Tracing the QCD pressure

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- idea relevant for many lattice-QCD calculations at T>0
- understand QCD where we think it is simple
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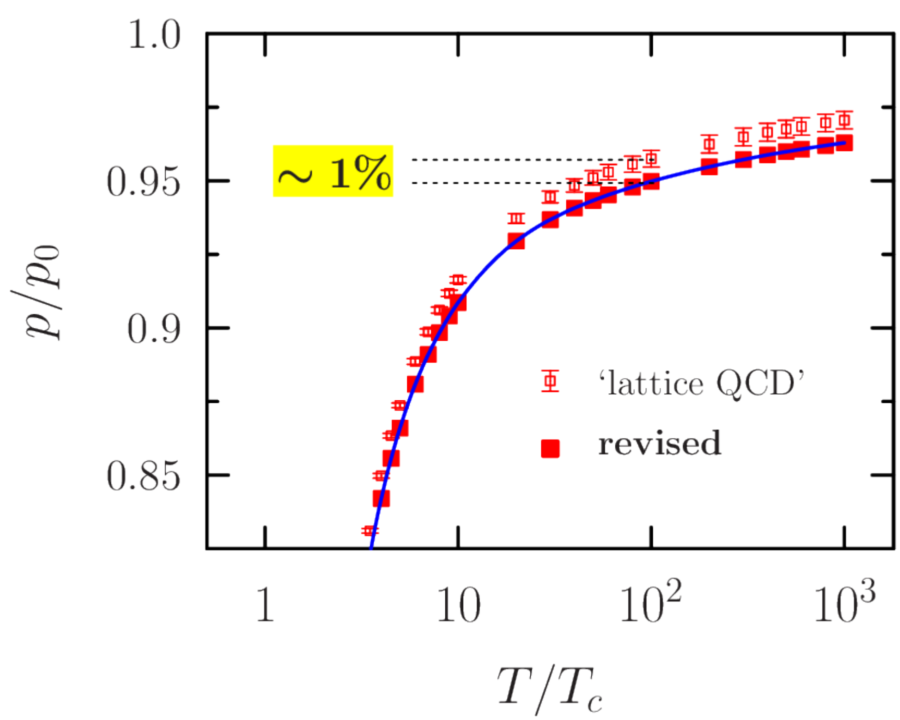

Methods

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \ldots \quad \text{where} \quad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

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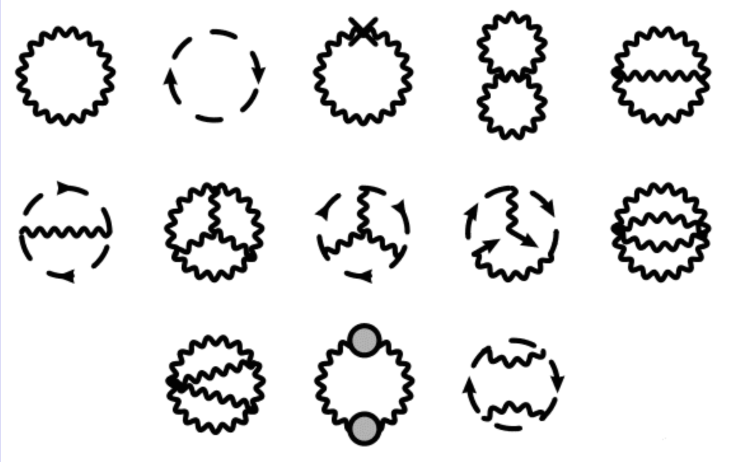
Perturbation theory



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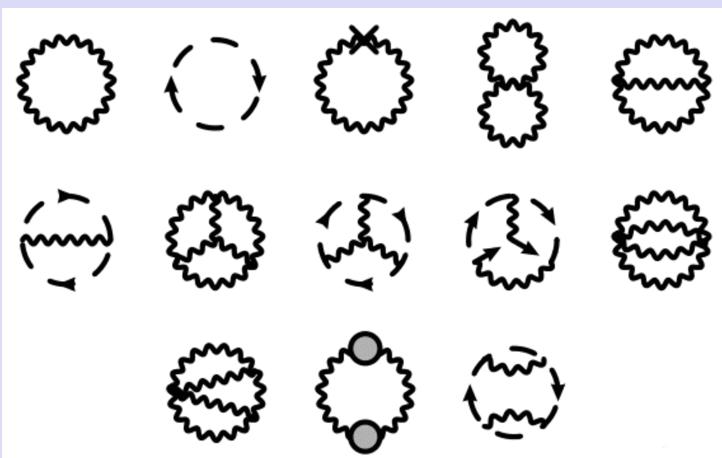

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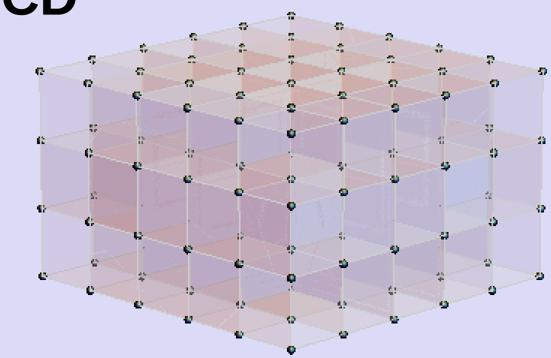
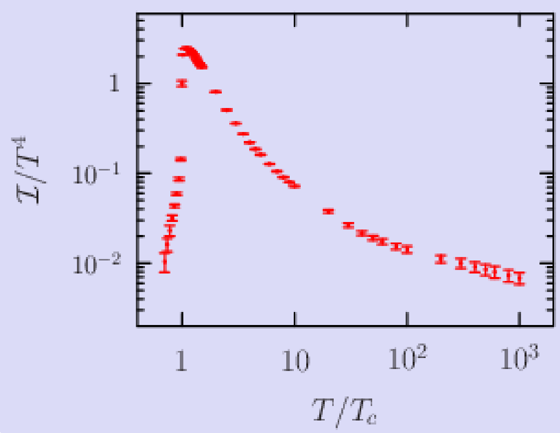
Lattice QCD





Perturbation theory CHALLENGES

$$\frac{p}{p_{\rm SB}} = 1 + c_2\alpha^{2/2} + c_3\alpha^{3/2} + (c_4 + \tilde{c}_4 \ln\alpha)\,\alpha^{4/2} + c_5\alpha^{5/2} + (c_6 + \tilde{c}_6 \ln\alpha)\,\alpha^{6/2} + \ldots$$

[Shuryak 1978]

⋮

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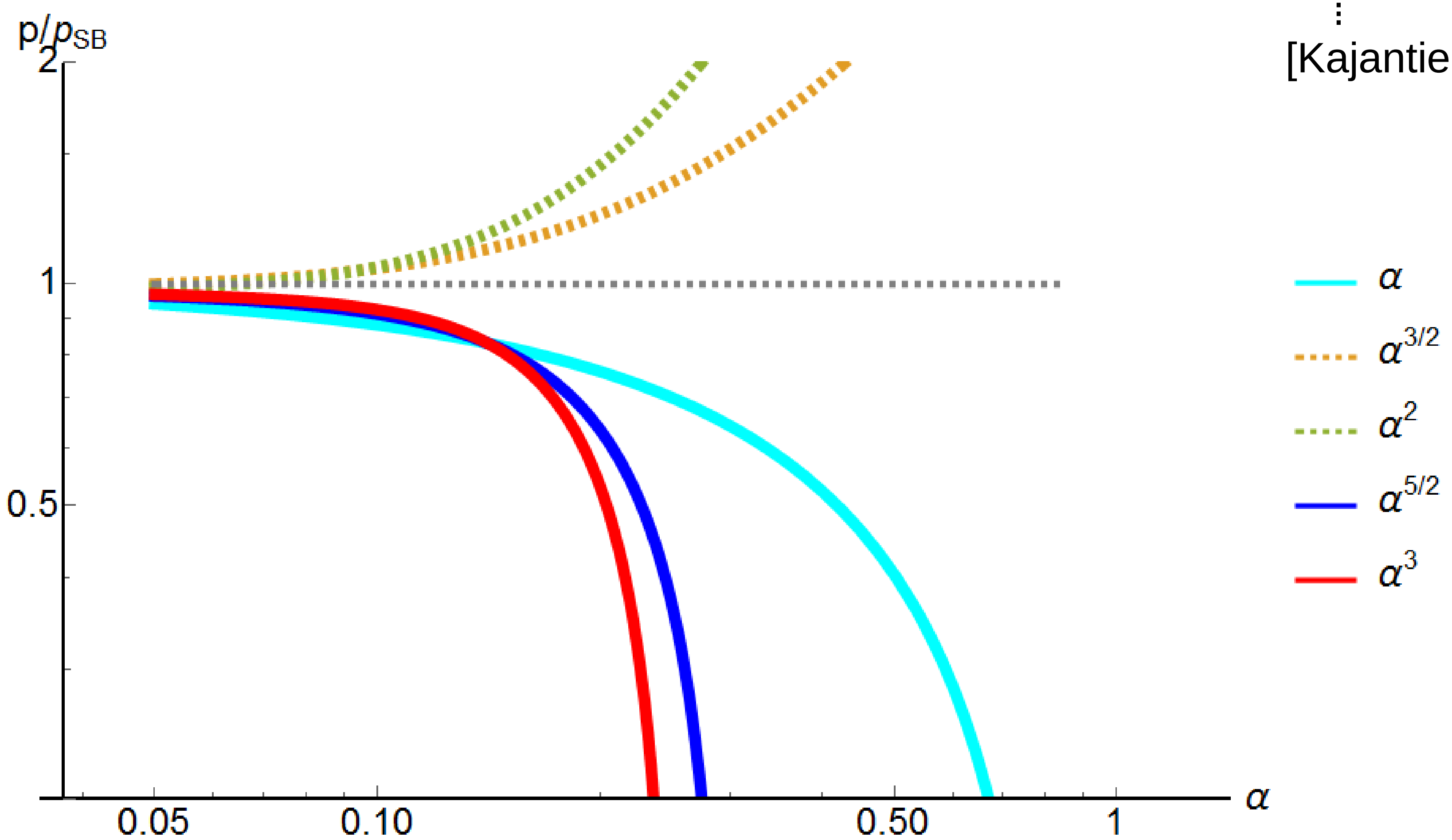

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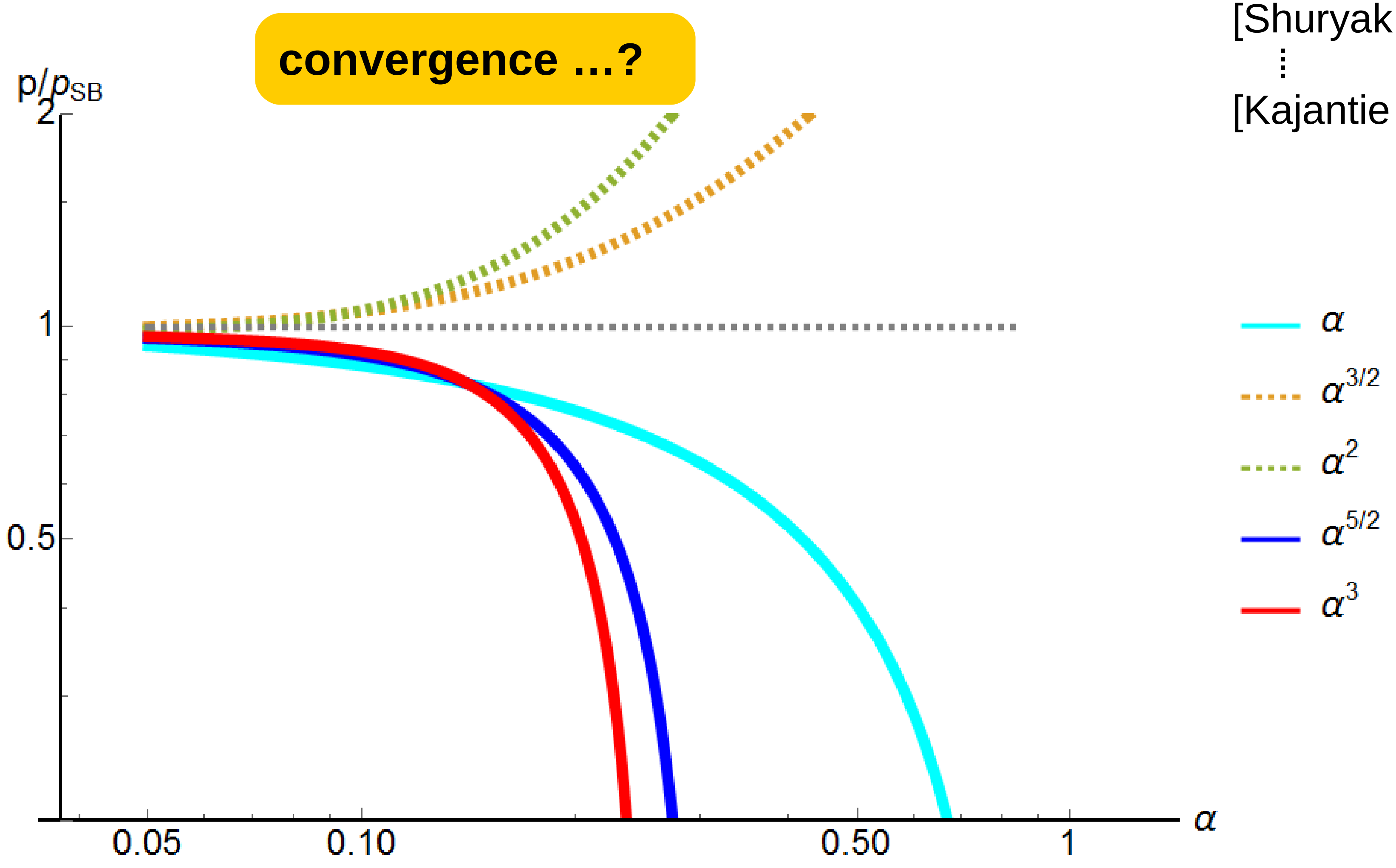

Perturbation theory "diverges"

toy model: scalar "QFT" in *d*=0 dimensions

- "Lagrangian" $L = \frac{1}{2}x^2 + \lambda x^4$ → "partition fnc" $Z(\lambda) = \int dx \exp(-L(\lambda))$
- perturbative expansion $Z(\lambda)/Z_0 = 1 - 3\lambda + \frac{1}{2}\,105\lambda^2 \mp \ldots$

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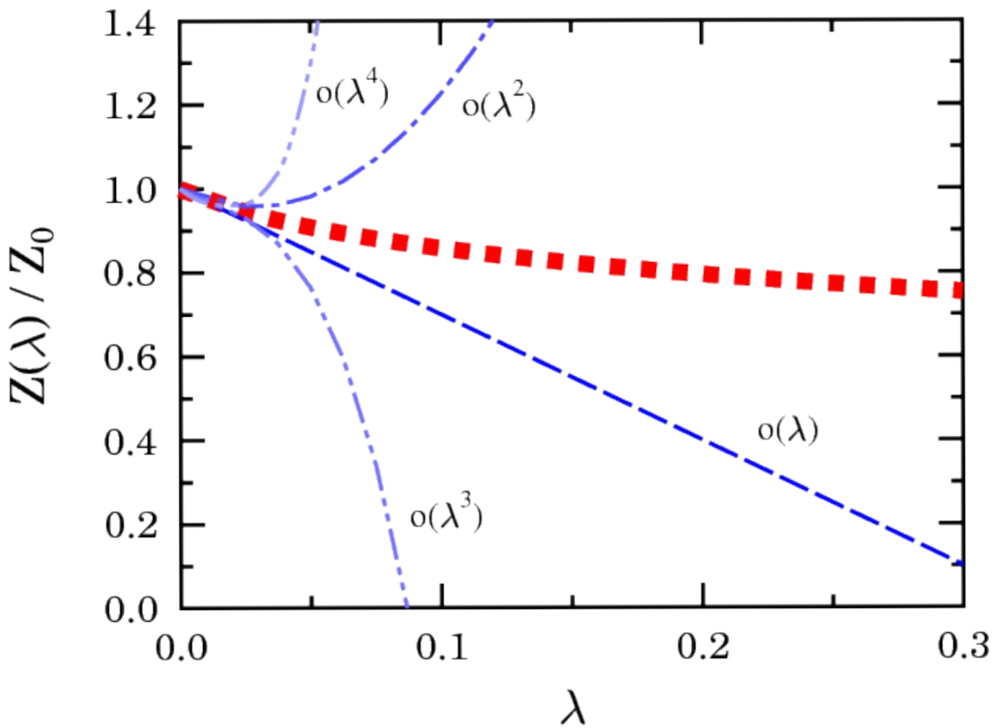

<u>lower</u> order better for larger coupling

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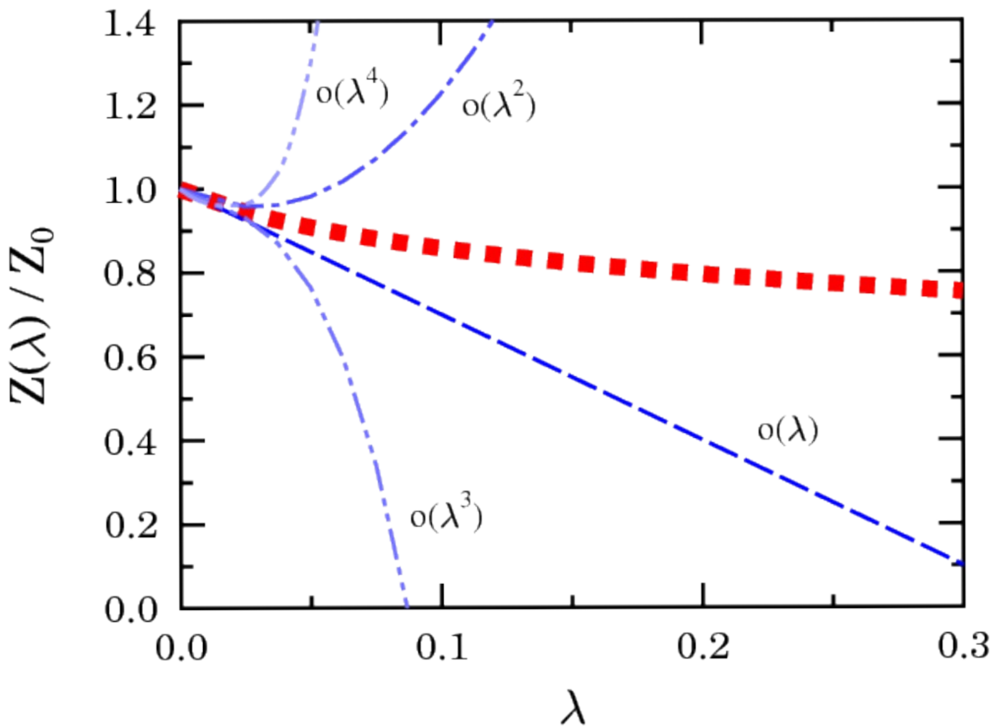

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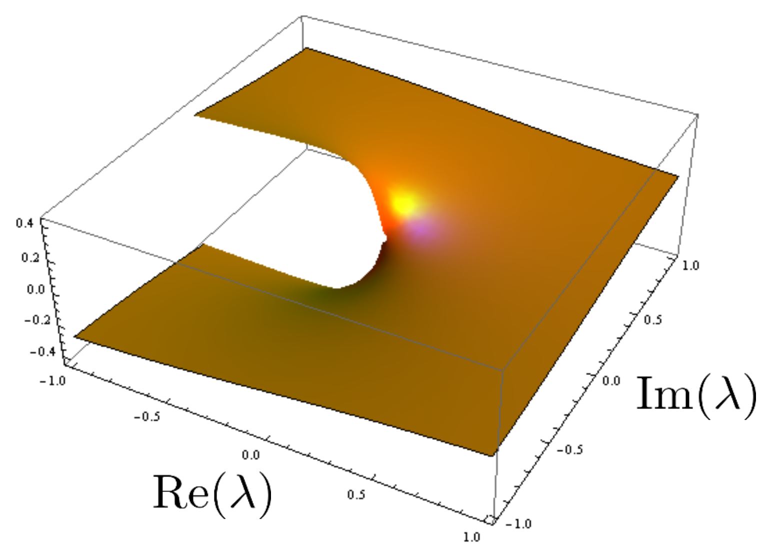
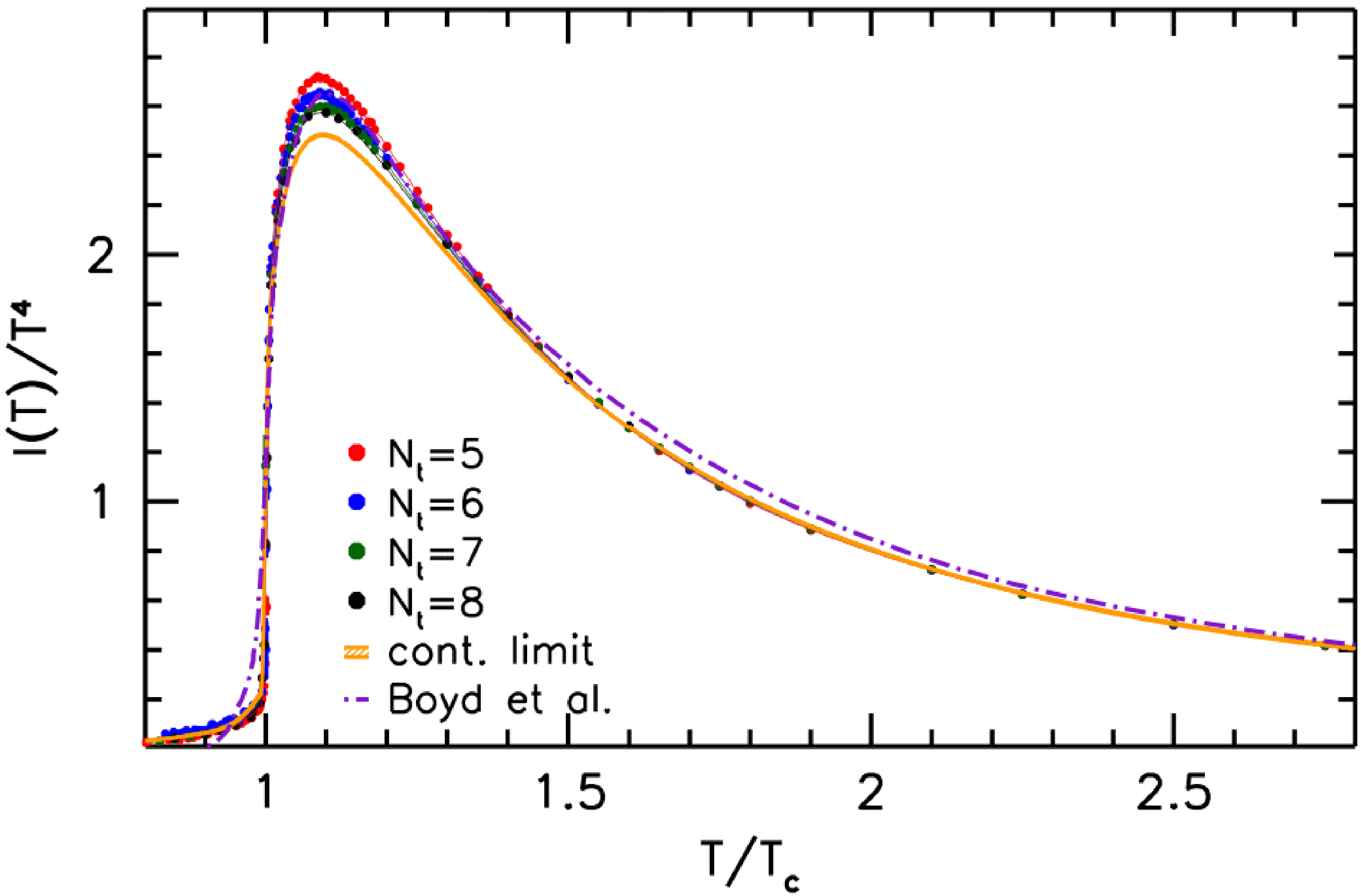
cut λ plane: convergence radius = 0



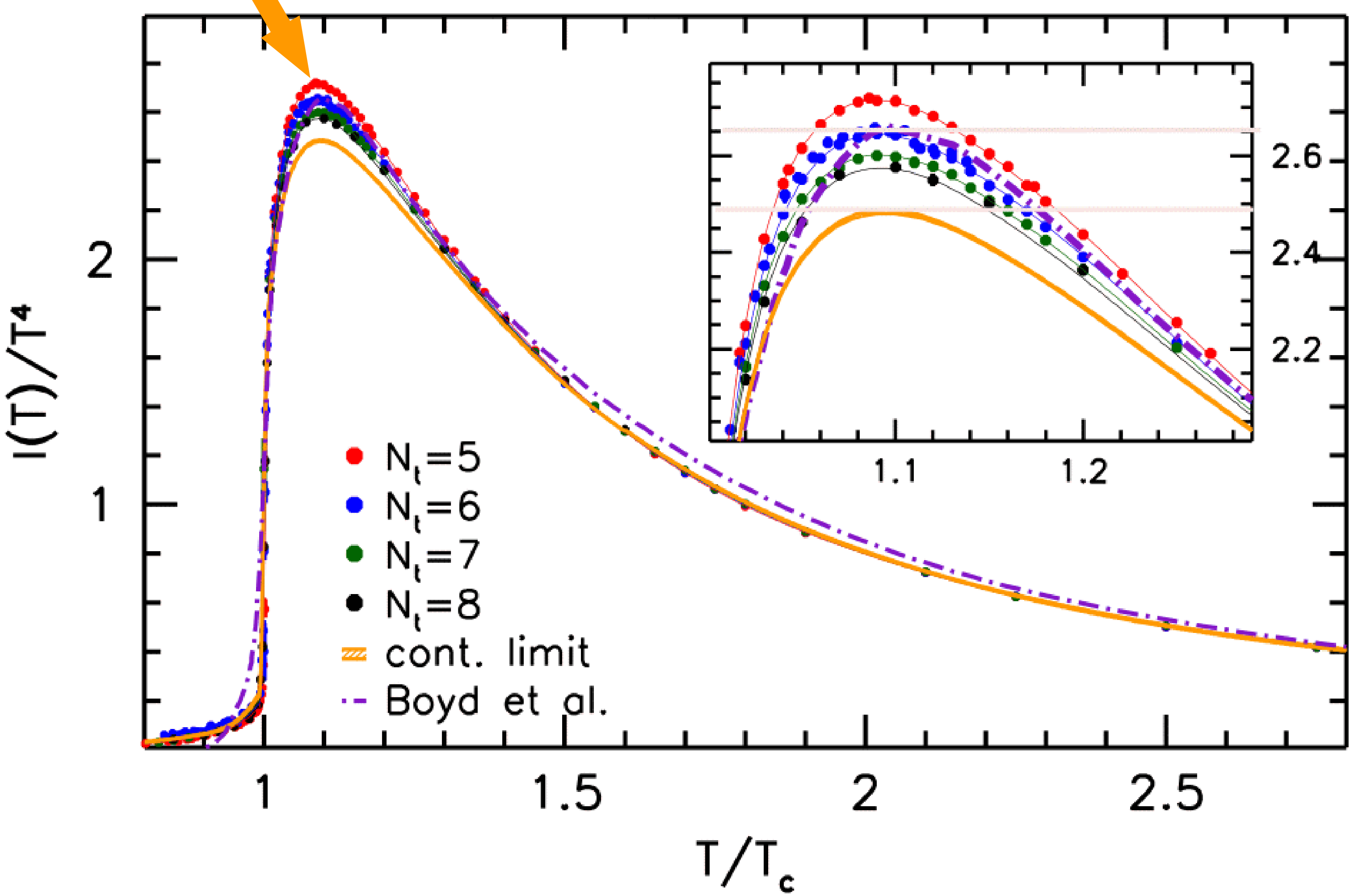
Lattice QCD CHALLENGES



[Borsanyi et al, 2012]

Lattice QCD **CHALLENGES**

finite-size artefacts in particular around T_c: correlations



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Lattice QCD

integral method: pressure from interaction measure $\mathcal{I} = e - 3p$

where $e = sT - p$

with $s = \partial p / \partial T$

$$\frac{p(T)}{T^4} = \sigma + \int_{T_0}^{T} \frac{dT'}{T'} \frac{\mathcal{I}(T')}{T'^4} \quad \text{where} \quad \sigma = \frac{p(T_0)}{T_0^4}$$

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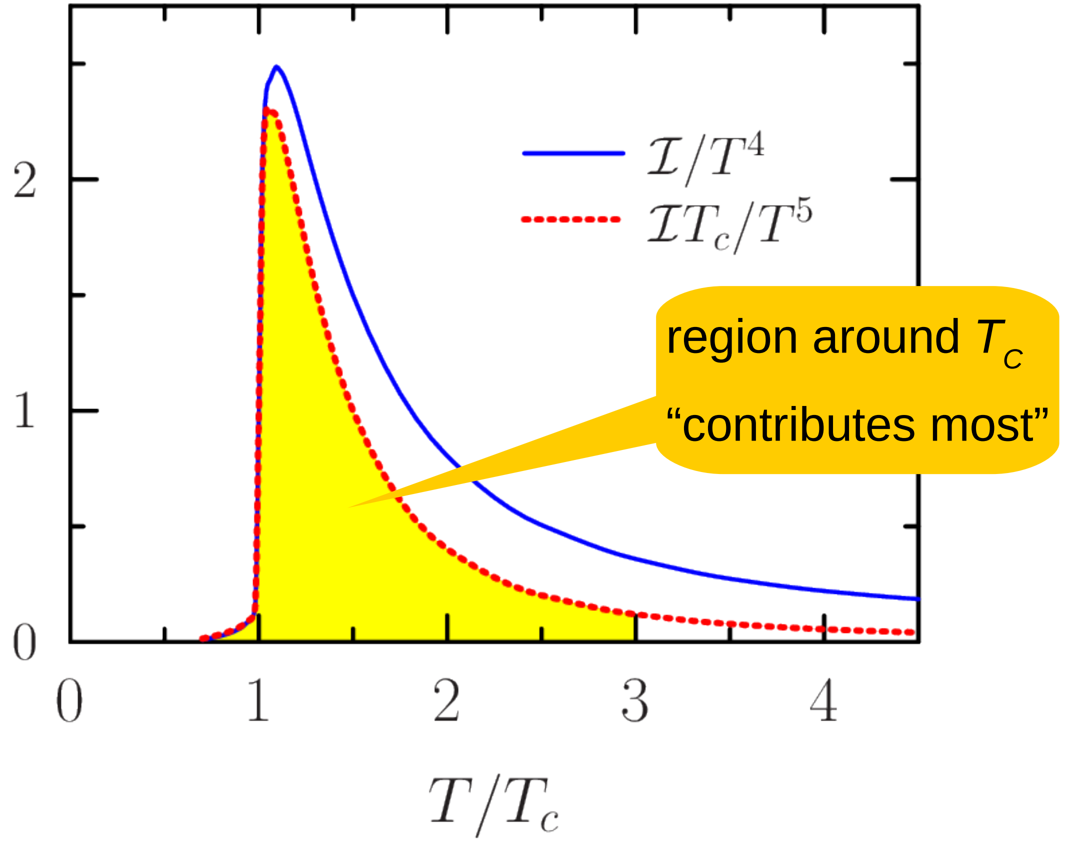
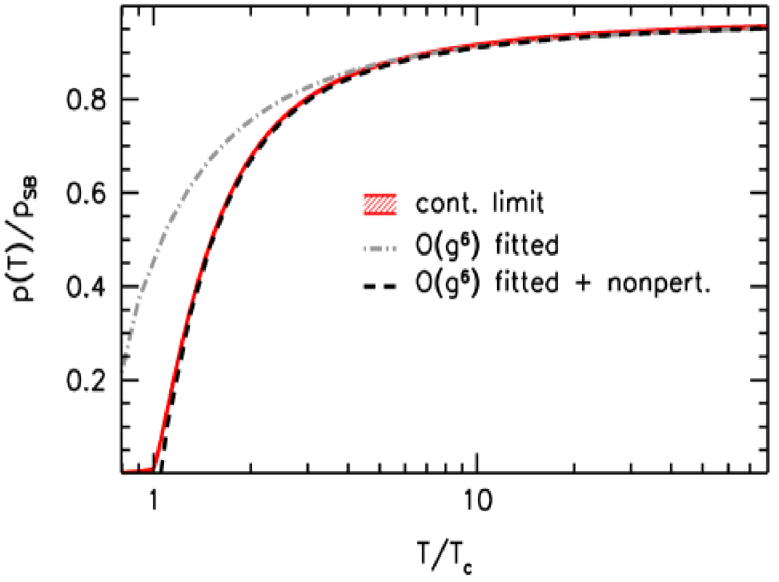
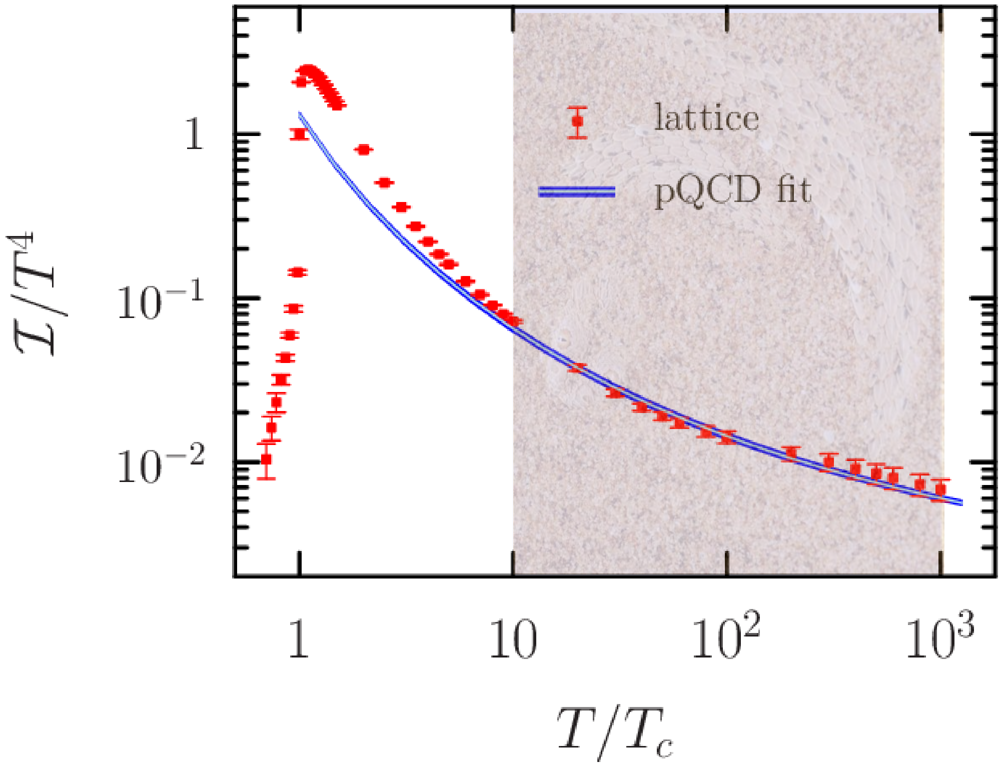




Scrutinize existing results **[Borsanyi et al, 2012]**



compare to pQCD $\boldsymbol{\Lambda = 0.73T_c}$ known

→ fit: $\boldsymbol{c_6 = -72 \pm 3}$

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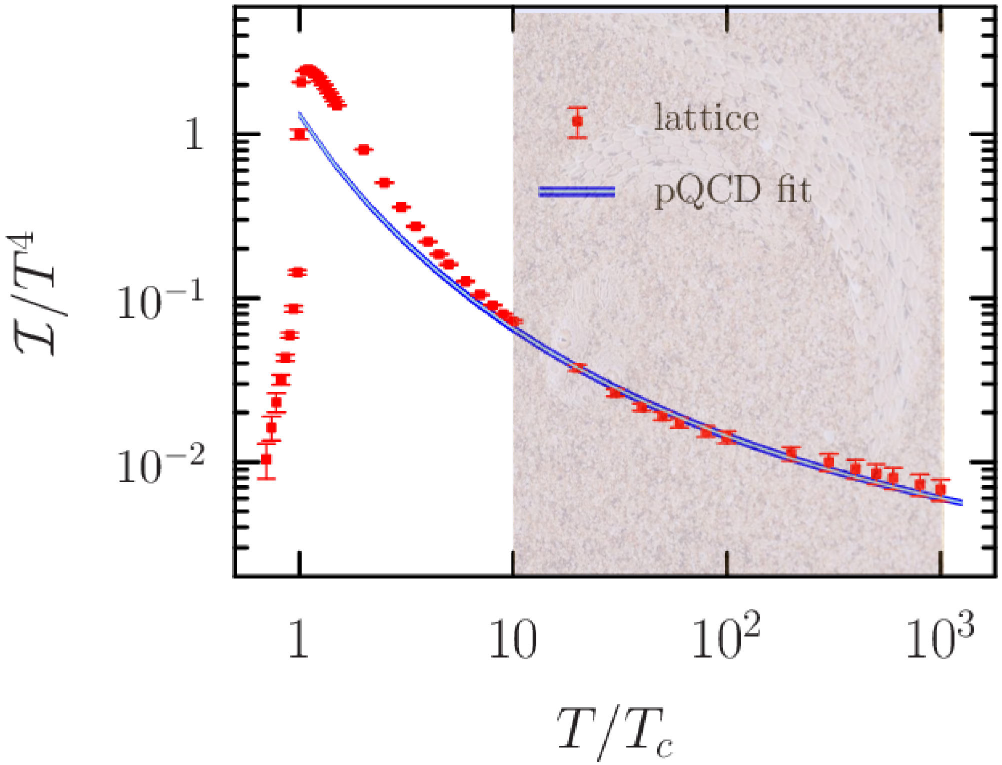

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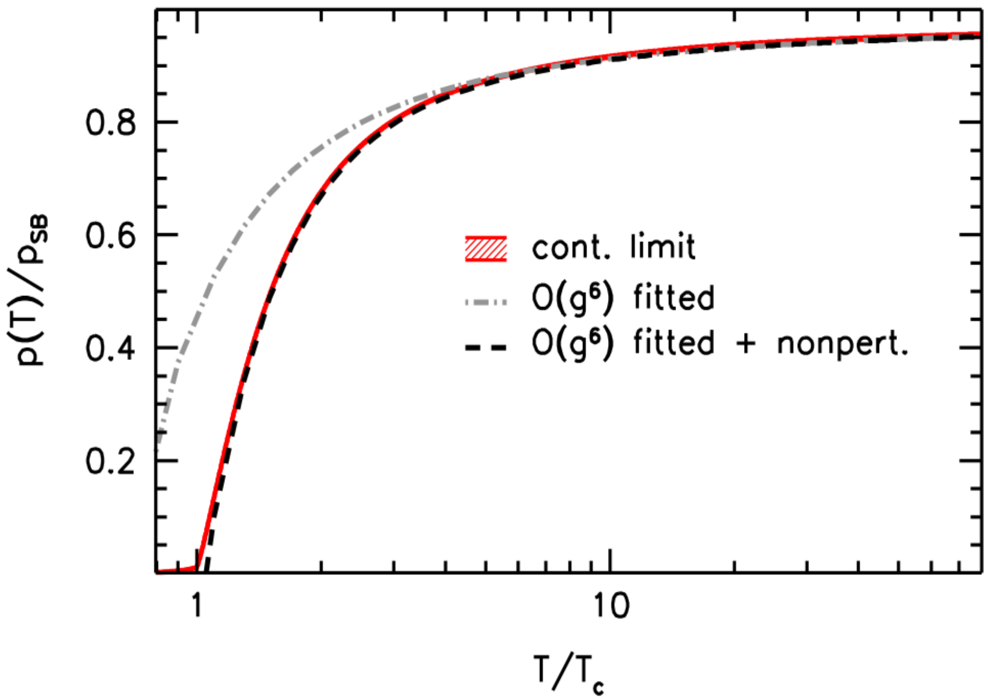

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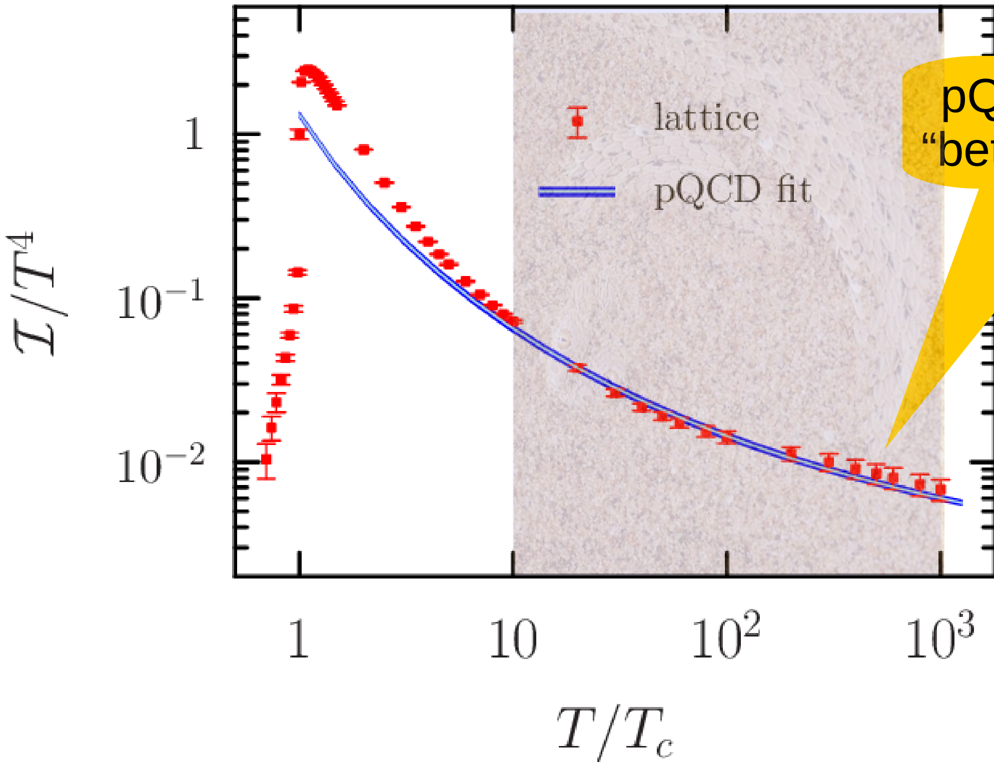

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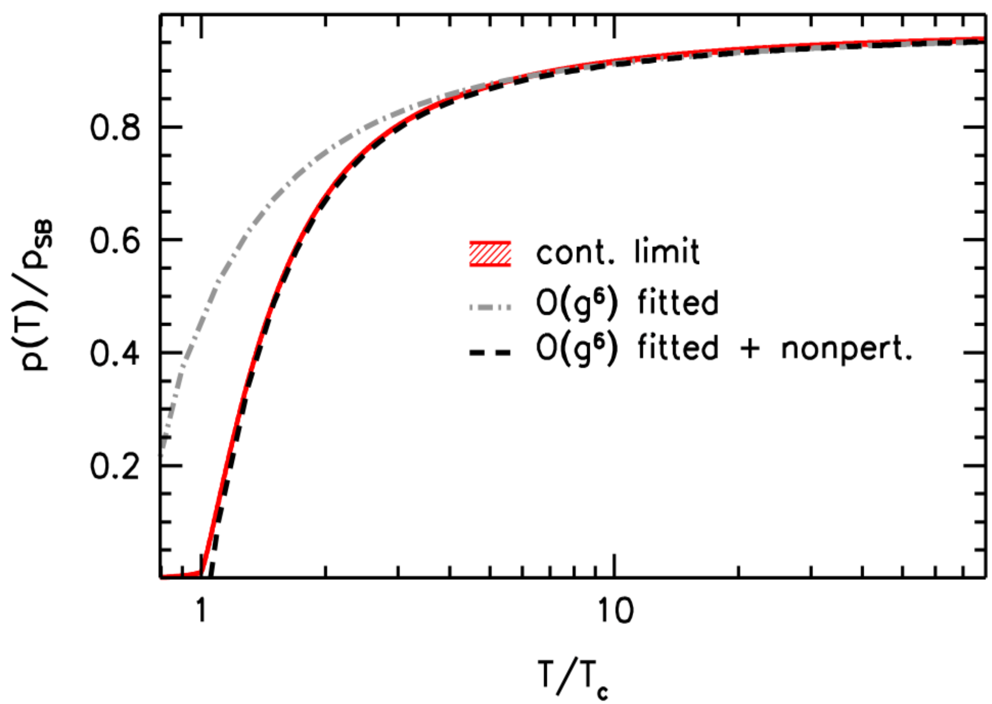

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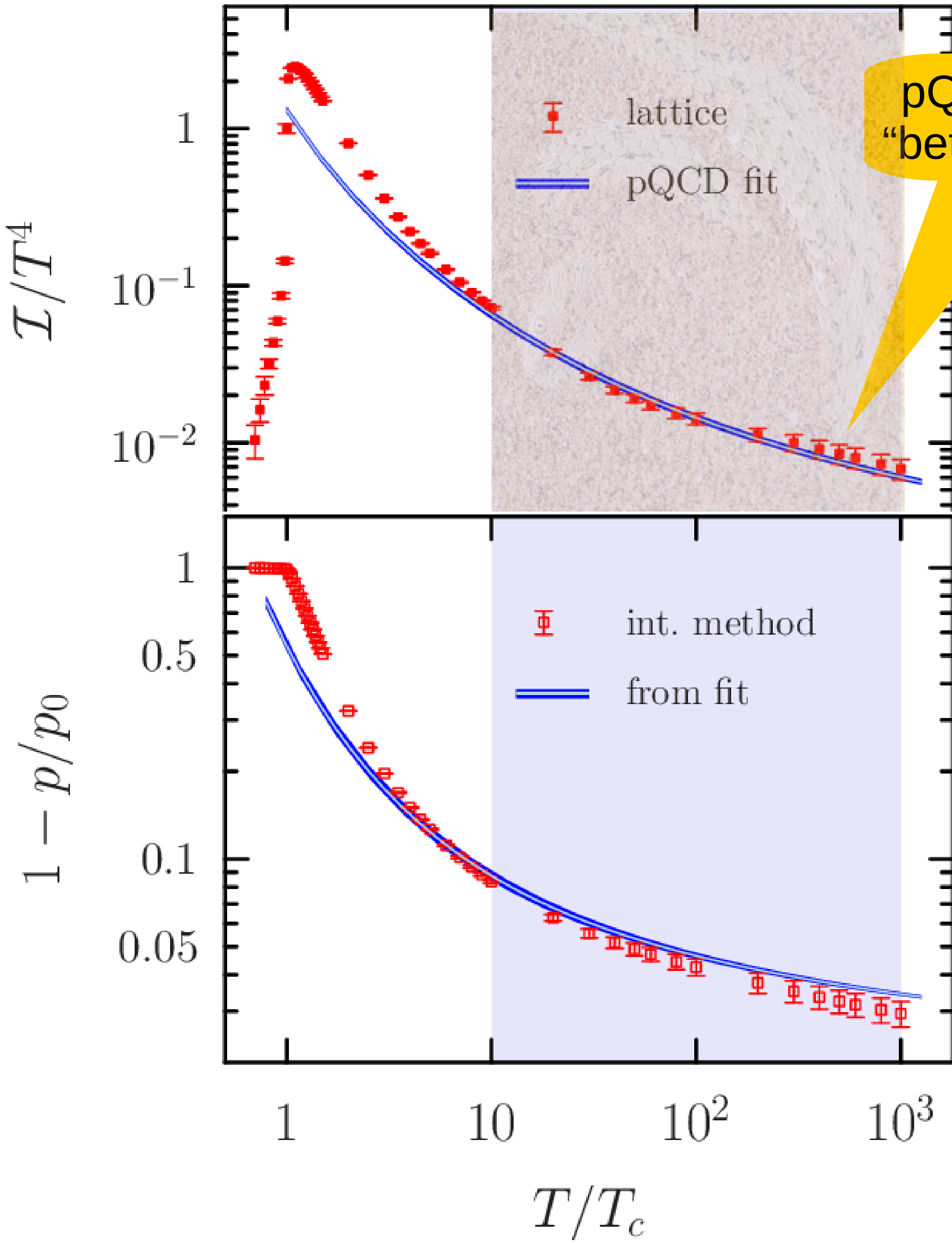

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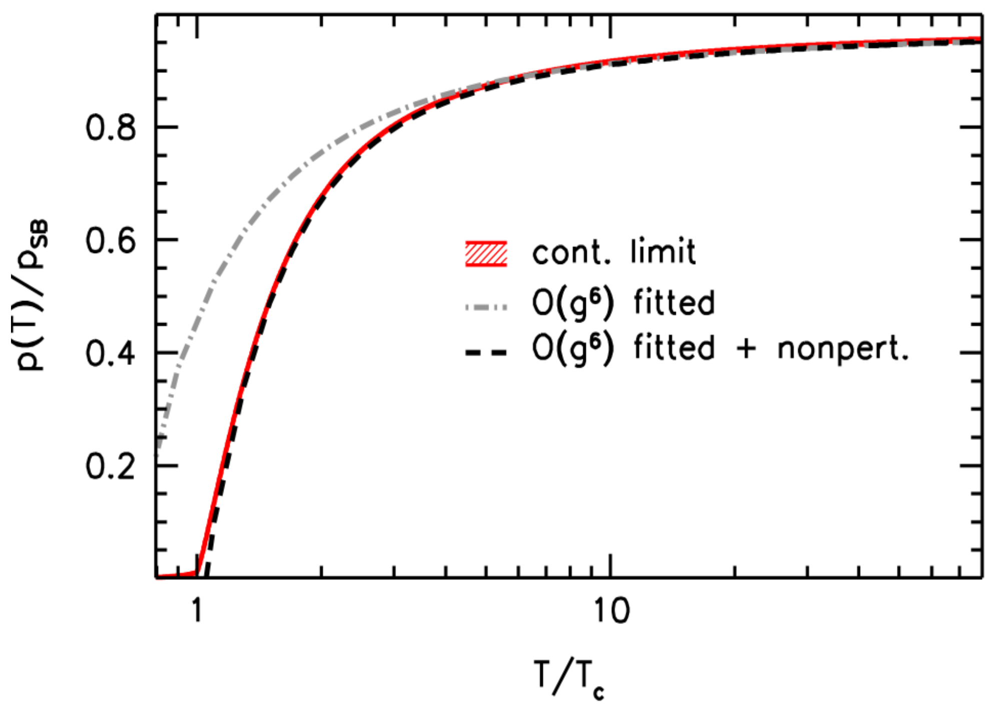

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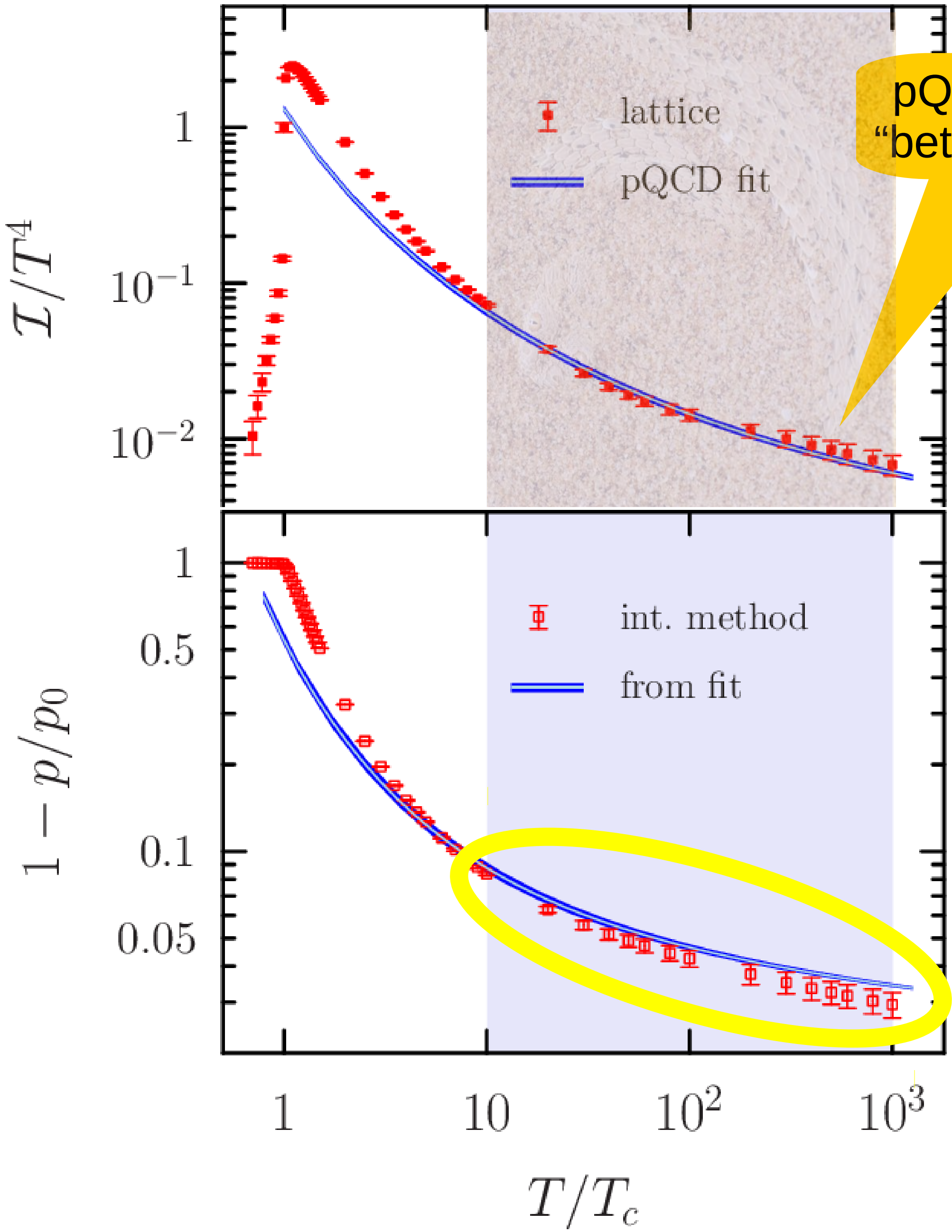

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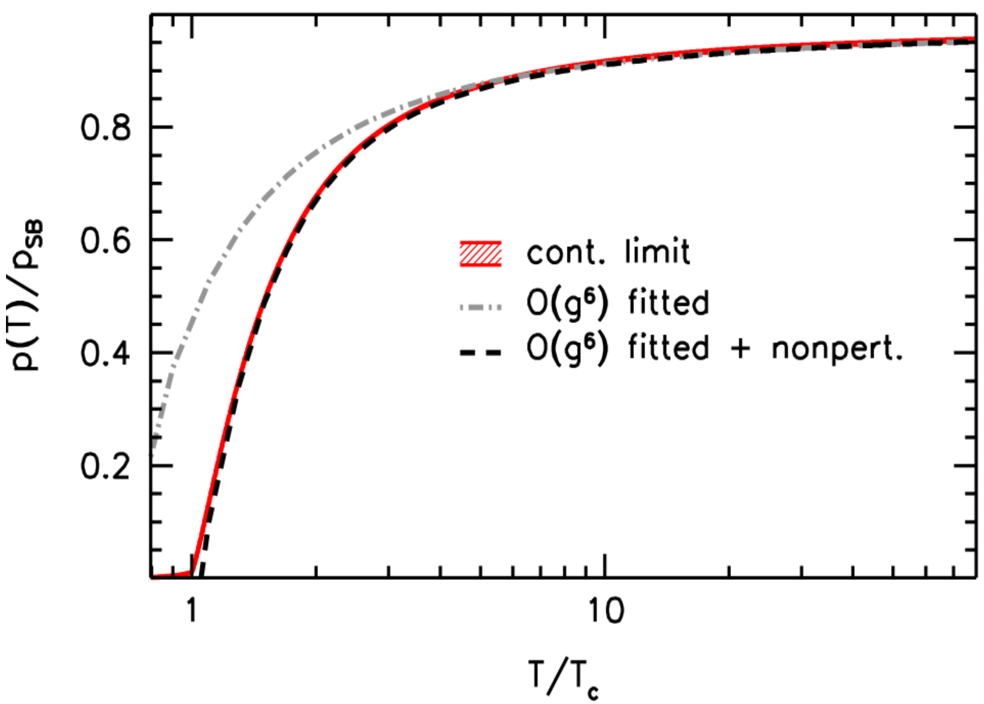
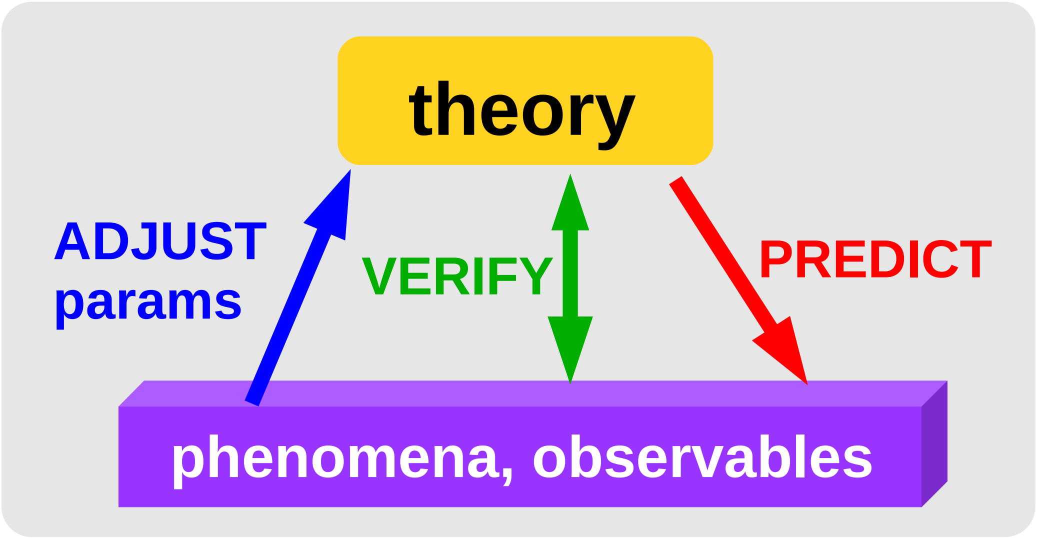


Theory & models



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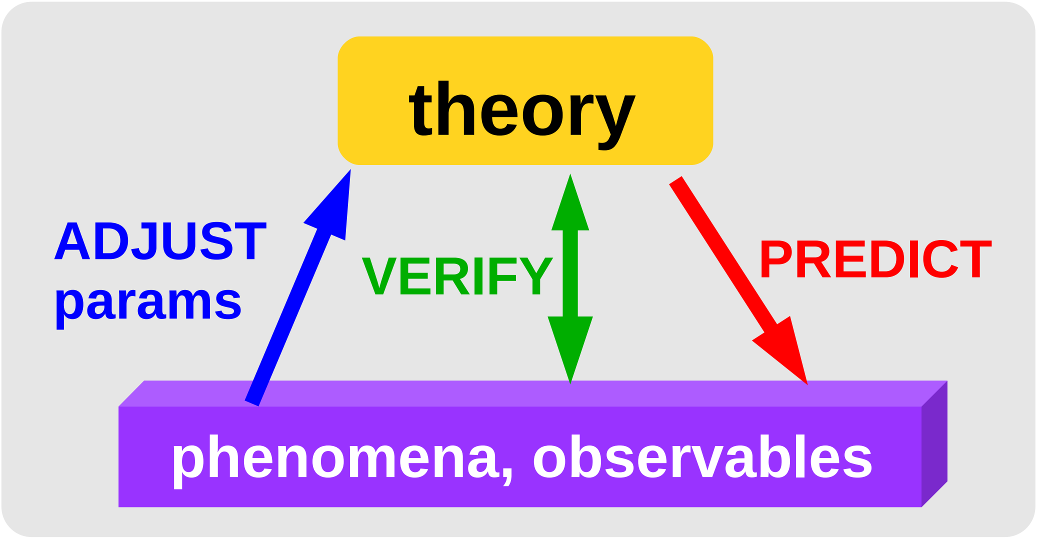

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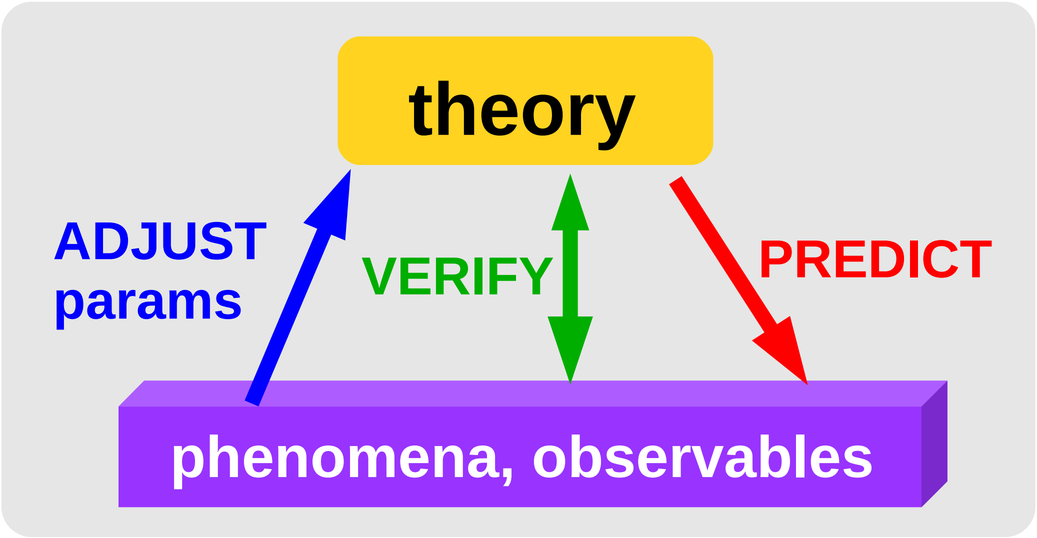

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- in QFT
  - adjusting parameters = **renormalization**

Our approach

- **thermodynamic renormalization:** match perturbative results to lattice data at sufficiently large “renormalization temperature” **to specify model parameter(s)**

  use interaction measure (being the actual lattice “observable”)

$$\mathcal{I}_{\rm model} = T^5 \frac{\partial \left(p_{\rm model}/T^4\right)}{\partial T}$$

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- make **predictions for other observables** in applicability range

($n|l$) models (n_f=0)

pressure (= thermodynamic potential) to order n:

$$p_{(n)} = p_0 \left[1 + \sum_{m=2}^{n} C_m \, \alpha^{m/2} \right] \quad \text{where} \quad p_0 = 8 \times 2 \frac{\pi^2}{90} T^4$$

$$\begin{aligned} C_2 &= -1.2 \\ C_3 &= +5.4 \\ C_4 &= 6.8 \ln \alpha + 16.2 \\ C_5 &= -45.7 \\ C_6 &= -36.6 \ln \alpha + c_6 \qquad (\text{for } \mu = 2\pi T) \end{aligned}$$

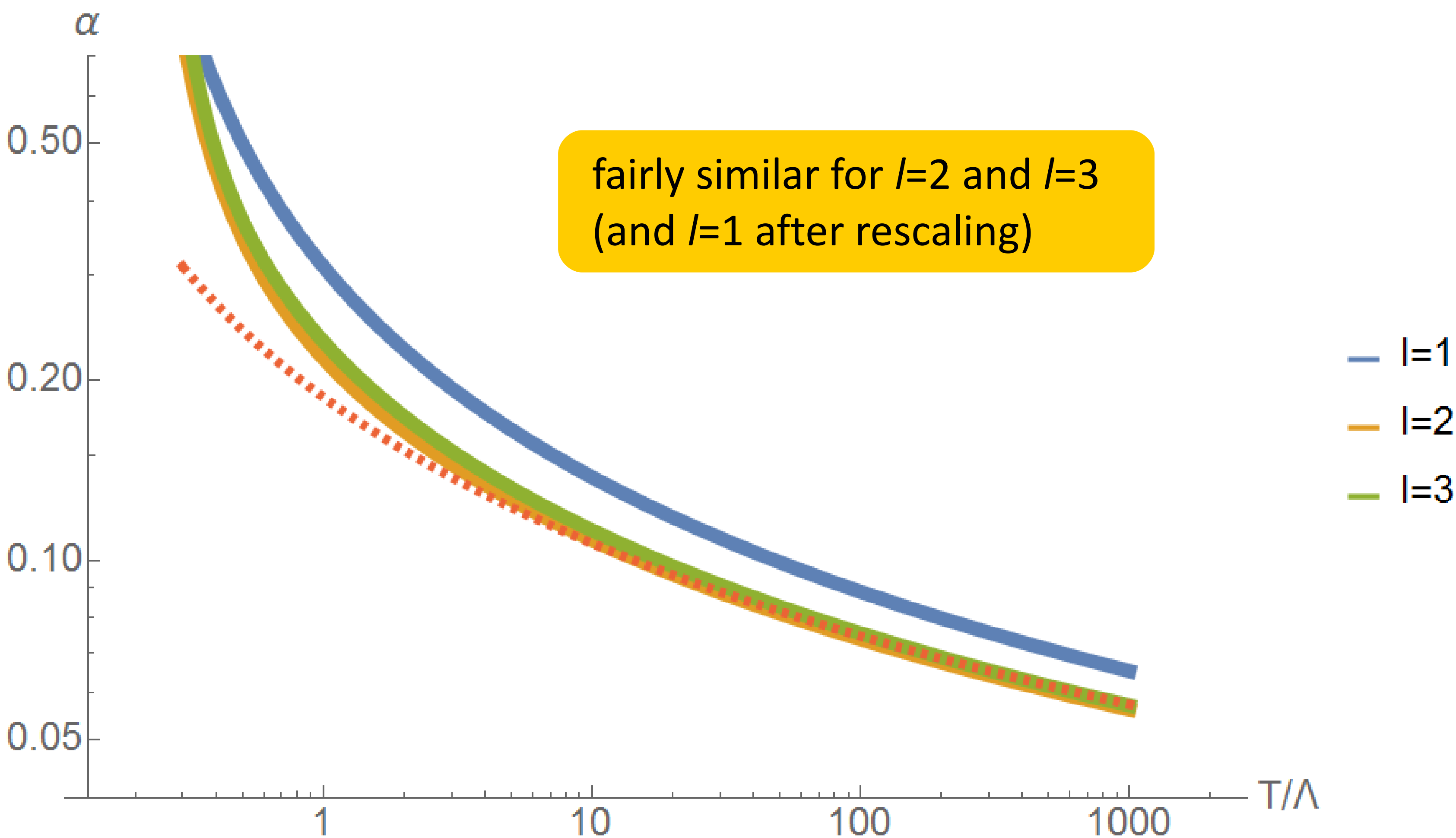
running coupling to order l:

$$\alpha_{(\ell)} = \sum_{k=1}^{\ell} a_k(L) L^{-k}$$

$$a_1 = 1.14, \; a_2 = -0.96 \ln L, \; a_3 = 0.41 + 0.81(\ln L - 1) \ln L$$

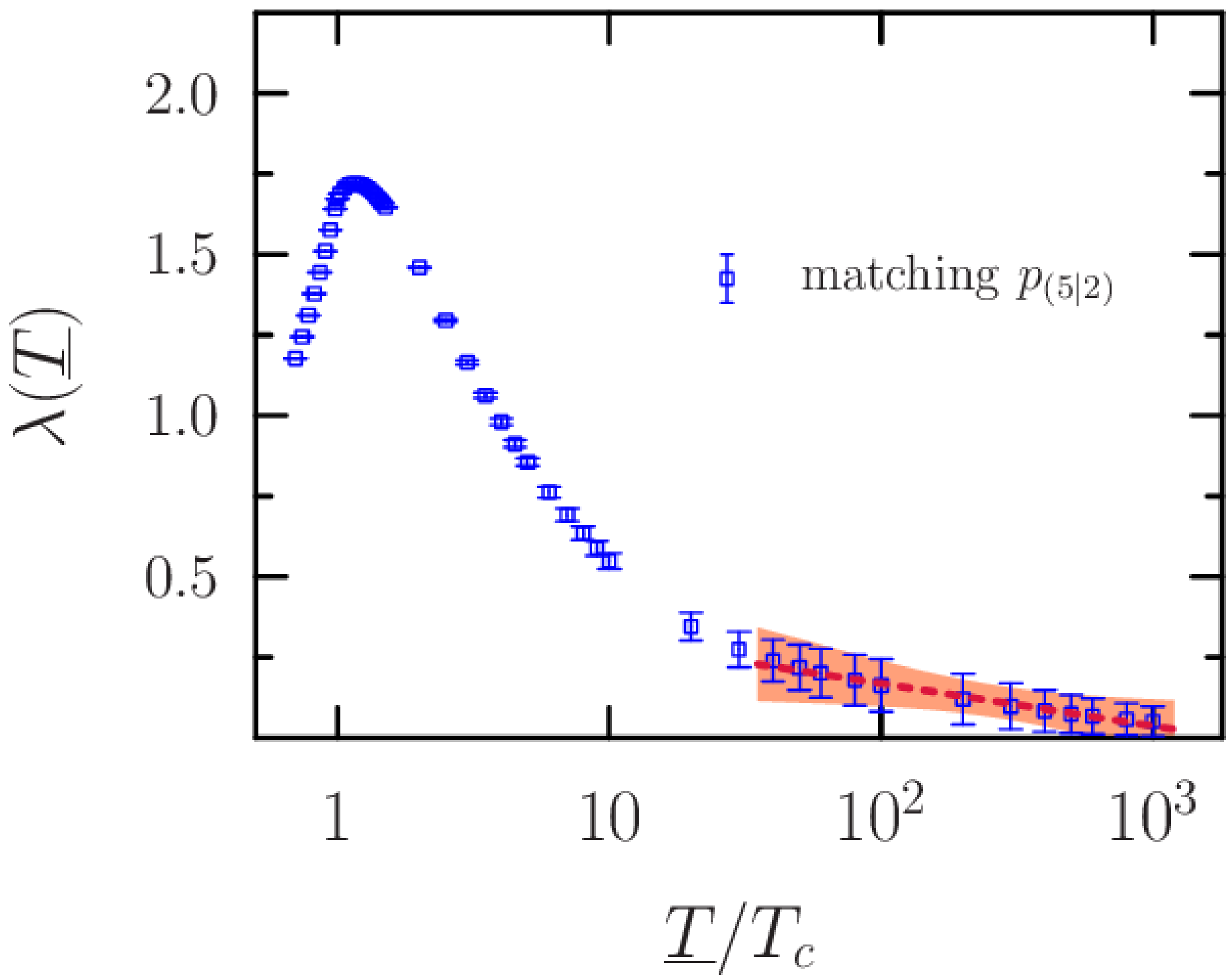
$$L(T) = \ln \left(\frac{2\pi}{\lambda} \frac{T}{T_c} \right)^2 \quad \text{where} \quad \lambda = \Lambda / T_c$$

Running coupling



fairly similar for *l*=2 and *l*=3
(and *l*=1 after rescaling)

(5|2)-model … how **NOT** to: *p*-scheme



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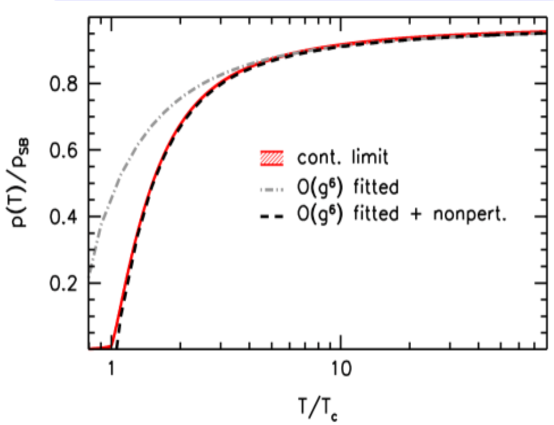

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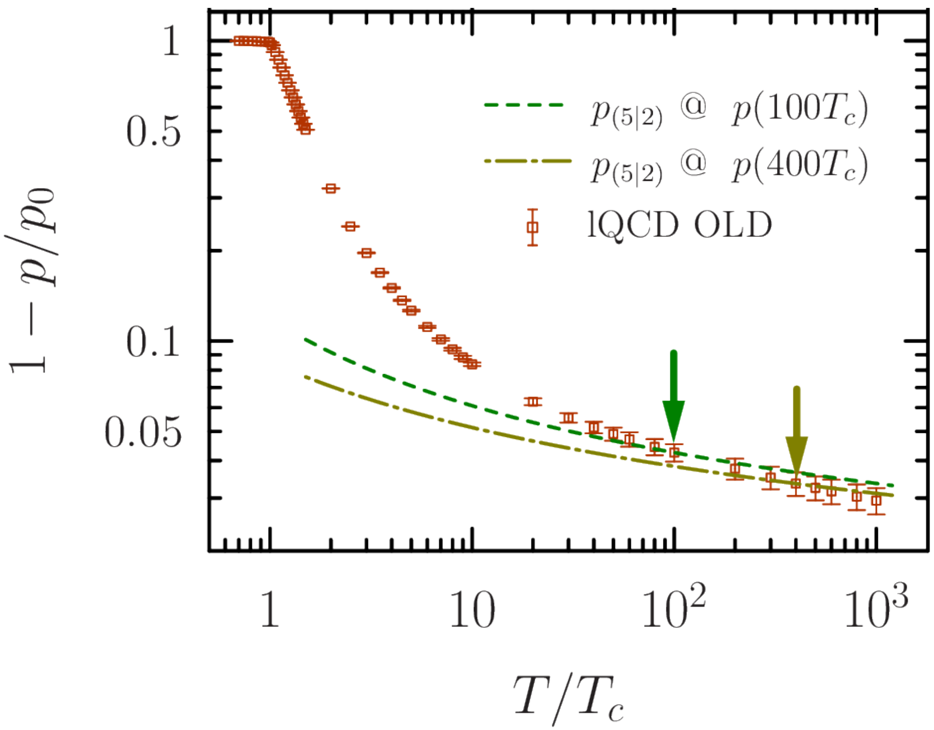

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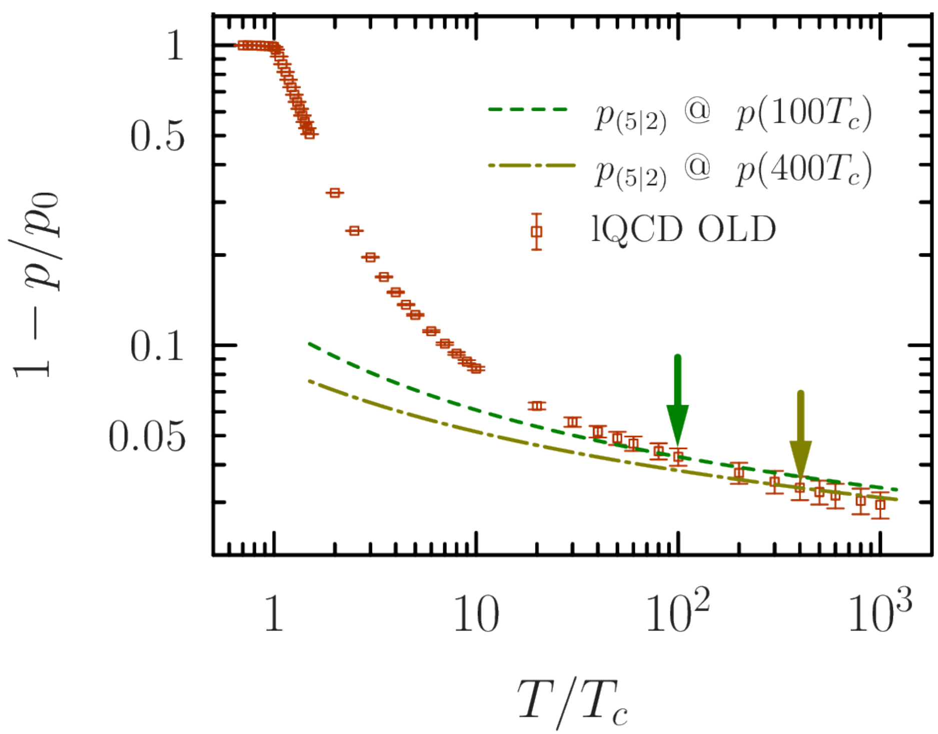

small applicability range,
not to $T \longrightarrow \infty$

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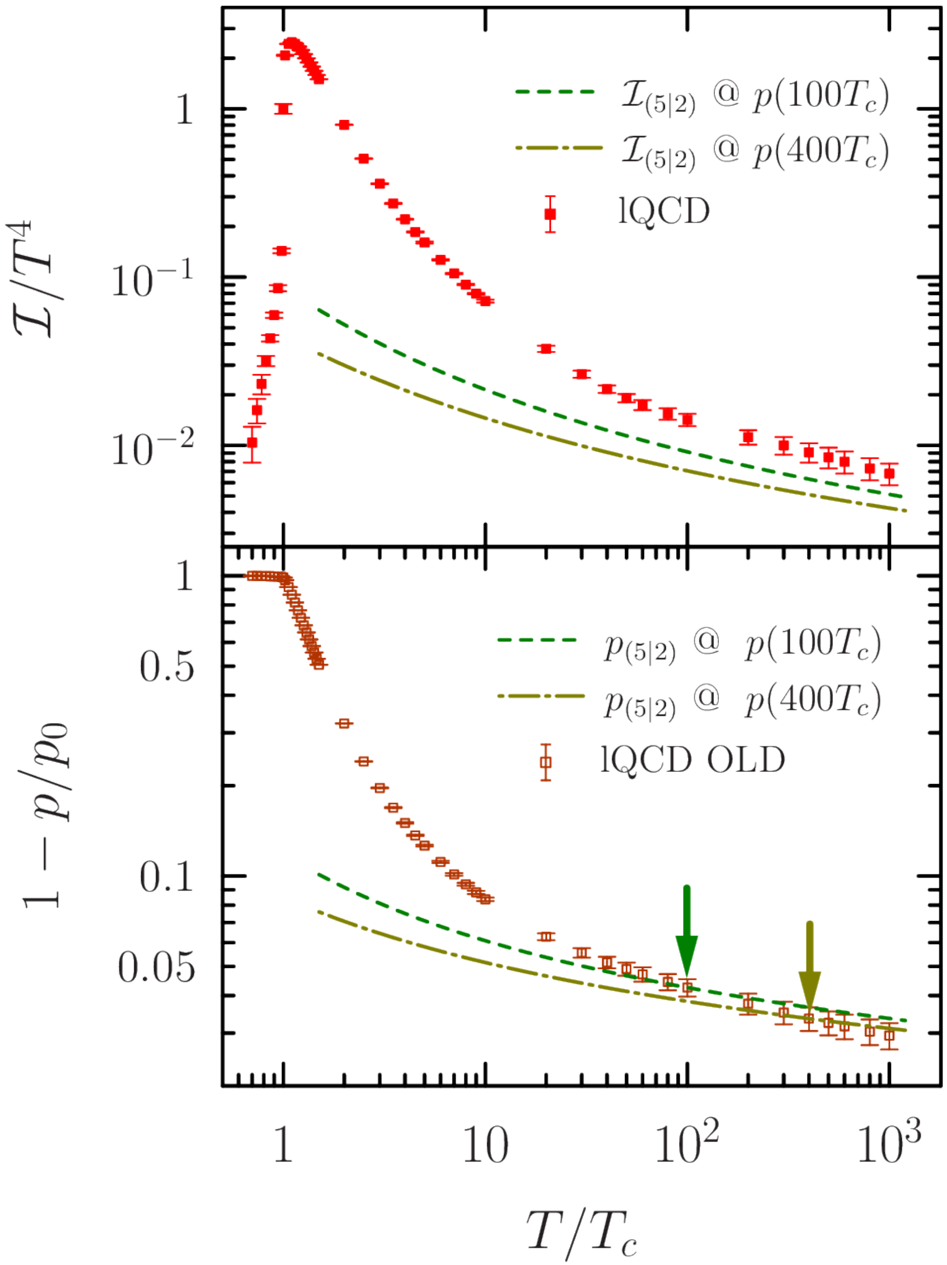
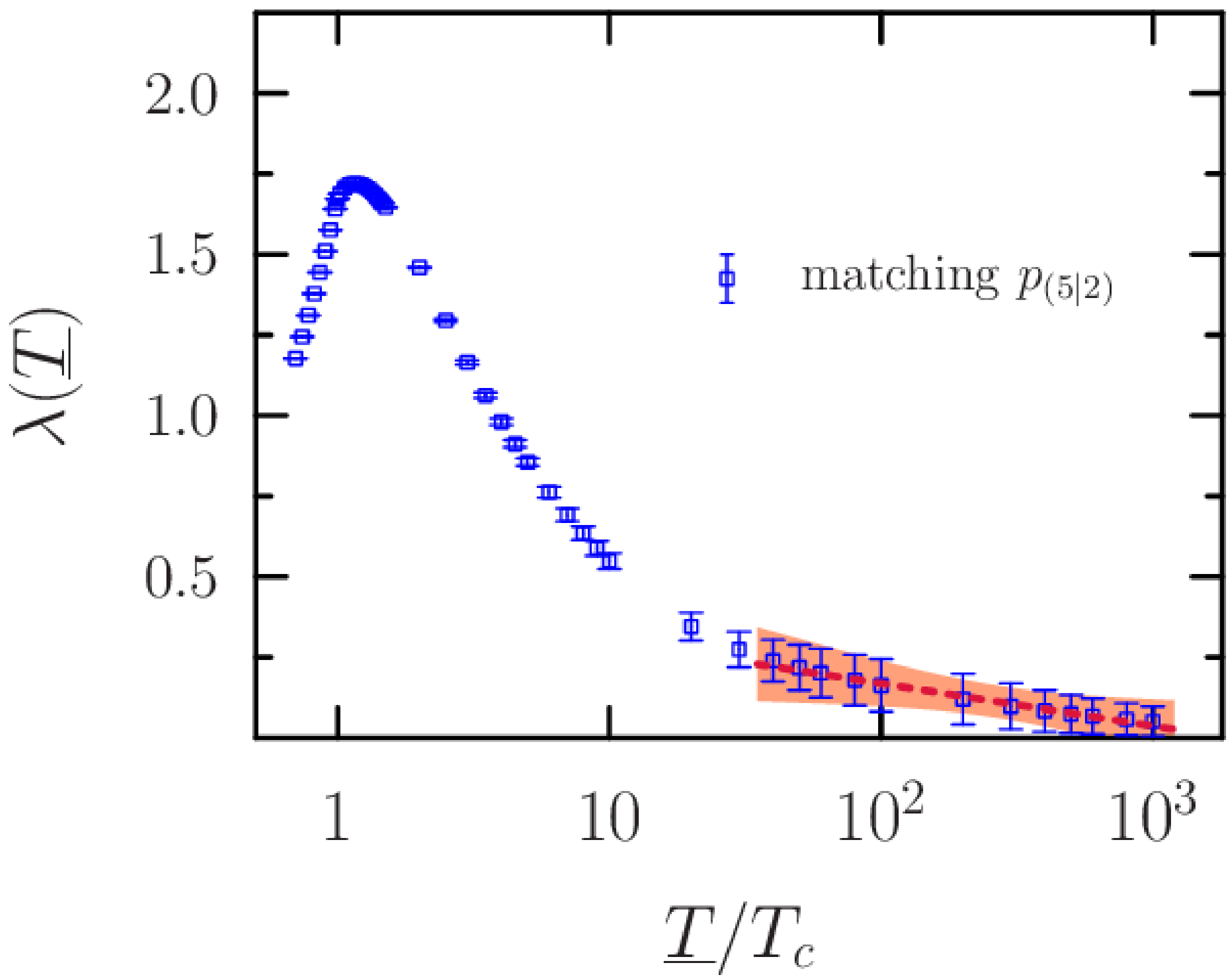


systematic discrepancy for interaction measure as actual lattice "observable"

small applicability range, not to $T \to \infty$

(5|2)-model



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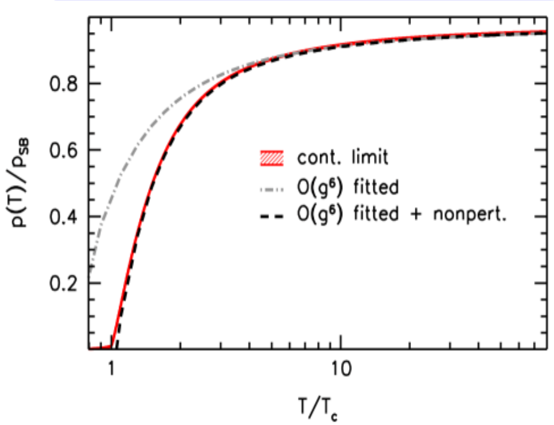

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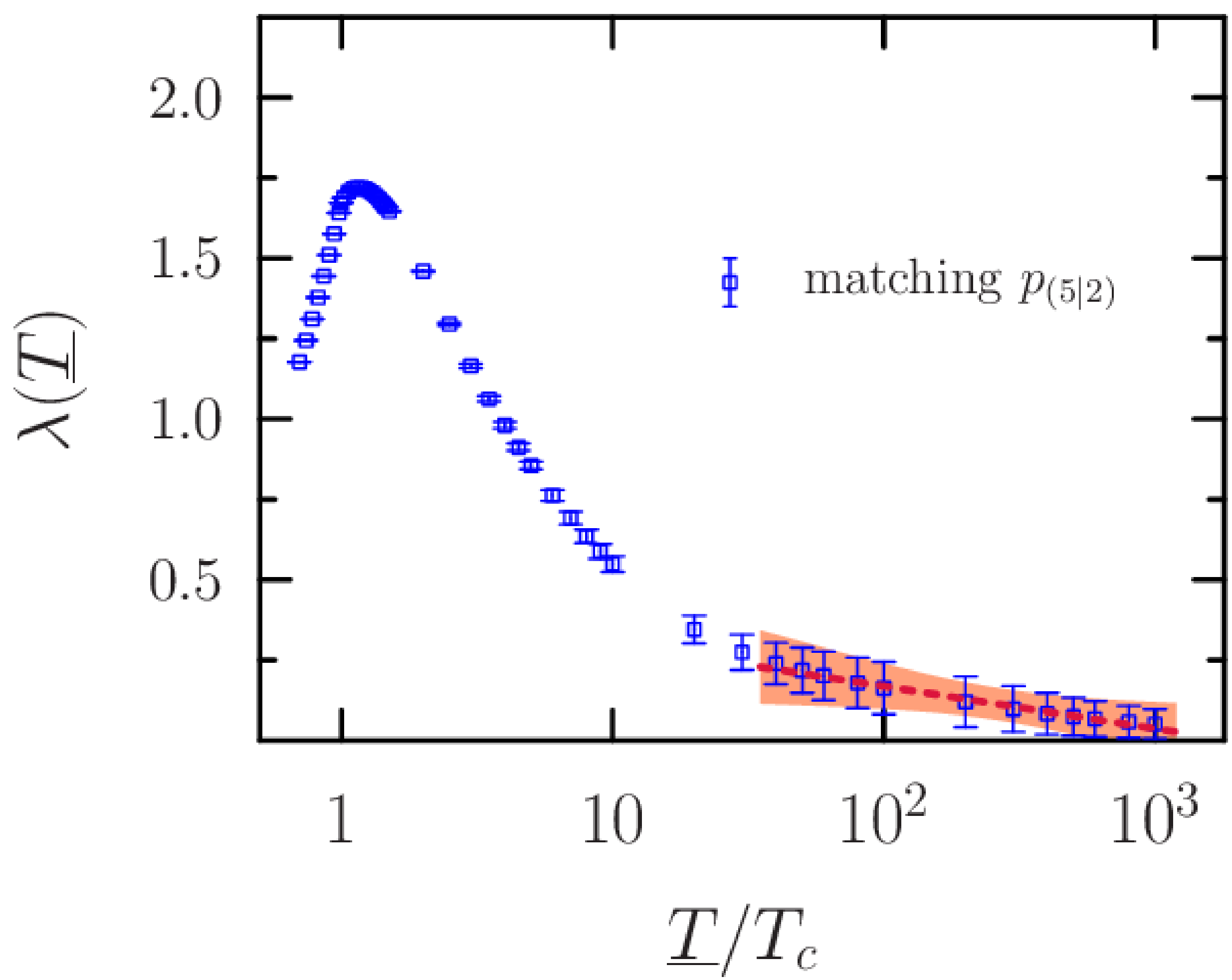

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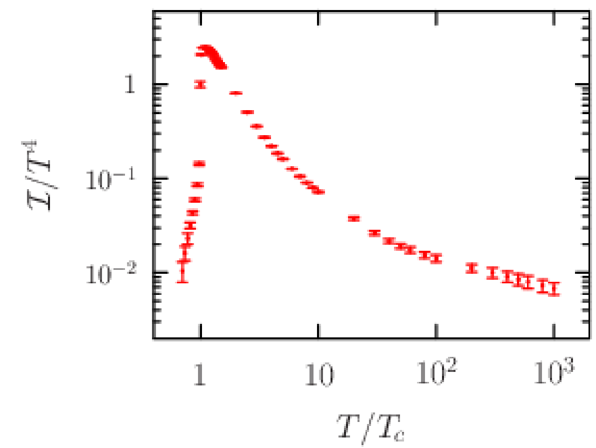

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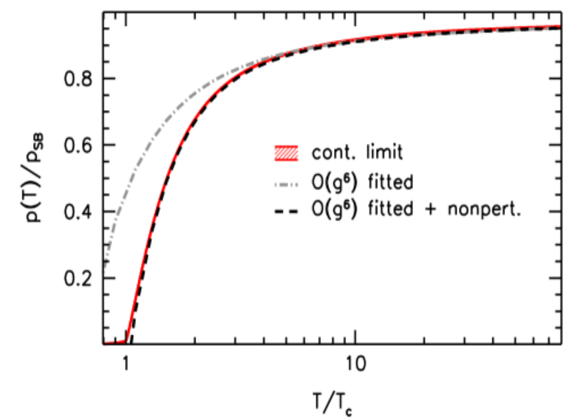

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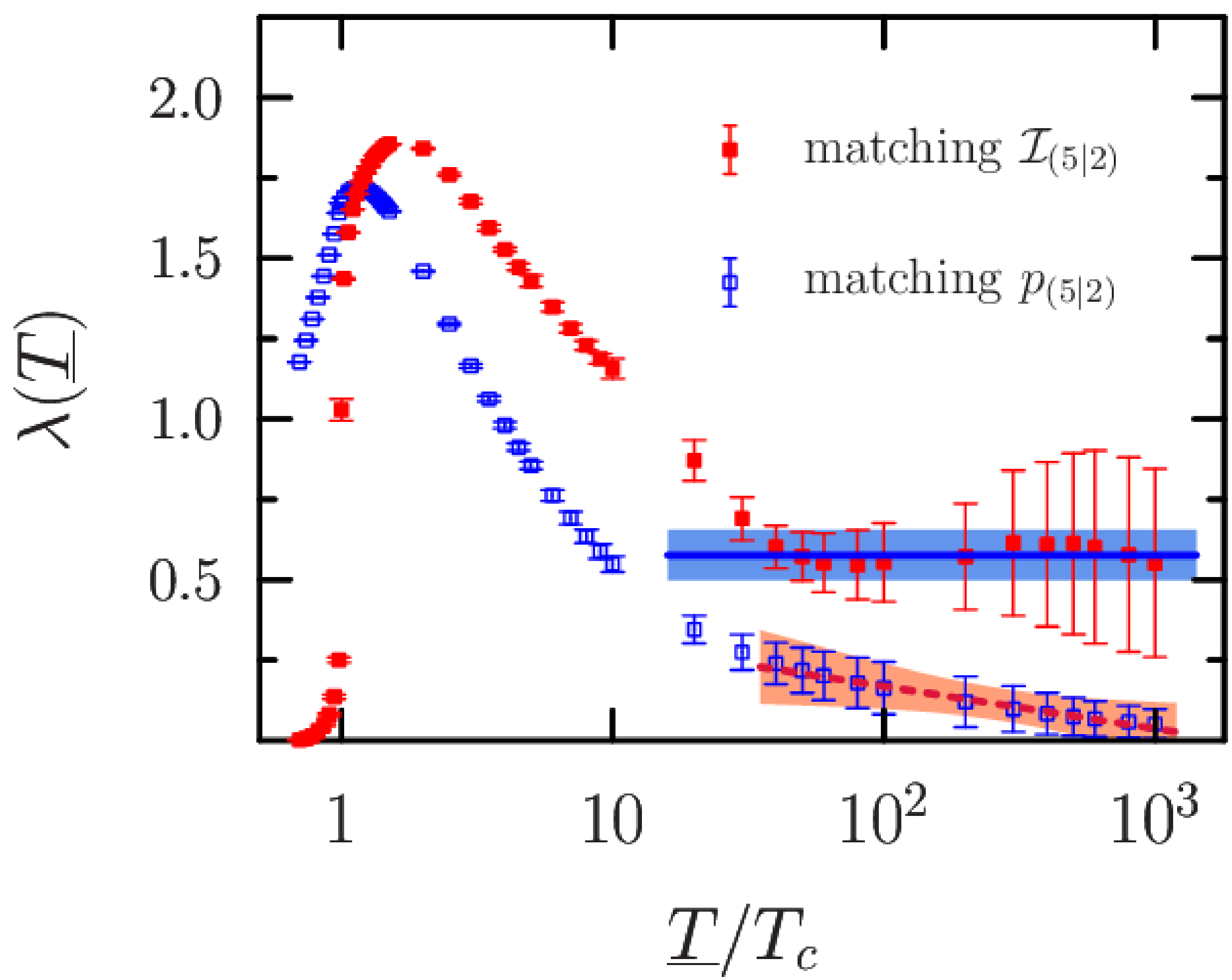

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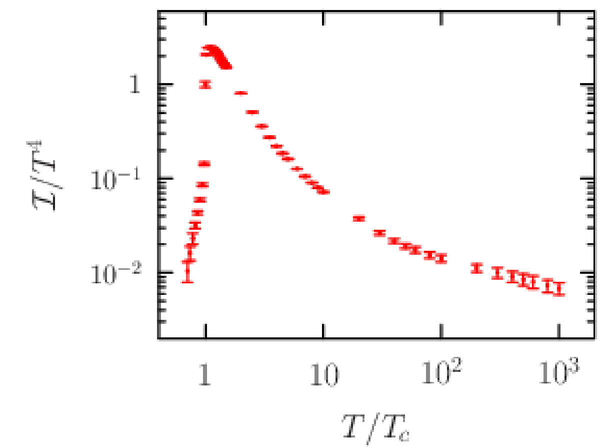

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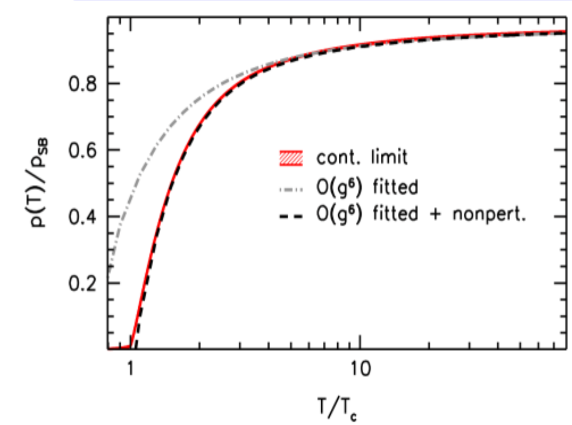
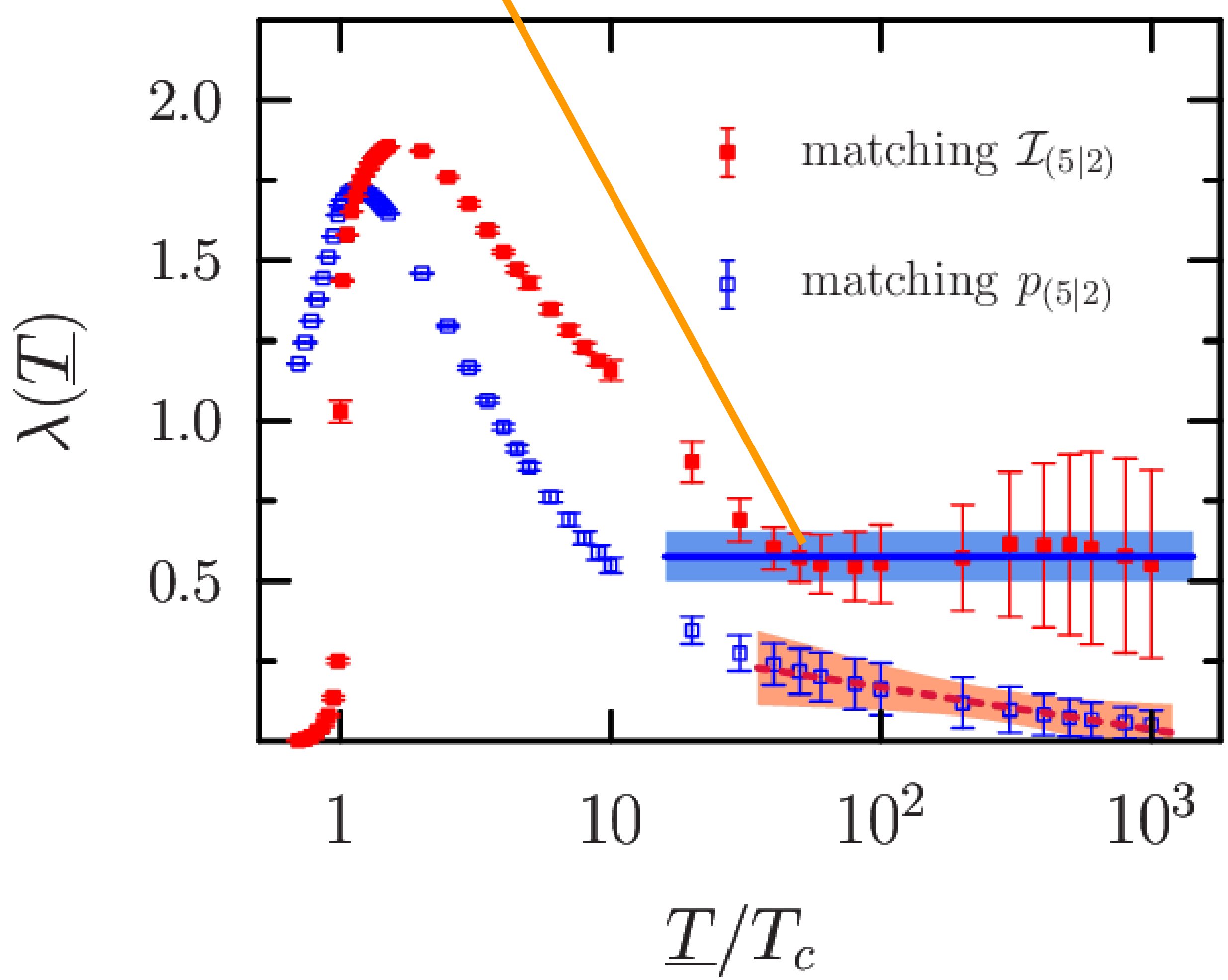


(5|2)-model

$\lambda^{\star}_{(5|2)} = 0.58 \pm 0.11$



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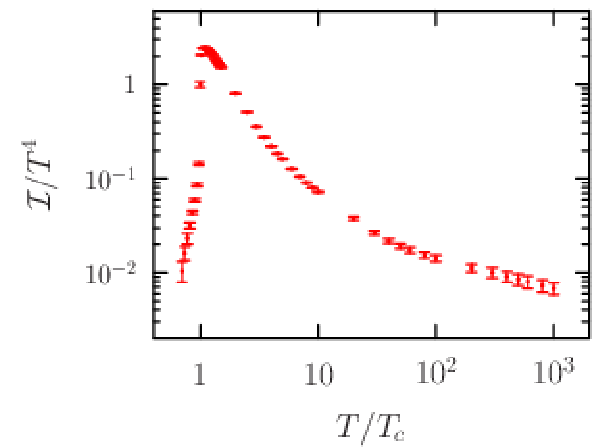

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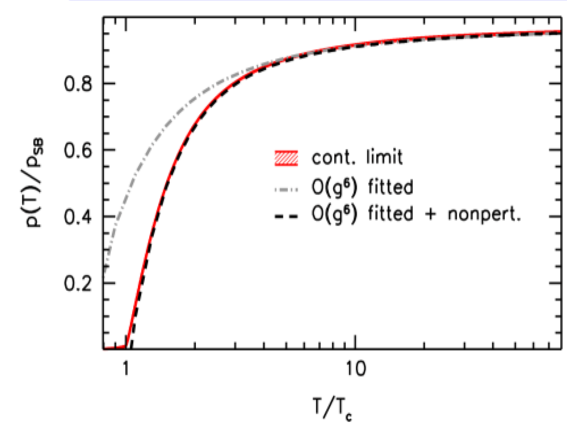

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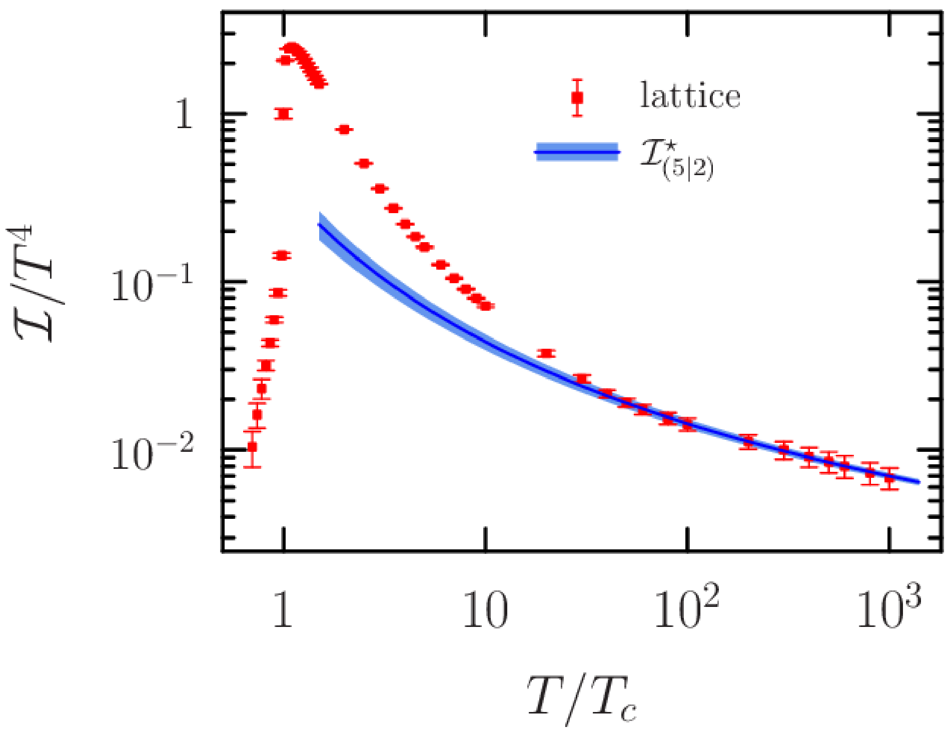

applicability range: $T > 40T_c$

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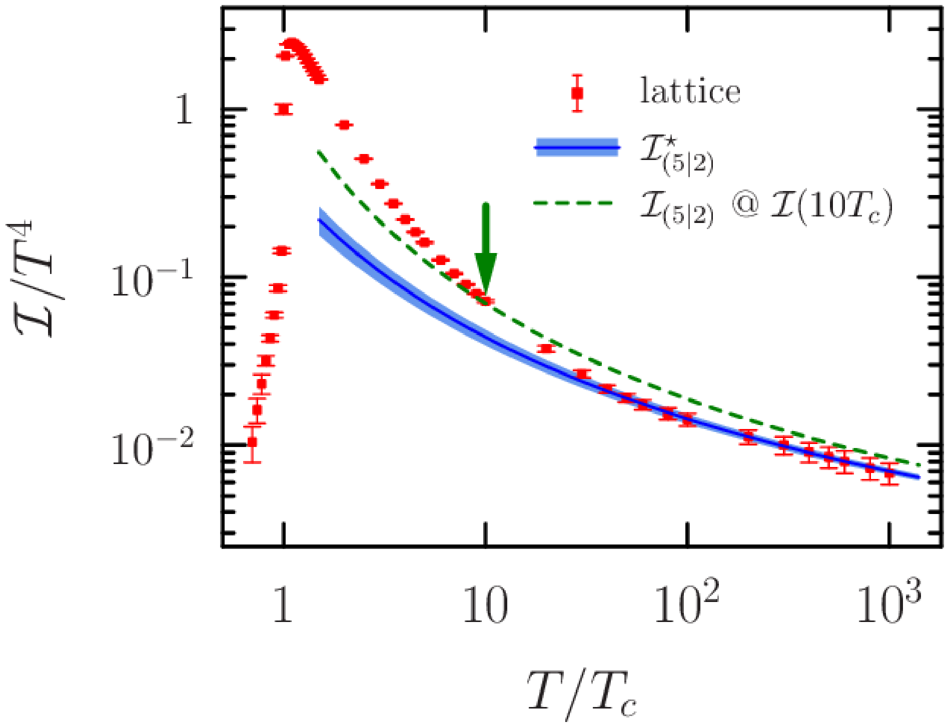

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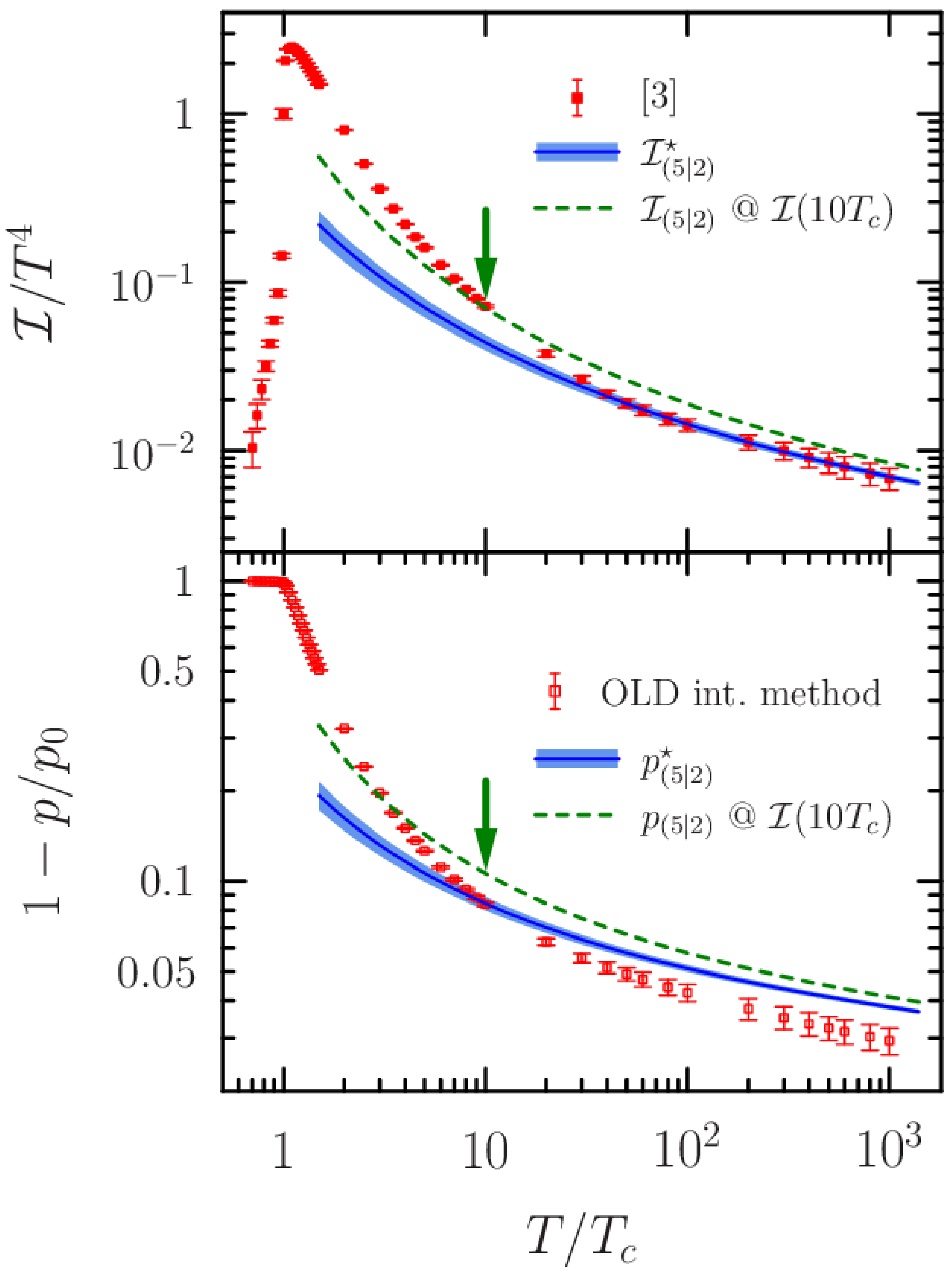

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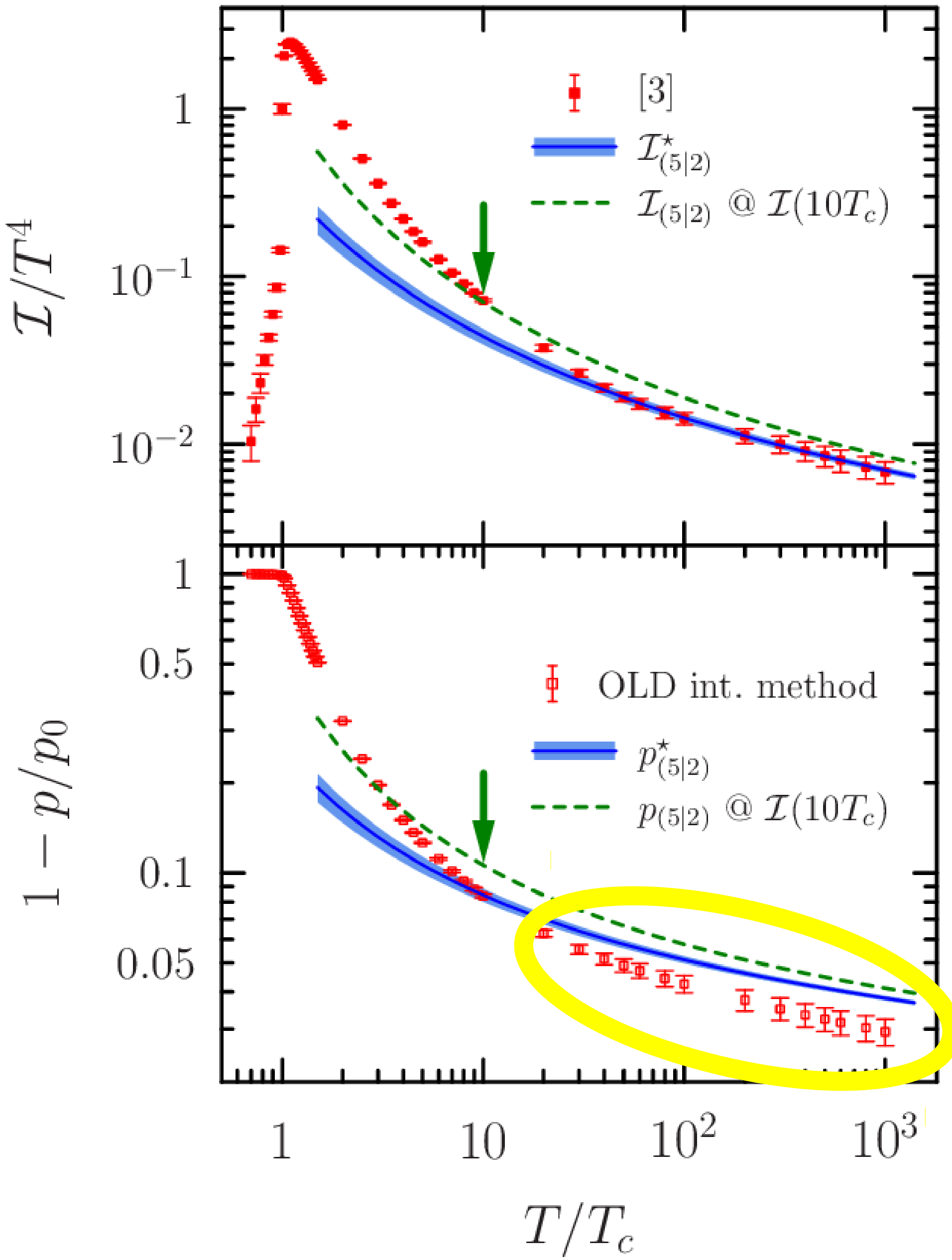

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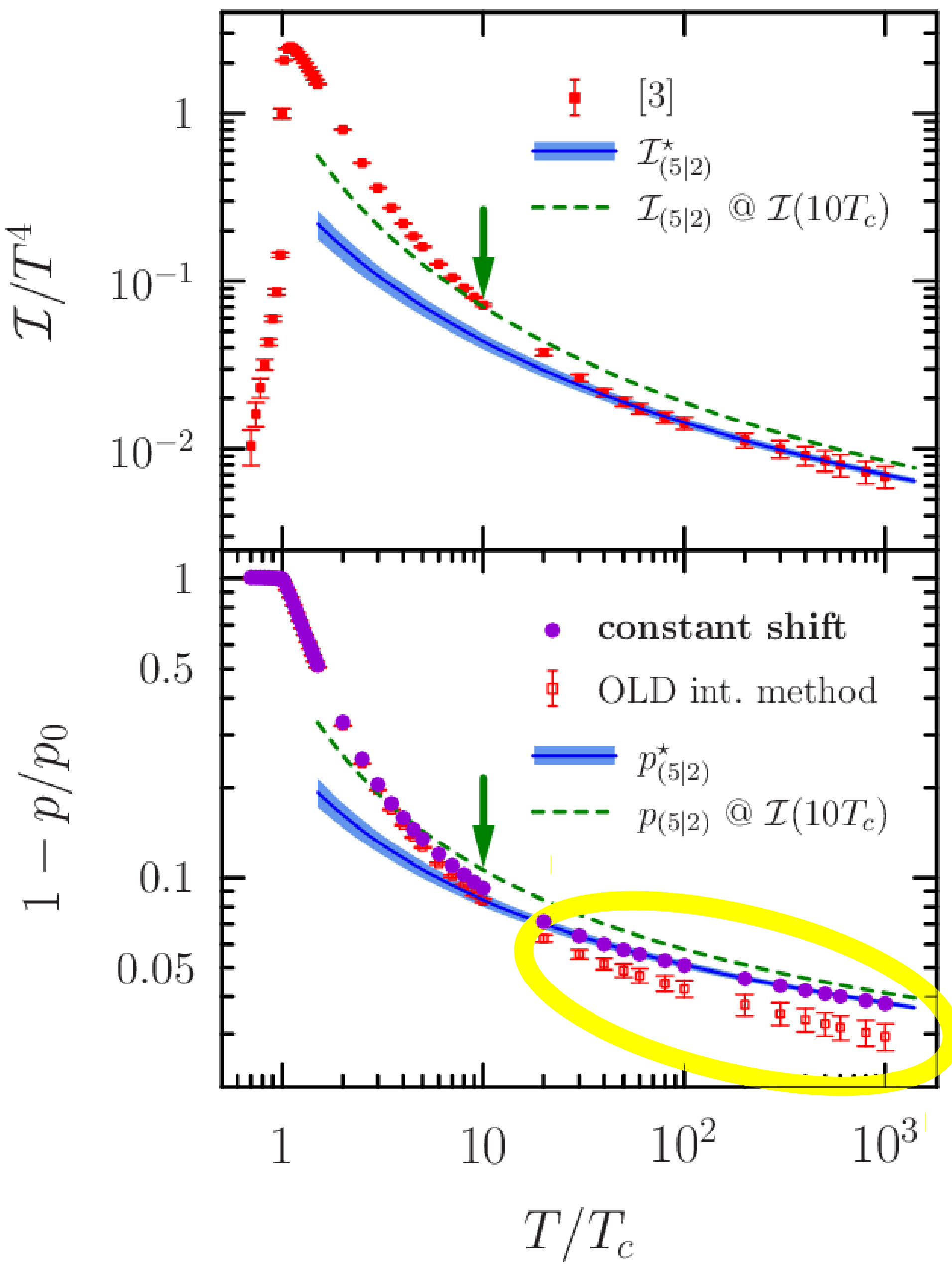

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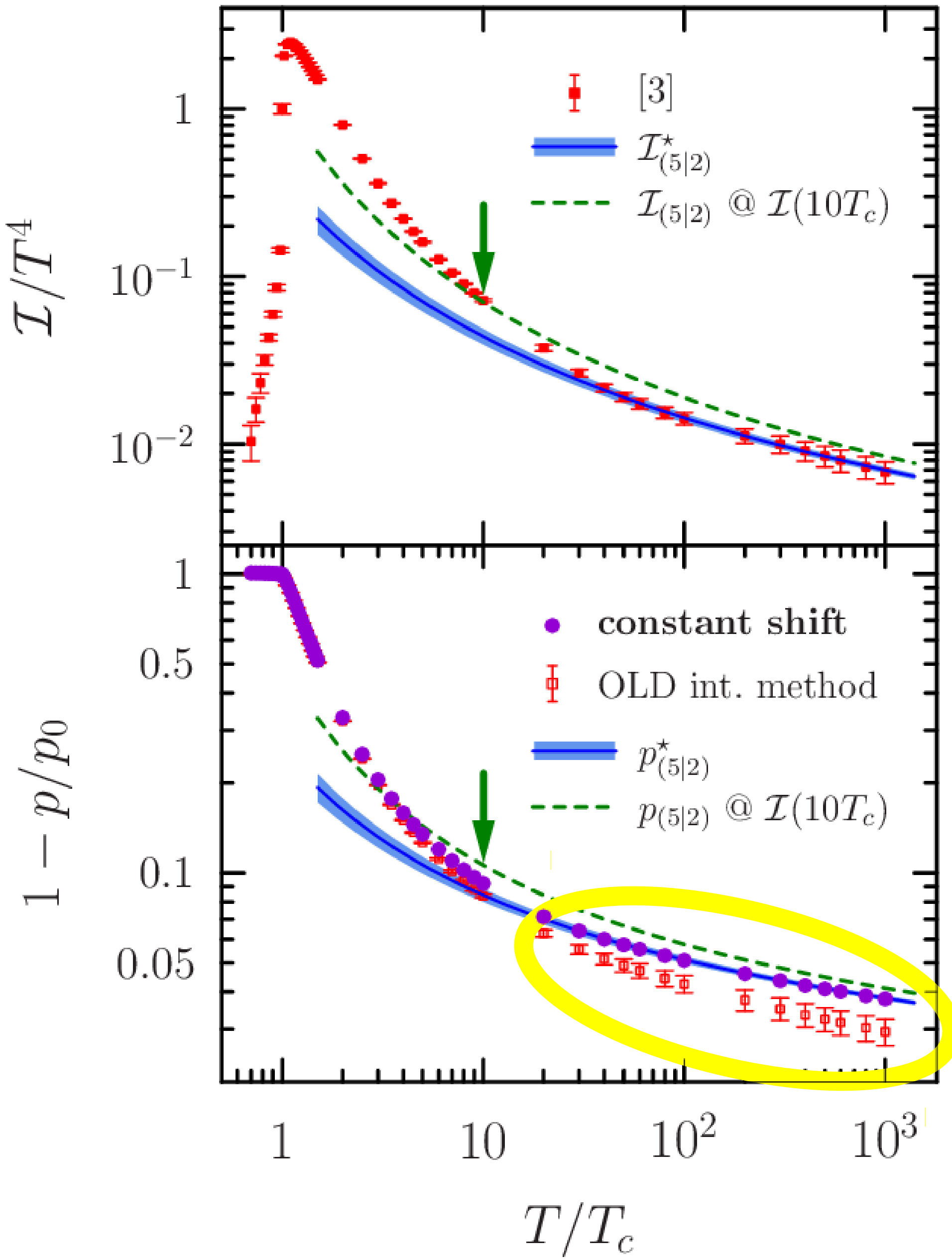

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$$\frac{p(T)}{T^4} = \sigma + \int_{T_0}^{T} \frac{dT'}{T'} \frac{\mathcal{I}(T')}{T'^4}$$

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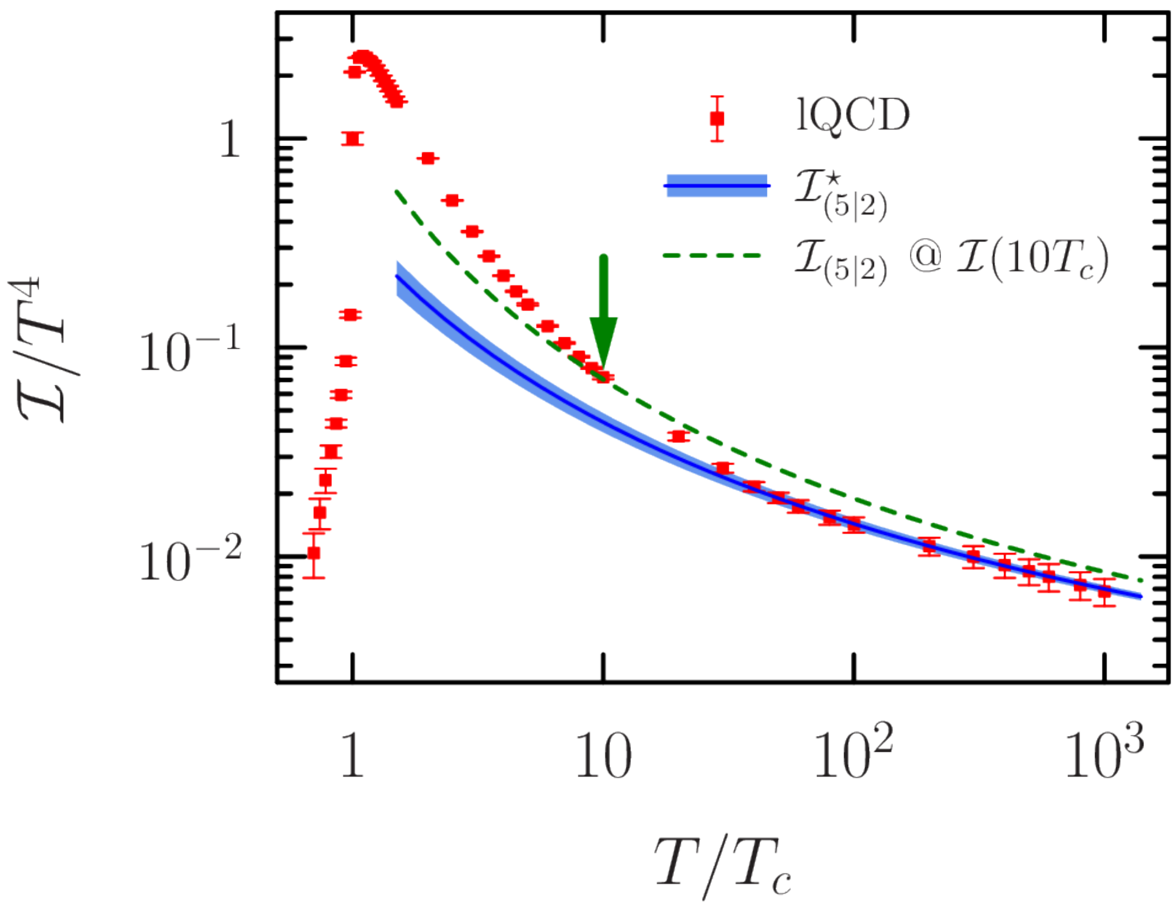
breakdown at ***T* ~ 40T_c*** because “coupling too large” …?



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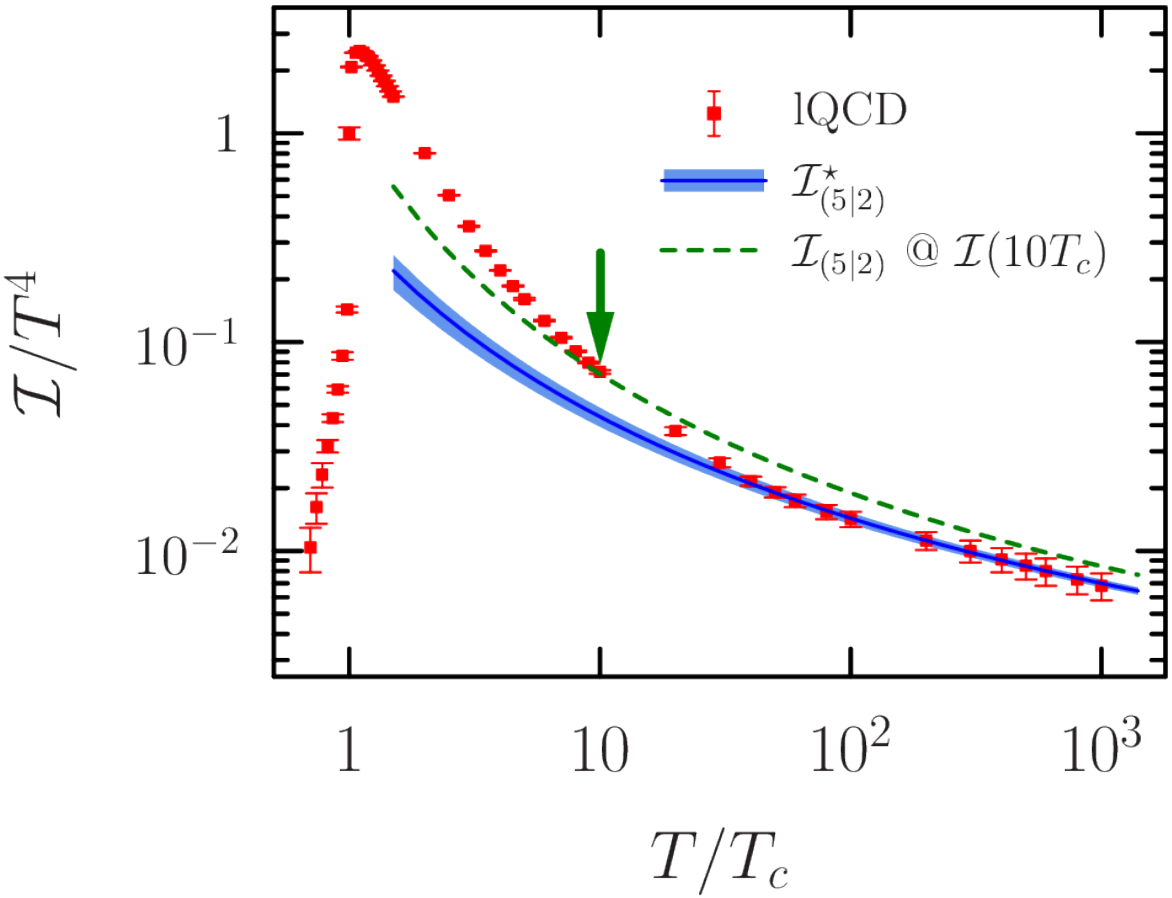

$$\begin{aligned} \alpha(10T_c) &= 0.10 \\ \alpha(40T_c) &= 0.08 \\ \alpha(400T_c) &= 0.06 \end{aligned}$$

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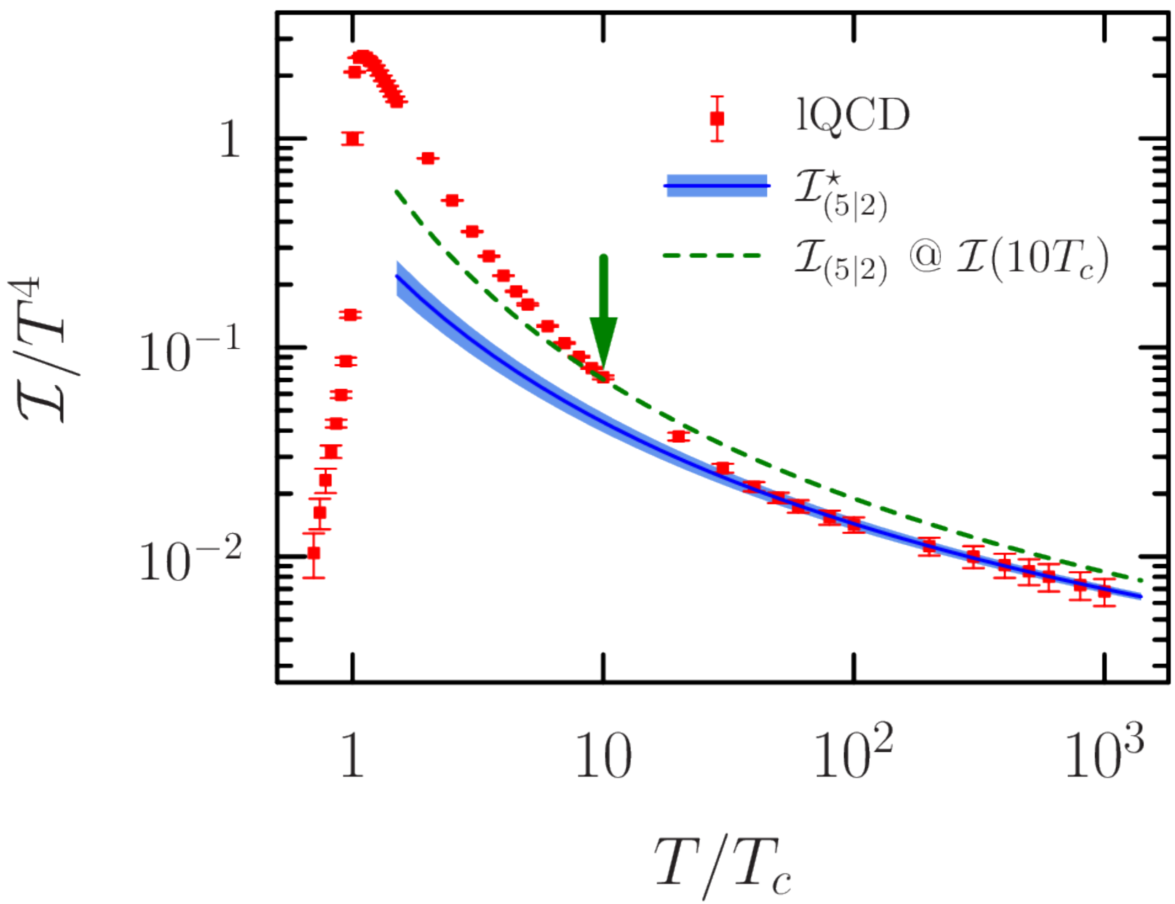

$$\begin{aligned}\alpha(10T_c) &= 0.10\\ \alpha(40T_c) &= 0.08\\ \alpha(400T_c) &= 0.06\end{aligned}$$

similar properties as **asymptotic series**

$$p^\star_{(5|2)}(40T_c) = p_0[1 - 0.09 + 0.12 - 0.01 - 0.08] \approx p_0[1 - \tfrac{1}{2}0.09]$$

(6|3) model

more difficult

- 2 parameters

  $\lambda_{(6|3)}$, c_6

- expect smaller applicability range

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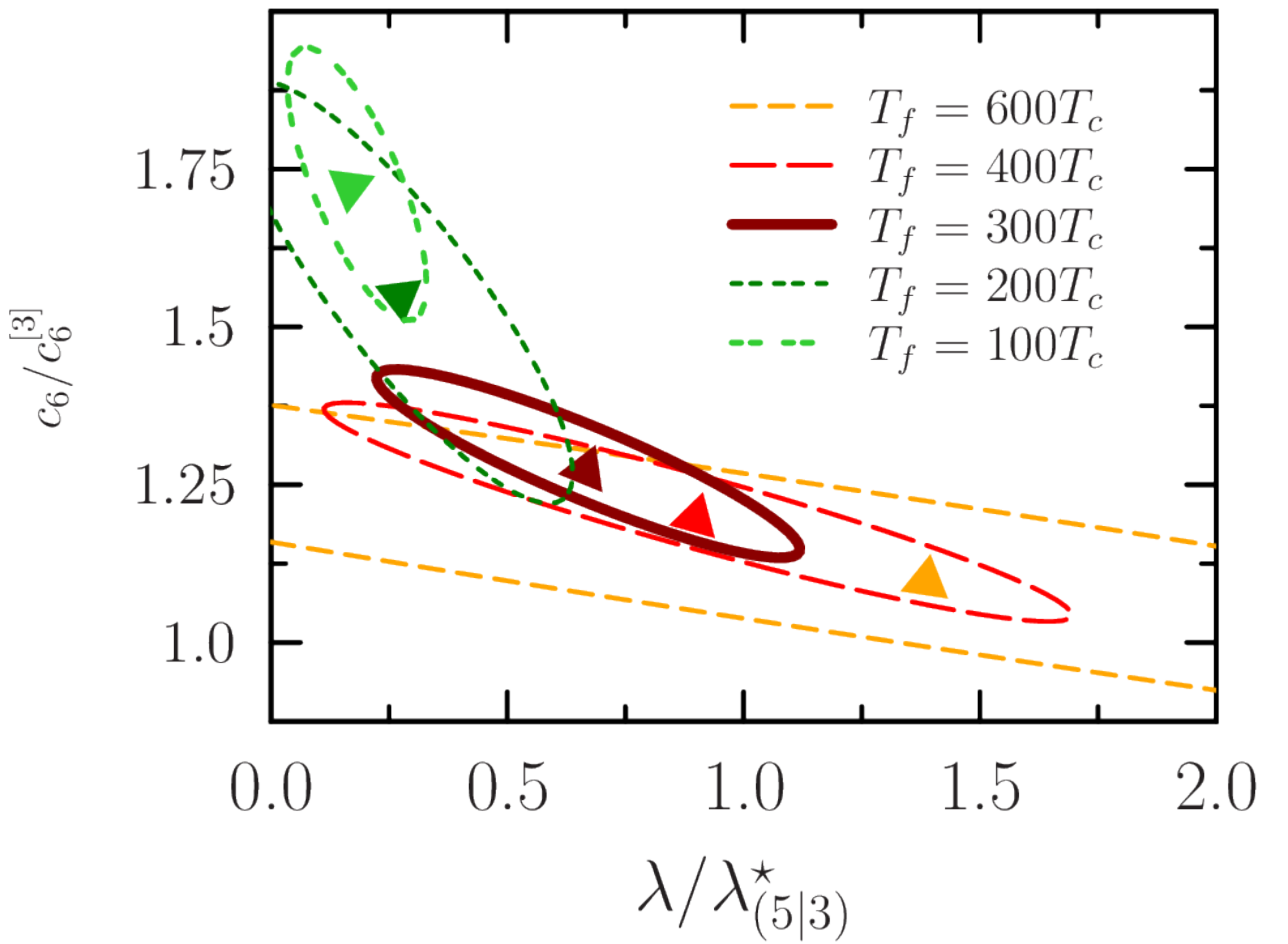
fit over interval [T_f, T_{max}]



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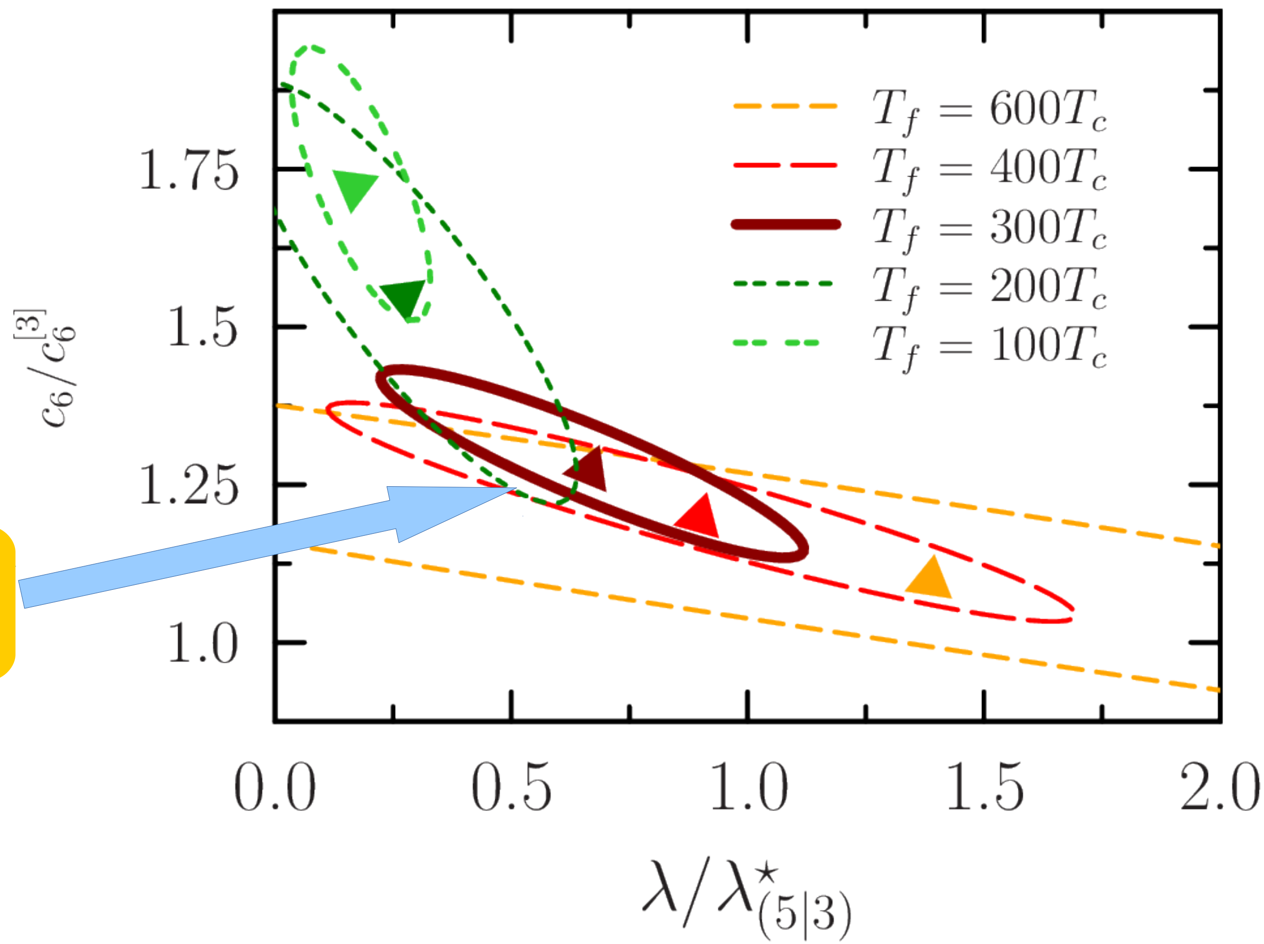
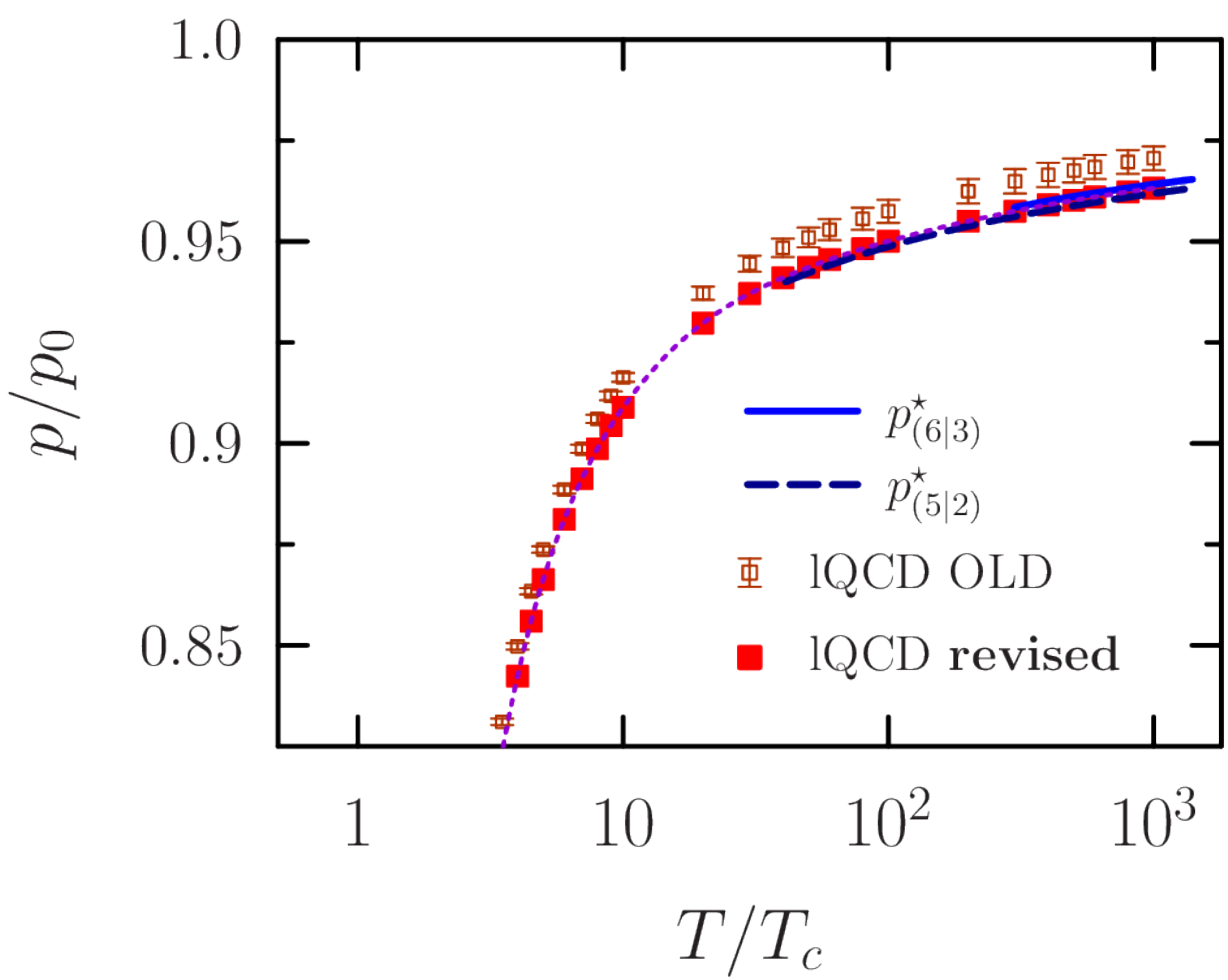


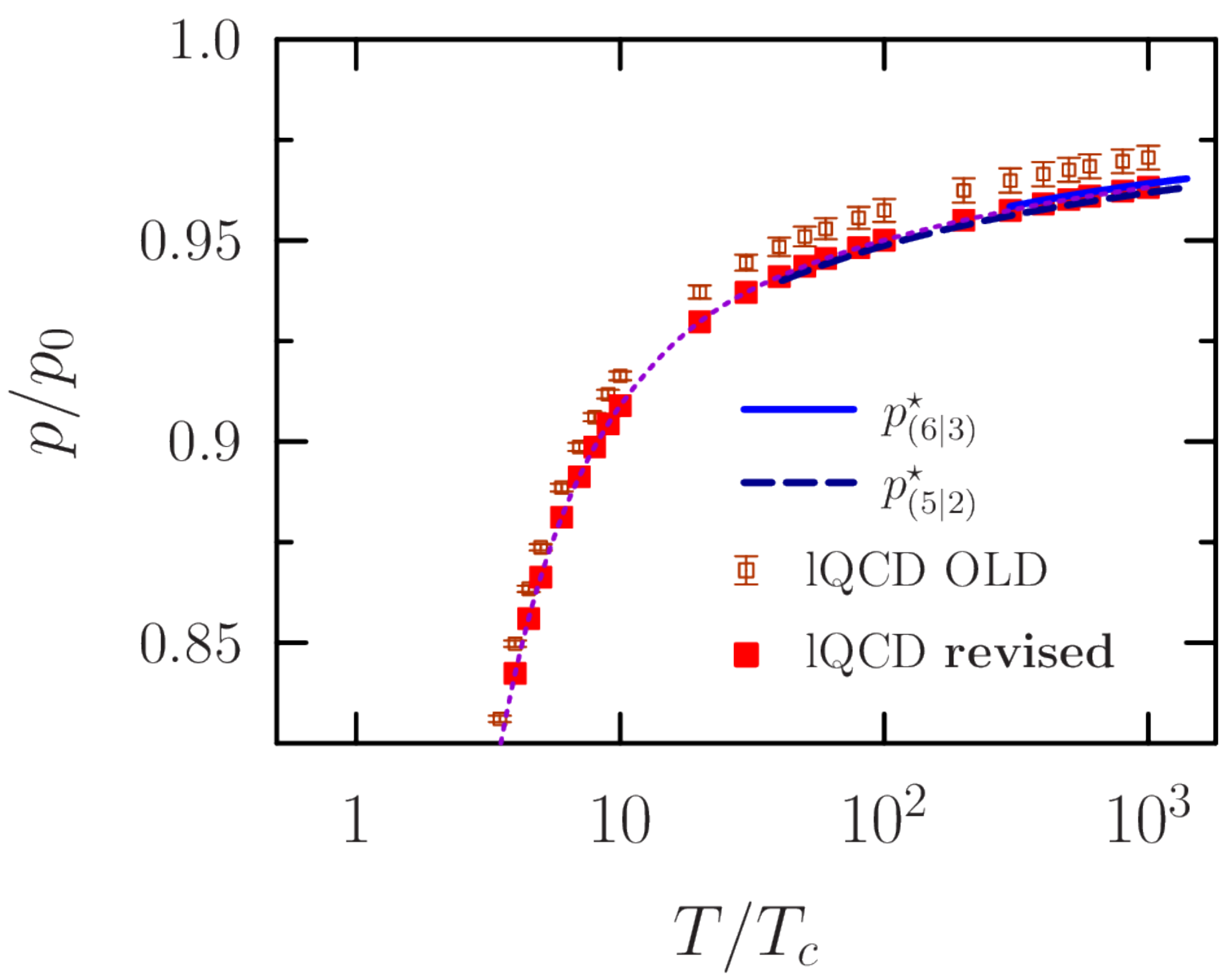
applicability range: $T > 300T_c$

Results



1.0
0.95
0.9
0.85
p/p_0
1
10
10^2
10^3
T/T_c
$p^\star_{(6|3)}$
$p^\star_{(5|2)}$
lQCD OLD
lQCD revised

Results



matching
interaction measure at large T

↓

“re-calibration” of
pressure at $T > 4T_C$

slower approach to free limit

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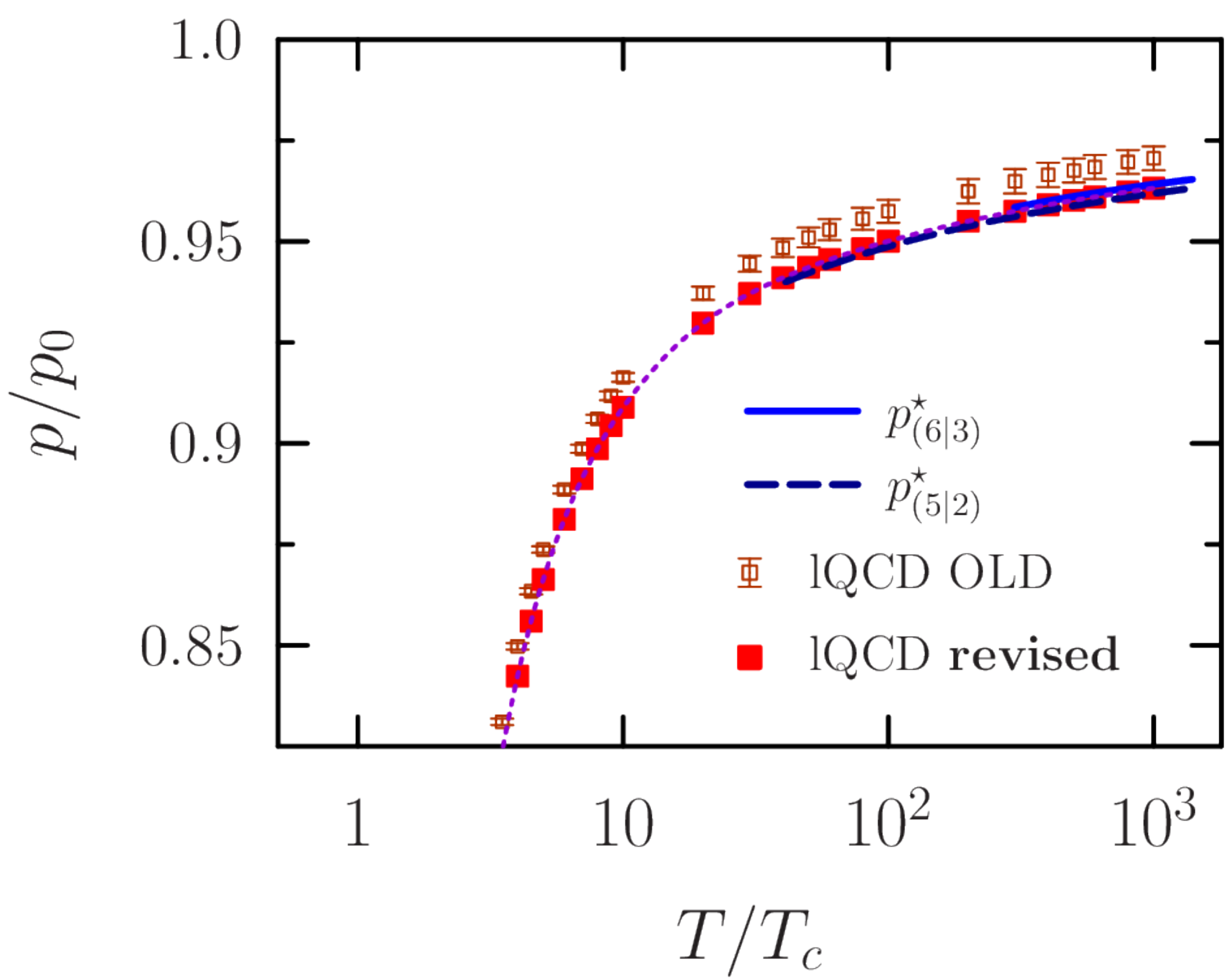

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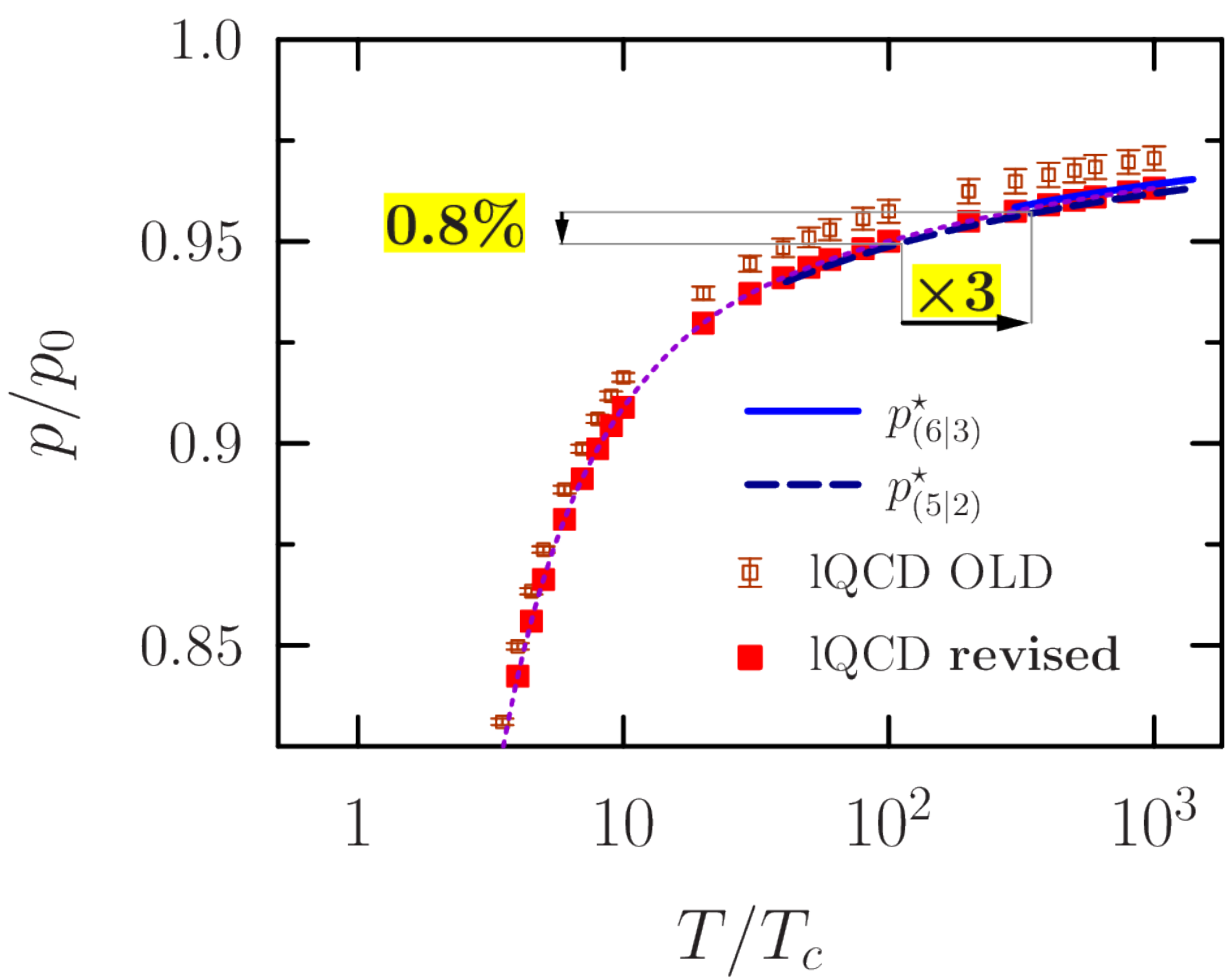

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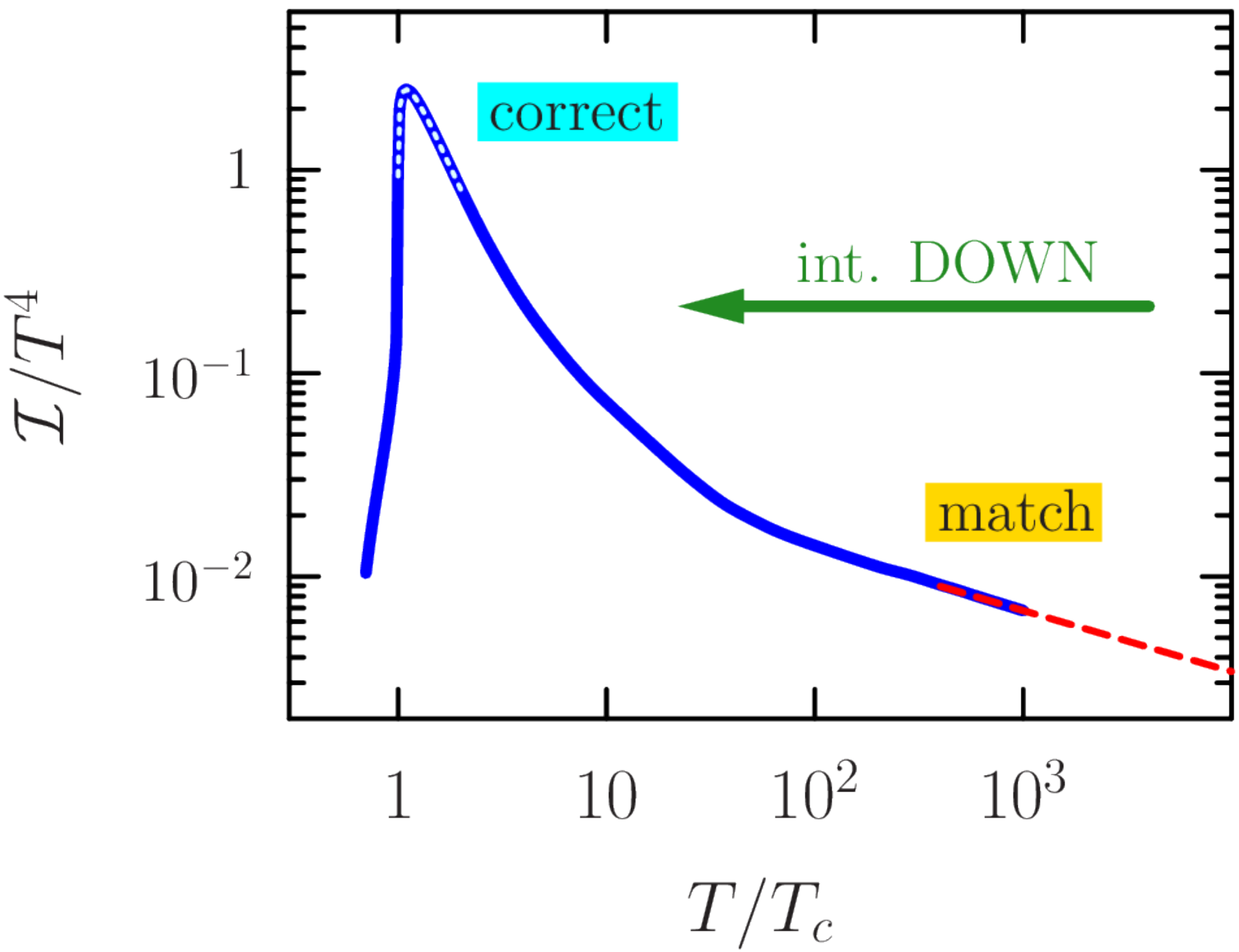
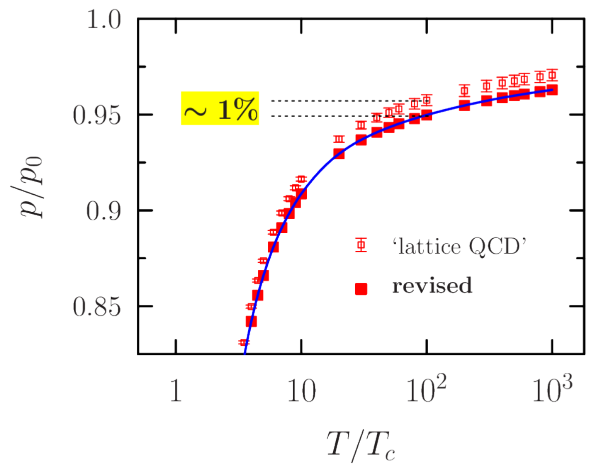
Resumé


correct
int. DOWN
match
1
10^{-1}
10^{-2}
$\mathcal{I}/T^4$
1
10
10^2
10^3
T/T_c


$\sim 1\%$
'lattice QCD'
revised
p/p_0
1.0
0.95
0.9
0.85
1
10
10^2
10^3
T/T_c

Resumé [arXiv:1610.08530]

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- outlook: **phenomenological implications for physical case**