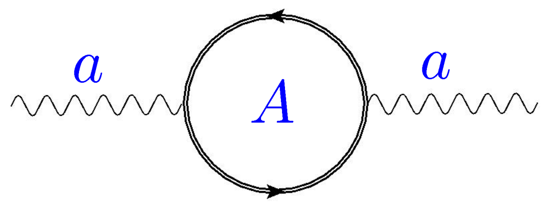
- Split $A \to A + a$: BG (laser) and fluctuation (probe)
  - Probe photons: 

  - Volkov electrons: ---------------
  - ‘dressed’ by laser photons:



- “Furry Picture” Diagrams:

Trees 



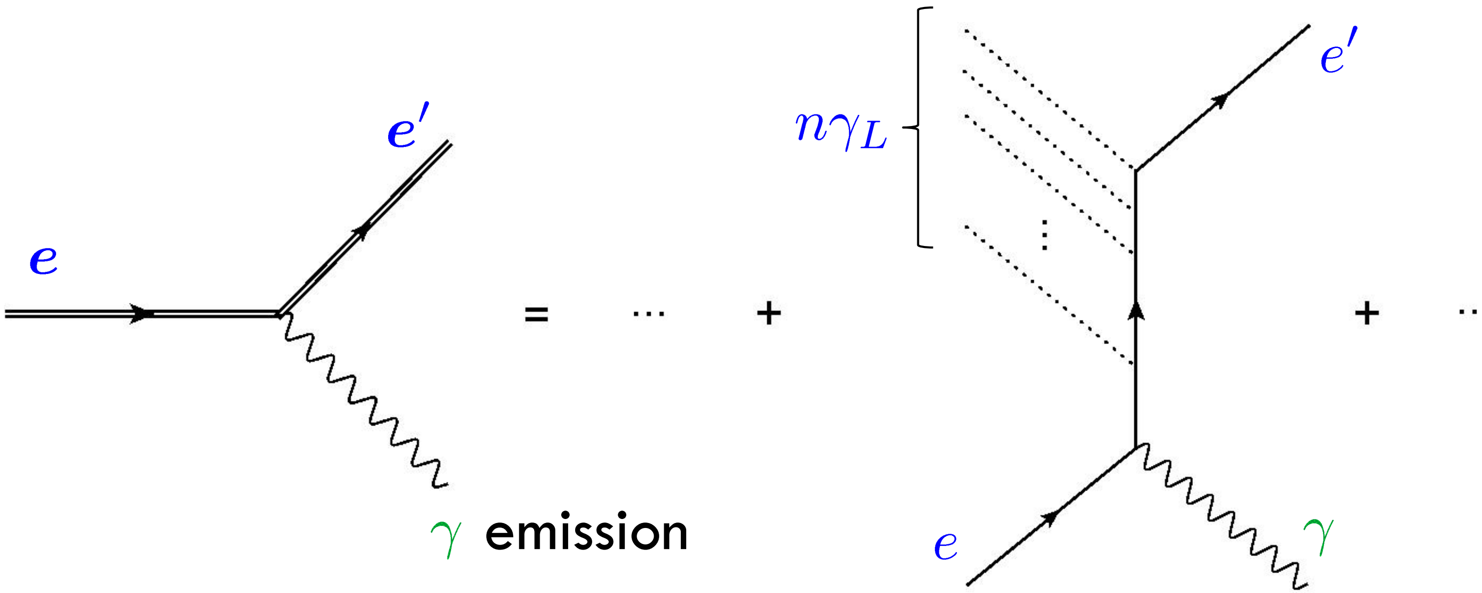
Loops

3. Strong Field QED

3.2 Trees

Nonlinear Compton (NLC) scattering

- Expand Furry picture Feynman diagram $\rightarrow$
- Sum over all processes of the type $e + n\gamma_L \rightarrow e' + \gamma$

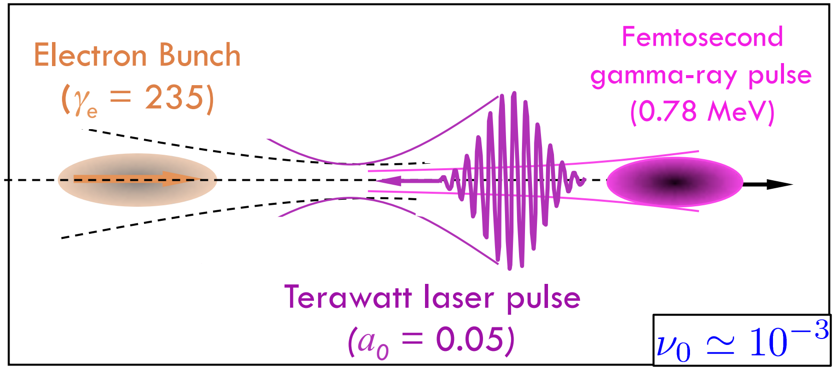


Schott 1912; Nikishov/Ritus 1964,
Brown/Kibble 1964, Goldman 1964

NLC: main features

- No energy threshold – can be done **now**!
- Classical limit: NL-Thomson ($\nu_0 \ll 1$)
- For $a_0 < O(1)$: frequency upshift $\omega'_{\mathrm{max}} \simeq 4\gamma_e^2\omega$
- Used for

  X-ray generation



T-REX, LLNL (2008)

- Nonlinearity:

$$N_\gamma \sim \sigma(a_0)\, N_e N_{\gamma_L}$$

NLC cont[d]

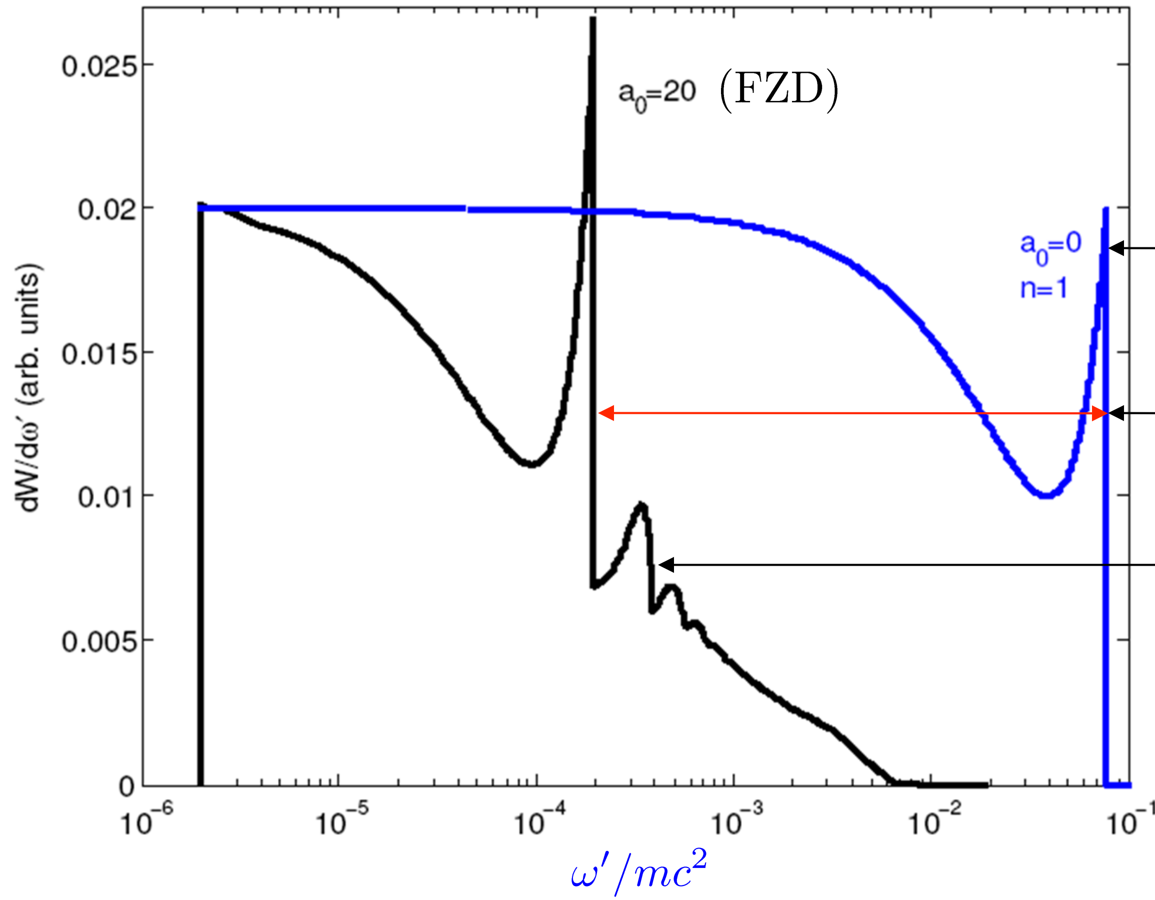
- For high intensity, $a_0 \gg 1$
- **modified** Compton edge due to mass shift

$$\omega'_{n,\mathrm{max}} \simeq 4\gamma_e^2 n\omega / a_0^2 \ , \quad n = 1, 2, \ldots$$

- In particular (for IPW):
  - Higher harmonics: $n > 1$ (Chen, Maksimchuk, Umstadter, Nature1998)
  - Overall blueshift maintained as long as $a_0 \lesssim 2\gamma_e$
  - **Redshift** of $n=1$ edge

$$\omega'_{\mathrm{max}} \simeq 4\gamma_e^2 \omega \longrightarrow 4\gamma_e^2 \omega / a_0^2$$

NLC spectrum for IPW



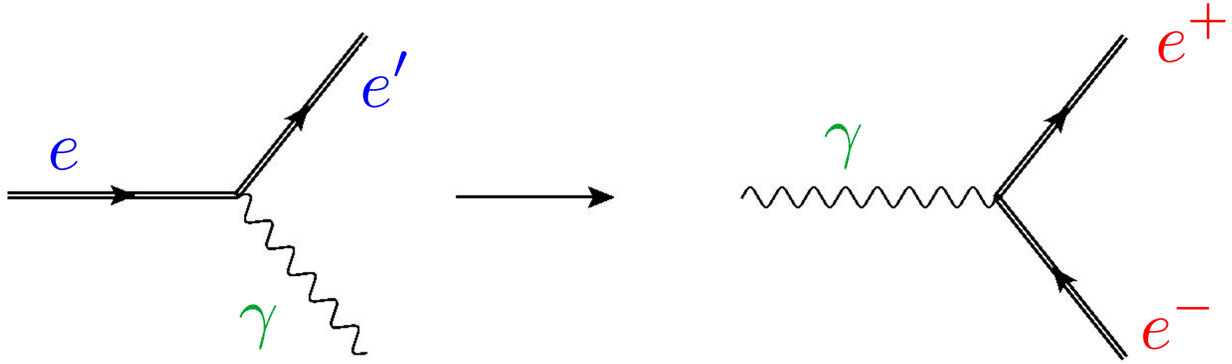
C. Harvey, TH, A. Ilderton, PRA **79**, 2009

But: strongly modified by finite size effects!

TH, B. Kämpfer, D. Seipt, PRA **81**, 2010

Stimulated Pair Production (PP)

- Obtained from NLC via crossing

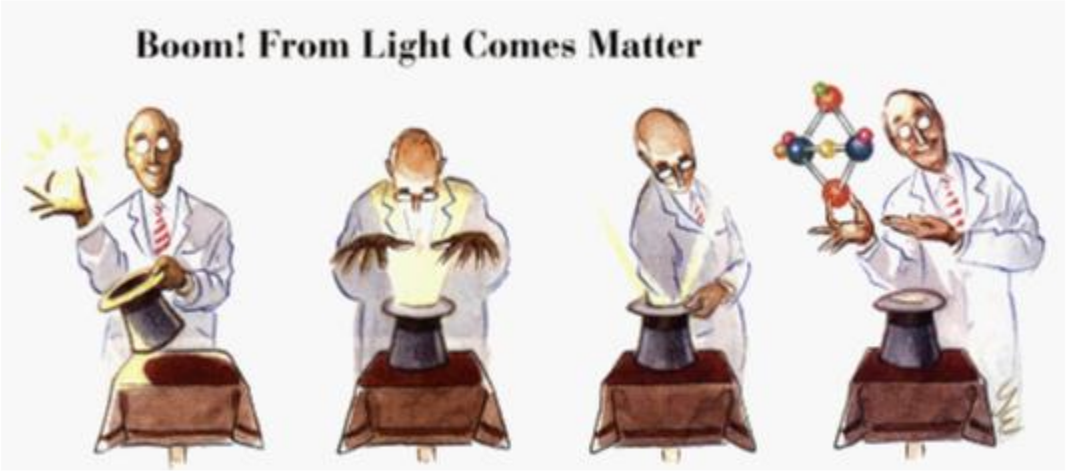


- Main new feature: energy threshold $2m_*^2$
- Experiment SLAC E-144 (1995): combine both processes @ $\nu_0 \gg 1$ ...

SLAC E-144 (Bula et al. ‘96, Burke et al. ’97)

- Two stages: $e + n\gamma_L \to e' + \gamma$ NLC

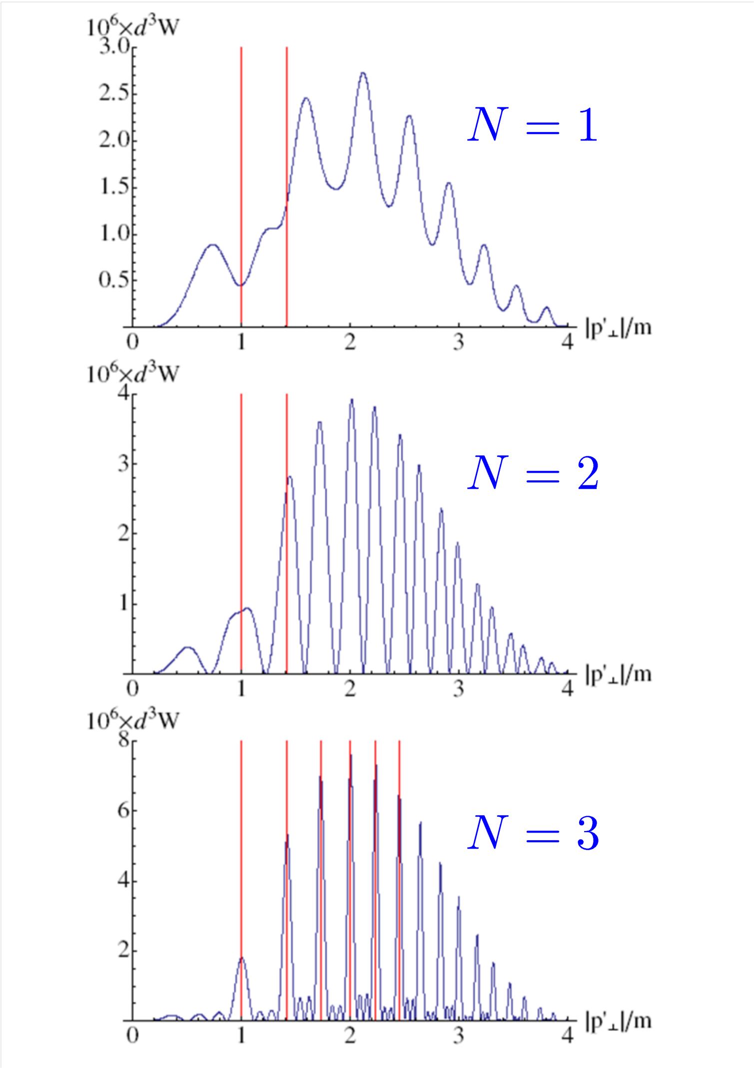
  $\gamma + n\gamma_L \to e^+ + e^-$ stimulated PP



Gil Eisner, Photonics Spectra 1997

$50\text{ GeV } e^- \to 30\text{ GeV } \gamma \to O(10)\text{ pairs}$ @ $a_0 \simeq 0.5$

Stimulated PP: finite-size effects



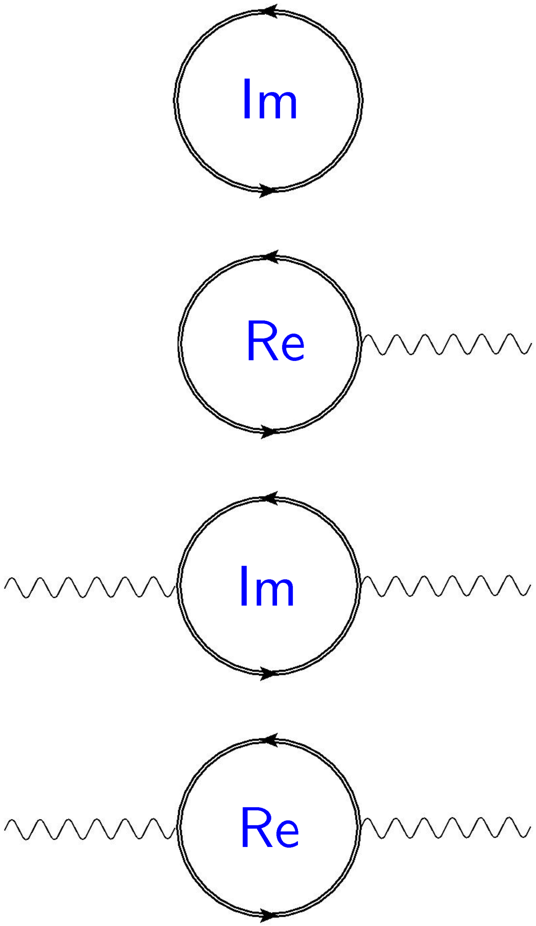
- IPW:
  - triple-diff rate = 'delta comb'
  - above threshold (m_*)
- PPW:
  - dependence on cycles per pulse, N
  - Sub-threshold signals
  - IPW approached for $N \gg 1$

TH, A. Ilderton, M. Marklund, PLB, 2010

3. Strong Field QED

3.2 Loops

Overview

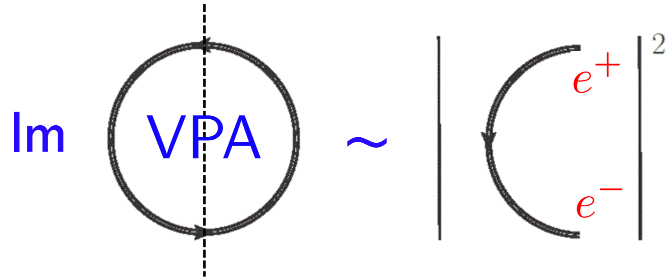


Exotic loop particles?
→ PVLAS (G. Zavattini et al.)

- Vacuum PP: ‘Schwinger effect’
  - zero for null fields
  - exponential suppression
- Vacuum emission
  - Power suppression
- ‘Stimulated’ PP
  - threshold ‘suppression’
- Vacuum Birefringence
  - Power suppression

Spontaneous (vacuum) PP

- Feynman diagram (optical theorem)



'vacuum breakdown'

- Identically **zero** for PWs as $\mathcal{S} = \mathcal{P} = 0$
- Substantial when

$$a = \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} + \mathcal{S}\right)^{1/2} \gtrsim E_S$$

'pair creativity'

  - otherwise: rate exponentially suppressed (Schwinger 1951)

$$\mathfrak{R} \sim \exp(-\pi E_S / a)$$

Vacuum PP cont[d]

- With lasers: **very difficult!**
- Need to fight both
  - Exponential suppression
  - Null field (plane wave) character ($a = 0$)
- Expect rate for e.g. Gaussian beams

$$\mathfrak{R} \sim \kappa^2 \coth(\pi B/E) \exp(-\pi E_S/\kappa E)$$

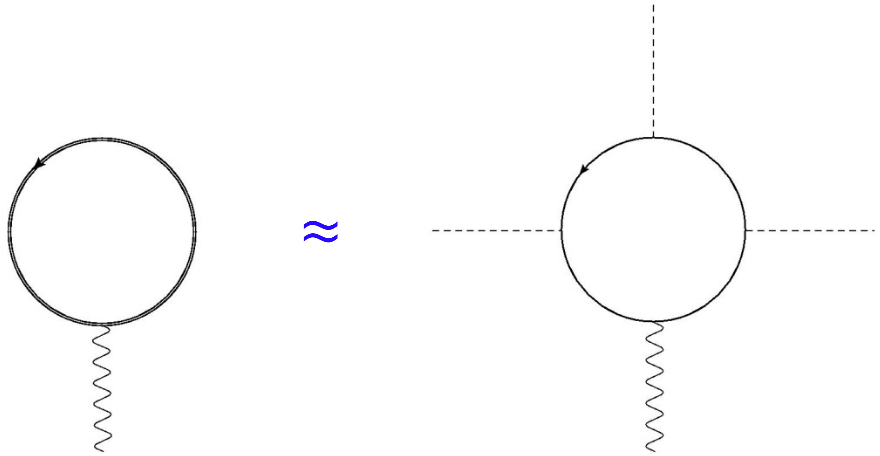
- Alternative: counter-propagating lasers (standing wave)?

$$\mathcal{S} \neq 0 \quad \text{and/or} \quad \mathcal{P} \neq 0$$

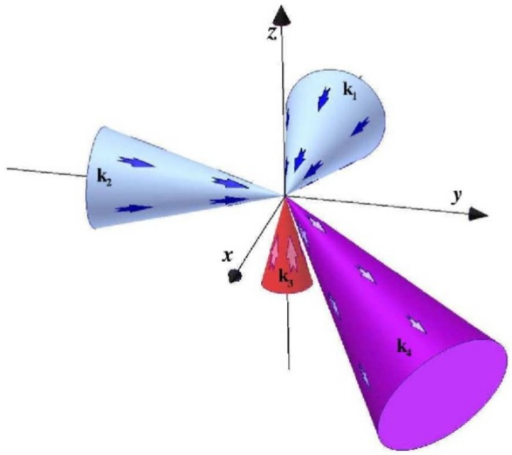
Light-by-light scattering

- Prediction: Halpern 1934, Euler/Kockel 1935, Euler; Heisenberg/Euler 1936
- But (for real γ's) never observed in lab!
- Idea: $3\gamma_L \to \gamma$ ('3-wave mixing') (Lundström et al., 2005) (Monden, Kodama, 2011)

Feynman diagrams:



Artistic view:



Light-by-light scattering cont^{d}

- Low-energy X-section ('Euler-Kockel' approx^{n}):

$$\sigma_{\gamma\gamma} = \frac{973}{10125\pi^2}\,\alpha^2\, r_e^2\, \nu_L^6 \simeq 10^{-67}\,\mathrm{cm}^2$$

- Laser photon density:

$$n_L \simeq 10^{14}\, a_0^2/\mu\mathrm{m}^3$$

- Photon number in focus volume $(10\,\mu\mathrm{m})^3$

$$N_\gamma \simeq 10^{17} a_0^2$$

- Number of scattered γ' s @ $a_0 \simeq 10^2$

$$N_{\gamma'} \simeq \frac{\sigma_{\gamma\gamma}}{(10\,\mu\mathrm{m})^2}\, N_\gamma^3 \simeq 10^{-4}$$

Vacuum birefringence

- Recall: for *single* plane wave BG $\text{Im VPA} = 0$
- But: for *two* plane waves, $A \to A + a$ (BG & probe) find eff. Lagrangian to 2nd order in fluctuation

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{\text{HE}} = \frac{1}{2} a_\mu \left(\Box g^{\mu\nu} + \Pi^{\mu\nu}[A] \right) a_\nu$$

- vac.pol. tensor $\Pi^{\mu\nu}(A; k) = C^{\mu\nu}{}_{\alpha\beta}(A)\, k^\alpha k^\beta$
- **Two** nontrivial eigenval.s $\Pi_\pm = c_\pm k_\mu T^{\mu\nu} k_\nu$
- **Two** dispersion relations ('deformed LC')

$$k^2 - \Pi_\pm = (g^{\mu\nu} - c_\pm T^{\mu\nu}) k_\mu k_\nu = 0$$

Vacuum birefringence cont[d]

Calcite crystal

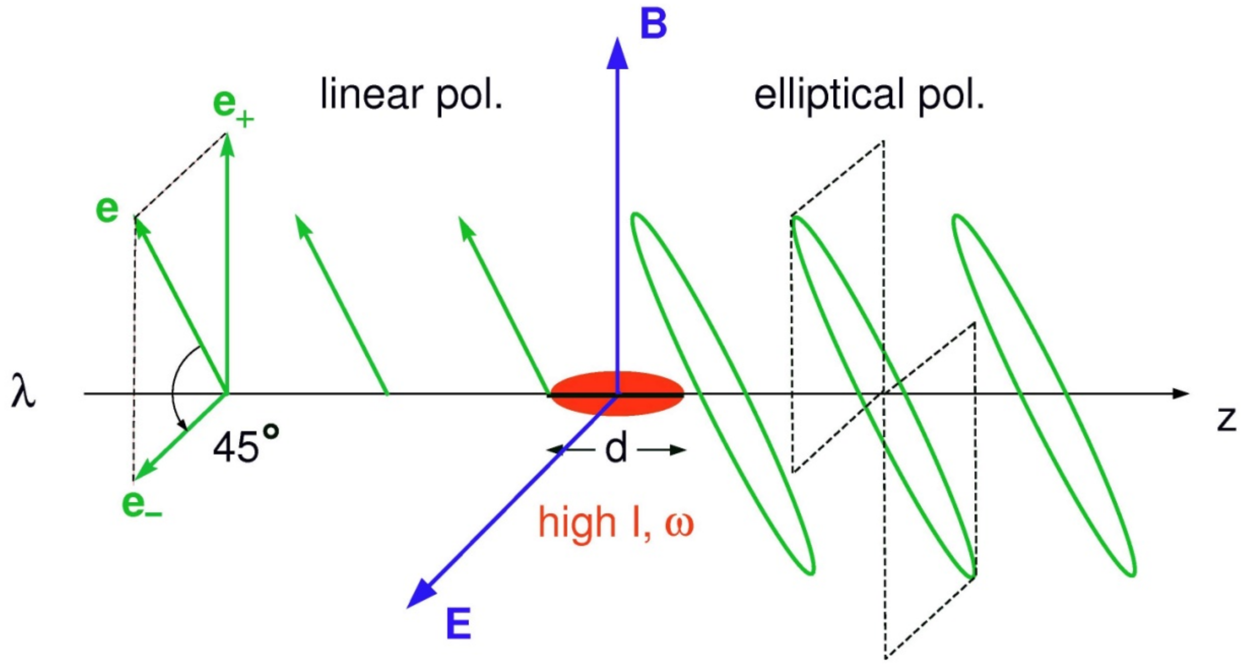
- **Two** indices of refraction (Toll 1952; Narozhny 1968; Brezin, Itzykson 1970)

$$n_{\pm} = 1 + \frac{\alpha}{45\pi}(11 \pm 3)\epsilon^2 \qquad \epsilon \equiv E/E_S$$

- Experimental signature: ellipticity (squared)

$$\delta^2 \sim (n_+ - n_-)^2$$

Experiment: measure ellipticity



Phase retardation of e_+

Analysis (TH et al., Opt. Commun., 2006)

- ellipticity (squared)

$$\delta^2 = 3.2 \times 10^5 \, \epsilon^2 \left(\frac{d}{\mu\text{m}} \epsilon\nu \right)^2 , \quad \epsilon\nu \ll 1 \qquad (\nu \equiv \omega_{\text{probe}}/m)$$

- Optimal scenario: XFEL ($\nu \simeq 10^{-2}$) & HP laser
  - PW laser ($\epsilon \simeq 10^{-4}$): $\delta^2 \simeq 10^{-11}$
  - ELI ($\epsilon \simeq 10^{-2}$): $\delta^2 \simeq 10^{-7} ... 10^{-4}$
  - X-ray polarimetry:
  - New record in polarisation purity: 1.5×10^{-9} @ 6 keV
    (Marx et al., Opt. Comm., 2010)
  - Recently (I. Uschmann) 2.4×10^{-10}

Beyond Heisenberg-Euler I

- For large probe frequency ($\nu > 1$) HE breaks down
- need full polarisation tensor
  - Crossed fields (Toll 1952, Narozhny 1968, Batalin/Shabad 1968)
  - Plane waves (Becker/Mitter 1975, Baier et al. 1975)
- for simpler case of crossed fields

$$n = 1 + \frac{2\alpha}{\pi}\epsilon^2 f(\epsilon\nu)$$

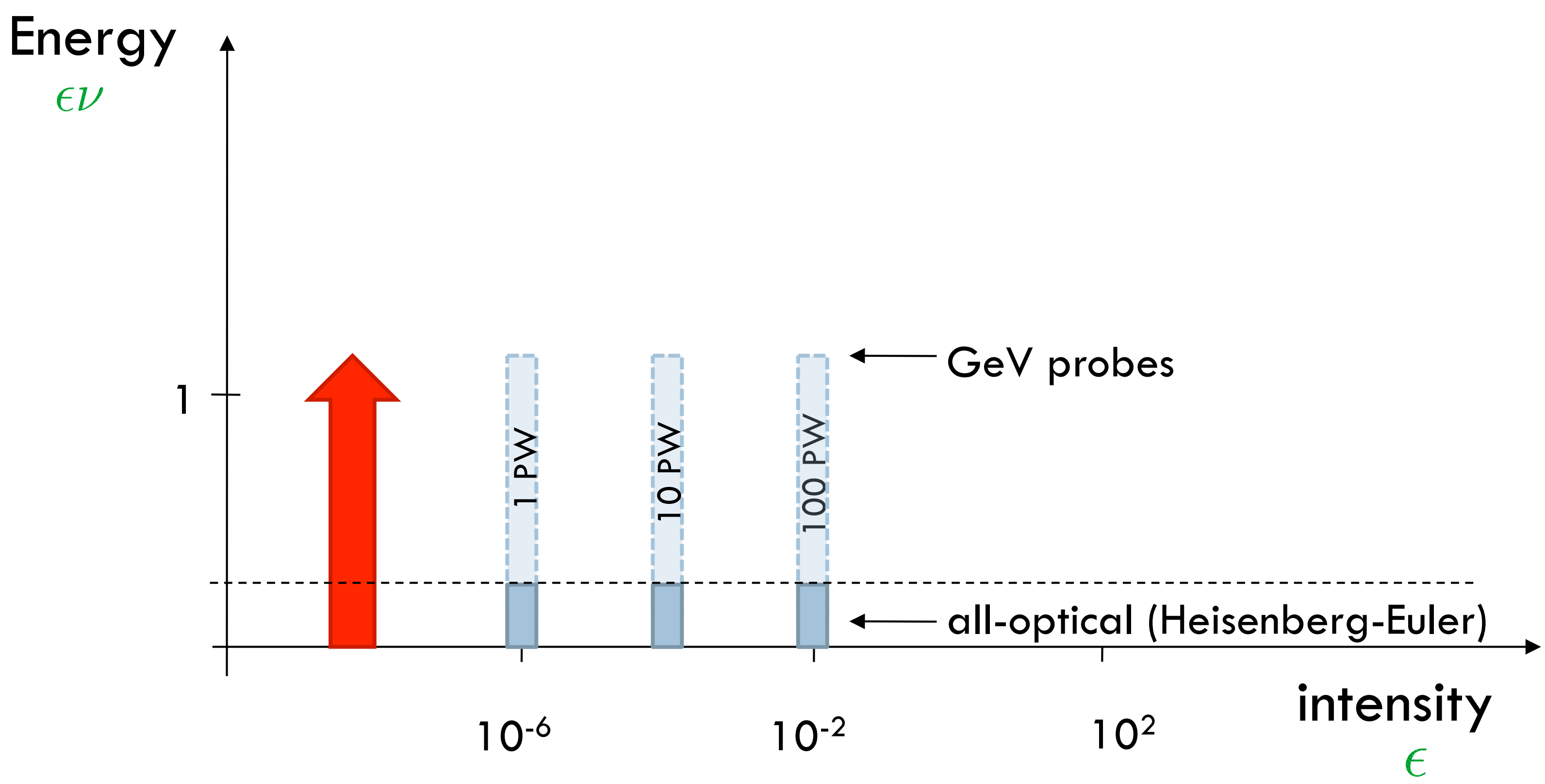
Toll 1952
TH, O. Schröder 2006
Shore 2007

- With *dispersive and absorptive* parts

$$f = f_R + i f_I$$

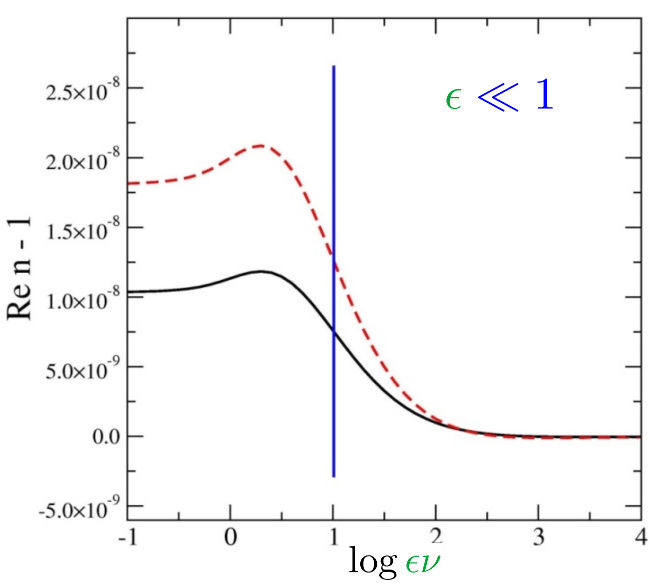
Beyond Heisenberg-Euler II

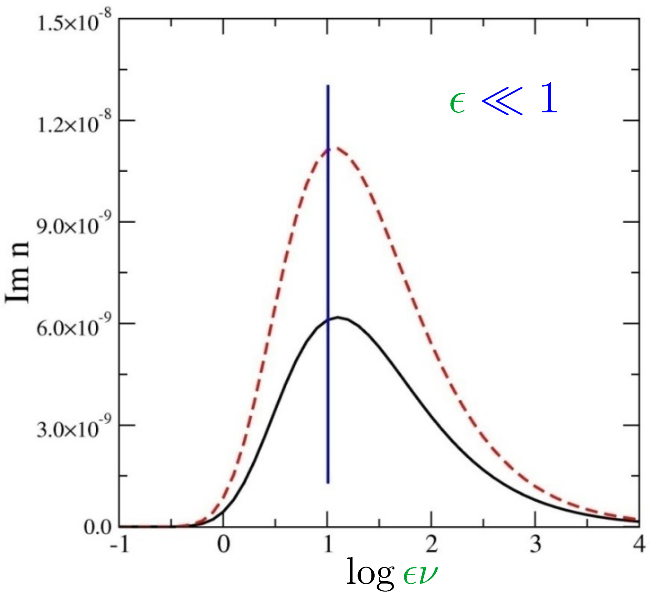
- Energy intensity diagram



Beyond Heisenberg-Euler III

- e.g. $\epsilon\nu \simeq 3$ from Compton backscattering (few GeV e^-)





(K. Langfeld)

Anomalous dispersion

Absorption → PP

Conclusion

- Laser power approaching exawatt regime
- Extreme field physics
  - **High-intensity QED** $\longrightarrow$ Sauter-Schwinger limit
  - Needs to be **tested** experimentally for $a_0 > 1$!
  - **N**ew physics (noncomm. FT, WISPs, ALPs,…)
- Theory ($\longrightarrow$ a_0 dependence)
  - Challenges:
    - Finite size effects: Pulse duration/extension
    - Beyond plane waves: Gaussian beams, …
    - Beyond ext. field approx: Back reaction

Thank you very much…

…for your attention

Appendix

Chirped Pulse Amplification (CPA)

- Problems @ ultra-high intensities:

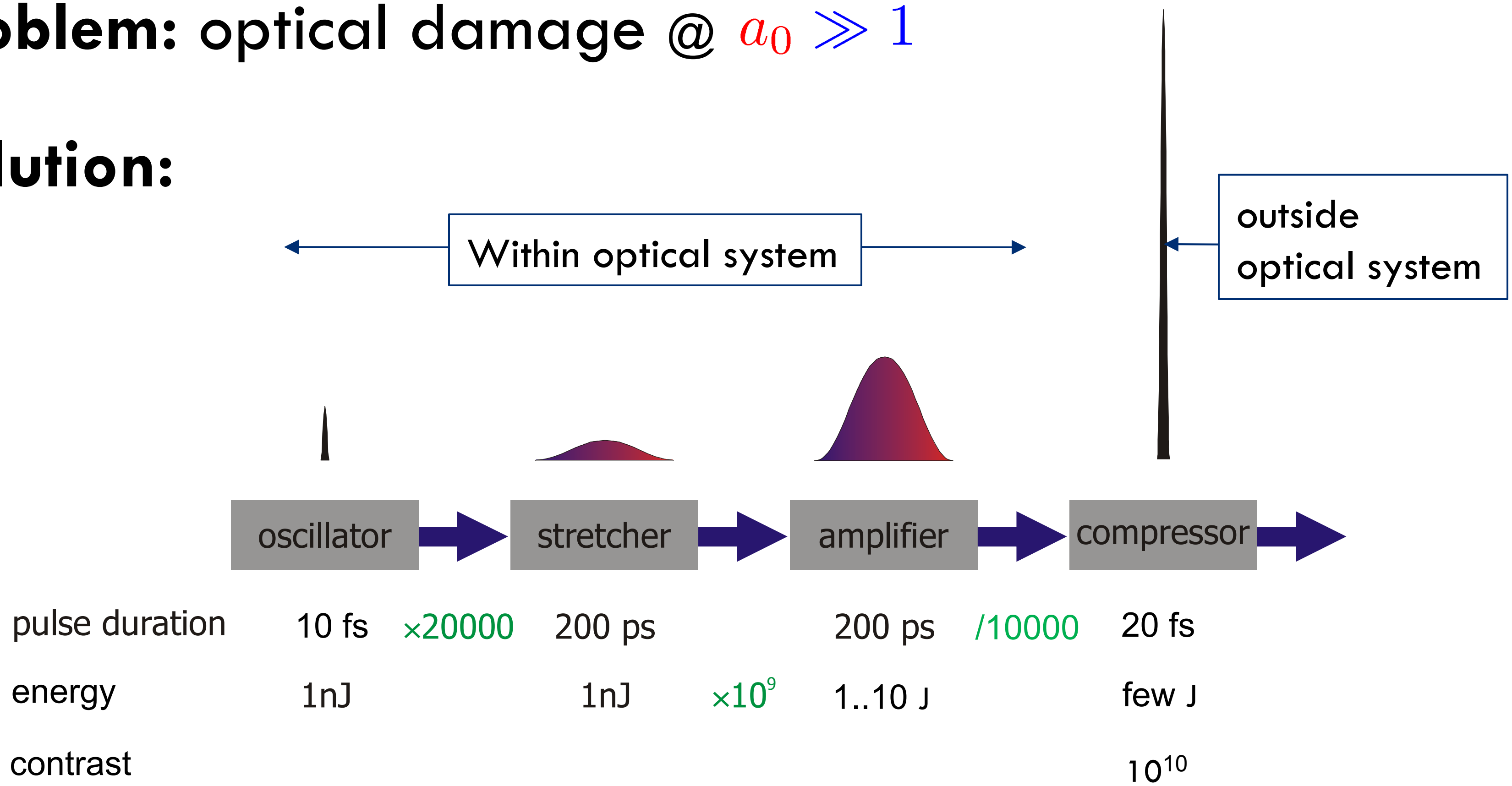
- Beam quality
  - Intensity dependent refractive index: phase delays
  - Self-focussing of beam
  - Beam collapse into beamlets ('filamentation')

- Most severe: optical damage to mirrors and lenses

Chirped Pulse Amplification (CPA)

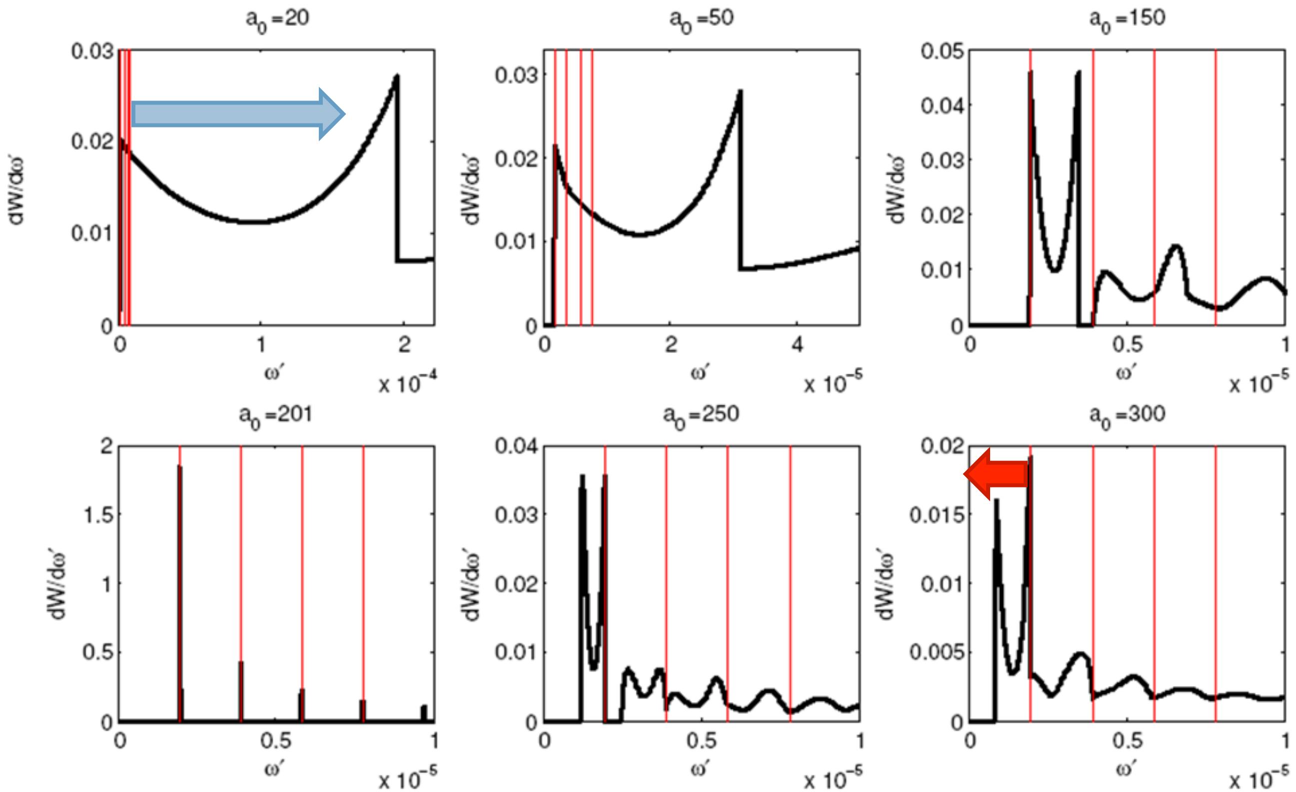
Problem: optical damage @ $a_0 \gg 1$

Solution:



Courtesy R. Sauerbrey

a_0 dependence (lab)



Tuning a_0 similar to changing frame: when $a_0 = a_{0c} \simeq 2\gamma$

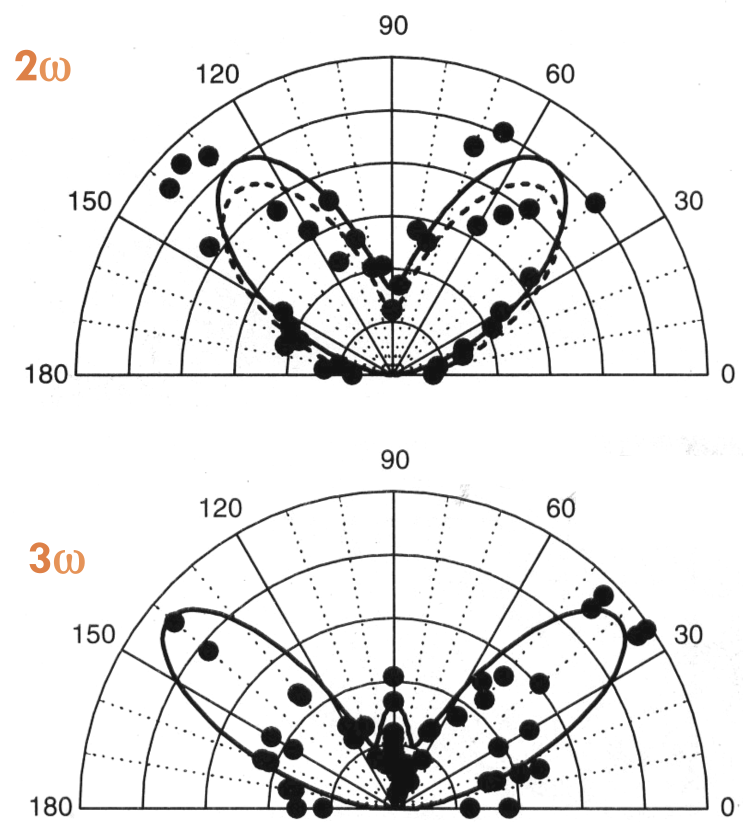
'inverse' Compton $\longrightarrow$ Compton

C. Harvey, TH, A. Ilderton, PRA **79**, 2009

Aside: Higher harmonics

- Harmonics $n=2$ and $n=3$ observed in 'relativistic Thomson scattering' using *linearly* polarised laser ($a_0=1.88$)
- Signal: quadrupole and sextupole pattern in angular distribution

(Chen, Maksimchuk, Umstadter, Nature, 1998)



$\theta = 90°$

Noncommutative (NC) QED

- Space-time noncommutativity

$$[x^\mu, x^\nu] = ic^{\mu\nu}/\Lambda^2 \ , \quad c^{\mu\nu} = O(1) \ , \ \Lambda \gtrsim 1\,\text{TeV}$$

- Ordinary product $\longrightarrow$ Moyal star product

$$f(x) \star g(x) \equiv f(x) \ \exp\left[i\partial_L \wedge \partial_R\right] g(x)$$

$$a \wedge b \equiv \tfrac{1}{2} a_\mu c^{\mu\nu} b_\nu / \Lambda^2$$

- Use in QED Lagrangian…

SF NCQED

- Main consequence: photons become self-interacting due to “star” vertices ($\longrightarrow$ new nonlinearity)

$$A \star A \star A \, , \quad A \star A \star A \star A$$

- As usual: split into strong BG + fluctuation

$$A \to A + \mathsf{A} \, , \quad \mathsf{A} = \mathsf{A}(\xi)$$

- NC Volkov electron:

$$\Psi_{\boldsymbol{p}} = \Psi_{\boldsymbol{p}}[\tilde{\mathsf{A}}] \, , \quad \tilde{\mathsf{A}} = \mathsf{A}(\xi + k \wedge p)$$

- Quasi-momentum unchanged as $\langle \tilde{\mathsf{A}} \rangle = \langle \mathsf{A} \rangle$

Dressed NC photon

- **Main result:** NC photon is dressed by laser BG, analogous to Volkov sol^{n}
- Acquires *quasi-momentum*

$$l'_\mu = k'_\mu + \frac{2a_0^2 m^2 \sin^2(k \wedge k')}{k \cdot k'} k_\mu$$

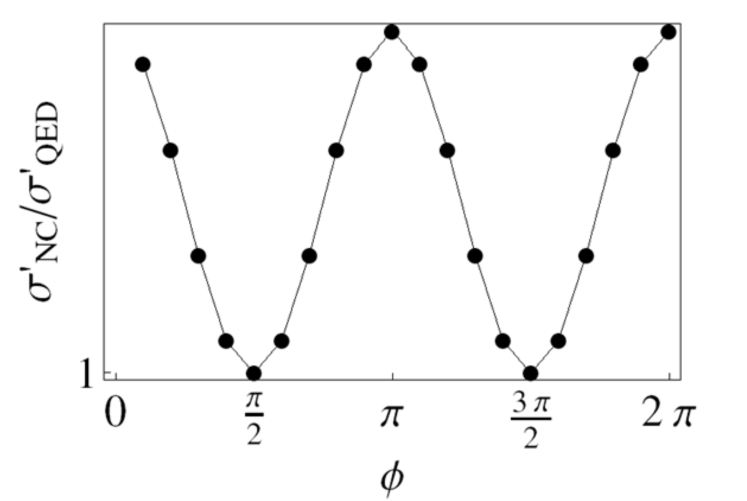
- hence an *effective photon mass* (or self-energy)

$$l'^2 = 4m^2 a_0^2 \sin^2(k \wedge k') \equiv 4m^2 \Delta^2$$

- **NB**: depends both on intensity *and* NC scale

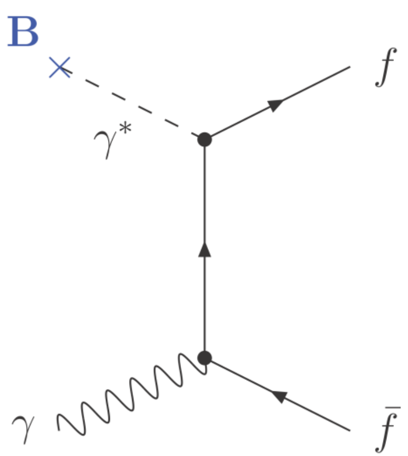
‘Photon mass’ effects

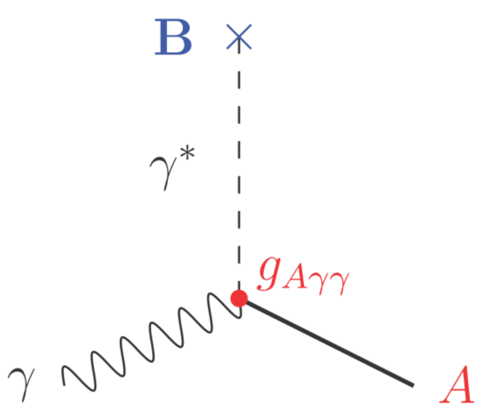
- SF NC parameter Δ extremely small
  - NLC: $\Delta \lesssim 10^{-11}$
  - Induced PP: $\Delta \lesssim 10^{-3}$
- Exp. signature: azimuthal dependence due to preferred direction
- Relative cross section:



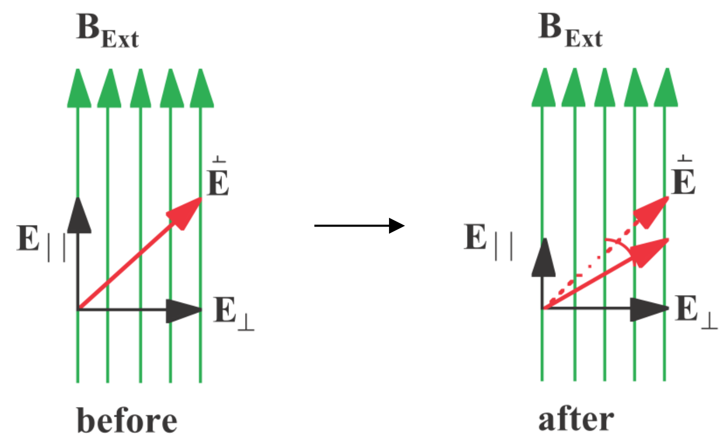
  - NLC: $\frac{\sigma'_{\rm NC}}{\sigma'_{\rm QED}} \simeq 10^{-27}$
  - PP: $\frac{\sigma'_{\rm NC}}{\sigma'_{\rm QED}} \simeq 10^{-11}$

Light fermions, ALPs





- ‘disappearing’ photons
- Absorption
- Coeff^s $\kappa_\pm$



- Rotation: $\Delta\theta \sim |\kappa_+ - \kappa_-|$

(A. Ringwald, hep-ph/07043195)