Institut für Theoretische Physik
Goethe-Universität, Frankfurt

Thermodynamics of quantum fields subject to a geometric confinement: When Casimir meets Linde

Dr. Sylvain Mogliacci

Main references:

SM, WA Horowitz, I Kolbé / arXiv:1701.XXXXX (PRD?)

SM, JO Andersen, M Strickland, N Su, A Vuorinen / arXiv:1307.8098 (JHEP)

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2. Bulk thermodynamics of the QGP
   - Experimental quests and theoretical challenges
   - Correlations and fluctuations of conserved charges
3. Finite-μ QCD EoS via resummed PT
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- More realistic description, properly accounting for the finite size!
- ...Input for a more quantitative description of jet quenching

Bulk thermodynamics of the QGP

Phase diagram and critical end point

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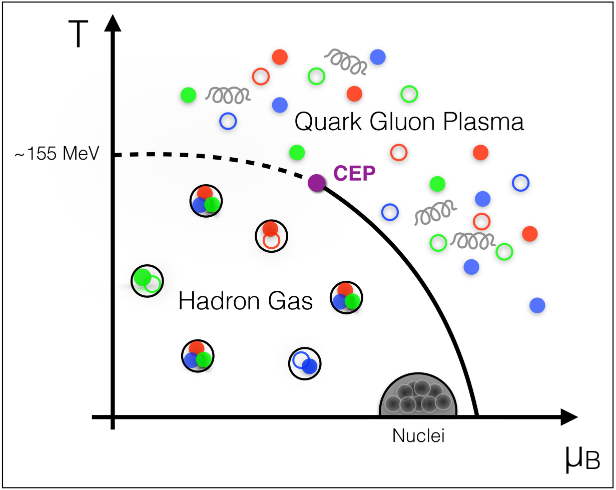

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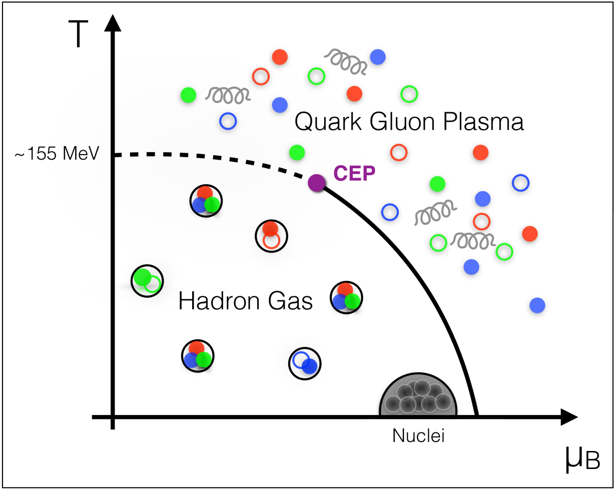

1GeV?

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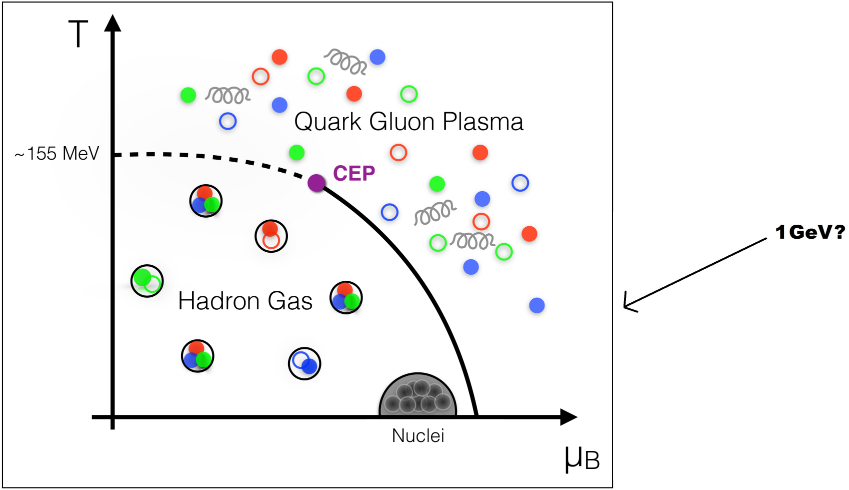

Figure from [Heng-Tong Ding et al., Int.J.Mod.Phys. E**24** (2015)]

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Lattice discretization of the theory
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Bielefeld's GPU (Germany)

500 Teraflops $\sim$ 10 000 PCs

(And $\approx$ EUR 1.1×10^6)

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$\Rightarrow$ Simulations (still) not (yet) feasible!

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$$p_{\mathrm{QCD}}\;\equiv\;\frac{T}{V}\;\log\mathcal{Z}_{\mathrm{QCD}}$$

$$\mathcal{S}\equiv\frac{\partial\,p_{\mathrm{QCD}}}{\partial T}\quad;\quad\mathcal{N}_f\equiv\frac{\partial\,p_{\mathrm{QCD}}}{\partial\mu_f}$$

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$$\chi_{u_i\,d_j\,s_k\,\ldots}(T) \equiv \frac{\partial^{\,i+j+k+\ldots}\ p(T,\{\mu_f\})}{\partial\mu_u^i\,\partial\mu_d^j\,\partial\mu_s^k\ldots}\Bigg|_{\{\mu_f\}=0}$$

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... But first, what about bare (not resummed) and conventional (infinite volume; no spatial compactification) perturbation theory...?

(massless) QCD with $N_f = 3$ and $\boldsymbol{\mu} = \mathbf{0}$:

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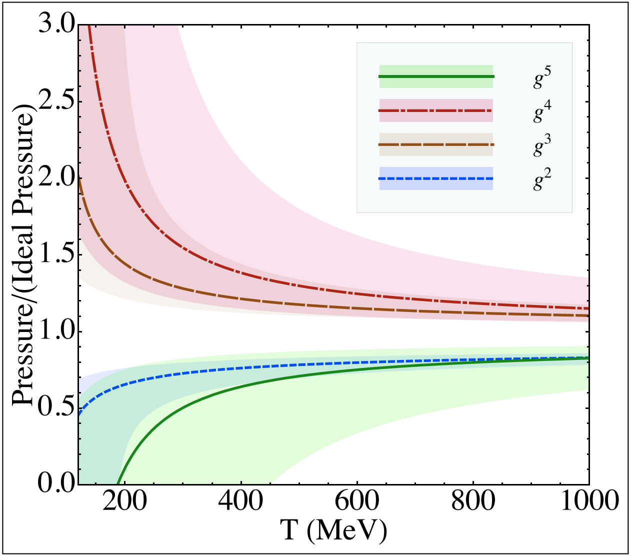

Finite density QCD Equation of State via resummed PT

Low order cumulants

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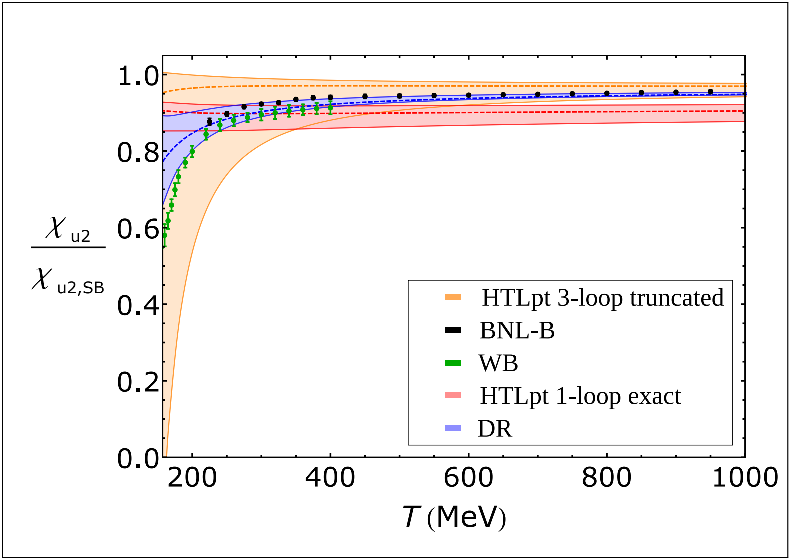

$$\chi_{B4} = \Big(\chi_{u4} + \chi_{d4} + \chi_{s4} + 4\,\chi_{u3\,d} + 4\,\chi_{u3\,s} + 4\,\chi_{d3\,u} + 4\,\chi_{d3\,s} + 4\,\chi_{s3\,u} + 4\,\chi_{s3\,d}$$
$$+6\,\chi_{u2\,d2} + 6\,\chi_{d2\,s2} + 6\,\chi_{u2\,s2} + 12\,\chi_{u2\,ds} + 12\,\chi_{d2\,us} + 12\,\chi_{s2\,ud}\Big)/81$$

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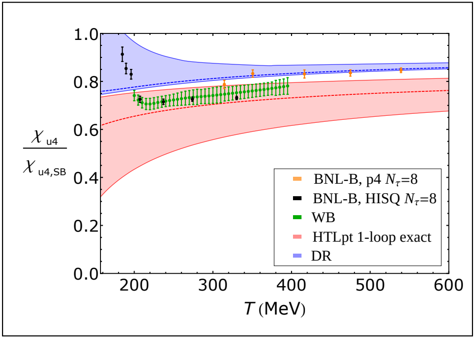

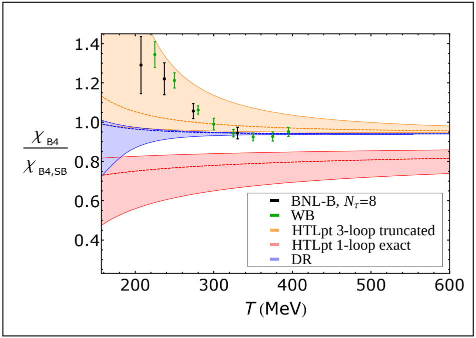

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Which is nothing but a Taylor series containing all order cumulantss:

$$\begin{aligned}
\Delta p(T) &= \sum_{i,j,k,\ldots=1}^{\infty} \left. \frac{\partial^{i+j+k+\cdots} p(T, \{\mu_u, \mu_d, \mu_s, \ldots\})}{\partial \mu_u^i \partial \mu_d^j \partial \mu_s^k \ldots} \right|_{\{\mu_f\}=0} \times \frac{\mu_u^i \mu_d^j \mu_s^k \ldots}{i!\ j!\ k! \ldots} \\
&= \sum_{i,j,k,\ldots=1}^{\infty} \chi_{u_i\, d_j\, s_k\, \ldots} \times \frac{\mu_u^i \mu_d^j \mu_s^k \ldots}{i!\ j!\ k! \ldots}
\end{aligned}$$

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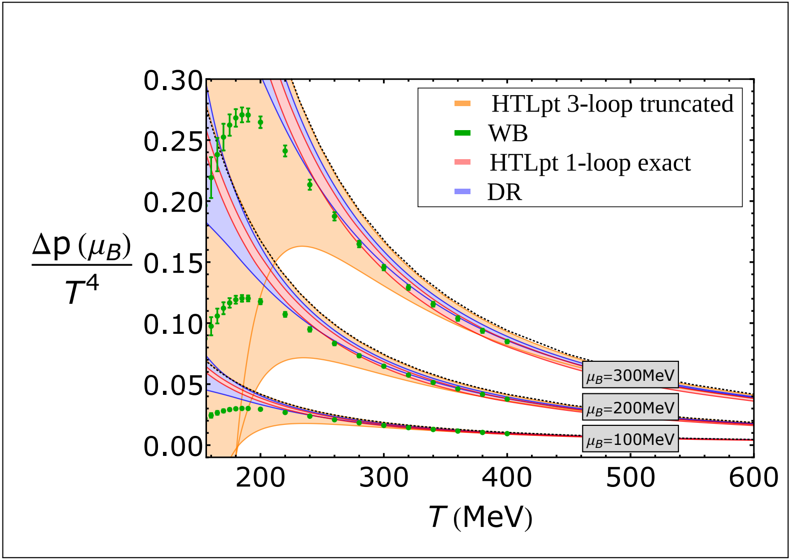

Geometric confinement and finite volume

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3. $\mathcal{X}_v(T,L_i) = -\sum_{i=1}^{D-1}\left[L_i \times \frac{\partial f(T,\{L_i\})}{\partial L_i}\right]$

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$$-\frac{T^{1+2\alpha}}{2\prod_{i=1}^{c}\left(L_i\right)}\times\left(\frac{\bar{\Lambda}^2 e^{\gamma_E}}{4\pi}\right)^{2-\frac{D}{2}}\times$$

$$\times\sum_{n\in\mathbb{Z}^1}\ \sum_{\boldsymbol{k}\in\mathbb{N}^c}\int\frac{\mathrm{d}^{\mathrm{D}-1-\mathrm{c}}\boldsymbol{p}}{(2\pi)^{D-1-c}}\left[\frac{1}{\left(\omega_n^2+\sum_{i=1}^{c}\omega_{k_i}^2+\boldsymbol{p}^2+m^2\right)^{\alpha}}\right]$$

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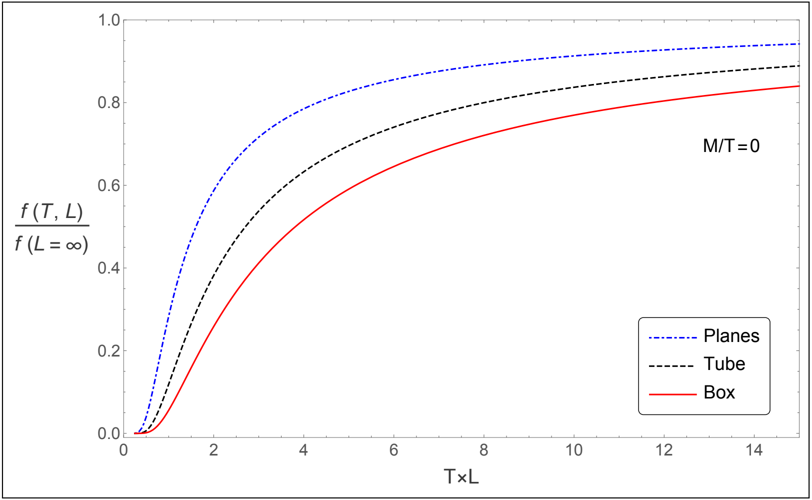
- Analytically continuing the above, say for $c=3$ and $m\neq 0$, gives such a (out of many different possible) representation(s) for the proper free-energy:

$$\bar{f}_{\rm R}^{(3)}(T,L_1,L_2,L_3;m_{\rm R}) = -\frac{T}{8L_1L_2L_3}\times\log\left(1-e^{-\frac{m_{\rm R}}{T}}\right) - \frac{m_{\rm R}T}{8\pi L_1L_2}\times\sum_{(s,s_1)\in\mathbb{Z}^2\setminus\{\mathbf{0}\}}{}^{'}\left[\frac{K_1\left(\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_3)^2s_1^2}\right)}{\sqrt{s^2+(2TL_3)^2s_1^2}}\right]$$
$$-\frac{m_{\rm R}T}{8\pi L_3}\times\sum_{(s,s_1)\in\mathbb{Z}^2\setminus\{\mathbf{0}\}}{}^{'}\left[\frac{K_1\left(\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_2)^2s_1^2}\right)}{L_1\sqrt{s^2+(2TL_2)^2s_1^2}}+\frac{K_1\left(\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_1)^2s_1^2}\right)}{L_2\sqrt{s^2+(2TL_1)^2s_1^2}}\right]$$
$$+\frac{T^3}{8\pi L_1}\times\sum_{(s,s_1,s_2)\in\mathbb{Z}^3\setminus\{\mathbf{0}\}}{}^{'}\left[\frac{e^{-\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_2)^2s_1^2+(2TL_3)^2s_2^2}}\left(1+\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_2)^2s_1^2+(2TL_3)^2s_2^2}\right)}{\left(s^2+(2TL_2)^2s_1^2+(2TL_3)^2s_2^2\right)^{3/2}}\right]$$
$$+\frac{T^3}{8\pi L_2}\times\sum_{(s,s_1,s_2)\in\mathbb{Z}^3\setminus\{\mathbf{0}\}}{}^{'}\left[\frac{e^{-\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_3)^2s_2^2}}\left(1+\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_3)^2s_2^2}\right)}{\left(s^2+(2TL_1)^2s_1^2+(2TL_3)^2s_2^2\right)^{3/2}}\right]$$
$$+\frac{T^3}{8\pi L_3}\times\sum_{(s,s_1,s_2)\in\mathbb{Z}^3\setminus\{\mathbf{0}\}}{}^{'}\left[\frac{e^{-\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2}}\left(1+\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2}\right)}{\left(s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2\right)^{3/2}}\right]$$
$$-\frac{m_{\rm R}^2T^2}{4\pi^2}\times\sum_{(s,s_1,s_2,s_3)\in\mathbb{Z}^4\setminus\{\mathbf{0}\}}{}^{'}\left[\frac{K_2\left(\frac{m_{\rm R}}{T}\sqrt{s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2+(2TL_3)^2s_3^2}\right)}{\left(s^2+(2TL_1)^2s_1^2+(2TL_2)^2s_2^2+(2TL_3)^2s_3^2\right)}\right], \qquad (66)$$

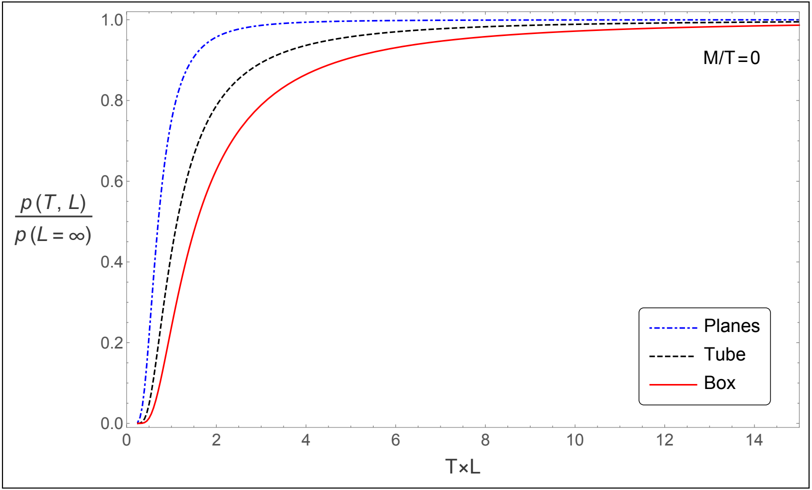
Now, finally, some new plots!

Scaling of the L_i-symmetric functions

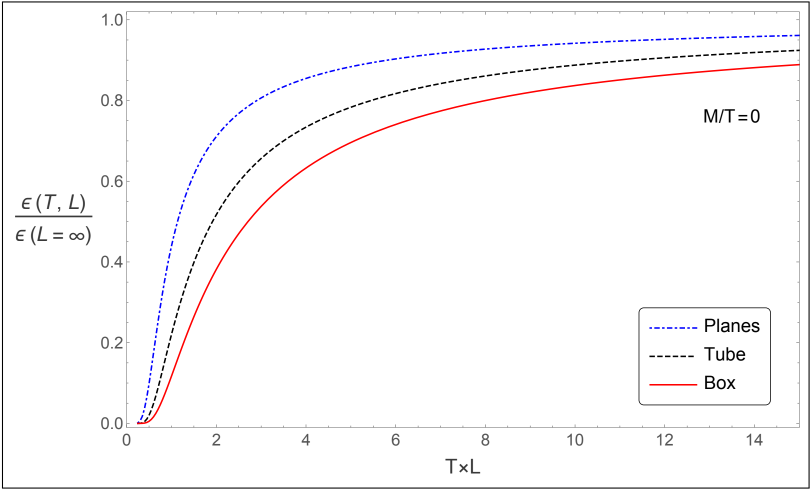
(1) Free-Energy density for $M/T = 0$ $(L_1 = L_2 = L_3 \equiv L)$



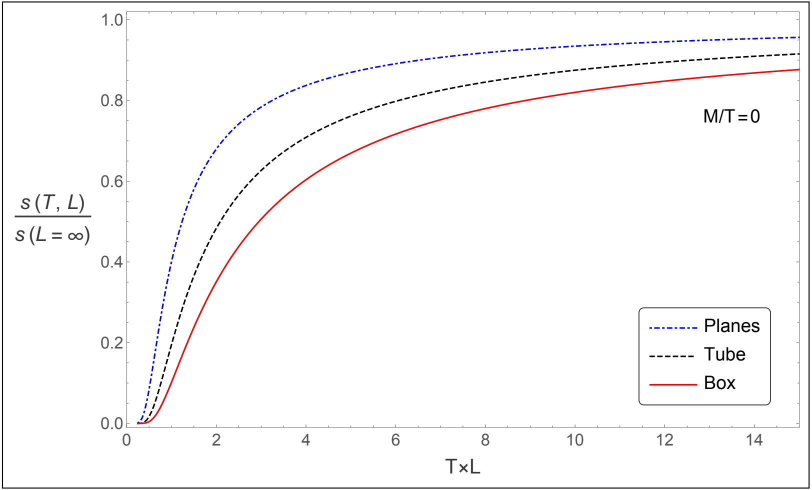
(1) Pressure for $M/T = 0$ $(L_1 = L_2 = L_3 \equiv L)$



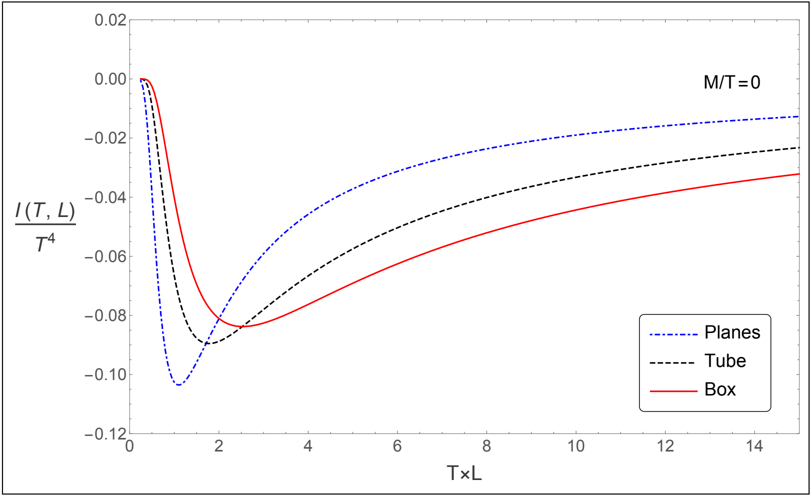
(1) Energy density for $M/T = 0$ ($L_1 = L_2 = L_3 \equiv L$)



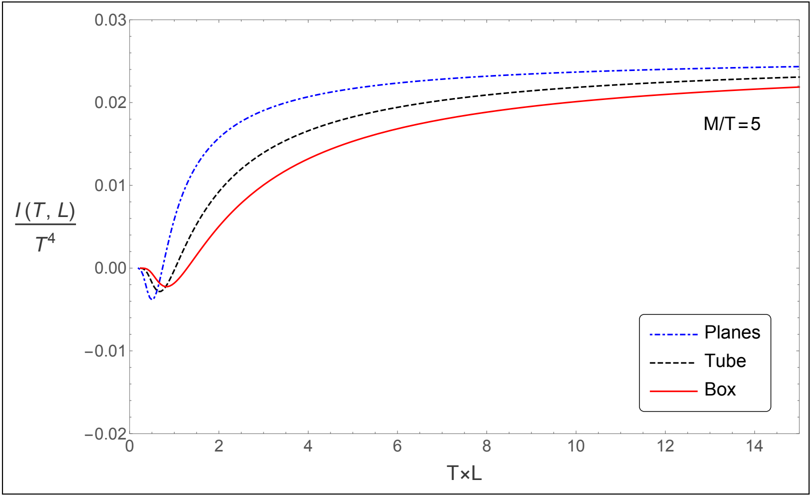
(1) Entropy density for $M/T = 0$ ($L_1 = L_2 = L_3 \equiv L$)



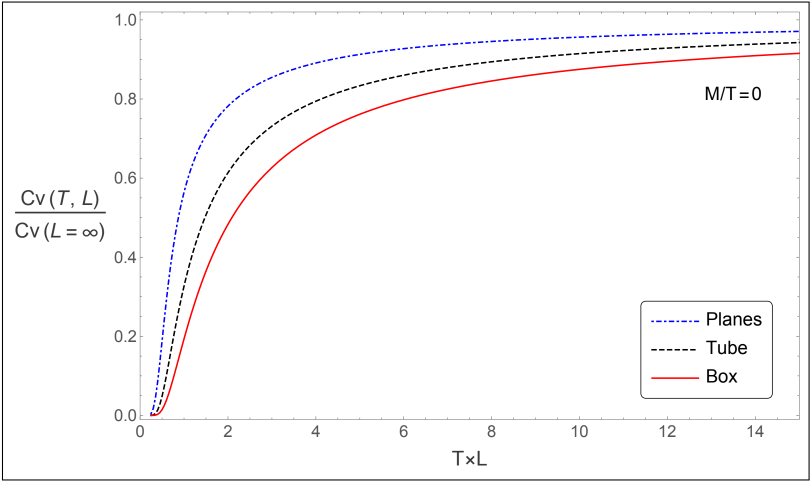
(1) Trace anomaly for $M/T = 0$ ($L_1 = L_2 = L_3 \equiv L$)



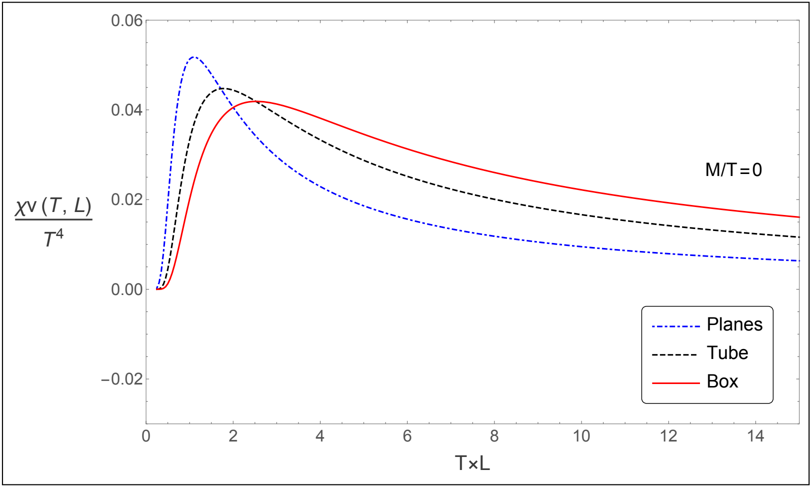
(1) Trace anomaly for $M/T = 5$ ($L_1 = L_2 = L_3 \equiv L$)



(1) Heat capacity for $M/T = 0$ ($L_1 = L_2 = L_3 \equiv L$)

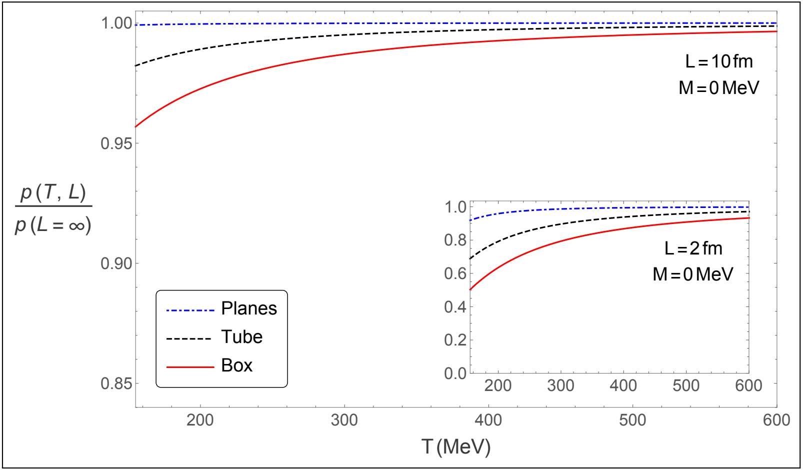


(1) Geometric suscep. for $M/T = 0$ $(L_1 = L_2 = L_3 \equiv L)$

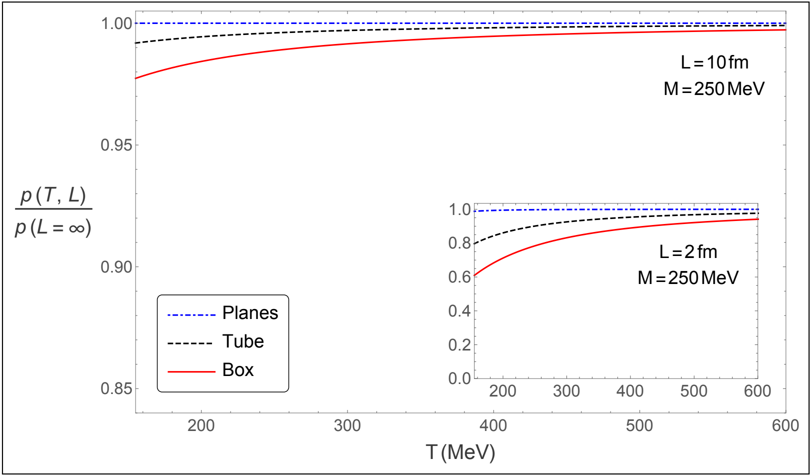


L_i-symmetric functions versus temperature

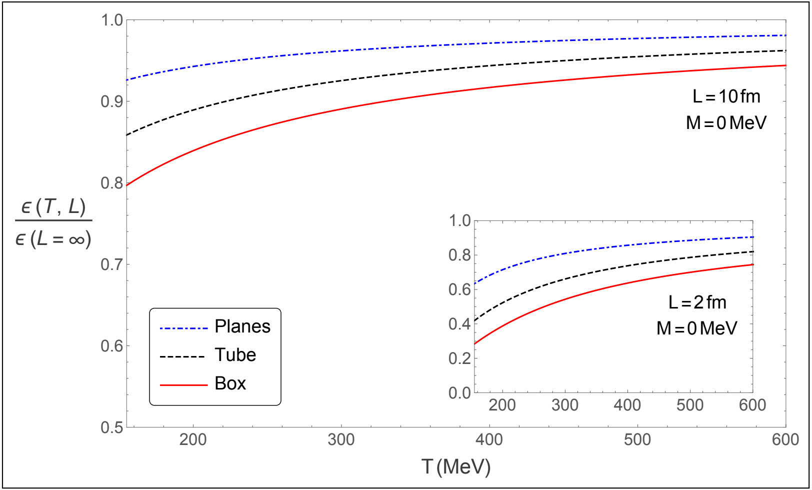
(2) Pressure for $M = 0$ MeV ($L_1 = L_2 = L_3 \equiv L$)



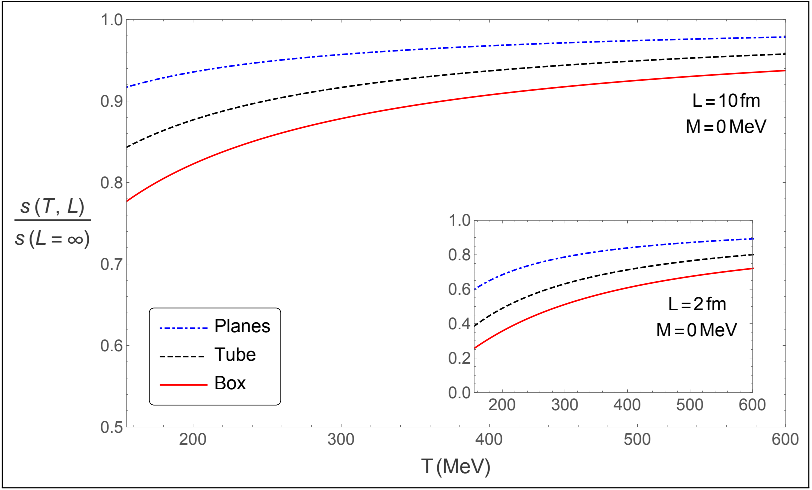
(2) Pressure for $M = 250$ MeV ($L_1 = L_2 = L_3 \equiv L$)



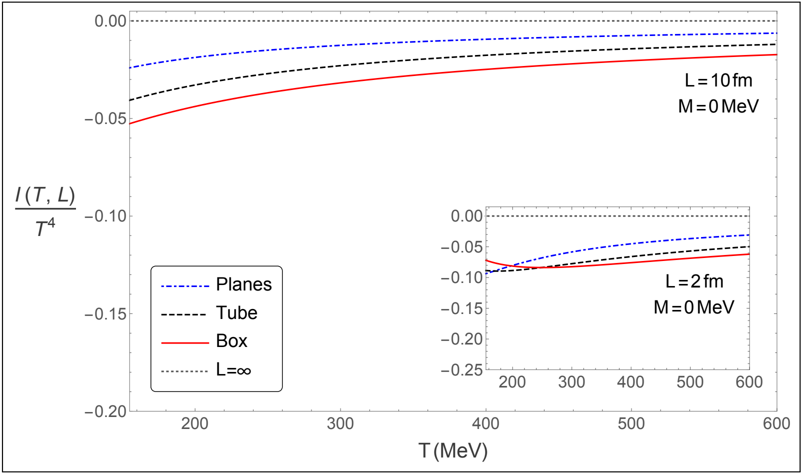
(2) Energy density for $M = 0$ MeV ($L_1 = L_2 = L_3 \equiv L$)



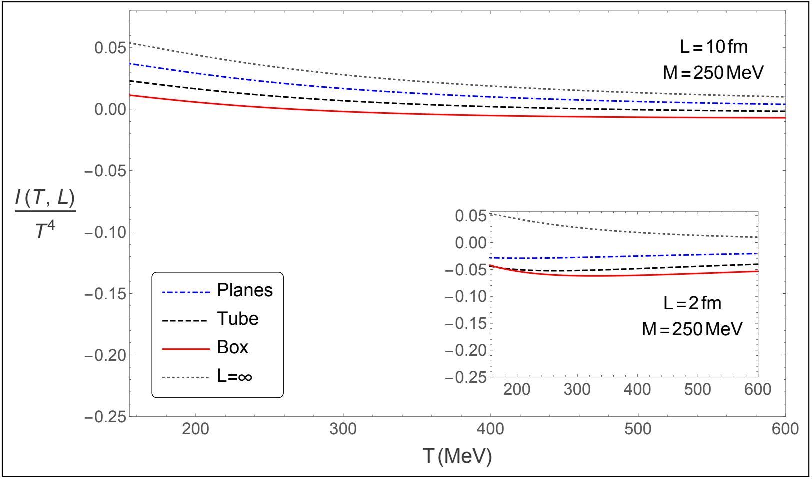
(2) Entropy density for $M = 0$ MeV ($L_1 = L_2 = L_3 \equiv L$)



(2) Trace anomaly for $M = 0$ MeV ($L_1 = L_2 = L_3 \equiv L$)

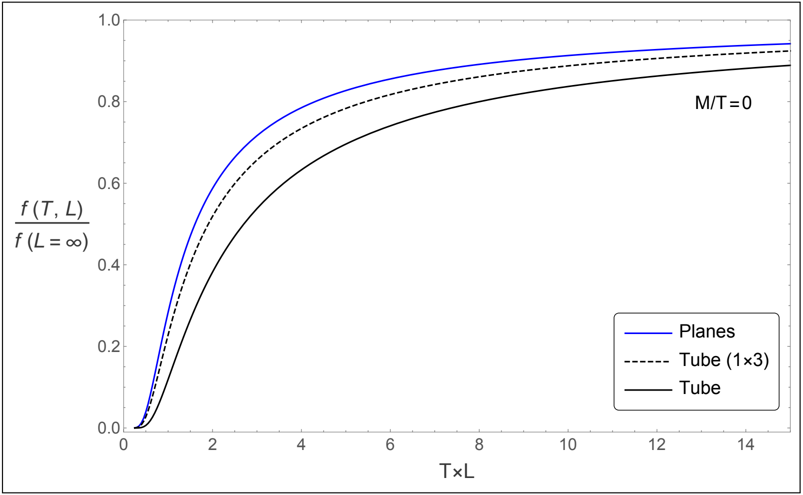


(2) Trace anomaly for $M = 250$ MeV ($L_1 = L_2 = L_3 \equiv L$)

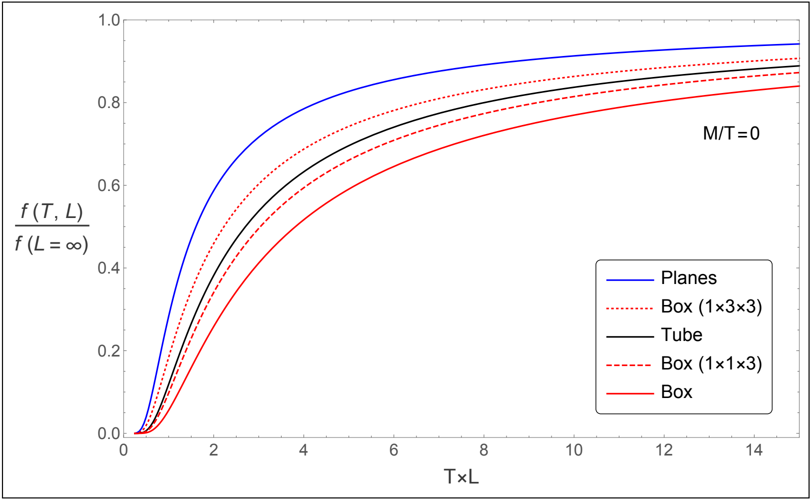


Scaling of the L_i-asymmetric functions

(3a) Assymmetric Free-Energy density for $M/T = 0$



(3b) Assymmetric Free-Energy density for $M/T = 0$



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THANKS A LOT FOR YOUR ATTENTION!

Backup slides

Backup: Some notation

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At one-loop, contributions coming from, e.g., the quarks read:

$$p_{q_f}(T,\boldsymbol{\mu}) = 2 \sum\!\!\!\!\!\!\!\!\int_{\{K\}} \log\left[A_S^2(i\widetilde{\omega}_n + \mu_f, k) - A_0^2(i\widetilde{\omega}_n + \mu_f, k)\right]$$

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With A_S and A_0:

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Where the HTL function $\widetilde{\mathcal{T}}_{\mathrm{K}}$ can be represented as:

$$\widetilde{\mathcal{T}}_{\mathrm{K}}(i\widetilde{\omega}_n+\mu_f,k) = {}_2F_1\left(\frac{1}{2},1;\frac{3}{2}-\epsilon;\frac{k^2}{(i\widetilde{\omega}_n+\mu_f)^2}\right)$$

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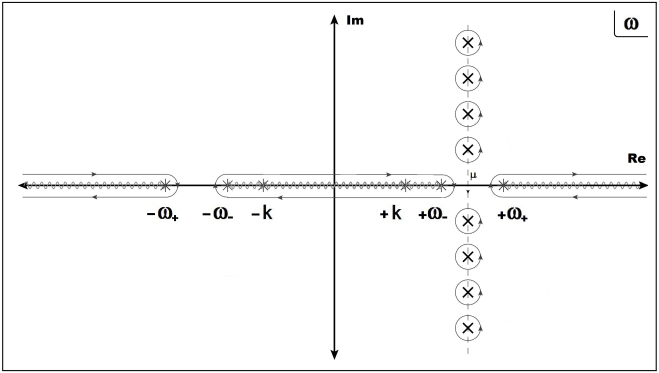

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- QCD scale: Matching the running to lattice value at a reference scale

  $\Rightarrow$ Gives $\Lambda_{\overline{\mathrm{MS}}}^{\mathrm{HTLpt/DR}} = 176/283 \pm 30$ MeV to be "conservative"

Backup: HTLpt/DR parameters and lattice data

- Relevant to nowadays experiments at RHIC [Tannenbaum, arXiv:1201.5900], LHC [Müller, ARNPS **62** (2012)], FAIR [Heuser, NPA **904-905** (2013)] and NICA [Kekelidze et al., NPA **904-905** (2013)]:

Three massless flavors and colors

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- Lattice data from:

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  WB [Borsányi et al., JHEP **01** (2012), PRL **111** (2013) and JHEP **08** (2012); Borsányi, NPA **904-905** (2013)]

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- Truncated 3-loop HTLpt results from:

  [Haque et al., PRD **89** (2014)]

Backup: HTLpt/DR higher order cumulant

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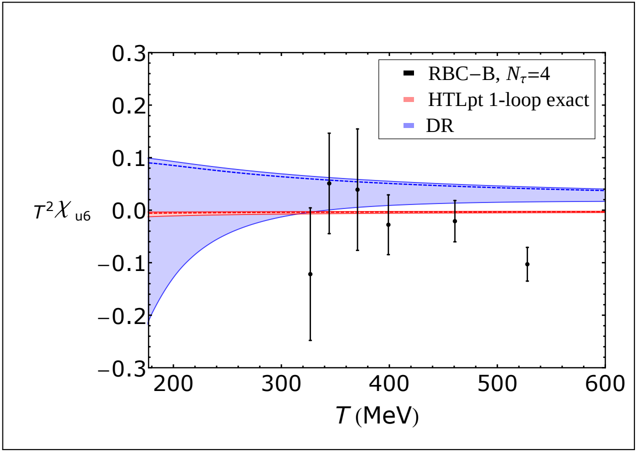

Backup: HTLpt/DR ratios of cumulants

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Recall that:

$$\chi_{B4} = \Big(\chi_{u4} + \chi_{d4} + \chi_{s4} + 4\,\chi_{u3\,d} + 4\,\chi_{u3\,s} + 4\,\chi_{d3\,u} + 4\,\chi_{d3\,s} + 4\,\chi_{s3\,u} + 4\,\chi_{s3\,d} + 6\,\chi_{u2\,d2} + 6\,\chi_{d2\,s2} + 6\,\chi_{u2\,s2} + 12\,\chi_{u2\,ds} + 12\,\chi_{d2\,us} + 12\,\chi_{s2\,ud}\Big)/81$$

$$\chi_{B2} = \Big(\chi_{u2} + \chi_{d2} + \chi_{s2} + 2\,\chi_{ud} + 2\,\chi_{ds} + 2\,\chi_{us}\Big)/9$$